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PREDICTIVE MATHEMATICAL OPTIMAL LOCATION MODEL BASED ON MULTI CRITERIA EVALUATION: VALIDATION BY AQUACULTURE

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ABSTRACT

Optimal locational modeling is a complex task, involving in identification of optimal sites that are economically, socially and environmentally sustainable and commercially practicable. Solving the problem of optimal location in a given context requires a mathematical technique, so that its complexity and multifaceted nature can be managed by means of an iterative search through the context of modeling. This paper describes the development of a mathematical model to select the optimal location based on the objective function, which is derived from Ideal Point method of Multi Criteria Evaluation. Model was implemented as a software tool, which will enhance the decision making capacity of anyone engaged in the design and construction of new / existing facilities. The developed model and its tool appeared to be confident and robust in proof-of-concept application for aquaculture in West Godavari district, Andhra Pradesh.

***KEYWORDS**: Optimal Location Model, Multi Criteria Evaluation, Ideal Point Method, Software Tool & Aquaculture*

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Original Article

1. INTRODUCTION

The role of optimal location is to select the best alternative from among the number of feasible alternatives (Repede, 1994). The selection of best alternative for a system/problem is not formulated just from one factor, but also multiple factors should be considered. Most of the systems / problems have an objective function of either the maximization / minimization of set of criterions. There exist some systems / problems, where the objective function is to be defined as a function of both maximization and minimization criterions. Such problems can be solved by a lead of computer based mathematical technique for finding a maximum or minimum value of a function of several variables, subject to a set of constraints.

Selection of optimal location involves making decisions on how to use available land to satisfy land users' needs. Mathematical and computer models are useful for assisting the decision making process (Stagnitti and Austin, 1998).The role of a decision support system is to assist the decision maker in selecting the best alternative from among the number of feasible alternatives (Jankowski, 1995). As there are multiple factors, there is a need for a function, which takes care of all the factors (Rao and Jayashree, 2004). In order to evaluate the multiple criteria, objective function is defined which is derived from the Multi Criteria Evaluation (MCE) technique. In this purpose, focus has been given on the formulation of a model for optimal decision problem by MCE technique, TOPSIS (Technique for Order Preference by Similarity to the Ideal Solution), which is one of the ideal point methods, validation of the model using aquaculture sector, followed by implementation of the model as a software tool and conclusion.

2. MODEL FORMULATION

As multiple factors are considered, the final objective function gives the ultimate result/weightage, considering all the factors/criteria. The objective function of any optimal location model depends upon the main-criteria (q) and each main-criterion depends on one or more influencing factors / criteria (n). The basic principle is to construct a matrix whose elements reflect the characteristics of a given set of choice possibilities determined by means of a given set of a criterion. The objective function is desired from such matrices. The final objective function (OF_i) (i = 1, 2, 3 ……… m; m is the number of alternatives) for each alternative function determined from MCE techniques is

$OF_i = \sum_j \alpha_j C_{ij+}$

Where, C_{ij+} is the relative closeness to the ideal point of the i^{th} alternative with respect to the j^{th} attribute (or main-criteria) and the weight α_j is a normalizedweight for the given main-criterion under consideration, so that $\sum\alpha_j = 1$.

2.1. Formulation of Relative Closeness (C_{ij+})

Ideal point method (IPM) orders a set of alternatives on the basis of their separation from ideal point. The alternative that is closest to the ideal point is the best alternative. The IPM based relative closeness calculation involves the following steps.

Step (1): Standardize each attribute by transforming the various attribute dimensions (b_{ij}) to unidimension attributes/Utility values (U_{ij}). Calculation of utility values is as follows:

Objective function in optimization would be either maximization or minimization value function, which are defined by

$$f(x) = \text{Max } [F(X)] \quad (1)$$

$$g(x) = \text{Max } [G(X)] \quad (2)$$

Let S_1, S_2, S_3 are three sets related to maximization value attributes, minimization value attributes or the combination of both value attributes respectively.

Let us consider, $S_1 = \{X_1, X_2, X_3,…………, X_{n1}\}$ where n1 is the number of maximization value attributes.

