Data Envelopment Analysis

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Data Envelopment Analysis: It is a powerful tool to optimise used to measure the efficiency of any sectorial unit in terms of both technical and allocative point of view. It compares a DMUs to a target on the frontier. The frontier is the best practice frontier based upon the current set.

DEA is multiple criteria decision making tools, before one applies DEA model one need to choose a set of peer units, DMU's (Decision Making Unit) in DEA, it means they are the units under evaluation ort benchmarking for eg. Hospitals, banks, universities, products, cities, government, airlines etc. ultimately DEA is intended as a method for performance evaluation and benchmarking against best-practice.

Efficient Frontier:

The line connecting the most efficient point is the efficient frontier called 'frontier line'. Efficient Frontier envelopes other data points, that's why it is called Data Envelopment Analysis (DEA).

- The frontier line displays the performance of the unit in the comparison.
- The efficiency of other unit can be measured by the deviation of the points from the frontier line. Efficiency of other stores is measured relative to the efficient frontier.
- Efficient frontier serves as Benchmark.

The efficiency of any firm or unit consists of two components:

1. **Technical efficiency:** It means the ability of a firm to obtain maximum output from a given set of inputs.
2. **Allocative efficiency:** It means the ability of a firm to use the inputs in optimal proportions, given their respective prices.

In DEA, the efficiency or performance of DMU's were measured in two ways:

1. **Input-oriented model:** It tries to see if the DMU can reduce its current input and still producing the same amount of outputs.
2. **Output-oriented model:** It tries to see if the DMU can increase its current output using current input level.

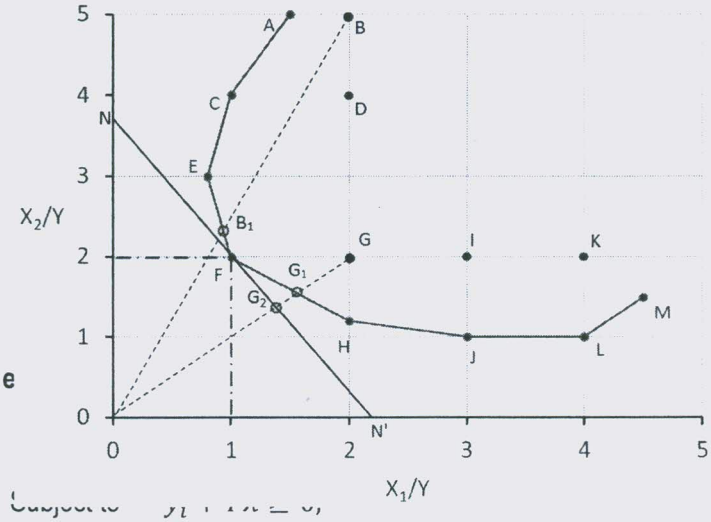
Return to scale (RTS) is often used to characterise the different DEA models. The different DEA frontier type are follows.

1. Constant RTS - CRS
2. Variable RTS : VRS (Constant, Increasing and Decreasing RTS)
3. Non-increasing RTS – NIRS
4. Non-decreasing RTS – NDRS

The term 'Efficiency' is the ability of a firm to obtain maximum (minimum) outputs (inputs) from a given set of inputs (outputs), whereas cost-efficiency requires achieving the lowest possible cost, given the current prices and firm outputs. In figure given below depicts the concept of cost efficiency where there are twelve DMUs labelled as A, B, C, . . .L and M with two inputs and a single constant output with their respective prices. The input price ratio is reflected by the slope of the iso-cost-line NN/ and the curve line connecting the points from 'A' to 'M' DMUs is the frontier line. The concept of frontier is especially

important for the analysis of efficiency, because we measure efficiency as the relative distance to the frontier. For instance, the DMUs like B, D, G, I and K that are technically inefficient, operate at points in the interior of the shaded region, while DMUs that are technically efficient, operate somewhere along the technology defined by the frontier. So every package of inputs along the frontier line is considered technically efficient, while any point above and to the right of the frontier is technically inefficient producer, i.e. the DMU produces the same amount of output, but with greater amounts of both inputs. For example, at point G the measure of radial efficiency identifies the two points, G_1 and G_2 (Farrell, 1957) and defines the 'technical efficiency', 'allocative efficiency' and 'cost efficiency' as $\frac{d(O,G_1)}{d(O,G)}$, $\frac{d(O,G_2)}{d(O,G_1)}$ and $\frac{d(O,G_2)}{d(O,G)}$, respectively. The multiplicative interaction of both technical efficiency (TE) and allocative efficiency (AE), termed as overall economic efficiency (EE), is calculated as per Equation (1):

$$\mathrm{EE} = \mathrm{TE} \times \mathrm{AE} = \frac{\mathrm{d(O,G_1)}}{\mathrm{d(O,G)}} \times \frac{\mathrm{d(O,G_2)}}{\mathrm{d(O,G_1)}} = \frac{\mathrm{d(O,G_2)}}{\mathrm{d(O,G)}} \quad \text{..... (1)}$$



Cost e **ented)**

Subject to

$$\theta x_i - X\lambda \geq 0,$$
$$\lambda \geq 0, \quad \text{.... (2)}$$

Where, θ is a scalar and λ is an Nx1 vector of constraints. This envelopment form involves fewer constraints than the multiplier form$[(K + M) < (N + 1)]$, the value of θ is the efficiency score for the ith DMU. It will satisfy$\theta \leq 1$, with a value of 1 indicating a point on the frontier and hence a technically efficient DMU, according to the Farrell (1957) definition.

