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ON CONSTRUCTION OF SYMMETRICAL AND ASYMMETRICAL FRACTIONAL FACTORIALS

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SUMMARY

This thesis contains two parts. In the first part a method of construction of symmetrical fractional factorials with blocking for factors each at three levels is given. The method is illustrated with an example. In the appendix a number of plans of various such designs from 7 factors to 15 factors using blocks of size 3^2 and 3^3 have been presented. The key block only of each design is given indicating the method of obtaining the rest of the blocks of the designs. In the second part a few methods of construction of asymmetrical fractional factorial designs are given with illustrations. Also a measure of dependence between any two affected interaction components of an asymmetrical fraction is discussed and the results verified through a particular example. Using the above technique an interaction of interest can be estimated after adjusting it for the effect of any other interaction mixed with it.

I. SYMMETRICAL FRACTIONAL FACTORIALS

1. INTRODUCTION

In factorial experiments, as the number of factors increases, the number of plots required for a complete replication of the experiment becomes larger and larger. Even a single replication may go beyond the resources of the experimenter. Not only that such an experiment becomes quite expensive but also brings about considerable organisational difficulties leading to various hazards in the experiment. For solving these difficulties, Finney (1945) proposed the use of factorial experiments with fractional replication. Much work pertaining to the construction and analysis of such designs has been done by Plackett and Burman (1946), Kempthorne (1947) and others.

One essential assumption to make such experiments useful is that the higher order interactions are negligible. Such designs are usually obtained by taking the key block of a suitable confounded design. Thus the elements of one block of a confounded design constitute the fraction. The set of interactions confounded to get the block is known as the ' identity group of interactions ' or the ' defining contrasts '. Again these treatments are divided into blocks by confounding further suitable interactions. The 'alias' of any factorial effect consists of its generalised interactions with the defining contrasts.

Though several methods of construction of symmetrical fractional factorials are available in literature, it appears that there has not been any attempt for obtaining such designs split into blocks following some systematic method of construction. Das (1964) introduced an alternative approach for construction of confounded design for symmetrical factorial experiments. This method eliminates to a great extent the trial and error operations for getting designs by saving desired interactions. The method consists in first writing the independent combinations in the key block and next

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the other properties including the interactions confounded are derived therefrom. The general method of construction is described below.

Consider the construction of an s^n design in blocks of size s^r where s is a prime or a prime power. Let the s elements of a G.F. denote the s levels of each of the factors. To start with, r independent combinations of r factors are taken such that they form the r columns of an r x r unit matrix where 0 and 1 are the first two levels of each factor. The remaining (n - r) factors are accommodated by introducing (n - r) further columns containing elements 0,1,... s-1 so as to form a scheme of r rows and n columns in such a way that if all k-factor interactions are to be saved, no column should be taken which is the sum of m^{th} multiple (where m is any nonzero element of the field) of one or more of the (k-1) previous columns. From the r independent treatment combinations thus obtained (s^r-1) treatment combinations can be generated through the usual method. These together with the control form the contents of the key block. Through a converse procedure the effects confounded can also be obtained.

In the present investigation a systematic method of constructing symmetrical fractional factorials with each factor at three levels when there is blocking is given utilising the method discussed. A method of getting interactions in the identity group is also given. Tables of such designs from 7 to 15 factors each at three levels in two block sizes 3^2 and 3^3 are also presented. It seems such an attempt has not been made earlier. We have already prepared similar tables for factors each at two levels alongwith the detailed discussion of the method.

While constructing confounded fractional factorial designs, it is desirable that the identity group does not contain

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any interaction with less than five factors nor any two-factor interaction should be confounded with the blocks. If any four-factor interaction or less is included it is likely that a number of two-factor interactions get lost being in the same alias group. On the other hand, confounding a few two-factor interactions might not be so serious in certain cases. The points discussed above have been taken care of while preparation of the tables.

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2. THE METHOD

Let there be n factors each at three levels. Let the levels of each factor be represented by the three elements 0, 1 and 2 G.F. (3). We shall discuss the construction of a $1/3^p$ th replicate of 3^n in blocks of size 3^r such that no interaction with less than five factors is present in the identity group and no interaction with less than three factors is confounded with the blocks. It is necessary that p independent interactions be chosen first to obtain the identity group and then another set of (n-p-r) interactions for the purpose of blocking. The p independent interactions in the identity group are chosen in the following way.

We start with the following (n-p) independent treatment combinations in (n-p) factors which forms the rows of an (n-p)x(n-p) unit matrix.

Factors

		F1	F2	F3	...	Fn-p
	1	1	0	0	...	0
Independent	2	0	1	0	...	0
treatment	3	0	0	1	...	0
combinations	.	-	-	-	...	-
	n-p	0	0	0	...	1

Next p more columns are taken to the right of the above unit matrix one by one, such that the sum of the m^{th} multiples (m = 1,2) of any $t \leq 3$ columns should not give any column which already exists or which is twice of any existing column. Once these p columns are filled as

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above, p independent interactions can be obtained from them by following the method of writing such interactions from columns given by Das (1964). These p interactions alongwith their generalised set constitute the identity group. None of these can contain less than 5 factors by virtue of the restrictions while choosing the columns.

After getting the identity group as above, the next problem is to confound (n-p-r) independent interactions and to get the corresponding key block of size 3^r. For this we shall take the following r independent treatment combinations of r factors in the form of an r x r unit matrix.

Basic Factors

		F1	F2	...	Fr
	1	1	0	...	0
Independent	2	0	1	...	0
treatment	.	-	-	...	-
combinations	r	0	0	...	1

The (n-r) remaining factors are introduced by adding (n-r) more columns to the above unit matrix. First (n-p-r) columns involving 0,1,2 are introduced one by one so as to confound (n-p-r) independent interactions such that no column is taken which has already occured or is a multiple. This will ensure saving of all main effects and two-factor interaction components. The last p columns starting with

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the $(n-p+1)^{th}$ are filled up one by one so as to confound the p independent interactions already chosen while obtaining the identity group.

We shall take one of the above independent interactions in the identity group first. It will have a new factor in it which has not come earlier. The column below the new factor is so taken that the interaction chosen is confounded. We shall adopt the following procedure to ensure such confounding.

Let n_i denote the power of the i^{th} factor in the interaction $(n_i = 0,1,2)$ when the factors are written in some order. Let the elements in the column below the i^{th} factor be denoted by x_i. Now the column x_j $(n-p < j \leq n)$ below the new factor in the interaction selected is so chosen that $\sum_{i=1}^{n-p} n_i x_i + x_j = 0$ (mod 3) for each treatment combination. Again a second interaction is chosen from the set of p independent interactions and the column corresponding to the new factor in this interaction is obtained in a similar way. For example, consider the following system.

Factors

		F1	F2	...	F_{r-1}	Fr	Fr+1	...	Fn-p	Fn-p+1	..	Fn-p+k
	1	1	0	...	0	0	1	...	0	1	..	0
Independent	2	0	1	...	0	0	2	...	2	1	..	2
treatment	.	-	-	...	-	-	-	...	-	-	..	-
combinations	r-1	0	0	...	1	0	0	...	2	1	..	2
	r	0	0	...	0	1	2	...	1	0	..	2

Contd.....7

Let the interaction chosen from the identity group be $F1\ F2^2\ F^2r{+}1\ Fn{-}p\ F^2n{-}p{+}k$. Here Σ mi xi for the first treatment combination is 1+0+2+1 = 3. So the corresponding element below the factor Fn-p+k should be 0 so that $\Sigma m_i\ x_i + x_j = 0$ (mod 3) in the first treatment combination. Similarly in the second combination, the level of the factor Fn-p+k should be 2. In this way the column below the new factor Fn-p+k is filled.

If any one or more of the p columns added thus happens to be a repetition or multiple of any column already taken, then the columns added for confounding have to be adjusted suitably and the process repeated.

The r independent treatment combinations in n factors generate the required key block of the design. The independent effects confounded are also obtained from the above set by inspection.

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3. DERIVATION OF OTHER BLOCKS

After obtaining the key block as above, the next problem is to get the rest of the blocks of the design. It is necessary that these blocks correspond to the set of interactions in the identity group as well as the same set of effects are confounded as in the key block. At the same time care should be taken that no block generated should be identical with any block already obtained. The following method is to be adopted to generate the rest of the blocks of the design.

We prepare a 'key' for obtaining the other blocks from the key block. The key consists of $k(= n-p-r)$ independent interactions when there are 3^k blocks. The key is prepared in the following way. The 'general sum' between two interactions is defined as the sum of the contributions from each of the common factors between these two. A common factor contributes 1 if it has the same power in both the interactions and 2, if the power is different. Thus the general sum of the two interactions AB^2CDF^2 and BC^2EF^2 is $2+2+1 = 5$. Each interaction in the key is so chosen that its general sum with each of the independent interactions in the identity group is zero (mod 3). This takes care of having the same identity group for the derived blocks.

