TREND FREE BLOCK DESIGNS BALANCED FOR INTERFERENCE EFFECTS FROM NEIGHBOURING EXPERIMENTAL UNITS

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Abstract

In this paper, we have considered block model with interference effects arising from the immediate neighbouring units on both sides and incorporating spatial trend component. The case of one-sided (say left) interference effects has been considered as a particular case. The information matrices for estimating direct as well as interference effects incorporating trend component have been derived. Further, the conditions for a block design with interference effects to be trend free have been obtained. Method of constructing trend free block design balanced for interference effects have been discussed with reference to both complete and incomplete blocks. The trend free block designs so obtained are totally balanced for estimating direct and interference effects of treatments.

Keywords: Block design, Interference, Neighbouring units, Totally balanced, Trend.

1. INTRODUCTION

In designing of scientific experiments, block designs can be an effective tool for controlling local variation over the experimental material as in such setup, the whole experimental material is divided into groups/blocks such that the experimental units are homogeneous within a block than within the experimental material as a whole. In classical block model, it is assumed that the response from a unit/plot to a particular treatment is not affected by the treatment applied on the neighbouring plots and the fertility associated with plots in a block is constant. However, in agricultural field experiments conducted in larger units with gaps, the estimates of treatment differences may deviate because of interference by the treatments applied in neighbouring units.

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For example, in varietal trials, the yields of shorter varieties may be depressed due to shading from taller neighbouring varieties (Kempton and Lockwood, 1984). In experimenting with field plant communities, the interference can occur with regard to differences in availability of light, nutrients, effect of wind, spread of diseases etc. Here, the neighbouring plots interfere (compete) with one another and induce serious source of bias in the evaluation of treatments. Hence, it is important to include the interference effects in the model for the proper specification. Designs balanced for interference effects from neighbours are used for these situations. Azais *et al.* (1993) obtained series of designs that are balanced in v-1 blocks of size v and v blocks of size v-1, where v is the number of treatments. Designs for one-sided neighbour effects have been studied and table of such designs with different block sizes have been presented by Bailey (2003). Tomar *et al.* (2005) constructed some totally balanced incomplete block designs for competition effects. Jaggi *et al.* (2006) obtained some methods for constructing partially balanced block designs for neighbouring competition effects. Pateria *et al.* (2007) proposed a series of incomplete non-circular block designs for competition effects.

In block design set up, spatial trend in the experimental material may affect the plots within the blocks. In such situations, the response may also depend on the spatial position of the experimental unit within a block. For example, in field experiments, when there is slope or while dealing with undulating land in hilly areas, if the land is irrigated the nutrients supplied by the fertilizers may not be equally distributed and a slope may cause a trend in experimental units. To overcome such situations, a suitable arrangement of treatments over plots within a block is required such that the arranged design is capable of completely eliminating the effects of defined components of a common trend. Such designs have been called as Trend Free Block (TFB) designs (Bradley and Yeh, 1980). These designs are constructed in such a manner that treatment effects and trend effects are orthogonal. Trend-free block designs permit elimination of effects of lower-order components of common within block trends over experimental units. Bradley and Yeh (1980) introduced the concept of a TFB design along with the necessary and sufficient condition for the existence of such designs. Yeh and Bradley (1983) discussed some results for the existence of a TFB design for specified trends in one or more dimensions under a homoscedastic model when each treatment is equally replicated and also gave some methods of constructing such designs.

In this article, we have considered block model with interference effect arising from the immediate left and right neighbouring experimental units incorporating trend component. The case of one-sided interference effects have been considered as a particular case. The experimental setup has been defined and the information matrices for estimating direct as well as interference effects incorporating trend component have been derived. Further, the conditions for a block design with interference effects to be trend free have been obtained. Methods of constructing complete/incomplete trend free block designs balanced for interference effects have been discussed and their characterization properties have been investigated.

2. EXPERIMENTAL SETUP

Consider a class of proper block designs with v treatments and $n = bk$ units that form b blocks each containing k units. Let Y_{ij} be the response from the i^{th} plot in the j^{th} block ($i = 1, 2,\ldots, k$; $j = 1, 2,\ldots, b$). It is assumed that the experiment is conducted in small plots in well separated linear blocks with no guard areas between the plots in a block. Further, the layout includes border plots at both left and right end of every block. The treatment on the left border plot is same as the treatment on the right end plot of the block and the treatment on the right border plot is same as the treatment on the left end plot of the block i.e. the design is *circular*. It is also assumed that trend effects also affect the plots within blocks and the within-block trend effects can be represented by orthogonal polynomial of p^{th} degree ($p < k$).