To calculate cost efficiency, prices of all the six inputs were used to study the behavioural objective, such as cost minimization or profit maximization. For this, the mathematical form of cost minimization DEA as represented in Equation (3) can be used:

$$min_{\lambda, x_i^*}\ w_i' x_i^*,$$

Subject to

$$-y_i + Y\lambda \geq 0,$$
$$x_i^* - X\lambda \geq 0,$$
$$\lambda \geq 0, \quad \text{.... (3)}$$

where, w_i is a vector of input prices for the ith DMU and x_i^* is the cost minimizing vector of input quantities for the ith DMU, given the input price w_i and the output level y_i. The total cost efficiency (CE) or economic efficiency of the ith DMU is calculated by Equation (4):

$$CE = w_i' x_i^*, / w_i' x_i. \quad \ldots.(4)$$

It is the ratio of minimum cost and observed cost. Using Equation (1) the allocative efficiency (AE) can be calculated as:

$$AE = CE/TE.$$

Steps in DEA analysis

a. Collect the pooled data on Output and Input quantities and their respective values for different DMU's
b. Download the open source DEAP software from the **Centre for Efficiency and Productivity Analysis** (CEPA) portal http://www.uq.edu.au/economics/cepa/deap.htm
c. Install the DEAP
d. Arrange the dataset as per the requirement of DEAP (output first followed by inputs) in *txt file
e. Modify the inbuilt instruction file and compute the Technical, Allocative and Economic efficiency

In that DEAP folder we have to create three type of txt file.

1. Instruction file - Eg1-ins.txt
2. Data file - Eg1-dta.txt
3. Output file - Eg1-out.txt

Instruction file: it can be modified according to our data taken for analysis whether it may single output with multiple input situation or multiple output with single input or multiple output with multiple input. For measuring allocative efficiency value of the inputs also have to mention.

```
eg3-dta.txt        DATA FILE NAME
eg3-out.txt        OUTPUT FILE NAME
5                  NUMBER OF FIRMS
1                  NUMBER OF TIME PERIODS
1                  NUMBER OF OUTPUTS
2                  NUMBER OF INPUTS
0                  0=INPUT AND 1=OUTPUT ORIENTATED
0                  0=CRS AND 1=VRS
1                  0=DEA (MULTI-STAGE), 1=COST-DEA, 2=MALMQUIST-DEA, 3=DEA(1-STAGE), 4=DEA(2-
STAGE)
```

DEA: Illustrated Example using DEAP software

Data has to be arrange in this format given below, here the 1st column represent output quantity, 2nd & 3rd column is input and 4th & 5th column represent input value

1 2 5 1 3

2 2 4 1 3
3 6 6 1 3
1 3 2 1 3
2 6 2 1 3

Results from DEAP Version 2.1

Instruction file = eg3-ins.txt
Data file = eg3-dta.txt

Cost efficiency DEA

Scale assumption: CRS

EFFICIENCY SUMMARY:

firm	te	ae	ce
1	0.500	0.706	0.353
2	1.000	0.857	0.857
3	0.833	0.900	0.750
4	0.714	0.933	0.667
5	1.000	1.000	1.000
mean	0.810	0.879	0.725

Note: te = technical efficiency
ae = allocative efficiency = ce/te
ce = cost efficiency

SUMMARY OF COST MINIMISING INPUT QUANTITIES:

firm	input:	1	2
1		3.000	1.000
2		6.000	2.000
3		9.000	3.000
4		3.000	1.000
5		6.000	2.000

Application of Data Envelopment Analysis (DEA) for determining economic efficiency of Ring seiners operated off Munambam Coast of Kerala

