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# Trend free second order neighbour balanced block designs

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**Abstract**

In agricultural experiments, under block design setup along with direct effect of treatments, neighbor effects may also affect the response of a particular plot. Neighbour effects may not only arise from the treatments applied to the immediate adjacent units but also from the treatments applied to the units at higher distance. Further, in block design set up, the response may also depend on the spatial position of the experimental units within a block, that is, trend may affect the plots within blocks. Hence, it is important to include both interference effects and trend effects in the block model for the proper specification. In this paper, we have considered block model with second order neighbour effects that is, effects up to distance 2 and incorporating trend component. The information matrices for estimating various effects have been derived. Further, the conditions for the existence of a trend free second order neighbour balanced block design have been obtained. Series of trend free second order neighbour balanced block designs have been obtained.

**Key Words** : *Block design,Circular, Neighbouring units, Strongly balanced, Totally balanced, Trend.*

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# 1 Introduction

In designing of scientific experiments, block designs can be an effective tool for controlling local variation over the experimental material as in such setup, the whole experimental material is divided into groups/blocks such that the experimental units are homogeneous within a block than within the experimental material as a whole. In classical block model, it is assumed that the response from a unit/ plot to a particular treatment is not affected by the treatment applied on the neighbouring plots and the fertility associated with plots in a block is constant. However, in agricultural field experiments conducted in larger units with gaps, the estimates of treatment differences may deviate because of interference by the treatments applied in neighbouring units. For example, in varietal trials, the yields of shorter varieties may be depressed due to shading from taller neighbouring varieties (Kempton and Lockwood, 1984). In experimenting with field plant communities, the interference can occur with regard to differences in availability of light, nutrients, effect of wind, spread of diseases etc. Here, the neighbouring plots interfere (compete) with one another and induce serious source of bias in the evaluation of treatments. Hence, it is important to include neighbour effects in the model for the proper specification. Neighbour balanced designs are used for these situations. A lot of work has been done on various aspects on Neighbour Balanced designs [see for details Azais, et al (1993), Bailey (2003), Tomar, et al (2005), Jaggi, et al (2006,) Pateria, et al (2007)] and Bhowmik, et al (2013, 2015).

In agricultural experiments, neighbor effects may not only arise from immediate adjacent units but also may arise from the treatments applied to the units at higher distance as with the spread of inoculum in disease screening trials (Kempton, 1992). Further, it may also operate on a block basis, as described by Pearce (1957) for experiments with fruit trees where a treatment applied to a branch (unit) affected the response of all other branches on the same tree (block). Neighbour balanced block designs at higher distance are thus needed [Iqbal, et al (2006), Mingyao, et al. (2007), Akhtar and Ahmed (2009)]. In block design set up, spatial trend in the experimental material may affect the plots within the blocks. In such situations, the response may also depend on the spatial position of the experimental unit within a block. For example, in field experiments, when there is slope or while dealing with undulating land in hilly areas, if the land is irrigated the nutrients supplied by the fertilizers may not be equally distributed and a slope may cause a trend in experimental units. To overcome such situations, a suitable arrangement of treatments over plots within a block is required such that the arranged design is capable of completely eliminating the effects of defined components of a common trend. Such designs have been called as Trend Free Block (TFB) designs (Bradley and Yeh, 1980). These designs are constructed in such a manner that treatment effects and trend effects are orthogonal. Trend-free block designs permit elimination of effects of lower-order components of common within block trends over experimental units. A good number of literature is available on the construction of Trend Free Block designs [Yeh and Bradley (1983), Dhall (1986) and Lal, et al. (2005)].

In this article, we have considered block model with second order neighbor effects incorporating trend component. The experimental setup has been defined and the information matrices for estimating direct as well as neighbour effects up to second order incorporating trend component have been derived. Further, the conditions for a block design with second order neighbor effects to be trend free have been obtained. Methods of constructing complete/ incomplete trend free second order neighbor balanced block designs have been discussed and their characterization properties have been investigated.