2.1. Block Model with Interference Effects Incorporating Trend Component

Based on the above experimental setup, following fixed effects additive model is considered for analyzing a block design with interference effects incorporating trend component:

$$\mathbf{Y} = \boldsymbol{\mu}\mathbf{1}+\boldsymbol{\Delta}'\boldsymbol{\tau}+\boldsymbol{\Delta}_1'\boldsymbol{\delta}+\boldsymbol{\Delta}_2'\boldsymbol{\gamma}+\mathbf{D}'\boldsymbol{\beta}+\mathbf{Zp}+\mathbf{e}, \qquad \ldots (2.1)$$

where, $\mathbf{Y}$ is a $n\times 1$ vector of observations, μ is the general mean, $\mathbf{1}$ is a $n\times 1$ vector of unity, $\boldsymbol{\Delta}'$ is a $n\times v$ matrix of observations versus direct treatments, $\boldsymbol{\tau}$ is a $v\times 1$ vector of direct treatment effects, $\boldsymbol{\Delta}_1'$ is a $n\times v$ matrix of observations versus interference effect from the left neighbour treatment, $\boldsymbol{\delta}$ is $v\times 1$ vector of left neighbour interference effects, $\boldsymbol{\Delta}_2'$ is a $n\times v$ matrix of observations versus interference effect from the right neighbour treatment, $\boldsymbol{\gamma}$ is $v\times 1$ vector of right neighbour interference effects, $\mathbf{D}'$ is a $n\times b$ incidence matrix of observations versus blocks, $\boldsymbol{\beta}$

is a $b\times 1$ vector of block effects, $\boldsymbol{\rho}$ is a $p\times 1$ vector representing the trend effects. The matrix $\mathbf{Z}$, of order $n\times p$, is the matrix of coefficients which is given by $\mathbf{Z}=\mathbf{1_b}\otimes\mathbf{F}$ where $\mathbf{F}$ is a $k\times p$ matrix with columns representing the (normalized) orthogonal polynomials and $\mathbf{e}$ is a $n\times 1$ vector of errors with $\mathrm{E}(\mathbf{e}) = 0$ and $\mathrm{D}(\mathbf{e}) = \sigma^2\ \mathbf{I}_n$. Without loss of generality, it can be assumed that the first k observations pertain to the first block, the next k observation pertain to the next block, and so on. Under this ordering, $\mathbf{D}'=\mathbf{I}_b\otimes\mathbf{1}_k$. Since $\mathbf{F}$ is a $k\times p$ matrix with columns representing the (normalized) orthogonal polynomials, thus $\mathbf{1'F}=\mathbf{0}$, $\mathbf{F'F}$ and hence $\mathbf{Z'Z}=b\mathbf{I_P}$.

Rewriting the model as follows by writing parameter of interest first:

$$\mathbf{Y} = \boldsymbol{\Delta}'\boldsymbol{\tau} + \boldsymbol{\Delta}_1'\boldsymbol{\delta} + \boldsymbol{\Delta}_2'\boldsymbol{\gamma} + \mu\mathbf{1} + \mathbf{D}'\boldsymbol{\beta} + \mathbf{Z}\boldsymbol{\rho} + \mathbf{e}, \qquad \ldots(2.2)$$

Equation (2.2) can also be written as:

$$\mathbf{Y} = \mathbf{X}_1\boldsymbol{\theta}_1 + \mathbf{X}_2\boldsymbol{\theta}_2 + \mathbf{e}, \qquad \ldots(2.3)$$

where,

$$\mathbf{X}_1 = \begin{bmatrix}\boldsymbol{\Delta}' & \boldsymbol{\Delta}_1' & \boldsymbol{\Delta}_2'\end{bmatrix};\ \mathbf{X}_2 = \begin{bmatrix}\mathbf{1} & \mathbf{D}' & \mathbf{Z}\end{bmatrix};\ \boldsymbol{\theta}_1 = \begin{bmatrix}\boldsymbol{\tau}' & \boldsymbol{\delta}' & \boldsymbol{\gamma}'\end{bmatrix};\boldsymbol{\theta}_2 = \begin{bmatrix}\mu & \boldsymbol{\beta}' & \boldsymbol{\rho}'\end{bmatrix}'.$$

Let,

$$\mathbf{R}_\tau = \boldsymbol{\Delta\Delta}',\ \mathbf{R}_\delta = \boldsymbol{\Delta}_1\boldsymbol{\Delta}_1',\ \mathbf{R}_\gamma = \boldsymbol{\Delta}_2\boldsymbol{\Delta}_2',\ \mathbf{M}_1 = \boldsymbol{\Delta}\boldsymbol{\Delta}_1',\ \mathbf{M}_2 = \boldsymbol{\Delta}\boldsymbol{\Delta}_2',\ \mathbf{M}_3 = \boldsymbol{\Delta}_1\boldsymbol{\Delta}_2',\ \mathbf{DD}' = \mathbf{K},$$

$$\mathbf{N}_1 = \boldsymbol{\Delta}\mathbf{D}',\ \mathbf{N}_2 = \boldsymbol{\Delta}_1\mathbf{D}',\ \mathbf{N}_3 = \boldsymbol{\Delta}_2\mathbf{D}'.$$

and

$$\mathbf{r} = \boldsymbol{\Delta}\mathbf{1},\ \mathbf{r}_1 = \boldsymbol{\Delta}_1\mathbf{1},\ \mathbf{r}_2 = \boldsymbol{\Delta}_2\mathbf{1},$$

where $\mathbf{r} = \begin{bmatrix}r_1 & r_2 & \ldots & r_v\end{bmatrix}'$ is the $v\times 1$ replication vector of direct treatments with r_s as the number of times s^{th} treatment appears in the design, $\mathbf{r}_1 = \begin{bmatrix}r_{11} & r_{12} & \ldots & r_{1v}\end{bmatrix}'$ is the $v\times 1$ replication vector of left neighbour treatment with r_{1s} as the number of times s^{th} treatment appears in the design as left neighbour and $\mathbf{r}_2 = \begin{bmatrix}r_{21} & r_{22} & \ldots & r_{2v}\end{bmatrix}'$ is the $v\times 1$ replication vector of right neighbour treatment with r_{2s} as the number of times s^{th} treatment appears in the design as right neighbour.