In this, $X_1, X_2, X_3,…………, X_{n1}$ are defined as

$X_1 = \{x_{11}, x_{12}, x_{13},…………, x_{1m}\}$

$X_2 = \{x_{21}, x_{22}, x_{23},…………, x_{2m}\}$

$X_{n1} = \{x_{n11}, x_{n12}, x_{n13},…………, x_{n1m}\}$

Where, m is the number of alternatives

$$f(x) = \text{Max } ((x_{ij})_{j=1}^{n1}\ _{i=1}^{m})$$

So, from equation (1)

Let the ideal value function and negative ideal value function be defined as I(x) and N(x), respectively. Here, I(x) is the corresponding minimum upper bound value for the maximization value attributes, and N(x) is the corresponding maximum lower bound value for the maximization value attributes. The algorithmic steps for transformation of b_{ij} to U_{ij} for maximization value attributes are given below.

for j = 1 to n1

{

for i = 1 to m

{

if ($b_{ij} \geq I(x)$) then $x_{ij} = 1$

elseif ($b_{ij} \leq N(x)$) then $x_{ij} = 0$

elseif ($N(x) < b_{ij} < I(x)$) then

{

$l = (b_{ij} - N(x)) / (I(x) - N(x))$

$x_{ij} = l$

}

}

}

Let us consider, $S_2 = \{Y_1, Y_2, Y_3, \ldots\ldots\ldots\ldots, Y_{n2}\}$ where n2 is the number of minimization value attributes.

In this $Y_1, Y_2, Y_3, \ldots\ldots\ldots\ldots, Y_{n2}$ are defined as

$Y_1 = \{y_{11}, y_{12}, y_{13}, \ldots\ldots\ldots\ldots, y_{1m}\}$

$Y_2 = \{y_{21}, y_{22}, y_{23}, \ldots\ldots\ldots\ldots, y_{2m}\}$

$Y_{n2} = \{y_{n21}, y_{n22}, y_{n23}, \ldots\ldots\ldots\ldots, y_{n2m}\}$

So, from equation (2)

$$g(y) = \mathrm{Min}\left(\left(Y_{ij}\right)\right)_{j=1\ i=1}^{n2\ \ m}$$

Here, I(x) is the corresponding maximum lower bound value for the minimization value attributes and N(x) is the corresponding minimum upper bound value for the minimization value attributes. The algorithmic steps for transformation of b_{ij} to U_{ij} for minimization value attributes are given below.

for j = 1 to n2

{

for i = 1 to m

{

if ($b_{ij} \le I(x)$) then $y_{ij} = 1$

elseif ($b_{ij} \ge N(x)$) then $y_{ij} = 0$

elseif ($N(x) > b_{ij} > I(x)$) then

{

$l = (N(x) - b_{ij}) / (N(x) - I(x))$

$y_{ij} = l$

}

}

}

Let us consider, $S_3 = \{Z_1, Z_2, Z_3, \ldots\ldots\ldots\ldots, Z_{n2}\}$ where n3 is the number of combination of minimization and maximization value attributes.

In this $Z_1, Z_2, Z_3, \ldots\ldots\ldots\ldots, Z_{n3}$ are defined as

$Z_2=\{z_{11}, z_{12}, z_{13}, \ldots\ldots\ldots\ldots, z_{1m}\}$

$Z_2 = \{z_{21}, z_{22}, z_{23}, \ldots\ldots\ldots\ldots, z_{2m}\}$

$Z_{n1} = \{z_{n31}, z_{n32}, z_{n33}, \ldots\ldots\ldots\ldots z_{n3m}\}$

Let us consider, the ideal value function and the negative ideal value function be defined as IP(X) and NP(X) respectively. IP(X) is the corresponding minimum ideal value (IP_1) and maximum ideal value (IP_2) for the combination of minimization and maximization value attributes (i.e). $IP(x) = (IP_1, IP_2)$. NP(x) is the corresponding minimum negative ideal value (NP_1) and maximum negative ideal value (NP_2) for the combination of minimization and maximization value attributes (i.e). $NP(x) = (NP_1, NP_2)$.

The algorithmic steps for transformation of b_{ij} to U_{ij} for the combination of minimization and maximization value attributes are given below:

for j = 1 to n3

{

for i = 1 to m

{

if ($IP_1 \le b_{ij} \le IP_2$) then $z_{ij} = 1$

elseif ($b_{ij} \le NP_1$) or ($b_{ij} \ge NP_2$) then $z_{ij} = 0$

elseif ($NP_1 < b_{ij} < IP_1$) then

{

$l = (b_{ij} - NP_1) / (IP_1 - NP_1)$

$z_{ij} = l$ }

elseif ($IP_2 < b_{ij} < NP_2$) then

{

$l = (NP_2 - bi_j) / (NP_2 - IP_2)$

$z_{ij} = l$

}

}

}

From the above step, the utility function (U_{ij}) is defined as the union of three sets S_1, S_2 and S_3, and the total number of criteria under consideration (n) is equal to the addition of n1, n2 and n3.

Step (2): Normalized weight (a_j) for the j^{th} criteria under consideration is given by (Malczewski, 1999)

for j = 1 to n

$a_j = (n - w_j + 1) / \Sigma (n - w_k + 1)$

Where, n is the number of criteria under consideration (k = 1,2, 3,……, n) and w_j is the rank position of the criterion.