Next we find the general sums of the interaction in the key with each of the independent interactions confounded. Thus we get a vector corresponding to each interaction in the key formed of the general sums of each interaction confounded. Such vectors for the different interactions in the key should be independent.

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This ensures that no block thus generated is a repetition of any block already obtained.

After getting the key as above, all their generalised interactions are obtained. Then take each interaction in the key and those generated from the key and connect it to the corresponding treatment combination by writing the level zero for the absent letters (factors) in it, 1 for the letters with power 1 and 2 for the factors with power 2. Next this treatment combination is added (mod 3) to each of the combinations in the key block following the usual procedures in the ordinary designs. The same procedure is adopted for each interaction in the key and its generalised set so that all the blocks are obtained. For example, if $AC^2 DE^2$ when there are five factors A,B,C,D & E; is one such interaction in the key, a corresponding block is obtained by adding the treatment combination (1 0 2 1 2) to each combination in the key block.

Thus by considering all the interactions in the key and those generated from it, we get the rest of the blocks of the design.

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4. THE TABLES

By following this method we have prepared a number of plans of various fractional factorials with blocking. These plans have been presented in the appendix.

The tables give the key block, the independent interactions in the identity group, the independent interactions confounded, the two-factor interactions saved and a key to the other blocks of the various fractional factorials for factors each at 3 levels ranging from 7 to 15 involving fractions from 1/3 to $1/3^7$. Each design has been split into two different blocks sizes viz. 3^2 and 3^3. Three main variables characterising each design are thus (i) n, the number of factors (ii) p, the fraction and (iii) r, the size of the block. The designs are constructed such that the identity group of interactions do not contain any interaction with less than 5 factors.

Two types of designs are included. The first consists designs keeping all the two-factor interactions free from block effects. In the second type a few of the two-factor interactions are confounded. The second type of designs are mostly optimum designs in the sense that the minimum number of two-factor interactions are being confounded with the blocks in any particular design.

First the independent interactions in the identity group are given for any particular design having specified values of n,p and r. The complete set of interactions in the identity group is to be obtained by generating out of the independent ones given in the usual way. Next the key block of the design is given followed by the independent interactions confounded. The complete set of confounded interactions also can be obtained as usual from the independent confounded interactions. The set of two-factor interactions saved are given next under the headline ' I – S(2) '.

If there is one or more two-factor interactions confounded, they have been mentioned as exception. Next the key for obtaining the other blocks has been presented. The key provides only a set of independent interactions out of which all possible interactions can be generated.

For a given number of factors and fraction size there are two designs — one with blocks of size 3^2 and the other with 3^3. Designs with both the block sizes have been obtained for 7 to 13 factors. Owing to the confounding of increased number of two-factor interactions, (using block size 3^2) when the number of factors is greater than 13, designs with only 3^3 as block size have been presented for 14 and 15 factors. It seems these tables exhaust the possible designs obtained under the restriction that the identity group does not contain any interaction with less than five factors.

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5. ILLUSTRATION

Let us consider for illustration the design $\frac{1}{3^2}(3^7)$ in blocks of size 3^2. Let the factors be represented by A,B,C,D,E,F and G.

Here 2 independent interactions are to be confounded for fractionation and 3 for blocking. To obtain the two independent interactions in the identity group, first a 5 x 5 unit matrix is taken to represent 5 treatment combinations in the first 5 factors. Next two more columns have been taken such that no column is repeated or no column is twice any of the existing columns. In addition each added column has at least 4 nonzero elements and the sum of the m^{th} multiples,(m = 1,2) of these two new columns is having at least 3 nonzero elements. We thus get the following scheme.

A	B	C	D	E	F	G
1	0	0	0	0	1	0
0	1	0	0	0	1	2
0	0	1	0	0	1	1
0	0	0	1	0	1	1
0	0	0	0	1	0	1

The two independent interactions in the identity group are thus $ABCDF^2$ and B^2CDEG^2. From these the following 4 interactions can be obtainedviz, $ABCDF^2$, B^2CDEG^2, $AC^2D^2F^2G^2$, $AB^2E^2F^2G$. The above interactions constitute the identity group given by

$I = ABCDF^2 = B^2CDEG^2 = AC^2D^2F^2G^2 = AB^2E^2F^2G$,

Contd.....13

Next a 2 x 2 unit matrix is taken (r being 2), and 3 (= n-p-r) columns are ' added ' to this so as to confound three independent interactions. Obviously, it is not possible to save all two-factor interactions in this case as there are 7 columns and only 4 independent columns are available. viz. $\binom{1}{0}$, $\binom{0}{1}$, $\binom{1}{1}$ and $\binom{1}{2}$. So necessarily three columns are to be taken repeated or double of any three of the above four. After filling up the first 5, the remaining 2 columns are filled up so as to confound the two independent interactions in the I.G. obtained above. First the interaction $ABCDF^2$ is chosen and the elements below the new factor F is taken such that the sum of elements below A,B,C,D together with twice the elements below F is 0 (Mod 3). Next the second interaction B^2CDEG^2 is taken and the elements below the new factor G is obtained in a similar way. Thus we get the two independent treatment combinations in the key block as

A	B	C	D	E	F	G
1	0	1	0	1	2	2
0	1	1	2	0	1	2

Out of the above two, (3^2-1) treatment combinations can be generated and together with the control treatment we get the key block of size 3^2. The three independent interactions confounded are ABC^2, B^2D^2 and BE^2.

It can be seen that there are more than one way of obtaining the same design by confounding different sets of three independent interactions.

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Next we select 3 (= n-p-r) independent interactions to form the key to the other blocks such that each of these will have its 'general sum' 0 (mod 3) with each of the independent interactions in the I.G. First let us select the interaction CFG. Its general sum with the above two independent interactions of the I.G. is 0 (mod 3). Thus we obtain the three interactions in the key as CFG, DFG and EG. Taking the 'general sum' between each of the above interactions in the key and each of the independent interactions confounded, we get the vectors $\begin{pmatrix} 2 \\ 0 \\ 0 \end{pmatrix}$, $\begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix}$ and $\begin{pmatrix} 0 \\ 0 \\ 2 \end{pmatrix}$ corresponding to CFG, DFG, and EG respectively. These are evidently independent. Thus the three independent interactions CFG, DFG, and EG constitute the key to other blocks.

On generating the interactions in the key, we get a set of 13 interactions. Each of these interactions is squared so as to generate another 13 interactions. Next we take a particular interaction in the key and write the corresponding treatment combinations and add this to each treatment combination in the key block. This gives a new block. There will be 26 such blocks and alongwith the key block already obtained we get the complete design. Two blocks are given below derived from the two key interactions EG & CDF^2G^2. They are obtained by adding to each treatment combination in the key, in mod (3), the combinations (0 0 0 0 1 0 1) and (0 0 1 1 0 2 2) respectively which correspond to the above two interactions.

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Key Block

A	B	C	D	E	F	G
0	0	0	0	0	0	0
1	0	1	0	1	2	2
0	1	1	2	0	1	2
1	1	2	2	1	0	1
1	2	0	1	1	1	0
2	0	2	0	2	1	1
0	2	2	1	0	2	1
2	2	1	1	2	0	2
2	1	0	2	2	2	0

Block
derived using E,G.

A	B	C	D	E	F	G
0	0	0	0	1	0	1
1	0	1	0	2	2	0
0	1	1	2	1	1	0
1	1	2	2	2	0	2
1	2	0	1	2	1	1
2	0	2	0	0	1	2
0	2	2	1	1	2	2
2	2	1	1	0	0	0
2	1	0	2	0	2	1

Block
Derived using CDF^2G^2

A	B	C	D	E	F	G
0	0	1	1	0	2	2
1	0	2	1	1	1	1
0	1	2	0	0	0	1
1	1	0	0	1	2	0
1	2	1	2	1	0	2
2	0	0	1	2	0	0
0	2	0	2	0	1	0
2	2	2	2	2	2	1
2	1	1	0	2	1	2

In a similar way the other blocks also can be obtained.

II. ASYMMETRICAL FRACTIONAL FACTORIALS

1. Introduction

Starting with the pioneering work of Yates (1937), the problem of construction of asymmetrical factorial designs has received considerable attention. Nair and Rao (1941, 1942 and 1948) gave some conditions of balance of the asymmetrical factorial designs. Later several methods of construction of confounded asymmetrical designs were given by Das (1960), Kishen and Srivastava (1959) and others. Chakravarti (1956) gave a technique of obtaining fractions of asymmetrical factorial designs by first considering fractions of symmetrical factorial designs and then combining suitably. But it seems that not much work has been done for construction and analysis of asymmetrical fractional factorials.