## 2 Experimental setup and model

Consider a class of proper block designs with $v$ treatments and $n = bk$ units that form $b$ blocks each containing k units. Let $Y_{ij}$ be the response from the $i^{\text{th}}$ plot in the $j^{\text{th}}$ block $(i = 1, 2, \cdots, k; j = 1, 2, \cdots, b)$. It is assumed that the experiment is conducted in small plots in well separated linear blocks with no guard areas between the plots in a block. Further, the layout includes border plots at both left and right end of every block upto distance 2. The blocks are circular i.e., the treatment on the immediate left border plot is same as the treatment on the right end inner plot of the block and treatment on the left border plot at distance 2 (leaving one plot from the first plot of the block) is same as the treatment on the second last inner plot from the right side. Similarly, treatments on the immediate right border plot is same as the treatment on the left end inner plot of the block and treatment on the right border plot at distance 2 (leaving one plot from the last plot of the block) is same as the treatment on the second last inner plot from the left side. It is also assumed that trend effects also affect the plots within blocks and the within-block trend effects can be represented by orthogonal polynomial of $p^{\text{th}}$ degree $(p < k)$.

Following fixed effects additive model is considered for analyzing a block design with second order neighbour effects incorporating trend component:

$$\mathbf{Y} = \mu\mathbf{1} + \mathbf{\Delta}'\tau + \mathbf{\Delta_1'}\delta + \mathbf{\Delta_2'}\gamma + \mathbf{\Delta_3'}\alpha + \mathbf{\Delta_4'}\eta + \mathbf{D}'\beta + \mathbf{Zp} + \mathbf{e}, \qquad (2.1)$$

where $\mathbf{Y}$ is a $n \times 1$ vector of observations, $\mu$ is the general mean, $\mathbf{1}$ is a $n \times 1$ vector of ones, $\mathbf{\Delta}'$ is a $n \times v$ matrix of observations versus direct treatments, $\tau$ is a $v \times 1$ vector of direct treatment effects, $\mathbf{\Delta_1'}$ is a $n \times v$ matrix of observations versus immediate left neighbour effects, $\delta$ is $v \times 1$ vector of immediate left neighbour effects, $\mathbf{\Delta_2'}$ is a $n \times v$ incidence matrix of observations versus immediate right neighbour effects, $\gamma$ is $v \times 1$ vector of immediate right neighbor effects, $\mathbf{\Delta_3'}$ is a $n \times v$ incidence matrix of observations versus second order left neighbour effects, $\alpha$ is $v \times 1$ vector of second order left neighbor effects, $\mathbf{\Delta}'_4$ is a $n \times v$ incidence matrix of observations versus second order right neighbour effects, $\eta$ is $v \times 1$ vector of second order right neighbour effects, $\mathbf{D}'$ is a $n \times b$ incidence matrix of observations versus blocks, $\beta$ is a $b \times 1$ vector of block effects, $\rho$ is a $p \times 1$ vector representing the trend effects. The matrix $\mathbf{Z}$, of order $n \times p$, is the matrix of coefficients which is given by $\mathbf{Z} = 1_b \otimes \mathbf{F}$ where $\mathbf{F}$ is a $k \times p$ matrix with columns representing the (normalized) orthogonal polynomials and $\mathbf{e}$ is a $n \times 1$ vector of errors with $E(\mathbf{e}) = 0$ and $D(\mathbf{e}) = \sigma^2 \mathbf{I}_n$. Without loss of generality, it can be assumed that the first $k$ observations pertain to the first block, the next $k$ observation pertain to the next block, and so on. Under this ordering, $\mathbf{D}' = \mathbf{I}_b \otimes \mathbf{I}_k$. Since $\mathbf{F}$ is a $k \times p$ matrix with columns representing the (normalized) orthogonal polynomials, thus $\mathbf{1}'\mathbf{F} = \mathbf{0}, \mathbf{F}'\mathbf{F} = \mathbf{I}_p$ and hence $\mathbf{Z}'\mathbf{Z} = b\mathbf{I}_p$.