Step (3): Weighted standardized (w_{ij}) function is defined as

for j= 1 to n

for i = 1 to m

$w_{ij}(x) = a_j * U_{ij}(x)$

Step (4): The ideal point (V_{+j}) is defined as,

$V_{+j}(x) = \max(w_{ij}(x))$, j=1,2,3………, n; i = 1,2,………, m.

The negative ideal point is given by

$V_{-j}(x) = \min(w_{ij}(x))$, j=1,2,3………, n; i = 1,2,………, m.

Step (5): Using a separation measure, calculate the distance between the ideal point and each alternative (S_{i+}) is given by (Malczewski, 1999)

$$S_{i+}(x) = (\Sigma_j (w_{ij} - V_{+j})^P)^{1/P} \quad j=1,2,3\ldots n;\ i = 1,2,\ldots, m \tag{3}$$

Using the same separation measure, determine the distance between the negative ideal point, and each alternative (S_{i-}) is given by (Malczewski, 1999)

$S_{i-}(x) = (\Sigma_j(w_{ij} - V_{-j})^P)^{1/P}$ j=1,2,3…, n; i = 1,2,…, m (4)

Here, P is a power parameter ranging from 1 to ∞. In this model both separations are calculated using Euclidean (or straight – line) distance metric. So,the equ (3) and equ (4)becomes

$S_{i+}(x) = (\Sigma_j(w_{ij} - V_{+j})^2)^{0.5}$ j=1,2,……n; i = 1,2,…., m

$S_{i-}(x) = (\Sigma_j(w_{ij} - V_{-j})^2)^{0.5}$ j=1, 2,…n; i = 1,2,…, m

Step (6): Relative closeness to the ideal point (C_{i*}) is given by

$$C_{i^*} = \frac{S_{i^-}}{S_{i^*} + S_{i^-}} \quad i = 1,2,3,\ldots\ldots\ldots, m$$

Subject to $0 < C_{i*} < 1$; that is an alternative is closer to the ideal point as C_{i*} approaches 1.

Step (7): Transfer the relative closeness to the ideal point (C_{i*}) values to some other relative closeness to the ideal point matrix ($C_{ij+;}$ j=1,2,3………, n; i = 1,2,………, m) for objective function calculation.

Step (8): Repeat the step (1) to step (7) until all the main-criteria under consideration were calculated.

2.2. Formulation of Normalized Weight (α_j)

Normalized weight for each main-criterion under consideration is defined by pairwise comparison method (Saaty, 1980). The normalized weight (α_j) is defined by using the normalized comparison matrix

for j= 1 to q

for i = 1 to q

$\alpha_j = \Sigma (a_{ji}) / q$

The normalized comparison matrix A = $\{a_{ij}\}$ (i = 1,2, 3…..,q; j = 1,2,3 ……..q)

2.3. Best Alternative

The objective function OF_i for each alternative is defined as,

for j= 1 to q

for i = 1 to m

$OF_i = \sum_j \alpha_j C_{ij+}$

Best alternative is selected according to the descending order of OF_i; the alternative with the highest value of OF_i is the best alternative.

3. IMPLEMENTATION AND VALIDATION OF THE MODEL

The proposed model is implemented as a computerized software tool (Figure 1), using visual basic programming language. Microsoft Access is selected for database management system. Using Microsoft Access, all information can be managed from a single database file. Decision Making Tool for Identification of Optimal Location in Aqua farming development (Mahalakshmi et al., 2012), is a decision making tool that allows the user to enter data for identification of an optimal location for aquaculture farming development through an interactive dialogue screen. The tool runs on a platform of Windows 95^{TM,} or above, is user-friendly and is best viewed at a screen resolution of 1366 by 768 pixels. There are five modules which are presented as tabs namely, Sub-Variables Weights, Main-Variable Weights, Relative Closeness, Best Alternatives, and Exit. The nested If Then, Else construct is extensively used as an interpretive algorithm for the generation of alternative decisions using the input information. Pop-up windows, button controls and mouse driven events are used for designing the Graphic User Interface (GUI) of the decision making tool.