Das (1960) introduced a general method of construction of confounded asymmetrical factorials by linking them with fractional replicates of symmetrical factorial designs. A few methods of construction of fractional factorials of asymmetrical designs are presented here.

It happens mostly in the case of confounded asymmetrical factorial designs that a number of lower order interactions get mixed with the confounded interactions and independent estimation of these effects is not possible. A study has been made here to examine the nature of such dependence and also to find out a suitable method of comparing fractionally replicated asymmetrical designs.

As we have to refer to the method of construction of confounded asymmetrical factorial designs given by Das (1960) it has been described briefly below.

The asymmetrical factorial design $s_1 \times s^m$ in blocks of $s_1 \times s^k$ plots is obtained through a fractional replicate of the symmetrical design (s^M, s^L) in s^L blocks with M factors each at s levels where L = m-k, M = p+m and p is such that $s^{p-1} < s_1 \leq s^p$. The set of P factors out of M corresponding to s_1 are called x-pseudo factors and the other set of m factors, the real factors.

To obtain the design (s^M, s^L), $\frac{s^L-1}{s-1}$ interactions of (s-1) d.f each will be confounded. The confounding set is so chosen such that no main effect or interaction of of the pseudo factors is confounded and this design is called the parent design. Now by omitting from each of the blocks of this design those treatment combinations containing a given number y, of the combinations of the pseudo factors we shall be left with s^L blocks each of size $(s^p-y)s^k$. Thus the total number of treatment combinations left will be $(s^p-y)s^m$. If y is so taken that $(s^p-y) = s_1$ and the remaining s_1 combinations of the x-pseudo factors be re-defined to be the s_1 levels of a factor called X, the remaining portion of the parent design gives a replicate of the asymmetrical design $s_1 \times s^m$ in blocks of $s_1 \times s^k$ plots. With the help of different confounding sets, different replicates are obtained. Thus a s_1/s^p fractional replicate of the parent design has been converted to a replicate of the asymmetrical design through redifinition of of s_1 of the combinations of the pseudo factors.

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If any interaction of the parent design contains a set of the pseudo factors together with some real factors it will correspond to that interaction of the asymmetrical design which is obtained from it by replacing the set of pseudo factors by the single factor X. Thus an interaction of the parent design corresponds to an interaction of the asymmetrical design if it contains only real factors.

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2. METHODS OF CONSTRUCTION

A few methods of construction of fractional replicates of asymmetrical factorial designs are presented below.

(i) Designs of the type $s_1 \times s^m$ in $s_1 \times s^r$ plots

By following the method given by Das (1960), first one replication of a confounded symmetrical factorial design is obtained. Next the key block of this design is taken as the required fraction. The identity group of interactions will consist of all these interactions which are confounded in the asymmetrical design. Similarly given any interaction say, P, of the asymmetrical design, we find its aliases by taking its generalised interactions with the set of interactions in the identity group of the asymmetrical design. While multiplying the interactions in the identity group by P, the square of any factor of assymetry in 2^n series or the cube of any such factor in 3^n series has to be taken as unity just as in the case of real factors.

In the case of symmetrical designs, the interactions in the alias group of any particular effect are completely dependent among themselves, but in asymmetrical designs, they are partially dependent so that they may be estimable through appropriate methods one of which has been described later.

As an illustration, let us consider the design $2^5 \times 3$ in $2^3 \times 3$ plots.

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Let the five real factors each at two levels 0 and 1, be denoted by A,B,C,D and E and the two pseudo factors by X_1 and X_2. The parent design is 2^7. We shall omit the combination (1 1) of the pseudo factors so that the remaining three combinations (0 0), (1 0) and (0 1) can be redefined as the three levels of the single factor X.

To get the fraction, we have to confound two independent interactions. Let them be $ABDEX_1$ and $BCDX_2$. By confounding the above two independent interactions in the parent design such that the combination (1 1) of X_1 and X_2 is not present in the design and by renaming the rest three levels we get the required fraction as given below.

X A B C D E	X A B C D E	X A B C D E
0 0 0 0 0 0	1 0 0 1 1 0	2 0 0 0 1 1
0 0 1 0 1 0	1 0 0 0 0 1	2 0 0 1 0 0
0 1 0 0 0 1	1 0 1 0 1 1	2 0 1 1 1 0
0 1 1 0 1 1	1 0 1 1 0 0	2 0 1 0 0 1
0 0 0 1 1 1	1 1 0 1 1 1	2 1 0 1 0 1
0 0 1 1 0 1	1 1 0 0 0 0	2 1 0 0 1 0
0 1 0 1 1 0	1 1 1 1 0 1	2 1 1 1 1 1
0 1 1 1 0 0	1 1 1 0 1 0	2 1 1 0 0 0

The identity group of the above fraction is

I = ABDEX = BCDX = ACEX = ABDE = BCD = ACE

The alias of the interaction say, BC is

BC = ACDEX = DX = ABEX = ACDE = D = ABE

Similarly the alias of AX is

AX = BDE = ABCD = CE = BDEX = ABCDX = CEX

Thus alias of any interaction can be obtained in a similar way.

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(ii) Mixed fractions of the type $s_1^{m1} \times s_2^{m2}$ in $s_1^{r1} \times s_2^{r2}$ plots, $s1 > s_2$.

It will be seen that through the method of construction described inthé previous section it is possible to get only fractions of the form 1/p where p is a prime or a prime power. We shall now describe a method by which a fraction of the form 1/q where q is a product of different primes or prime powers can be obtained. For this purpose we shall first get a design which is a fraction $1/q_1$ where q_1 is the greater factor which is either a prime or a prime power through the method discussed already. Next we consider only one of the two symmetrical sets having factors of the same level say, s_2 where s_2 is a prime or a prime power. A proper identity group is chosen among these factors and all the treatment combinations in the previous design are fractionated with the help of this identity group through an identical method.

This method can be easily generalised to get fractions of the design $s_1^{m1} \times s_2^{m2} \times s_3^{m3}$ and so on.

We shall discuss the construction of the design $\frac{1}{2 \times 3}(2^3 \times 3^3)$ in $2^2 \times 3^2$ plots.

Let X,Y,Z denote the factors each at two levels and A,B,C, the factors each at three levels. First we get a 1/3 replication of the above design by the method discussed in the earlier section by taking XYZABC as the identity group. Next a half-replicate of the above fraction is obtained by confounding the interaction XYZ. We get the design in 36 combinations as given below.

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X Y Z	A B C
	0 0 0
	0 1 2
	0 2 1
	1 2 0
0 0 0	1 0 2
	1 1 1
	2 0 1
	2 1 0
	2 2 2

X Y Z	A B C
	0 1 0
	0 0 1
	0 2 2
0 1 1	1 1 2
1 0 1	1 2 1
1 1 0	1 0 0
	2 0 2
	2 2 0
	2 1 1

The combinations of XYZ are taken with each of the combinations of A B C given to the right.

The identity group of interactions of the above design is

$I = ABC = ABCXYZ = XYZ = ABCX = ABCY$

$= ABCZ = ABCXY = ABCXZ = ABCYZ$

The alias of an interaction, say, A X is given by

$AX = A^2BCX = A^2BCYZ = AYZ = A^2BC = A^2BCXY$

$= A^2BCXZ = A^2BCY = A^2BCZ = A^2BCXYZ$

Similarly the alias of A B is

$AB = A^2B^2C = A^2B^2CXYZ = ABXYZ = A^2B^2CX = A^2B^2CY$

$= A^2B^2CZ = A^2B^2CXY = A^2B^2CXZ = A^2B^2CYZ$

In a similar way, the alias of any other interaction can be obtained. These aliases show the type of dependence among the different interaction components.

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3. A MEASURE OF DEPENDENCE

In the case of asymmetrical fractionally replicated designs, the lower order interactions may sometimes get mixed up among themselves. As mentioned earlier, the different interactions in the alias group are partially dependent. In such cases the independent estimation of such interactions is not possible. Because of this fact comparison of two or more designs cannot be done in an effective way, unless a measure of the degree of dependence of such interactions of interest can be obtained. An attempt has been made here to investigate for a measure of the degree of dependence among the dependent interactions.

It often happens that a fractionally replicated asymmetrical design can be constructed by two or more methods. The contents as well as the nature of dependence of different interaction components are usually different for the different designs, even though their sizes remain the same. In such cases it becomes necessary to evolve a criterian for the comparison of such designs.