Therefore,

$$\mathbf{X}_1'\mathbf{X}_1 = \begin{bmatrix} \boldsymbol{\Delta\Delta}' & \boldsymbol{\Delta\Delta}_1' & \boldsymbol{\Delta\Delta}_2' \\ \boldsymbol{\Delta}_1\boldsymbol{\Delta}' & \boldsymbol{\Delta}_1\boldsymbol{\Delta}_1' & \boldsymbol{\Delta}_1\boldsymbol{\Delta}_2' \\ \boldsymbol{\Delta}_2\boldsymbol{\Delta}' & \boldsymbol{\Delta}_2\boldsymbol{\Delta}_1' & \boldsymbol{\Delta}_2\boldsymbol{\Delta}_2' \end{bmatrix} = \begin{bmatrix} \mathbf{R}_\tau & \mathbf{M}_1 & \mathbf{M}_2 \\ \mathbf{M}_1' & \mathbf{R}_\delta & \mathbf{M}_3 \\ \mathbf{M}_2' & \mathbf{M}_3' & \mathbf{R}_\gamma \end{bmatrix},$$

$$\mathbf{X}_1'\mathbf{X}_2 = \begin{bmatrix} \boldsymbol{\Delta}\mathbf{1} & \boldsymbol{\Delta}\mathbf{D}' & \boldsymbol{\Delta}\mathbf{Z} \\ \boldsymbol{\Delta}_1\mathbf{1} & \boldsymbol{\Delta}_1\mathbf{D}' & \boldsymbol{\Delta}_1\mathbf{Z} \\ \boldsymbol{\Delta}_2\mathbf{1} & \boldsymbol{\Delta}_2\mathbf{D}' & \boldsymbol{\Delta}_2\mathbf{Z} \end{bmatrix} = \begin{bmatrix} \mathbf{r} & \mathbf{N}_1 & \boldsymbol{\Delta}\mathbf{Z} \\ \mathbf{r}_1 & \mathbf{N}_2 & \boldsymbol{\Delta}_1\mathbf{Z} \\ \mathbf{r}_2 & \mathbf{N}_3 & \boldsymbol{\Delta}_2\mathbf{Z} \end{bmatrix}$$

and

$$\mathbf{X}_2'\mathbf{X}_2 = \begin{bmatrix} \mathbf{1}'\mathbf{1} & \mathbf{1}'\mathbf{D}' & \mathbf{1}'\mathbf{Z} \\ \mathbf{D1} & \mathbf{DD}' & \mathbf{DZ} \\ \mathbf{Z}'\mathbf{1} & \mathbf{Z}'\mathbf{D}' & \mathbf{Z}'\mathbf{Z} \end{bmatrix} = \begin{bmatrix} \mathrm{n} & \mathrm{k}\mathbf{1}' & \mathbf{0} \\ \mathrm{k}\mathbf{1} & \mathrm{k}\,\mathbf{I}_\mathrm{b} & \mathbf{0} \\ \mathbf{0} & \mathbf{0} & \mathrm{b}\mathbf{I}_\mathrm{p} \end{bmatrix}.$$

The $3v \times 3v$ symmetric, nonnegative definite, information matrix for estimating the direct effects, interference effects from the left neighbouring units and right neighbouring units is obtained as:

$$\mathbf{C} = \begin{bmatrix} \mathbf{R}_\tau - \mathbf{N}_1\mathbf{K}^{-1}\mathbf{N}_1' - \frac{1}{\mathrm{b}}\boldsymbol{\Delta}\mathbf{Z}\mathbf{Z}'\boldsymbol{\Delta}' & \mathbf{M}_1 - \mathbf{N}_1\mathbf{K}^{-1}\mathbf{N}_2' - \frac{1}{\mathrm{b}}\boldsymbol{\Delta}\mathbf{Z}\mathbf{Z}'\boldsymbol{\Delta}_1' & \mathbf{M}_2 - \mathbf{N}_1\mathbf{K}^{-1}\mathbf{N}_3' - \frac{1}{\mathrm{b}}\boldsymbol{\Delta}\mathbf{Z}\mathbf{Z}'\boldsymbol{\Delta}_2' \\ \mathbf{M}_1' - \mathbf{N}_2\mathbf{K}^{-1}\mathbf{N}_1' - \frac{1}{\mathrm{b}}\boldsymbol{\Delta}_1\mathbf{Z}\mathbf{Z}'\boldsymbol{\Delta}' & \mathbf{R}_\delta - \mathbf{N}_2\mathbf{K}^{-1}\mathbf{N}_2' - \frac{1}{\mathrm{b}}\boldsymbol{\Delta}_1\mathbf{Z}\mathbf{Z}'\boldsymbol{\Delta}_1' & \mathbf{M}_3 - \mathbf{N}_2\mathbf{K}^{-1}\mathbf{N}_3' - \frac{1}{\mathrm{b}}\boldsymbol{\Delta}_1\mathbf{Z}\mathbf{Z}'\boldsymbol{\Delta}_2' \\ \mathbf{M}_2' - \mathbf{N}_3\mathbf{K}^{-1}\mathbf{N}_1' - \frac{1}{\mathrm{b}}\boldsymbol{\Delta}_2\mathbf{Z}\mathbf{Z}'\boldsymbol{\Delta}' & \mathbf{M}_3' - \mathbf{N}_3\mathbf{K}^{-1}\mathbf{N}_2' - \frac{1}{\mathrm{b}}\boldsymbol{\Delta}_2\mathbf{Z}\mathbf{Z}'\boldsymbol{\Delta}_1' & \mathbf{R}_\gamma - \mathbf{N}_3\mathbf{K}^{-1}\mathbf{N}_3' - \frac{1}{\mathrm{b}}\boldsymbol{\Delta}_2\mathbf{Z}\mathbf{Z}'\boldsymbol{\Delta}_2' \end{bmatrix}$$