Let $\mathbf{r} = (r_1, r_2, \cdots, r_v)'$ be the $v \times 1$ replication vector of direct treatments with $\mathbf{r}_s$ $(s = 1, 2, \cdots, v)$ being the number of times the $s^{\text{th}}$ treatment appears in the design. $\mathbf{r}_1 = (r_{11}, r_{12}, \cdots, r_{1v})'$ be the $v \times 1$ replication vector of the immediate left neighbour treatments with $r_{1s}$ being the number of times the treatments in the design has $s^{\text{th}}$ treatment as immediate left neighbour. $\mathbf{r}_2 = (r_{21}, r_{22}, \cdots, r_{2v})'$ be the $v \times 1$ replication vector of the immediate right neighbour treatments with $r_{2s}$ being the number of times the treatments in the design has $s^{\text{th}}$ treatment as immediate right neighbour. $\mathbf{r}_3 = (r_{31}, r_{32}, \cdots, r_{3v})'$ be the $v \times 1$ replication vector of the second order left neighbour treatments with $r_{3s}$ being the number of times the treatments in the design has $s^{\text{th}}$ treatment as second order left neighbour. $\mathbf{r}_4 = (r_{41}, r_{42}, \cdots, r_{4v})'$ be the $v \times 1$ replication vector of the second order right neighbour treatments with $r_{4s}$ being the number of times the treatments in the design has $s^{\text{th}}$ treatment as second order right neighbour. Further,

$$\begin{aligned}
\Delta\Delta' &= \mathbf{R}_\tau = diag(r_1, r_2, \cdots, r_v), \quad \Delta_1\Delta_1' = \mathbf{R}_\delta = diag(r_{11}, r_{12}, \cdots, r_{1v}), \\
\Delta_2\Delta_2' &= \mathbf{R}_\gamma = diag(r_{21}, r_{22}, \cdots, r_{2v}), \quad \Delta_3\Delta_3' = \mathbf{R}_\alpha = diag(r_{31}, r_{32}, \cdots, r_{3v}),
\end{aligned}$$

$$\begin{aligned}
\Delta_4\Delta_4' &= \mathbf{R}_\eta = diag(r_{41}, r_{42}, \cdots, r_{4v}), \quad \mathbf{DD}' = \mathbf{K} = diag(k_1, k_2, , \cdots, k_b),\\
\Delta\Delta_1' &= \mathbf{M}_1, \Delta\Delta_2' = \mathbf{M}_2, \Delta_1\Delta_2' = \mathbf{M}_3, \Delta\Delta_3' = \mathbf{M}_4, \Delta\Delta_4' = \mathbf{M}_5,\\
\Delta_1\Delta_3' &= \mathbf{M}_6, \Delta_1\Delta_4' = \mathbf{M}_7, \Delta_2\Delta_3' = \mathbf{M}_8, \Delta_2\Delta_4' = \mathbf{M}_9, \Delta_3\Delta_4' = \mathbf{M}_{10},\\
\Delta\mathbf{D}' &= \mathbf{N}_1, \Delta_1\mathbf{D}' = \mathbf{N}_2, \Delta_2\mathbf{D}' = \mathbf{N}_3, \Delta_3\mathbf{D}' = \mathbf{N}_4, \Delta_4\mathbf{D}' = \mathbf{N}_5,
\end{aligned}$$

where, $\mathbf{M}_1$ is $v \times v$ incidence matrix of direct treatments versus immediate left neighbour treatments, $\mathbf{M}_2$ is a $v \times v$ incidence matrix of direct treatments versus immediate right neighbour treatments, $\mathbf{M}_3$ is a $v \times v$ incidence matrix of immediate left neighbour treatments versus immediate right neighbour treatments, $\mathbf{M}_4$ is a $v \times v$ incidence matrix of direct treatments versus second order left neighbour treatments, $\mathbf{M}_5$ is a $v \times v$ incidence matrix of direct treatments versus second order right neighbour treatments, $\mathbf{M}_6$ is a $v \times v$ incidence matrix of immediate left neighbour treatments versus second order left neighbour treatments, $\mathbf{M}_7$ is a $v \times v$ incidence matrix of immediate left neighbour treatments versus second order right neighbour treatments, $\mathbf{M}_8$ is a $v \times v$ incidence matrix of immediate right neighbour treatments versus second order left neighbour treatments, $\mathbf{M}_9$ is a $v \times v$ incidence matrix of immediate right neighbour treatments versus second order right neighbour treatments, $\mathbf{M}_{10}$ is a $v \times v$ incidence matrix of second order left neighbour treatments versus second order right neighbour treatments. $\mathbf{N}_1$ is a $v \times b$ incidence matrix of direct treatments versus blocks. $\mathbf{N}_2$ is a $v \times b$ incidence matrix of immediate left neighbour treatments versus blocks. $\mathbf{N}_3$ is a $v \times b$ incidence matrix of immediate right neighbour treatments versus blocks. $\mathbf{N}_4$ is a $v \times b$ incidence matrix of second order left neighbour treatments versus blocks and $\mathbf{N}_5$ is a $v \times b$ incidence matrix of second order right neighbour treatments versus blocks.