Suppose two interaction components are dependent. We can then form a two-way table corresponding to the levels of these two interactions. The cell frequencies of this table will be unequal. With the help of the frequency table we can adjust the S.S. due to one interaction for the other. The loss of information of one interaction due to adjustment for the other can be taken as a measure of the degree of their dependence.

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This method becomes particularly helpfull when a main effect is dependent on a two-factor interaction component or one two-factor interaction component is dependent on another two-factor interaction component and the other dependent interactions are of higher order and can be as such neglected.

It may be mentioned here that the loss of information of the interaction say, A when adjusted for the interaction B has been observed to be the same as the loss of B when it is adjusted for A.

The loss of information ofthe interaction A is obtained for each orthogonal contrast representing A by subtracting from unity the ratio of the variance of the contrast of A when adjusted to that of A when unadjusted. The total loss of A is the sum of the losses for the individual contrasts.

There is another method for getting a measure of the degree of dependence of two interaction components. We shall introduce this concept through a particular case and it would appear that its generalisation is immediate. The second method has been introduced particularly because it leads to the same result as obtainable from the first.

Let AB and CD be two interactions which are dependent in a fractionally replicated asymmetrical design with the factors A, B, C and D each at two levels. We know that a contrast of the observations or treatment combinations gives an estimate of AB. A similar contrast is there for CD. With reference to each of these contrasts each of the treatment combinations available in the fraction can be associated with a frequency in the following way.

Contd......25

We shall consider a particular treatment combination. Let it occur in the contrast of AB with the coefficient C_1. Then we allot a frequency c_1 to this observation with respect to the contrast of AB. Similarly the same observation will have another frequency say, c_2 allotted to it with respect to the contrast of CD. In this way each of the treatment combinations in the fraction will be allotted two frequencies corresponding to the two interaction contrasts. A correlation coefficient among these frequencies can then be obtained by taking the ratio of the sum of products of the frequencies and the product of the square roots of individual sum of squares of the two sets. When any one or both of the interaction components has got more than one degree of freedom, we can define mutually orthogonal contrasts of each interaction and then form all possible pairs of such contrasts of the two interactions under consideration. The correlation coefficient for each such pair of contrasts is then obtained.

It has been found that the sum of squares of these correlation coefficients is equal to the loss of information described earlier.

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4. ILLUSTRATION

As an illustration to verify the results discussed in the previous section, let us consider a half-replicate of the design ($2^2 \times 5^2$). We shall denote the two real factors by A and B, each at five levels, and the three X-Pseudo factors by X_1, X_2, X_3 and the three Y-pseudo factors by Y_1, Y_2 and Y_3. The parent design is thus a 2^8 design. The combinations (1 1 1), (1 1 0) and (0 1 1) of the two sets of pseudo factors are omitted. By choosing the interaction in the identity group to be of the highest order viz. $ABX_1X_2X_3Y_1Y_2Y_3$ in the parent design and then renaming the combinations (0 0 0), (1 0 0), (0 1 0), (0 0 1) and (1 0 1) of the pseudo factors of X and Y as 0, 1, 2 , 3 and 4 respectively we obtain the design in 50 combinations as given below.

A B	X Y
	0 0
	1 1
	2 1
	3 1
	1 2
0 0	2 2
1 1	3 2
	1 3
	2 3
	3 3
	4 4
	0 4
	4 0

A B	X Y
	1 0
	2 0
	3 0
	1 4
	2 4
1 0	3 4
0 1	4 2
	0 2
	0 1
	0 3
	4 1
	4 3

Contd......27

The combinations (0 0) and (1 1) of AB are taken with each of the 12 combinations of X and Y given right and (1 0) and (0 1) with each of the 12 combinations of X and Y presented to the right of them forming thus the complete design.

The identity group of interactions in the asymmetrical designs is thus

$I = ABXY = ABX = ABY = AB = XY$

We are interested to know the extent to which any particular interaction of interest is mixed up with any other such interaction or interactions so that we can estimate that interaction after adjusting for the other. As the fraction was obtained by confounding $ABX_1X_2\ X_3Y_1Y_2Y_3$ i.e ABXY, AB is completely mixed up with XY so that no information will be left when it is adjusted for XY and vice-versa. Also it is found that the main effect X is not independent of AB. We can however estimate X after adjusting for AB through the principle of fitting constants. The frequency table for the purpose is shown below.

Levels of X

		X_0	X_1	X_2	X_3	X_4	Marginal totals
Levels	$(AB)_0$	4	6	6	6	4	26
of AB	$(AB)_1$	6	4	4	4	6	24
Marginal totals		10	10	10	10	10	50

We have two treatments say, $(AB)_0$ and $(AB)_1$, one way and five treatments say, X_0, X_1, X_2, X_3 and X_4 the other way. We shall write the normal equations of either adjusted for the other. Such normal equations for AB are shown below.

$$\left(26 - \frac{140}{10}\right)(AB)_0 - \frac{120}{10}(AB)_1 = Q_0$$
$$\left(24 - \frac{120}{10}\right)(AB)_1 - \left(\frac{120}{10}\right)(AB)_0 = Q_1$$

By solving these equations in the usual way, we get the estimates as

$$\hat{(AB)}_0 = \frac{Q_0}{24}$$
$$\hat{(AB)}_1 = \frac{Q_1}{24}$$

The adjusted sum of squares can also be obtained through the usual method of analysis of non-orthogonal data through the method of fitting constants. i.e from $\sum_{i=1}^{2} (AB)_i Q_i$. Thus the variance of the adjusted difference of the two effects is

$$V\left[\hat{(AB)}_0 - \hat{(AB)}_1\right] = \frac{2}{24}\sigma^2$$

The variance of the difference when unadjusted for X is

$$V\left[(AB)_0 - (AB)_1\right] = \left(\frac{1}{26} + \frac{1}{24}\right)\sigma^2 = \frac{50}{26x24}\sigma^2$$

Thus the loss of AB due to adjustment for X is

$$1 - \frac{\text{Variance adjusted}}{\text{Variance unadjusted}} = \frac{1}{26}$$

Let us now consider the loss of X after adjusting for AB. The five normal equations are given below.

Contd......29

$$\left(10 - \frac{16}{26} - \frac{36}{24}\right) X_0 - \left(\frac{24}{26} + \frac{24}{24}\right)(X_1+X_2+X_3) - \left(\frac{16}{26}+\frac{36}{24}\right)(X_4) = Q_0 \ldots\ldots(1)$$

$$\left(10 - \frac{36}{26} - \frac{16}{24}\right) X_1 - \left(\frac{24}{26} + \frac{24}{24}\right)(X_0+X_4) - \left(\frac{36}{26} + \frac{16}{24}\right)(X_2+X_3) = Q_1 \ldots(2)$$

$$\left(10-\frac{36}{26} - \frac{16}{24}\right) X_2 - \left(\frac{24}{26} + \frac{24}{24}\right)(X_0+X_4) - \left(\frac{36}{26} + \frac{16}{24}\right)(X_1 + X_3) = Q_2 \ldots\ldots(3)$$

$$\left(10-\frac{36}{26} - \frac{16}{24}\right) X_3 - \left(\frac{24}{26} + \frac{24}{24}\right)(X_0+X_4) - \left(\frac{36}{26} + \frac{16}{24}\right)(X_1+X_2) = Q_3 \ldots\ldots(4)$$

$$\left(10-\frac{16}{26} - \frac{36}{24}\right) X_4 + \left(\frac{24}{26} + \frac{24}{24}\right)(X_1+X_2+X_3) - \left(\frac{16}{26}+\frac{36}{24}\right)(X_0) = Q_4 \ldots\ldots(5)$$

Through these equations we get the following estimates of the four mutually orthogonal contrasts c_1, c_2, c_3 and c_4 as shown below.

$$\hat{c}_1 = \hat{X}_0 - \hat{X}_4 = \frac{Q_0 - Q_4}{10}$$

$$\hat{c}_2 = \hat{X}_1 - \hat{X}_2 = \frac{Q_1 - Q_2}{10}$$

$$\hat{c}_3 = \hat{X}_1 + \hat{X}_2 - 2\hat{X}_3 = \frac{Q_1 + Q_2 - 2Q_3}{10}$$

and $$\hat{c}_4 = \frac{\hat{X}_0 + \hat{X}_4}{2} - \frac{\hat{X}_1 + \hat{X}_2 + \hat{X}_3}{3} = \left(\frac{13}{250} + \frac{13}{375}\right)(Q_0 + Q_4)$$

Now the variances of the above four orthogonal contrasts c_1, c_2, c_3 and c_4 adjusted for AB are given by $\frac{2}{10}, \frac{2}{10}, \frac{6}{10}$ and 13/150 and the variances unadjusted, by 2/10, 2/10, 6/10 and 5/60 respectively. The losses corresponding to the four contrasts are thus 0, 0, 0, and 1/26 respetively. Thus the total loss of information of X due to adjustment for AB is 1/26 which is the same as the loss of AB when adjusted for X obtained earlier.