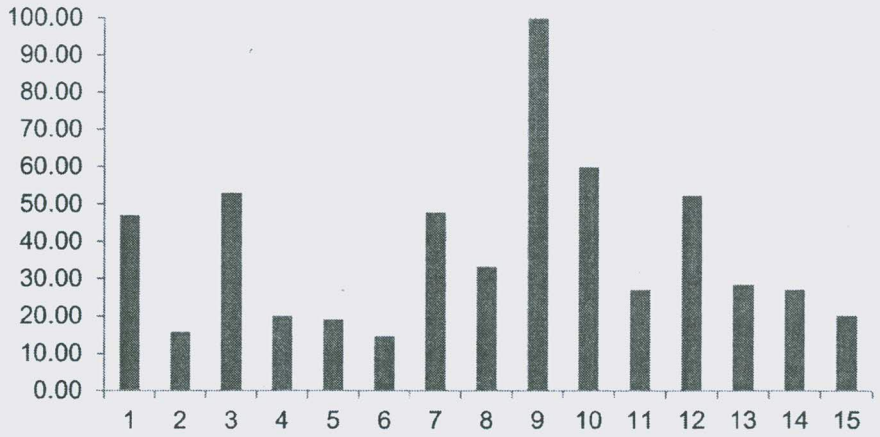
Data Envelopment Analysis (DEA) is an optimization technique is being widely used for measuring efficiency of individual decision making units (DMU). An attempt was made to use DEA for studying the economic efficiency of ring seiners operating off the coast of Munamabam in Ernakulam district of Kerala. DEA is a non-parametric method of measuring the efficiency of and the economic efficiency includes the product of technical as well as allocative efficiency. In DEA the efficiency is worked out within a particular group or sample. The group here is the sampled ring seiners. The technical efficiency is the maximum output that can be obtained from the given level of physical inputs. IN DEA this is with reference to the group and here the technical efficiency is the possible way the given inputs (craft, gear, crew, fuel etc.) is converted into outputs relative to the best DMU in the group . The best unit will be operating at 100% technical efficiency and the DMU with less technical efficiency will be working at percentages below 100%. The costs of inputs do not play any part in technical efficiency and this is factored in the allocative efficiency. Allocative efficiency is related to the cost of inputs in relation to the output, and equilibrium where marginal cost is equal to average revenue. A DMU allocative efficiency is with regard to the allocation of inputs *vis a vis* its price for a given level of output, so as to minimize the costs of production. Also expressed in percentages the higher the percentage the better is the cost minimization with respect to the particular DMU. Cost efficiency refers to the product of technical and allocative efficiencies, expressed as percentage.

Table 1: Technical and Allocative Efficiency of Ring Seiners operating off Munambam coast, Ernkaulam, Kerala

DMU's	Technical Efficiency	Allocative Efficiency
1	1.00	0.47
2	0.26	0.61
3	1.00	0.53
4	0.80	0.25
5	0.28	0.68
6	0.17	0.86
7	0.52	0.92
8	0.37	0.90
9	1.00	1.00
10	0.74	0.81
11	0.30	0.90
12	0.53	0.99
13	0.35	0.81
14	0.36	0.75
15	0.22	0.92
Overall	**0.53**	**0.76**

Data collected during a period July 2009 to June 2012 from 15 individual ring seines operating off Munambam fishing harbour were used for the analysis. Each of the 15 crafts were considered as one DMU. The analysis revealed that the overall technical efficiency of ring seiners operating in Munambam was 52.6% (Table 1) This indicates that the crafts were operating at almost 50% inefficiency that can be attributed to technical factors and the crafts are using comparatively more inputs than necessary which includes the size of the craft, crew, fuel etc. in relation to the quantity of catch (production). The technical efficiency of individual DMUs ranged from 17% to 100%.Allocative efficiency is with regard to the allocation of inputs *vis a vis* its price, so as to minimize the costs of production, and it was 76.0% overall, with individual efficiencies ranging from 25% to 100%. The cost efficiency ranged from 14.62 % to 100% for individual DMUs (Fig. 1).

Fig 1: Cost efficiency of Ring Seiners opertaing off Munambam coast, Ernakulam, Kerala



Further reading:

Charnes A, Cooper W.W and Rhodes E. (1978) Measuring the efficiency of decision making units, European Journal Operational Research, 2, 429 - 444.

Charnes, A., Cooper, W., Lewin, A., Y., and Seinfoud, L., M. (Eds) (1994) Data Envelopment Analysis: Theory, methodology and applications. Boston, Kluwer. Data Envelopment Analysis.

Coelli, T.J. (1996) A guide to DEAP Version 2.1: a data envelopment analysis (computer) program. CEPA Working Papers – No. 8/96. Department of Econometrics, University of New England, Armidale, Australia.

Coelli, T.J. (2008). A Guide to DEAP Version 2.1: A Data Envelopment Analysis (Computer) Program (available at http://www.owlnet.rice.edu/~econ380/DEAP.PDF)

Cooper, W.W., Seiford, L.M., Tone, K. (2007) Data envelopment analysis: A comprehensive text with models, applications, references and DEA-solver software. Kluwer Academic Publishers, Boston, 318 pp.

Farrell, M.J. (1957) The measurement of productivity efficiency. Journal of the Royal Statistical Society 120(3), 253-290.

Johansson, H. (2005) Technical, allocative, and economic efficiency in Swedish dairy farms: the data envelopment analysis versus the stochastic frontier approach. Poster background paper at 11th International congress of the European Association of Agricultural Economists (EAAE), Copenhagen, Denmark, August 24-27, 2005, 17pp.

Ramanathan, R. (2003) An Introduction to Data Envelopment Analysis: A Tool for Performance Measurement. Sage Publications India Pvt. Ltd, New Delhi.

Subhash, C.R. (2004) Data envelopment analysis: Theory and techniques for economics and operational research. Cambridge University Press, UK, 14p.