The $5v \times 5v$ symmetric, nonnegative definite joint information matrix $\mathbf{C}$ for estimating the direct effects of treatment and neighbor effects up to distance 2 is obtained as follows.

$$C = \begin{bmatrix} C_{11} & C_{12} \\ C_{21} & C_{22} \end{bmatrix},$$

where

$$C_{11}=\begin{bmatrix} \mathbf{R}_\tau-\mathbf{N}_1\mathbf{K}^{-1}\mathbf{N}'-\frac{1}{b}\Delta\mathbf{ZZ}'\Delta' & \mathbf{M}_1-\mathbf{N}_1\mathbf{K}^{-1}\mathbf{N}_2'-\frac{1}{b}\Delta\mathbf{ZZ}'\Delta_1' \\ \mathbf{M}_1'-\mathbf{N}_2\mathbf{K}^{-1}\mathbf{N}_1'-\frac{1}{b}\Delta_1\mathbf{ZZ}'\Delta' & \mathbf{R}_\delta-\mathbf{N}_2\mathbf{K}^{-1}\mathbf{N}_2'-\frac{1}{b}\Delta\mathbf{ZZ}'\Delta_1' \end{bmatrix},$$

$$C_{12}=\begin{bmatrix} \mathbf{M}_2-\mathbf{N}_1\mathbf{K}^{-1}\mathbf{N}_3'-\frac{1}{b}\Delta\mathbf{ZZ}'\Delta_2' & \mathbf{M}_4-\mathbf{N}_1\mathbf{K}^{-1}\mathbf{N}_4'-\frac{1}{b}\Delta\mathbf{ZZ}'\Delta_3' & \mathbf{M}_5-\mathbf{N}_1\mathbf{K}^{-1}\mathbf{N}_5'-\frac{1}{b}\Delta\mathbf{ZZ}'\Delta_4' \\ \mathbf{M}_3-\mathbf{N}_2\mathbf{K}^{-1}\mathbf{N}_3'-\frac{1}{b}\Delta_1\mathbf{ZZ}'\Delta_2' & \mathbf{M}_6-\mathbf{N}_2\mathbf{K}^{-1}\mathbf{N}_4'-\frac{1}{b}\Delta_1\mathbf{ZZ}'\Delta_3' & \mathbf{M}_7-\mathbf{N}_2\mathbf{K}^{-1}\mathbf{N}_5'-\frac{1}{b}\Delta_1\mathbf{ZZ}'\Delta_4' \end{bmatrix},$$

$$C_{21}=C_{12}' \qquad \text{and}$$

$$C_{22}=\begin{bmatrix} \mathbf{R}_\gamma-\mathbf{N}_3\mathbf{K}^{-1}\mathbf{N}_3'-\frac{1}{b}\Delta_2\mathbf{ZZ}'\Delta_2' & \mathbf{M}_8-\mathbf{N}_3\mathbf{K}^{-1}\mathbf{N}_4'-\frac{1}{b}\Delta_2\mathbf{ZZ}'\Delta_3' & \mathbf{M}_9-\mathbf{N}_3\mathbf{K}^{-1}\mathbf{N}_5'-\frac{1}{b}\Delta_2\mathbf{ZZ}'\Delta_4' \\ \mathbf{M}_8'-\mathbf{N}_4\mathbf{K}^{-1}\mathbf{N}_3'-\frac{1}{b}\Delta_3\mathbf{ZZ}'\Delta_2' & \mathbf{R}_\alpha-\mathbf{N}_4\mathbf{K}^{-1}\mathbf{N}_4'-\frac{1}{b}\Delta_3\mathbf{ZZ}'\Delta_3' & \mathbf{M}_{10}-\mathbf{N}_4\mathbf{K}^{-1}\mathbf{N}_5'-\frac{1}{b}\Delta_3\mathbf{ZZ}'\Delta_4' \\ \mathbf{M}_9'-\mathbf{N}_5\mathbf{K}^{-1}\mathbf{N}_3'-\frac{1}{b}\Delta_4\mathbf{ZZ}'\Delta_2' & \mathbf{M}_{10}'-\mathbf{N}_5\mathbf{K}^{-1}\mathbf{N}_4'-\frac{1}{b}\Delta_4\mathbf{ZZ}'\Delta_3' & \mathbf{R}_\eta-\mathbf{N}_5\mathbf{K}^{-1}\mathbf{N}_5'-\frac{1}{b}\Delta_4\mathbf{ZZ}'\Delta_4' \end{bmatrix}.$$