This illustrates the first result that the loss of information is the same when any one of the two interactions is adjusted for the other.

Contd......30

For a 2 x k frequency table the same loss can be obtained by the following relationship.

$$\text{Loss} = 1 - \frac{n}{n_{\cdot 1}\, n_{\cdot 2}} \sum_{i=1}^{k} \frac{n_{i1}\, n_{i2}}{n_{i\cdot}}$$

Where n_{ij} is the frequency corresponding to the i^{th} level of one factor and the j^{th} level of the second; $i = 1$ to k, $j = 1,2$ and $n = \sum_{i,j} n_{ij}$, $n_{i\cdot} = \sum_j n_{ij}$, $n_{\cdot j} = \sum_i n_{ij}$

Applying the above formula we have the loss = $1 - \frac{50}{26 \times 24} \times \frac{24}{10} \times 5 = \frac{1}{26}$

Next we shall verify the results that the total loss is equal to the sum of squares of the correlation coefficients. For this we shall write down first all the 50 treatment combinations. For each combination we associate certain frequencies as discussed earlier, corresponding to the contrast $(AB)_0 + (AB)_1$ of AB denoted by S, and the four orthogonal contrasts of the levels of X. For example the combination (0 1 1 4) will have frequencies - 1/24, 0, 1, 1, and - 1/3 corresponding to the contrasts S, c_1, c_2, c_3 and c_4 respectively. Next we calculate the correlation coefficients between S and each of the four contrasts of the levels of X. If x_{i1} and a_i denote the frequencies corresponding to the contrasts c_1 and S respectively of the i^{th} treatment combination, then the correlation coefficient of the two contrasts is given by

$$r_1 = \text{cor}\,(S, c_1) = \frac{\sum x_{i1} a_i}{\sqrt{\sum x_{i1}^2} \cdot \sqrt{\sum a_i^2}}, \quad i = 1 \text{ to } 50$$

The correlation coefficients thus calculated are given below.

$$r_1 = \text{cor}\,(S, c_1) = 0$$
$$r_2 = \text{cor}\,(S, c_2) = 0$$
$$r_3 = \text{cor}\,(S, c_3) = 0$$
$$r_4 = \text{cor}\,(S, c_4) = \sqrt{1/26}$$

It can be easily verified that the total loss is the sum

Contd...31

of squares of the correlation coefficients.

The above results holds good for any set of 4 mutually orthogonal contrasts of the levels of X. Let us consider another set of mutually orthogonal contrasts namely $X_0 - X_1$, $X_0 + X_1 - 2X_2$, $X_0+X_1+X_2-3X_3$, $X_0 + X_1 + X_2 + X_3 - 4X_4$ denoted by m_1, m_2, m_3, m_4 respectively. By writing down the corresponding frequencies of the five contrasts and calculating the correlation between S and each of the contrasts of the levels of X, we obtain the following four correlation coefficients.

$$r'_1 = \text{cor}\ (S, m_1) = \frac{\sqrt{5}}{\sqrt{26 \times 12}}$$

$$r'_2 = \text{cor}\ (S, m_2) = \frac{\sqrt{5}}{\sqrt{26 \times 36}}$$

$$r'_3 = \text{cor}\ (S, m_3) = \frac{\sqrt{5}}{\sqrt{26 \times 72}}$$

$$r'_4 = \text{cor}\ (S, m_4) = \frac{\sqrt{3}}{\sqrt{26 \times 8}}$$

It can be verified that

$$r'^2_1 + r'^2_2 + r'^2_3 + r'^2_4 = \frac{1}{26}$$

The results discussed above seem to be true in general though pointed out through a particular example.

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APPENDIX

Notations:

N, the number of factors

P, the fraction

R, the size of the block

1. DESIGN WITH N= 7, P= 1, R=2
IDENTITY GROUP OF INTERACTIONS

I=ABCDEFG

KEY BLOCK

A	B	C	D	E	F	G
0	0	0	0	0	0	0
1	0	1	1	1	0	2
0	1	1	2	0	1	1
1	1	2	0	1	1	0
1	2	0	2	1	2	1
2	0	2	2	2	0	1
0	2	2	1	0	2	2
2	2	1	0	2	2	0
2	1	0	1	2	1	2

INDEPENDENT INTERACTIONS CONFOUNDED

$ABC^2, AB^2D^2, AE^2, BF^2$

I.S(2)-ALL EXCEPT AE^2, BF^2, DG

KEY TO OTHER BLOCKS

C^2G, D^2G, E^2G, F^2G

2. DESIGN WITH N= 7, P= 2, R=2
IDENTITY GROUP OF INTERACTIONS

$I=ABCDF^2=B^2CDEG^2$

KEY BLOCK

A	B	C	D	E	F	G
0	0	0	0	0	0	0
1	0	1	0	1	2	2
0	1	1	2	0	1	2
1	1	2	2	1	0	1
1	2	0	1	1	1	0
2	0	2	0	2	1	1
0	2	2	1	0	2	1
2	2	1	1	2	0	2
2	1	0	2	2	2	0

INDEPENDENT INTERACTIONS CONFOUNDED

ABC^2, BD, AE^2

I.S(2)-ALL EXCEPT AE^2, BD, CG

KEY TO OTHER BLOCKS

CFG, DFG, EG

3. DESIGN WITH N= 8, P= 1, R=2

IDENTITY GROUP OF INTERACTIONS

$I=ABCDEF^{2}GH$

KEY BLOCK

A	B	C	D	E	F	G	H
0	0	0	0	0	0	0	0
1	0	1	1	1	0	1	1
0	1	1	0	2	1	2	1
1	1	2	1	0	1	0	2
1	2	0	1	2	2	2	0
2	0	2	2	2	0	2	2
0	2	2	0	1	2	1	2
2	2	1	2	0	2	0	1
2	1	0	2	1	1	1	0

INDEPENDENT INTERACTIONS CONFOUNDED

$ABC^{2}, AD^{2}, AB^{2}E^{2}, BF^{2}, AB^{2}G^{2}$

I.S(2)-ALL EXCEPT $AD^{2}, BF^{2}, CH, EG^{2}$

KEY TO OTHER BLOCKS

$CH^{2}, DH^{2}, FH, GH^{2}$

4. DESIGN WITH N= 8, P= 2, R=2

IDENTITY GROUP OF INTERACTIONS

$I=ABCDG^{2}=B^{2}CDEH^{2}$

KEY BLOCK

A	B	C	D	E	F	G	H
0	0	0	0	0	0	0	0
1	0	1	1	2	2	0	1
0	1	2	1	1	2	1	0
1	1	0	2	0	1	1	1
1	2	2	0	1	0	2	1
2	0	2	2	1	1	0	2
0	2	1	2	2	1	2	0
2	2	0	1	0	2	2	2
2	1	1	0	2	0	1	2

INDEPENDENT INTERACTIONS CONFOUNDED

$AB^{2}C^{2}, ABD^{2}, AB^{2}E, ABF$

I.S(2)-ALL EXCEPT AH^{2}, BG^{2}, CE, DF

KEY TO OTHER BLOCKS

CGH, DGH, EH, F

5. DESIGN WITH N= 8, P= 3, R=2
IDENTITY GROUP OF INTERACTIONS

$I=ABCDF^2=B^2CDEG^2=A^2BDE^2H^2$

KEY BLOCK

A	B	C	D	E	F	G	H
0	0	0	0	0	0	0	0
1	0	1	1	2	0	1	1
0	1	2	1	1	1	0	1
1	1	0	2	0	1	1	2
1	2	2	0	1	2	1	0
2	0	2	2	1	0	2	2
0	2	1	2	2	2	0	2
2	2	0	1	0	2	2	1
2	1	1	0	2	1	2	0

INDEPENDENT INTERACTIONS CONFOUNDED

AB^2C^2, ABD^2, AB^2E

I.S(2)-ALL EXCEPT AG^2, BF^2, CE, DH^2

KEY TO OTHER BLOCKS

$CFG, DFGH, EGH^2$

6. DESIGN WITH N= 9, P= 1, R=2
IDENTITY GROUP OF INTERACTIONS

I=ABCDEFGHI

KEY BLOCK

A	B	C	D	E	F	G	H	I
0	0	0	0	0	0	0	0	0
1	0	1	1	2	1	0	1	2
0	1	1	2	1	0	2	1	1
1	1	2	0	0	1	2	2	0
1	2	0	2	1	1	1	0	1
2	0	2	2	1	2	0	2	1
0	2	2	1	2	0	1	2	2
2	2	1	0	0	2	1	1	0
2	1	0	1	2	2	2	0	2