$$= \begin{bmatrix} \mathbf{C}_{11} & \mathbf{C}_{12} \\ \mathbf{C}_{21} & \mathbf{C}_{22} \end{bmatrix}. \qquad \text{...(2.4)}$$

The information matrix for estimating the direct effects can be obtained as follows:

$$\mathbf{C}_\tau = \mathbf{C}_{11} - \mathbf{C}_{12}\mathbf{C}_{22}^{-}\mathbf{C}_{21}, \qquad \text{... (2.5)}$$

where $\mathbf{C}^{-}_{22}$ is the g-inverse of $\mathbf{C}_{22}$.

Similarly, the information matrix for estimating the interference effects from left and right neighbouring units can be obtained.

2.2. Block Model with One-sided Interference Effects Incorporating Trend Component

Sometimes we may come across some situations where when there is an interference effect from one side only. For example, in pesticide or fungicides experiments, part of the experiment may spread to the plot immediately down wards. In such situations, one has to consider one-sided interference effects from the neighbouring unit. Following model is considered for analyzing a block design with one-sided interference effect incorporating trend component under the above experimental setup:

$$\mathbf{Y} = \mu\mathbf{1} + \mathbf{\Delta}'\boldsymbol{\tau} + \mathbf{\Delta}_1'\boldsymbol{\delta} + \mathbf{D}'\boldsymbol{\beta} + \mathbf{Z}\boldsymbol{\rho} + \mathbf{e}, \qquad \ldots (2.6)$$

where all the symbols have their same meaning as defined earlier.

The $2v\times 2v$ symmetric, nonnegative definite, information matrix for estimating the direct and interference effect from the immediate neighbouring unit is obtained as:

$$\mathbf{C} = \begin{bmatrix} \mathbf{R}_\tau - \frac{1}{k}\mathbf{N}_1\mathbf{N}_1' - \frac{1}{b}\mathbf{\Delta}\mathbf{Z}\mathbf{Z}'\mathbf{\Delta}' & \mathbf{M}_1 - \frac{1}{k}\mathbf{N}_1\mathbf{N}_2' - \frac{1}{b}\mathbf{\Delta}\mathbf{Z}\mathbf{Z}'\mathbf{\Delta}_1' \\ \mathbf{M}_1' - \frac{1}{k}\mathbf{N}_2\mathbf{N}_1' - \frac{1}{b}\mathbf{\Delta}_1\mathbf{Z}\mathbf{Z}'\mathbf{\Delta}' & \mathbf{R}_\delta - \frac{1}{k}\mathbf{N}_2\mathbf{N}_2' - \frac{1}{b}\mathbf{\Delta}_1\mathbf{Z}\mathbf{Z}'\mathbf{\Delta}_1' \end{bmatrix} \qquad \ldots (2.7)$$

The information matrix for estimating the direct effects can be obtained as follows:

$$\mathbf{C}_\tau = \mathbf{C}_{11} - \mathbf{C}_{12}\mathbf{C}_{22}^{-}\mathbf{C}_{21}, \qquad \ldots (2.8)$$

where $\mathbf{C}^{-}_{22}$ is the g-inverse of $\mathbf{C}_{22}$.

Similarly, the information matrices for estimating the interference effects from left and right neighbouring units can be obtained.

2.3. Definitions

Following are some general definitions associated with the block design with interference effects incorporating trend component:

Definition 2.1: A block design is *balanced for interference effects* from the neighbouring units if every treatment has every other treatment appearing as both left and right neighbour constant number of times (say, μ_1).

Definition 2.2: A block design with interference effects incorporating trend component, is called a *trend-free* design if the adjusted treatment sum of squares arising from direct effects of treatments and interference effects of treatments under the corresponding model is same as the adjusted treatment sum of squares under the usual block model with interference effects without trend component.

Definition 2.3: A trend-free block design with two-sided interference effects is called *variance balanced* if the variance of any estimated elementary contrast among the direct effects is constant (say V_1), the variance of any estimated elementary contrast among the interference effects arising from the left neighboring units is constant (say V_2) and the variance of any estimated elementary contrast among the interference effects arising from the right neighboring units is constant (say V_3). A block design with interference effects is *totally balanced* if $V_1 = V_2 = V_3$.