The information matrix for estimating the direct effects and neighbour effects up to second order can be obtained based on the joint information matrix.

**Definition 2.1**: A block design is said to be second order neighbor balance if every treatment has every other treatment appearing as both left and right neighbour up to distance 2 constant number of times (say $\mu_1$). Further, a block design with both sided interference effects is strongly balanced if each treatment has every treatment, including itself, appearing as both left and right neighbours up to second order a constant number of times (say $\mu_2$).

**Definition 2.2**: A block design with second order neighbour effects incorporating trend component, is called a trend-free design if the adjusted treatment sum of squares arising from direct effects of treatments and neighbour effects of treatments under the corresponding model is same as the adjusted treatment sum of squares under the usual block model with second order neighbour effects without trend component.

**Definition 2.3**: A trend-free block design with second order neighbour effects called variance balanced if the variance of any estimated elementary contrast among the direct effects is constant (say $V_1$), the variance of any estimated elementary contrast among the immediate left neighbour effects is constant (say $V_2$), the variance of any estimated elementary contrast among the immediate right neighbour effects is constant (say $V_3$), the variance of any estimated elementary contrast among the second order left neighbour effects is constant (say $V_4$) and the variance of any estimated elementary contrast among the second order right neighbour effects is constant (say $V_5$). A block design is totally balanced if $V_1 = V_2 = V_3 = V_4 = V_5$.

# 3 Conditions for the block design with second order neighbour effects to be trend free

The conditions for the block design with second order neighbour effects to be trend free have been obtained here so that the treatment (direct, neighbour effects from left and right neighbouring units up to second order) effects and trend effects are orthogonal and the analysis of the design could then be done in the usual manner, as if no trend effect was present. Such designs are known as trend free designs. We now derive a necessary and sufficient condition for a block design with interference effects to be trend free.

**Theorem 3.1**: A block design with second order neighbour effects from left and right neighbouring units and incorporating trend component is said to be trend free iff $\Delta\mathbf{Z} = \mathbf{0}, \Delta_1\mathbf{Z} = \mathbf{0}, \Delta_2\mathbf{Z} = \mathbf{0}, \Delta_3\mathbf{Z} = \mathbf{0}$ and $\Delta_4\mathbf{Z} = \mathbf{0}$, where the symbols have their usual meaning as defined earlier.

**Proof** : Let $\mathbf{X_1} = [\mathbf{1} \quad \mathbf{D}' \quad \mathbf{Z}]$. Let $\mathbf{X}_2 = [\mathbf{1} \quad \mathbf{D}']$.

We define,

$$\mathbf{A}_u = \mathbf{I}_n - \mathbf{X}_u(\mathbf{X}_u'\mathbf{X}_u)^- \ \mathbf{X}_u' \quad (u = 1, 2)$$

$$\begin{aligned} \mathbf{Q}_{u\tau} &= \Delta\mathbf{A}_u\Delta' \\ \mathbf{Q}_{u\delta} &= \Delta_1\mathbf{A}_u\Delta_1' \\ \mathbf{Q}_{u\gamma} &= \Delta_2\mathbf{A}_u\Delta_2' \\ \mathbf{Q}_{u\alpha} &= \Delta_3\mathbf{A}_u\Delta_3' \\ \mathbf{Q}_{u\eta} &= \Delta_4\mathbf{A}_u\Delta_4' \end{aligned} \tag{3.1}$$

On the lines of Bradley and Yeh (1980), the conditions for a block design with second order neighbour effects from left and right neighbouring units and incorporating trend component is said to be trend free if $\Delta\mathbf{Z} = \mathbf{0}, \Delta_1\mathbf{Z} = \mathbf{0}, \Delta_2\mathbf{Z} = \mathbf{0}, \Delta_3\mathbf{Z} = \mathbf{0}$ and $\Delta_4\mathbf{Z} = \mathbf{0}$ and vice versa.