INDEPENDENT INTERACTIONS CONFOUNDED

$ABC^2, AB^2D^2, AB^2E, AF^2, BG, ABH^2$

I.S(2)-ALL EXCEPT $AF, BG, CH^2, DI, EI^2, DE$

KEY TO OTHER BLOCKS

$CI^2, DI^2, EI^2, FI^2, GI^2, HI^2$

7. DESIGN WITH N= 9, P= 2, R=2

IDENTITY GROUP OF INTERACTIONS

$I=ABCDH^2=B^2CDEI^2$

KEY BLOCK

A	B	C	D	E	F	G	H	I
0	0	0	0	0	0	0	0	0
1	0	1	1	2	1	2	0	1
0	1	2	1	1	0	2	1	0
1	1	0	2	0	1	1	1	1
1	2	2	0	1	1	0	2	1
2	0	2	2	1	2	1	0	2
0	2	1	2	2	0	1	2	0
2	2	0	1	0	2	2	2	2
2	1	1	0	2	2	0	1	2

INDEPENDENT INTERACTIONS CONFOUNDED

$AB^2C^2, ABD^2, AB^2E, AF^2, ABG$

I .S(2) ALL EXCEPT $AF^2, AI^2, BH^2, CE, DG^2, FI^2$

KEY TO OTHER BLOCKS

CHI, DHI, EI, F, G

8. DESIGN WITH N= 9, P= 3, R=2

IDENTITY GROUP OF INTERACTIONS

$I=ABCDG^2=B^2CDEH^2=A^2BDE^2I^2$

KEY BLOCK

A	B	C	D	E	F	G	H	I
0	0	0	0	0	0	0	0	0
1	0	1	1	2	1	0	1	1
0	1	2	1	1	1	1	0	1
1	1	0	2	0	2	1	1	2
1	2	2	0	1	0	2	1	0
2	0	2	2	1	2	0	2	2
0	2	1	2	2	2	2	0	2
2	2	0	1	0	1	2	2	1
2	1	1	0	2	0	1	2	0

INDEPENDENT INTERACTIONS CONFOUNDED

$AB^2C^2, ABD^2, AB^2E, ABF^2$

I.S(2) ALL EXCEPT $AH^2, BG^2, CE, DI^2, DF^2, FI^2$

KEY TO OTHER BLOCKS

$CGH, DGHI, EHI^2, F$

9. DESIGN WITH N= 9, P= 4, R=2

IDENTITY GROUP OF INTERACTIONS

$I=ABCDF^2=B^2CDEG^2=A^2BDE^2H^2=ABC^2EI^2$

KEY BLOCK

A	B	C	D	E	F	G	H	I
0	0	0	0	0	0	0	0	0
1	0	1	1	2	0	1	1	2
0	1	2	1	1	1	0	1	0
1	1	0	2	0	1	1	2	2
1	2	2	0	1	2	1	0	2
2	0	2	2	1	0	2	2	1
0	2	1	2	2	2	0	2	0
2	2	0	1	0	2	2	1	1
2	1	1	0	2	1	2	0	1

INDEPENDENT INTERACTIONS CONFOUNDED

AB^2C^2, ABD^2, AB^2E

I.S(2)-ALL EXCEPT AG^2, AI, GI, BF^2, CE, DH^2

KEY TO OTHER BLOCKS

$CFGI^2$, DFGH, EGH^2I

10. DESIGN WITH N=10, P= 1, R=2

IDENTITY GROUP OF INTERACTIONS

$I=A^2BCDEF^2GHIJ$

KEY BLOCK

A	B	C	D	E	F	G	H	I	J
0	0	0	0	0	0	0	0	0	0
1	0	1	1	2	1	0	1	1	2
0	1	1	2	1	0	2	1	2	2
1	1	2	0	0	1	2	2	0	1
1	2	0	2	1	1	1	0	2	0
2	0	2	2	1	2	0	2	2	1
0	2	2	1	2	0	1	2	1	1
2	2	1	0	0	2	1	1	0	2
2	1	0	1	2	2	2	0	1	0

INDEPENDENT INTERACTIONS CONFOUNDED

ABC^2, AB^2D^2, AB^2E, AF^2, BG, ABH^2, AB^2I^2

I.S(2) ALL EXCEPT AF^2, BG^2, CH^2, CJ^2, HJ^2, DE, DI^2, EI

KEY TO OTHER BLOCKS

CJ^2, DJ^2, EJ^2, FJ^2, GJ^2, HJ^2, IJ^2

11. DESIGN WITH N=10, P= 2, R=2

IDENTITY GROUP OF INTERACTIONS

$I=ABCDEI^{2}=CD^{2}E^{2}FGJ^{2}$

KEY BLOCK

A	B	C	D	E	F	G	H	I	J
0	0	0	0	0	0	0	0	0	0
1	0	1	2	2	1	1	1	0	2
0	1	1	1	1	2	1	0	1	2
1	1	2	0	0	0	2	1	1	1
1	2	0	1	1	2	0	1	2	0
2	0	2	1	1	2	2	2	0	1
0	2	2	2	2	1	2	0	2	1
2	2	1	0	0	0	1	2	2	2
2	1	0	2	2	1	0	2	1	0

INDEPENDENT INTERACTIONS CONFOUNDED

$ABC^{2}, AB^{2}D, AB^{2}E, AB^{2}F^{2}, ABG^{2}, AH^{2}$

I.S(2)-ALL EXCEPT $AH^{2}, BI^{2}, CG^{2}, CJ, GJ, DE^{2}, DF, EF$

KEY TO OTHER BLOCKS

CIJ, EIJ^{2}, FJ, GJ, H

12. DESIGN WITH N=10, P= 3, R=2

IDENTITY GROUP OF INTERACTIONS

$I=ABCDH^{2}=B^{2}CDEI^{2}=A^{2}BDE^{2}J^{2}$

KEY BLOCK

A	B	C	D	E	F	G	H	I	J
0	0	0	0	0	0	0	0	0	0
1	0	1	1	2	1	0	0	1	1
0	1	2	1	1	1	1	1	0	1
1	1	0	2	0	2	1	1	1	2
1	2	2	0	1	0	2	2	1	0
2	0	2	2	1	2	0	0	2	2
0	2	1	2	2	2	2	2	0	2
2	2	0	1	0	1	2	2	2	1
2	1	1	0	2	0	1	1	2	0

INDEPENDENT INTERACTIONS CONFOUNDED

$AB^{2}C^{2}, ABD^{2}, AB^{2}E, ABF^{2}, BG^{2}$

I.S(2)-ALL EXCEPT $AI^{2}, BG^{2}, BH^{2}, GH^{2}, CE, DF^{2}, DJ^{2}, FJ^{2}$

KEY TO OTHER BLOCKS

$CHI, DHIJ, EIJ^{2}, F, G$

13. DESIGN WITH N=10, P= 4, R=2
IDENTITY GROUP OF INTERACTIONS

$I=ABCDG^2=B^2CDEH^2=A^2BDE^2I^2=ABC^2EJ^2$

KEY BLOCK

A	B	C	D	E	F	G	H	I	J
0	0	0	0	0	0	0	0	0	0
1	0	1	1	2	0	0	1	1	2
0	1	2	1	1	1	1	0	1	0
1	1	0	2	0	1	1	1	2	2
1	2	2	0	1	2	2	1	0	2
2	0	2	2	1	0	0	2	2	1
0	2	1	2	2	2	2	0	2	0
2	2	0	1	0	2	2	2	1	1
2	1	1	0	2	1	1	2	0	1

INDEPENDENT INTERACTIONS CONFOUNDED

$AB^2C^2, ABD^2, AB^2E, BF^2$

I.S(2)-ALL EXCEPT $AH^2, AJ, HJ, BF^2, BG^2, FG^2, CE, DI^2$

KEY TO OTHER BLOCKS

$CGHJ^2, DGHI, EHI^2J, F$

14. DESIGN WITH N=10, P= 5, R=2
IDENTITY GROUP OF INTERACTIONS

$I=A^2C^2DEF^2=ABCDG^2=B^2CDEH^2=A^2BDE^2I^2=ABC^2EJ^2$

KEY BLOCK

A	B	C	D	E	F	G	H	I	J
0	0	0	0	0	0	0	0	0	0
1	0	1	1	2	1	0	1	1	2
0	1	2	1	1	0	1	0	1	0
1	1	0	2	0	1	1	1	2	2
1	2	2	0	1	1	2	1	0	2
2	0	2	2	1	2	0	2	2	1
0	2	1	2	2	0	2	0	2	0
2	2	0	1	0	2	2	2	1	1
2	1	1	0	2	2	1	2	0	1