3. CONDITIONS FOR THE BLOCK DESIGN WITH INTERFERENCE EFFECTS TO BE TREND FREE

The conditions for the block design have been obtained here so that the treatment (direct, interference from left and right neighbouring units) effects and trend effects are orthogonal and the analysis of the design could then be done in the usual manner, as if no trend effect was present. Such designs are known as trend free designs. We now derive a necessary and sufficient condition for a block design with interference effects to be trend free.

Theorem 3.1: A block design with interference effects from left and right neighbouring units and incorporating trend component is said to be trend free if $\mathbf{\Delta Z = 0}$, $\mathbf{\Delta_1 Z = 0}$ and, $\mathbf{\Delta_2 Z = 0}$, where the symbols have their usual meaning as defined earlier.

***Proof*:** As defined in (2.3), $\mathbf{X}_2 = [\mathbf{1} \quad \mathbf{D}' \quad \mathbf{Z}]$. Let $\mathbf{X}_3 = [\mathbf{1} \quad \mathbf{D}']$.

We define,

$$\begin{aligned} \mathbf{A}_u &= \mathbf{I}_n - \mathbf{X}_u \left(\mathbf{X}_u'\mathbf{X}_u\right)^{-} \mathbf{X}_u' \quad (u = 2,3) \\ \mathbf{Q}_{u\tau} &= \boldsymbol{\Delta}\mathbf{A}_u\boldsymbol{\Delta}' \\ \mathbf{Q}_{u\delta} &= \boldsymbol{\Delta}_1\mathbf{A}_u\boldsymbol{\Delta}_1' \\ \mathbf{Q}_{u\gamma} &= \boldsymbol{\Delta}_2\mathbf{A}_u\boldsymbol{\Delta}_2' \end{aligned} \qquad \ldots (3.1)$$

Thus, $$\mathbf{X}_2'\mathbf{X}_2 = \begin{bmatrix} n & k\mathbf{1}' & \mathbf{0} \\ k\mathbf{1} & k\mathbf{I}_b & \mathbf{0} \\ \mathbf{0} & \mathbf{0} & b\mathbf{I}_p \end{bmatrix} \text{ and } \mathbf{X}_3'\mathbf{X}_3 = \begin{bmatrix} n & k\mathbf{1}' \\ k\mathbf{1} & k\mathbf{I}_b \end{bmatrix}. \qquad \ldots (3.2)$$

A g-inverse of $\mathbf{X}_2'\mathbf{X}_2$ and $\mathbf{X}_3'\mathbf{X}_3$ is given, respectively, by

$$\left(\mathbf{X}_2'\mathbf{X}_2\right)^{-} = \begin{bmatrix} 0 & \mathbf{0} & \mathbf{0} \\ \mathbf{0} & \frac{1}{k}\mathbf{I}_b & \mathbf{0} \\ \mathbf{0} & \mathbf{0} & \frac{1}{b}\mathbf{I}_p \end{bmatrix} \text{ and } \left(\mathbf{X}_3'\mathbf{X}_3\right)^{-} = \begin{bmatrix} 0 & \mathbf{0} \\ \mathbf{0} & \frac{1}{k}\mathbf{I}_b \end{bmatrix}. \qquad \ldots (3.3)$$

Hence,

$$\begin{aligned} \mathbf{A}_2 &= \mathbf{I}_n - \mathbf{X}_2 \left(\mathbf{X}_2'\mathbf{X}_2\right)^{-} \mathbf{X}_2' \\ &= \mathbf{I}_n - \frac{1}{k}\mathbf{D}'\mathbf{D} - \frac{1}{b}\mathbf{Z}\mathbf{Z}' \end{aligned} \qquad \ldots (3.4)$$

and

$$\begin{aligned} \mathbf{A}_3 &= \mathbf{I}_n - \mathbf{X}_3 \left(\mathbf{X}_3'\mathbf{X}_3\right)^{-} \mathbf{X}_3' \\ &= \mathbf{I}_n - \frac{1}{k}\mathbf{D}'\mathbf{D}. \end{aligned} \qquad \ldots (3.5)$$

Now we first prove the necessary part. Let the block design with interference effects incorporating trend component is a trend free design. Thus, we have to prove $\mathbf{\Delta Z = 0}$, $\mathbf{\Delta_1 Z = 0}$ and $\mathbf{\Delta_2 Z = 0}$. Let T_z and T_0 be the adjusted treatment sum of squares arising from the direct effect of treatments under Model (2.1) and under the usual block model with interference effects without trend effect respectively. Further let T_{zL} and T_{0L} be the adjusted treatment sum of squares arising from interference effects of treatments in the left neighbouring units under Model (2.1) and under the usual block model with interference effects without trend effect respectively and T_{zR} and T_{0R} be the adjusted treatment sum of squares arising from interference effects of treatments from the right neighbouring units under Model (2.1) and under the usual block model with two-sided interference effects without trend effect respectively. Since the design is assumed to be trend free, thus by definition (2.2), we can write $T_z = T_0$, $T_{zL} = T_{0L}$ and $T_{zR} = T_{0R}$ i.e.