**Corollary 3.1**: For a trend free block design with second order neighbour effects, the information matrix for estimating the direct effects as well as the information matrix for estimating the neighbour effects from left and right neighbouring units up to second order with trend is same as the information matrix for estimating the direct effects as well as the information matrix for estimating the neighbour effects from left and right neighbouring units up to second order without trend component.

# 4 Trend free designs

In this section, methods for construction of trend free second order neighbour balanced block designs have been described. In all the cases, it is assumed that the designs are circular. We choose $\mathbf{F}$ as a $k \times 1$ vector with columns representing the (normalized) orthogonal polynomials and Z can be obtained based on F as defined earlier in such a way that $\Delta\mathbf{Z} = \mathbf{0}, \Delta_1\mathbf{Z} = \mathbf{0}$, $\Delta_2\mathbf{Z} = \mathbf{0}, \Delta_3\mathbf{Z} = \mathbf{0}$ and $\Delta_4\mathbf{Z} = \mathbf{0}$.

## 4.1 Trend free second order neighbour balanced complete block designs

Let there be $v$ (prime) treatments labeled as $0, 1, 2, \cdots, v-1$. Develop an array of size $v \times (2v-1)$ by generating the following block for all $q = 0, 1, \cdots, (v-1)$ and for each value of $p = 1, 2, \cdots, (v-1)/2$:

$$q, q+p, q+2p, \cdots, q+(v-2)p, q+(v-1)p, q+(v-2)p, \cdots, q+2p, q+p, q \ \ (mod\, v).$$

Then, append each array of size $v(2v-1)$ [obtained for each values of $p$, i.e., $p = 1, 2, \cdots, (v-1)/2$] one below the other resulting in an array of size $v(v-1)/2 \times (2v-1)$. Now treating rows as blocks, a series of complete block design strongly balanced for interference effects up to distance 2 can be obtained with parameters $v$, $b = v(v-1)/2$, $r = (v-1)(2v-1)/2$, $k = 2v-1$, $\mu_1 = v-1$ and $\mu_2 = (v-1)/2$.

For this class of designs,

$$\mathbf{R}_\tau = \mathbf{R}_\delta = \mathbf{R}_\gamma = \mathbf{R}_\alpha = \mathbf{R}_\eta = \frac{(v-1)(2v-1)}{2}\mathbf{I}_v, \mathbf{K} = k\mathbf{I}_b = (2v-1)\mathbf{I}_b,$$

$$\mathbf{M}_1 = \mathbf{M}_2 = \mathbf{M}_3 = \mathbf{M}_4 = \mathbf{M}_5 = \mathbf{M}_6 = \mathbf{M}_7 = \mathbf{M}_8 = \mathbf{M}_9 = \mathbf{M}_{10} = \frac{(v-1)}{2}(2\mathbf{11'} - \mathbf{I}_v),$$

$$\mathbf{N}_u\mathbf{N}'_{u'} = \frac{(v-1)}{2}\left[\mathbf{I}_v + 4(v-1)\mathbf{11'}\right], \quad u, u' = 1, 2, \cdots, 5. \tag{4.1}$$

The joint information matrix for estimating the direct as well as neighbour effects up to second order is obtained and the information matrix for estimating the direct effects is

$$\mathbf{C}_\tau = \frac{2v(v-1)(v-3)}{(2v-5)}\left(\mathbf{I}_v - \frac{\mathbf{11'}}{v}\right), \quad v > 3. \tag{4.2}$$

The variance of estimated elementary contrast pertaining to direct effects of treatments is

$$V(\hat{\tau}_s - \hat{\tau}_{s'}) = V_1 = \sigma^2\frac{(2v-5)}{v(v-1)(v-3)}; \quad \forall \quad s, s' = 0, 1, \cdots, v-1. \tag{4.3}$$