INDEPENDENT INTERACTIONS CONFOUNDED

AB^2C^2, ABD^2, AB^2E

I.S(2)-ALL EXCEPT $AF^2, AH^2, AJ, FH^2, FJ, HJ, BG^2, CE, DI^2$

KEY TO OTHER BLOCKS

$CF^2GHJ^2, DFGHI, EFHI^2J$

15. DESIGN WITH N=11, P= 1, R=2
IDENTITY GROUP OF INTERACTIONS

I=ABCDEFGHIJK

KEY BLOCK

A	B	C	D	E	F	G	H	I	J	K
0	0	0	0	0	0	0	0	0	0	0
1	0	1	1	2	1	0	1	1	2	2
0	1	1	2	1	0	2	1	2	2	0
1	1	2	0	0	1	2	2	0	1	2
1	2	0	2	1	1	1	0	2	0	2
2	0	2	2	1	2	0	2	2	1	1
0	2	2	1	2	0	1	2	1	1	0
2	2	1	0	0	2	1	1	0	2	1
2	1	0	1	2	2	2	0	1	0	1

INDEPENDENT INTERACTIONS CONFOUNDED

$ABC^2, AB^2D^2, AB^2E, AF^2, BG, ABH^2, AB^2I^2, ABJ$

I.S(2)-ALL EXCEPT $AF^2, AK, FK, BG, CH^2, CJ, HJ, DE, DI^2, EI$

KEY TO OTHER BLOCKS

$CK^2, DK^2, EK^2, FK^2, GK^2, HK^2, IK^2, JK^2$

16. DESIGN WITH N=11, P= 2, R=2
IDENTITY GROUP OF INTERACTIONS

$I=ABCDJ^2=B^2CDEK^2$

KEY BLOCK

A	B	C	D	E	F	G	H	I	J	K
0	0	0	0	0	0	0	0	0	0	0
1	0	1	1	2	1	1	0	1	0	1
0	1	2	1	1	1	1	1	0	1	0
1	1	0	2	0	2	2	1	1	1	1
1	2	2	0	1	0	0	2	1	2	1
2	0	2	2	1	2	2	0	2	0	2
0	2	1	2	2	2	2	2	0	2	0
2	2	0	1	0	1	1	2	2	2	2
2	1	1	0	2	0	0	1	2	1	2

INDEPENDENT INTERACTIONS CONFOUNDED

$AB^2C^2, ABD^2, AB^2E, ABF^2, ABG^2, BH^2, AI^2$

I.S(2)-ALL EXCEPT $AI^2, AK^2, IK^2, BH^2, BJ^2, HJ^2, CE, DF^2, DG^2, FG^2$

KEY TO OTHER BLOCKS

CJK, DJK, EK, F, G, H, I

17. DESIGN WITH N=11, P= 3, R=2

IDENTITY GROUP OF INTERACTIONS

$I=ABCDI^2=B^2CDEJ^2=A^2BDE^2K^2$

KEY BLOCK

A	B	C	D	E	F	G	H	I	J	K
0	0	0	0	0	0	0	0	0	0	0
1	0	1	1	2	1	0	1	0	1	1
0	1	2	1	1	1	1	0	1	0	1
1	1	0	2	0	2	1	1	1	1	2
1	2	2	0	1	0	2	1	2	1	0
2	0	2	2	1	2	0	2	0	2	2
0	2	1	2	2	2	2	0	2	0	2
2	2	0	1	0	1	2	2	2	2	1
2	1	1	0	2	0	1	2	1	2	0

INDEPENDENT INTERACTIONS CONFOUNDED

$AB^2C^2, ABD^2, AB^2E, ABF^2, BG^2, AH^2$

I.S(2)-ALL EXCEPT $AH^2, AJ^2, HJ^2, BG^2, BI^2, GI^2, CE, DF^2, DK^2, FK^2$

KEY TO OTHER BLOCKS

$CIJ, DIJK, EJK^2, F, G, H$

18. DESIGN WITH N=11, P= 4, R=2

IDENTITY GROUP OF INTERACTIONS

$I=ABCDH^2=B^2CDEI^2=A^2BDE^2J^2=ABC^2EK^2$

KEY BLOCK

A	B	C	D	E	F	G	H	I	J	K
0	0	0	0	0	0	0	0	0	0	0
1	0	1	1	2	0	1	0	1	1	2
0	1	2	1	1	1	1	1	0	1	0
1	1	0	2	0	1	2	1	1	2	2
1	2	2	0	1	2	0	2	1	0	2
2	0	2	2	1	0	2	0	2	2	1
0	2	1	2	2	2	2	2	0	2	0
2	2	0	1	0	2	1	2	2	1	1
2	1	1	0	2	1	0	1	2	0	1

INDEPENDENT INTERACTIONS CONFOUNDED

$AB^2C^2, ABD^2, AB^2E, BF^2, ABG^2$

I.S(2)-ALL EXCEPT $AI^2, AK, IK, BF^2, BH^2, FH^2, CE, DG^2, DJ^2, GJ^2$

KEY TO OTHER BLOCKS

$CHIK^2, DHIJ, F, G$

19. DESIGN WITH N=11, P= 5, R=2
IDENTITY GROUP OF INTERACTIONS

$I=A^2C^2DEG^2=ABCDH^2=B^2CDEI^2=A^2BDE^2J^2=ABC^2EK^2$

KEY BLOCK

A	B	C	D	E	F	G	H	I	J	K
0	0	0	0	0	0	0	0	0	0	0
1	0	1	1	2	0	1	0	1	1	2
0	1	2	1	1	1	0	1	0	1	0
1	1	0	2	0	1	1	1	1	2	2
1	2	2	0	1	2	1	2	1	0	2
2	0	2	2	1	0	2	0	2	2	1
0	2	1	2	2	2	0	2	0	2	0
2	2	0	1	0	2	2	2	2	1	1
2	1	1	0	2	1	2	1	2	0	1

INDEPENDENT INTERACTIONS CONFOUNDED

$AB^2C^2, ABD^2, AB^2E, BF^2$

I.S(2)-ALL EXCEPT $AI^2, AK, IK, BF^2, BH^2, GH^2, CE, AG^2, GI^2, GK, DJ^2$

KEY TO OTHER BLOCKS

$CGHIK^2, DGHIJ, F$

20. DESIGN WITH N=12, P= 1, R=2
IDENTITY GROUP OF INTERACTIONS

I=ABCDEFGHIJKL

KEY BLOCK

A	B	C	D	E	F	G	H	I	J	K	L
0	0	0	0	0	0	0	0	0	0	0	0
1	0	1	1	1	0	1	1	1	0	1	1
0	1	1	2	0	1	1	2	0	1	1	2
1	1	2	0	1	1	2	0	1	1	2	0
1	2	0	2	1	2	0	2	1	2	0	2
2	0	2	2	2	0	2	2	2	0	2	2
0	2	2	1	0	2	2	1	0	2	2	1
2	2	1	0	2	2	1	0	2	2	1	0
2	1	0	1	2	1	0	1	2	1	0	1

INDEPENDENT INTERACTIONS CONFOUNDED

$ABC^2, AB^2D^2, AE^2, BF^2, ABG^2, AB^2H^2, AI^2, BJ^2, ABK^2$

I.S(2) ALLEXCEPT $AE^2, AI^2, EI^2, BF^2, BJ^2, FJ^2,$

$CG^2, CK^2, DH^2, DL^2,$ ~~EL~~ HL^2

KEY TO OTHER BLOCKS

$CL^2, DL^2, EL^2, FL^2, GL^2, HL^2, IL^2, JL^2, KL^2$

21. DESIGN WITH N=12, P= 2, R=2

IDENTITY GROUP OF INTERACTIONS

$I=ABCDK^2=B^2CDEL^2$

KEY BLOCK

A	B	C	D	E	F	G	H	I	J	K	L
0	0	0	0	0	0	0	0	0	0	0	0
1	0	1	1	2	1	1	0	1	1	0	1
0	1	2	1	1	1	1	1	0	2	1	0
1	1	0	2	0	2	2	1	1	0	1	1
1	2	2	0	1	0	0	2	1	2	2	1
2	0	2	2	1	2	2	0	2	2	0	2
0	2	1	2	2	2	2	2	0	1	2	0
2	2	0	1	0	1	1	2	2	0	2	2
2	1	1	0	2	0	0	1	2	1	1	2