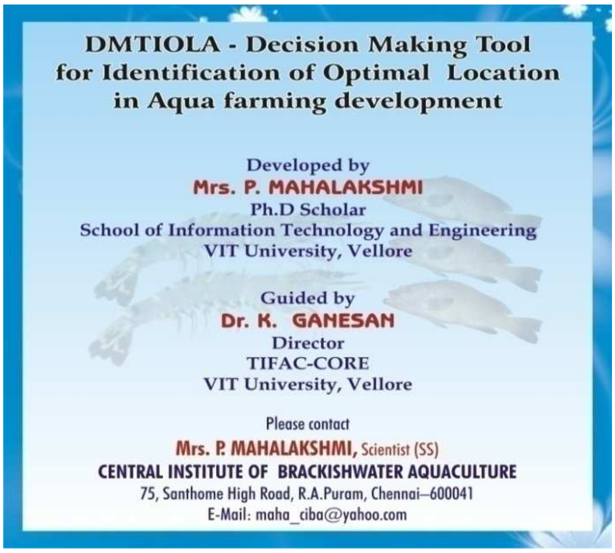

Figure 1: Decision Making Tool: Implementationof the Mathematical Model

The model was validated using Spearman rank correlation method. Spearman rank correlation Kothari (2002) was used to determine the measure of association between ranks obtained by model and the observed yield for the last crop. In this, null hypothesis (H_0) and alternative hypothesis (H_1) were defined as

H_0: There is strong association between ranks obtained by model and ranks obtained based on yield.

H_1: There is no association between ranks obtained by model and ranks obtained based on yield.

If X and Y denote the ranks achieved by 2 different methods for the same alternative m, then coefficient R is defined as

$$R = 1 - \left(\frac{6\sum D_m^2}{M\left(M^2 - 1\right)} \right)$$

Where, m stands for the number of alternatives (m = 1,2,…., M); D_m is the difference between the ranks X and Y.

Using the developed model, selection of optimal location for 80 aquaculture sites dispersed over West Godavari district, Andhra Pradesh, which is the west part of the Godavari delta and it lies between $16^o 15'$ to $17^o 30'$ Northern latitude and $80^o 55'$ Eastern longitude, has been done. The 80 sites are randomly selected from eight different areas. The identified six main-criteria such as water, soil, support, infrastructure, input and risk factor related data used in this study were collected from 15 randomly selected aqua sites in the study area such as Vempa, Bhimavaram, West Godavari, Andhra Pradesh.

Table 1: Relative Closeness and OF for Aquasites

S. NO	Relative Closeness Values						Objective Function
	Water	Soil	Support	Infrastructure	Input	Risk Factor	
1	0.275	0.221	0.056	0.033	0.012	0.018	0.615
2	0.182	0.232	0.000	0.000	0.012	0.018	0.444
3	0.134	0.155	0.000	0.021	0.012	0.018	0.340
4	0.070	0.137	0.009	0.015	0.012	0.000	0.244
5	0.152	0.154	0.021	0.016	0.012	0.018	0.373
6	0.174	0.144	0.056	0.010	0.036	0.000	0.420
7	0.356	0.176	0.009	0.000	0.000	0.000	0.542
8	0.199	0.216	0.000	0.000	0.000	0.018	0.434
9	0.192	0.129	0.000	0.044	0.036	0.036	0.437
10	0.158	0.167	0.042	0.054	0.000	0.014	0.435
11	0.138	0.200	0.000	0.032	0.018	0.015	0.405
12	0.080	0.181	0.000	0.000	0.009	0.031	0.301
13	0.261	0.175	0.000	0.009	0.018	0.023	0.487
14	0.105	0.181	0.051	0.000	0.036	0.027	0.400
15	0.076	0.183	0.000	0.000	0.027	0.023	0.309

Relative closeness and objective function (OF) for Vempa area, Bhimavaram, West Godavari is calculated as per the previous section calculations (Table 1). Normalized weight for water, soil, support, infrastructure, inputs and risk factors were calculated as 0.448, 0.357, 0.054, 0.0361 and 0.0361, respectively. The ranking order for Vempa area is given in Table 2. Based on observed yield value, site 7 was identified as best alternative followed by the alternatives 2, 1, and 13 (Table 2). Table 2 shows that there is slight change in the ranking pattern between model and observed yield. In both methods, alternative 4 was identified as the worst alternative. Correlation coefficient between the ranks obtained by model and ranks obtained based on yields was 0.96 very close to 1, which suggests that there is a strong association between model and observed yield. This shows that, the results of model and observed yield followed the same pattern.

Table 2: Rank for Aquasites Based on Model and Observed Yield

Site no	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
Rank (Model)	1	4	12	15	11	8	2	7	5	6	9	14	3	10	13
Rank (Yield)	3	2	12	15	11	10	1	6	5	7	9	13	4	8	14

4. CONCLUSIONS

A simple-to-use optimal location model and its tool has been developed for selecting one set of alternatives from among a large set of alternatives. This model will enhance the decision making capacity of anyone engaged in the design and construction of new / existing facilities. Case study application and presented results show that such an approach is comprehensive and confident in concept and relatively simple in computation. The model is designed in a flexible and modular fashion, and consequently can be easily applied to some other fields such as agriculture, forest and engineering field.

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