Similarly, the information matrices for estimating the immediate left neighbour effects, immediate right neighbour effects, second order left neighbour effects and second order right neighbour effects are obtained as

$$\mathbf{C}_\delta = \mathbf{C}_\gamma = \mathbf{C}_\alpha = \mathbf{C}_\eta = \frac{2v(v-1)(v-3)}{(2v-5)}\left(\mathbf{I}_v - \frac{\mathbf{11'}}{v}\right), \quad v > 3. \tag{4.4}$$

The design is thus variance balanced for estimating the contrast pertaining to direct effects of treatments and neighbour effects up to second order. Also, since $V_1 = V_2 = V_3 = V_4 = V_5$ the series of design obtained is totally balanced for estimating the contrasts pertaining to direct effects of treatments and neighbour effects up to second order.

**Remark 4.1**: For the above class of design, when neighbour effects from only one side, i.e. left neighbouring units are considered, the information matrices for estimating direct effects, immediate left neighbour effects and the second order left neighbour effects are obtained as

$$\mathbf{C}_\tau = \mathbf{C}_\delta = \mathbf{C}_\gamma = \frac{2v(v-1)(v-2)}{(2v-3)}\left(\mathbf{I}_v - \frac{\mathbf{11}'}{v}\right), \quad v > 2.$$

**Example 4.1**: For $v = 5$, following is a strongly balanced trend free second order neighbour balanced complete block design with $v = 5$, $b = 10$, $r = 18$, $k = 9$, $\mu_1 = 4$, $\mu_2 = 2$. Here every treatment appears in every position in the design two times.

| −4 | −3 | −2 | −1 | 0 | 1 | 2 | 3 | 4 |
|---|---|---|---|---|---|---|---|---|
| 0 | 1 | 2 | 3 | 4 | 3 | 2 | 1 | 0 |
| 1 | 2 | 3 | 4 | 0 | 4 | 3 | 2 | 1 |
| 2 | 3 | 4 | 0 | 1 | 0 | 4 | 3 | 2 |
| 3 | 4 | 0 | 1 | 2 | 1 | 0 | 4 | 3 |
| 4 | 0 | 1 | 2 | 3 | 2 | 1 | 0 | 4 |
| 0 | 2 | 4 | 1 | 3 | 1 | 4 | 2 | 0 |
| 1 | 3 | 0 | 2 | 4 | 2 | 0 | 3 | 1 |
| 2 | 4 | 1 | 3 | 0 | 3 | 1 | 4 | 2 |
| 3 | 0 | 2 | 4 | 1 | 4 | 2 | 0 | 3 |
| 4 | 1 | 3 | 0 | 2 | 0 | 3 | 1 | 4 |

Orthogonal trend component of degree one without normalization [Fisher and Yates (1957)] is given in the upper row.

## 4.2 Trend free second order neighbour balanced incomplete block designs

For $v$ prime, a series of incomplete block design strongly balanced for interference effects can also be obtained by developing the blocks of the design as follows for all $q = 0, 1, \cdots, (v-1)$ and $p = 1, 2, \cdots, (v-1)/2$:

$$q, q{+}p, q{+}2p, \cdots, q{+}(v{-}3)p, q{+}(v{-}2)p, q{+}(v{-}3)p, \cdots, q{+}2p, q{+}p, q \ \ (mod\, v).$$

The parameters of this class of designs are $v$, $b = v(v-1)/2$, $r = (v-1)(2v-3)/2$, $k = 2v-3$, $\mu_1 = v-2$ and $\mu_2 = (v-1)/2$. Here, every treatment appears in every position in the design same number of times i.e. $(v-1)/2$. For this class of designs,

$$\mathbf{R}_\tau = \mathbf{R}_\delta = \mathbf{R}_\gamma = \mathbf{R}_\alpha = \mathbf{R}_\eta = \frac{(v-1)(2v-3)}{2}\mathbf{I}_v, \mathbf{K} = k\mathbf{I}_b = (2v-3)\mathbf{I}_b,$$

$$\begin{aligned}&\mathbf{M}_1 = \mathbf{M}_2 = \mathbf{M}_3 = \mathbf{M}_4 = \mathbf{M}_5 = \mathbf{M}_6 = \mathbf{M}_7 = \mathbf{M}_8 = \mathbf{M}_9 = \mathbf{M}_{10}\\ &= (v-2)\mathbf{11'} - \frac{v-3}{2}\mathbf{I}_v,\end{aligned}$$

$$\mathbf{N}_u\mathbf{N}'_{u'} = 2(v-2)^2\mathbf{11'} + \frac{(5v-9)}{2}\mathbf{I}_v, \quad u, u' = 1, 2, \cdots, 5. \tag{4.5}$$