$$\mathbf{Y'A_2\Delta'Q_{2\tau}^{-}\Delta A_2Y = Y'A_3\Delta'Q_{3\tau}^{-}\Delta A_3Y}, \qquad \dots (3.6)$$

$$\mathbf{Y'A_2\Delta_1'Q_{2\delta}^{-}\Delta_1 A_2Y = Y'A_3\Delta_1'Q_{3\delta}^{-}\Delta_1 A_3Y} \qquad \dots (3.7)$$

and

$$\mathbf{Y'A_2\Delta_2'Q_{2\gamma}^{-}\Delta_2 A_2Y = Y'A_3\Delta_2'Q_{3\gamma}^{-}\Delta_2 A_3Y}. \qquad \dots (3.8)$$

Thus, from Equation (3.6),

$$\begin{aligned}&\mathbf{A_2\Delta'Q_{2\tau}^{-}\Delta A_2 = A_3\Delta'Q_{3\tau}^{-}\Delta A_3}\\ \Rightarrow\ &\mathbf{\Delta A_2\Delta'Q_{2\tau}^{-}\Delta A_2\Delta' = \Delta A_3\Delta'Q_{3\tau}^{-}\Delta A_3\Delta'}\\ \Rightarrow\ &\mathbf{\Delta(A_2 - A_3)\Delta' = 0}.\end{aligned} \qquad \dots (3.8)$$

Similarly using Equation (3.7) and Equation (3.8),

$$\begin{aligned}&\mathbf{A_2\Delta_1'Q_{2\delta}^{-}\Delta_1A_2 = A_3\Delta_1'Q_{3\delta}^{-}\Delta_1A_3}\\ \Rightarrow\ &\mathbf{\Delta_1A_2\Delta_1'Q_{2\delta}^{-}\Delta_1A_2\Delta_1' = \Delta_1A_3\Delta_1'Q_{3\delta}^{-}\Delta_1A_3\Delta_1'}\\ \Rightarrow\ &\mathbf{\Delta_1(A_2 - A_3)\Delta_1' = 0},\end{aligned} \qquad \dots (3.9)$$

and

$$\begin{aligned}
&\mathbf{A}_2\boldsymbol{\Delta}_2'\mathbf{Q}_{2\gamma}^{-}\boldsymbol{\Delta}_2\mathbf{A}_2 = \mathbf{A}_3\boldsymbol{\Delta}_2'\mathbf{Q}_{3\gamma}^{-}\boldsymbol{\Delta}_2\mathbf{A}_3\\
\Rightarrow\ &\boldsymbol{\Delta}_2\mathbf{A}_2\boldsymbol{\Delta}_2'\mathbf{Q}_{2\gamma}^{-}\boldsymbol{\Delta}_2\mathbf{A}_2\boldsymbol{\Delta}_2' = \boldsymbol{\Delta}_2\mathbf{A}_3\boldsymbol{\Delta}_2'\mathbf{Q}_{3\gamma}^{-}\boldsymbol{\Delta}_2\mathbf{A}_3\boldsymbol{\Delta}_2'\\
\Rightarrow\ &\boldsymbol{\Delta}_2\left(\mathbf{A}_2-\mathbf{A}_2\right)\boldsymbol{\Delta}_2' = \mathbf{0}.
\end{aligned}$$

Substituting the value of $\mathbf{A_2}$ and $\mathbf{A_3}$ from Equation (3.4) and (3.5) into Equation (3.8) and (3.9) respectively and then solving the corresponding equations we get

$$\boldsymbol{\Delta}\mathbf{Z}=\mathbf{0},\ \boldsymbol{\Delta}_1\mathbf{Z}=\mathbf{0} \text{ and } \boldsymbol{\Delta}_2\mathbf{Z}=\mathbf{0}. \qquad \dots (3.10)$$

To prove the sufficiency, we assume that the condition given in the above theorem is true i.e. $\boldsymbol{\Delta}\mathbf{Z}=\mathbf{0},\ \boldsymbol{\Delta}_1\mathbf{Z}=\mathbf{0}$ and $\boldsymbol{\Delta}_2\mathbf{Z}=\mathbf{0}$. Pre-multiplying and post-multiplying both sides of Equation (3.4) and (3.5) by $\boldsymbol{\Delta}$ and $\boldsymbol{\Delta}'$ respectively and using (3.1) we get:

$$\mathbf{Q}_{2\tau}=\boldsymbol{\Delta}\mathbf{A}_2\boldsymbol{\Delta}'=\boldsymbol{\Delta}\left[\mathbf{I}_n-\frac{1}{k}\mathbf{D}'\mathbf{D}-\frac{1}{b}\mathbf{Z}\mathbf{Z}'\right]\boldsymbol{\Delta}'=\mathbf{R}_\tau-\frac{1}{k}\mathbf{N}_1\mathbf{N}_1'\ \left[\because \boldsymbol{\Delta}\mathbf{Z}=\mathbf{0}\right] \qquad \dots (3.11)$$

and

$$\mathbf{Q}_{3\tau}=\boldsymbol{\Delta}\mathbf{A}_3\boldsymbol{\Delta}'=\boldsymbol{\Delta}\left[\mathbf{I}_n-\frac{1}{k}\mathbf{D}'\mathbf{D}\right]\boldsymbol{\Delta}'=\mathbf{R}_\tau-\frac{1}{k}\mathbf{N}_1\mathbf{N}_1' \qquad \dots (3.12)$$

As, $\mathbf{Q}_{2r}$ = $\mathbf{Q}_{3r}$, thus it is obvious that $T_z = T_0$. Similarly, we can prsove $T_{zL} = T_{0L}$ and $T_{zR} = T_{0R}$. Hence the condition given in the above theorem is both necessary and sufficient.

Corollary 3.1: For a trend free block design with interference effects, the information matrix for estimating the direct effects as well as the information matrix for estimating the interference effects from left and right neighbouring units with trend is same as the information matrix for estimating the direct effects as well as the information matrix for estimating the interference effects from the immediate left and right neighbouring units without trend component.

4. TREND FREE DESIGNS

In this section, methods for construction of trend free balanced block designs with interference effects have been described. In all the cases, it is assumed that the designs are circular. We choose **F** as a $k\times 1$ vector with columns representing the (normalized) orthogonal polynomials and **Z** can be obtained based on **F** as defined earlier in such a way that $\boldsymbol{\Delta}\mathbf{Z}=\mathbf{0}$, $\boldsymbol{\Delta}_1\mathbf{Z}=\mathbf{0}$ and $\boldsymbol{\Delta}_2\mathbf{Z}=\mathbf{0}$.

4.1. Trend Free Complete Block Designs

For v prime, the contents of the v-1 complete blocks of the design balanced for interference effect from neighbouring units can be obtained by writing the treatments in systematic order within a block with a difference of $1,2,\ldots,v$-1 between the treatments (modulo v) in the consecutive blocks. The first block is formed by taking the difference of one between treatments, the second block by taking the difference of two and so on the $(v\text{-}1)^{\text{th}}$ block by taking the difference of (v-1). Considering these (v-1) blocks as initial blocks and developing them modulo v will result in a series of trend free totally balanced complete block design with parameters v, $b = v(v\text{-}1) = r$, $\mu_1 = v$ and every treatment appears in every position in the design same number of times i.e. v-1.

For this class of designs,

$$\begin{aligned}
&\mathbf{R}_\tau = \mathbf{R}_\delta = \mathbf{R}_\gamma = \mathrm{v}(\mathrm{v}\text{-}1)\mathbf{I}_\mathrm{v},\ \mathbf{DD}' = \mathrm{v}\,\mathbf{I}_\mathrm{b},\\
&\mathbf{N}_\mathrm{u}\mathbf{N}'_{\mathrm{u}'} = \mathrm{v}(\mathrm{v}\text{-}1)\mathbf{11}'\ \ [\mathrm{u},\mathrm{u}'=1,2,3]\\
&\mathbf{M}_1 = \mathbf{M}_2 = \mathbf{M}_3 = \mathrm{v}(\mathbf{11}' - \mathbf{I}_\mathrm{v}).
\end{aligned} \qquad \ldots(4.1)$$

The joint information matrix for estimating the direct as well as interference effect from the neighbouring units is obtained and the information matrix for estimating the direct effects is

$$\mathbf{C}_\tau = \frac{\mathrm{v}^2(\mathrm{v}\text{-}3)}{(\mathrm{v}\text{-}2)}\left[\mathbf{I}_\mathrm{v} - \frac{\mathbf{11}'}{\mathrm{v}}\right]. \qquad \ldots(4.2)$$

Similarly, the information matrices for estimating the interference effects from left and right neighbouring units are obtained as

$$\mathbf{C}_{\delta} = \mathbf{C}_{\gamma} = \frac{v^2(v\text{-}3)}{(v\text{-}2)}\left[\mathbf{I}_v - \frac{\mathbf{11}'}{v}\right]. \qquad \text{... (4.3)}$$

Thus, the designs so obtained are totally balanced for estimating the contrasts pertaining to direct effects of treatments and interference effects arising from the left and right neighbouring units.

Remark 4.1: For the above class of design, when interference effects from only one side, i.e. left neighbouring units is considered, the information matrices for estimating direct effects and the interference effects from the left neighbouring units are obtained as

$$\mathbf{C}_{\tau} = \mathbf{C}_{\delta} = \frac{v^2(v\text{-}2)}{(v\text{-}1)}\left[\mathbf{I}_v - \frac{\mathbf{11}'}{v}\right]. \qquad \text{... (4.4)}$$

Example 4.1: For v = 5, the block design balanced for interference effects from the immediate left and right neighbouring units is

0	1	2	3	4	0	1
4	1	3	0	2	4	1
3	1	4	2	0	3	1
2	1	0	4	3	2	1

Considering these four blocks as initial blocks and developing them modulo 5 will result in a trend free totally balanced complete block design with parameters v = 5, b = 20 = r, μ_1 = 5 and every treatment appears in every position in the design four times.

	-2	-1	0	1	2	
0	1	2	3	4	0	1
1	2	3	4	0	1	2
2	3	4	0	1	2	3
3	4	0	1	2	3	4
4	0	1	2	3	4	0
4	1	3	0	2	4	1
0	2	4	1	3	0	2
1	3	0	2	4	1	3
2	4	1	3	0	2	4
3	0	2	4	1	3	0
3	1	4	2	0	3	1
4	2	0	3	1	4	2
0	3	1	4	2	0	3
1	4	2	0	3	1	4
2	0	3	1	4	2	0
2	1	0	4	3	2	1
3	2	1	0	4	3	2
4	3	2	1	0	4	3
0	4	3	2	1	0	4
1	0	4	3	2	1	0