The information matrix for estimating the direct effects is

$$\mathbf{C}_\tau = \frac{2v(v-2)(v-4)}{(2v-7)}\left(\mathbf{I}_v - \frac{\mathbf{11'}}{v}\right), \quad v > 4. \tag{4.6}$$

The variance of estimated elementary contrast pertaining to direct effects of treatments is

$$V(\hat{\tau}_s - \hat{\tau}_{s'}) = V_1 = \sigma^2\frac{(2v-7)}{v(v-2)(v-4)}; \quad \forall \quad s, s' = 0, 1, \cdots, v-1. \tag{4.7}$$

Similarly, the information matrices for estimating the immediate left neighbour effects, immediate right neighbour effects, second order left neighbour effects and second order right neighbour effects are obtained as

$$\mathbf{C}_\delta = \mathbf{C}_\gamma = \mathbf{C}_\alpha = \mathbf{C}_\eta = \frac{2v(v-2)(v-4)}{(2v-7)}\left(\mathbf{I}_v - \frac{\mathbf{11'}}{v}\right), \quad v > 4. \tag{4.8}$$

The design is thus variance balanced for estimating the contrast pertaining to direct effects of treatments and neighbour effects up to second order. Also, since $V_1 = V_2 = V_3 = V_4 = V_5$ the series of design obtained is totally balanced for estimating the contrasts pertaining to direct effects of treatments and neighbour effects up to second order.

**Remark 4.2**: For the above class of design, when neighbour effects from only one side, i.e. left neighbouring units are considered, the information matrices for estimating direct effects, immediate left neighbour effects and the second order left neighbour effects are obtained as

$$\mathbf{C}_\tau = \mathbf{C}_\delta = \mathbf{C}_\gamma = \frac{2v(v-2)(v-3)}{(2v-5)} \left( \mathbf{I}_v - \frac{\mathbf{11'}}{v} \right), \quad v > 3.$$

**Example 4.2**: For $v = 5$, following is a strongly balanced trend free second order neighbour balanced incomplete block design with $v = 5, b = 10, r = 14$, $k = 7$, $\mu_1 = 3$, $\mu_2 2 = 2$. Here every treatment appears in every position in the design two times.

| −3 | −2 | −1 | 0 | 1 | 2 | 3 |
|---|---|---|---|---|---|---|
| 0 | 1 | 2 | 3 | 2 | 1 | 0 |
| 1 | 2 | 3 | 4 | 3 | 2 | 1 |
| 2 | 3 | 4 | 0 | 4 | 3 | 2 |
| 3 | 4 | 0 | 1 | 0 | 4 | 3 |
| 4 | 0 | 1 | 2 | 1 | 0 | 4 |
| 0 | 2 | 4 | 1 | 4 | 2 | 0 |
| 1 | 3 | 0 | 2 | 0 | 3 | 1 |
| 2 | 4 | 1 | 3 | 1 | 4 | 2 |
| 3 | 0 | 2 | 4 | 2 | 0 | 3 |
| 4 | 1 | 3 | 0 | 3 | 1 | 4 |

Orthogonal trend component of degree one without normalization [Fisher and Yates (1957)] is given in the upper row.

**Remark 4.3**: It has also been seen that both the above classes of designs so obtained are trend free up to $p^{\text{th}}$ degree ($p < k$).

# 5 Conclusions

Neighbour effects are very common in field experiments. Neighbour effects may arise from the immediate neighbouring units or it may extend further. But, when there is slope or while dealing with undulating land in hilly areas, slope may cause a trend in experimental units. To overcome such situations, trend free block designs balanced for neighbour effects up to second order or distance 2 have been obtained. The designs so obtained are totally balanced for estimating direct and neighbour effects of treatments and are capable of completely eliminating the effects of a common trend.

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