Orthogonal trend component of degree one without normalization (Fisher and Yates, 1957) is given in the upper row and

$$\mathbf{F} = \begin{bmatrix} \frac{-2}{\sqrt{10}} & \frac{-1}{\sqrt{10}} & 0 & \frac{1}{\sqrt{10}} & \frac{2}{\sqrt{10}} \end{bmatrix}' = \begin{bmatrix} -0.63 & -0.31 & 0 & 0.31 & 0.63 \end{bmatrix}'$$

4.2. Trend Free Incomplete Block Designs

Let $v = sm + 1$ be a prime or prime power ($m > 3$), then series of trend free incomplete block design balanced for interference effects from the neighbouring units can be obtained by developing following initial blocks modulo v and augmenting the whole set of blocks generated from each initial block one after another:

$$x^{w+(m-1)s} | x^{w}, x^{w+s}, x^{w+2s}, \ldots x^{w+(m-1)s} | \text{for } w = 0,1,\ldots, s-1$$

where x is the primitive element of $GF\ (v)$. The design so obtained is a trend free block design balanced for interference effect with parameters $v = sm + 1$,

$b = sv$, $r = sm$, $k = m$, $\mu_1 = 1$ and every treatment appears in every position in the design s number of times.

For this class of designs,

$$\mathbf{R}_\tau = \mathbf{R}_\delta = \mathbf{R}_\gamma = \mathrm{r}\,\mathbf{I}_\mathbf{v},\ \mathbf{DD}' = \mathrm{k}\,\mathbf{I}_\mathbf{v},$$

$$\mathbf{N}_\mathrm{u}\mathbf{N}'_{\mathrm{u}'} = (\mathrm{v}\text{-}\mathrm{k})\mathbf{I}_\mathrm{v} + (\mathrm{k}\text{ - }1)\mathbf{11}'\ \left[\mathrm{u},\mathrm{u}'=1,2,3\right],$$

$$\mathbf{M}_1 = \mathbf{M}_2 = \mathbf{M}_3 = (\mathbf{11}' - \mathbf{I}_\mathrm{v}) \qquad \text{...(4.5)}$$

The information matrix for estimating the direct effects is

$$\mathbf{C}_\tau = \frac{\mathrm{v}(\mathrm{k}\text{-}3)}{(\mathrm{k}\text{-}2)}\left[\mathbf{I}_\mathrm{v} - \frac{\mathbf{11}'}{\mathrm{v}}\right]. \qquad \text{... (4.6)}$$

Similarly, the information matrices for estimating the interference effects from left and right neighbouring units are obtained as

$$\mathbf{C}_\delta = \mathbf{C}_\gamma = \frac{\mathrm{v}(\mathrm{k}\text{-}3)}{(\mathrm{k}\text{-}2)}\left[\mathbf{I}_\mathrm{v} - \frac{\mathbf{11}'}{\mathrm{v}}\right]. \qquad \text{... (4.7)}$$

Thus, the designs so obtained are totally balanced for estimating the contrasts pertaining to direct effects of treatments and interference effects arising from the left neighbouring unit.

Remark 4.2: For the above class of design, when interference effects from only left neighbouring units is considered, the information matrices for estimating direct effects and the interference effects are obtained as

$$\mathbf{C}_\tau = \mathbf{C}_\delta = \frac{\mathrm{v}(\mathrm{k}\text{-}2)}{(\mathrm{k}\text{-}1)}\left[\mathbf{I}_\mathrm{v} - \frac{\mathbf{11}'}{\mathrm{v}}\right]. \qquad \text{... (4.8)}$$

Example 4.2: Let $m = 5$, $s = 2$, then we get following two initial blocks modulo 11 for $w = 0$ and $w = 1$:

3 | 1 4 5 9 3 | 1 and 6 | 2 8 10 7 6 | 2

Developing these blocks, we obtain the following linear trend free totally balanced incomplete block design with $v = 11$, $b = 22$, $r = 10$, $k = 5$, $\mu_1 = 1$ and every treatment appears in every position in the design two times. **F** is same as in Example 4.1.

	-2	-1	0	1	2	
3	1	4	5	9	3	1
4	2	5	6	10	4	2
5	3	6	7	0	5	3
6	4	7	8	1	6	4
7	5	8	9	2	7	5
8	6	9	10	3	8	6
9	7	10	0	4	9	7
10	8	0	1	5	10	8
0	9	1	2	6	0	9
1	10	2	3	7	1	10
2	0	3	4	8	2	0
6	2	8	10	7	6	2
7	3	9	0	8	7	3
8	4	10	1	9	8	4
9	5	0	2	10	9	5
10	6	1	3	0	10	6
0	7	2	4	1	0	7
1	8	3	5	2	1	8
2	9	4	6	3	2	9
3	10	5	7	4	3	10
4	0	6	8	5	4	0
5	1	7	9	6	5	1

Remark 4.3: It has also been seen that the both the above class of designs so obtained are trend free up to pth degree ($p < k$).

5. CONCLUSIONS

Interference effects are very common in field experiments. But, when there is slope or while dealing with undulating land in hilly areas, slope may cause a trend in experimental units. To overcome such situations, trend free block designs balanced for interference effects from the neighbouring units have been obtained. The designs so obtained are totally balanced for estimating direct and interference effects of treatments and are capable of completely eliminating the effects of a common trend.

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