INDEPENDENT INTERACTIONS CONFOUNDED

$AB^2C^2, ABD^2, AB^2E, ABF^2, ABG^2, BH^2, AI^2, AB^2J^2$

I.S(2) ALL EXCEPT $AI^2, AL^2, IL^2, BH^2, BK^2, HK^2, CE, CJ^2, EJ, DF^2, DG^2, FG^2$

KEY TO OTHER BLOCKS

CKL,DKL,EL,F,G

22. DESIGN WITH N=12, P= 3, R=2

IDENTITY GROUP OF INTERACTIONS

$I=ABCDJ^2=B^2CDEK^2=A^2BDE^2L^2$

KEY BLOCK

A	B	C	D	E	F	G	H	I	J	K	L
0	0	0	0	0	0	0	0	0	0	0	0
1	0	1	1	2	1	0	1	1	0	1	1
0	1	2	1	1	1	1	0	2	1	0	1
1	1	0	2	0	2	1	1	0	1	1	2
1	2	2	0	1	0	2	1	2	2	1	0
2	0	2	2	1	2	0	2	2	0	2	2
0	2	1	2	2	2	2	0	1	2	0	2
2	2	0	1	0	1	2	2	0	2	2	1
2	1	1	0	2	0	1	2	1	1	2	0

INDEPENDENT INTERACTIONS CONFOUNDED

$AB^2C^2, ABD^2, AB^2E, ABF^2, BG^2, AH^2, AB^2I^2$

I.S(2)-ALL EXCEPT $AH^2, AK^2, HK^2, BG^2, BJ^2, GJ^2, CE, CI^2, EI, DF^2, DL^2, FL^2$

KEY TO OTHER BLOCKS

$CJK, DJKL, EKL^2, F, G, H, I$

23. DESIGN WITH N=12, P= 4, R=2

IDENTITY GROUP OF INTERACTIONS

$I=ABCDI^2=B^2CDEJ^2=A^2BDE^2K^2=ABC^2EL^2$

KEY BLOCK

A	B	C	D	E	F	G	H	I	J	K	L
0	0	0	0	0	0	0	0	0	0	0	0
1	0	1	1	2	0	1	1	0	1	1	2
0	1	2	1	1	1	1	2	1	0	1	0
1	1	0	2	0	1	2	0	1	1	2	2
1	2	2	0	1	2	0	2	2	1	0	2
2	0	2	2	1	0	2	2	0	2	2	1
0	2	1	2	2	2	2	1	2	0	2	0
2	2	0	1	0	2	1	0	2	2	1	1
2	1	1	0	2	1	0	1	1	2	0	1

INDEPENDENT INTERACTIONS CONFOUNDED

$AB^2C^2, ABD^2, AB^2E, BF^2, ABG^2, AB^2H^2$

I.S(2)-ALL EXCEPT $AJ^2, AL, JL, BF^2, BI^2, FI^2, CE, CH^2, EH, DG^2, DK^2$

$, GK^2$

KEY TO OTHER BLOCKS

$CIJL^2, DIJK, F, G, H$

24. DESIGN WITH N=12, P= 5, R=2

IDENTITY GROUP OF INTERACTIONS

$I=A^2C^2DEH^2=ABCDI^2=B^2CDEJ^2=A^2BDE^2K^2=ABC^2EL^2$

KEY BLOCK

A	B	C	D	E	F	G	H	I	J	K	L
0	0	0	0	0	0	0	0	0	0	0	0
1	0	1	1	2	0	1	1	0	1	1	2
0	1	2	1	1	1	1	0	1	0	1	0
1	1	0	2	0	1	2	1	1	1	2	2
1	2	2	0	1	2	0	1	2	1	0	2
2	0	2	2	1	0	2	2	0	2	2	1
0	2	1	2	2	2	2	0	2	0	2	0
2	2	0	1	0	2	1	2	2	2	1	1
2	1	1	0	2	1	0	2	1	2	0	1

INDEPENDENT INTERACTIONS CONFOUNDED

$AB^2C^2, ABD^2, AB^2E, BF^2, ABG^2$

I.S(2)-ALL EXCEPT $AH^2, AJ^2, AL, HJ^2, HL, JL, BF^2, BI^2, CE, DG^2, DK^2$

$, GK^2$

KEY TO OTHER BLOCKS

$CH^2IJL^2, DHIJK, F, G$

25. DESIGN WITH N=13, P= 1, R=2

IDENTITY GROUP OF INTERACTIONS

I=ABCDEFGHIJKLM

KEY BLOCK

A	B	C	D	E	F	G	H	I	J	K	L	M
0	0	0	0	0	0	0	0	0	0	0	0	0
1	0	1	1	1	0	1	1	1	0	1	2	2
0	1	1	2	0	1	1	2	0	1	1	1	1
1	1	2	0	1	1	2	0	1	1	2	0	0
1	2	0	2	1	2	0	2	1	2	0	1	1
2	0	2	2	2	0	2	2	2	0	2	1	1
0	2	2	1	0	2	2	1	0	2	2	2	2
2	2	1	0	2	2	1	0	2	2	1	0	0
2	1	0	1	2	1	0	1	2	1	0	2	2

INDEPENDENT INTERACTIONS CONFOUNDED

$ABC^2, AB^2D^2, AE^2, BF^2, ABG^2, AB^2H^2, AI^2, BJ^2, ABK^2, AB^2M$,

I.S(2)-ALL EXCEPT $AI^2, AE^2, EI^2, BF^2, BJ^2, FJ^2, CG^2, GK^2, DH^2$, DL, DM, HL, HM, LM^2

KEY.TO OTHER BLOCKS

$CM^2, DM^2, EM^2, FM^2, GM^2, HM^2, IM^2, JM^2, KM^2, LM^2$

26. DESIGN WITH N=13, P= 2, R=2

IDENTITY GROUP OF INTERACTIONS

$I=ABCDL^2=B^2CDEM^2$

KEY BLOCK

A	B	C	D	E	F	G	H	I	J	K	L	M
0	0	0	0	0	0	0	0	0	0	0	0	0
1	0	1	1	2	1	1	0	1	1	0	0	1
0	1	2	1	1	1	1	1	0	2	1	1	0
1	1	0	2	0	2	2	1	1	0	1	1	1
1	2	2	0	1	0	0	2	1	2	2	2	1
2	0	2	2	1	2	2	0	2	2	0	0	2
0	2	1	2	2	2	2	2	0	1	2	2	0
2	2	0	1	0	1	1	2	2	0	2	2	2
2	1	1	0	2	0	0	1	2	1	1	1	2

INDEPENDENT INTERACTIONS CONFOUNDED

$AB^2C^2, ABD^2, AB^2E, ABF^2, ABG^2, BH^2, AI^2, AB^2J^2, BK^2$

I.S(2)-ALL EXCEPT $AI^2, AM^2, IM^2, BH^2, BK^2, BL^2, HK^2, HL^2, KL^2$, CE, CJ^2, EJ, DF^2, DG^2, FG^2

KEY TO OTHER BLOCKS

CLM, DLM, EM, F, G, H, I, J, K

27. DESIGN WITH N=13, P= 3, R=2

IDENTITY GROUP OF INTERACTIONS

$I=ABCDK^2=B^2CDEL^2=A^2BDE^2M^2$

KEY BLOCK

A	B	C	D	E	F	G	H	I	J	K	L	M
0	0	0	0	0	0	0	0	0	0	0	0	0
1	0	1	1	2	1	0	1	1	0	0	1	1
0	1	2	1	1	1	1	0	2	1	1	0	1
1	1	0	2	0	2	1	1	0	1	1	1	2
1	2	2	0	1	0	2	1	2	2	2	1	0
2	0	2	2	1	2	0	2	2	0	0	2	2
0	2	1	2	2	2	2	0	1	2	2	0	2
2	2	0	1	0	1	2	2	0	2	2	2	1
2	1	1	0	2	0	1	2	1	1	1	2	0

INDEPENDENT INTERACTIONS CONFOUNDED

$AB^2C^2, ABD^2, AB^2E, ABF^2, BG^2, AH^2, AB^2I^2, BJ^2$

I.S(2)-ALL EXCEPT $AH^2, AL^2, HL^2, BG^2, BJ^2, BK^2, GJ^2, GK^2, JK^2, CE, CI^2, EI, DF^2, DM^2, FM^2$

KEY TO OTHER BLOCKS

$CKL, DKLM, FLM^2, F, G, H, I, J$

28. DESIGN WITH N=13, P= 4, R=2

IDENTITY GROUP OF INTERACTIONS

$I=ABCDJ^2=B^2CDEK^2=A^2BDE^2L^2=ABC^2EM^2$

KEY BLOCK

A	B	C	D	E	F	G	H	I	J	K	L	M
0	0	0	0	0	0	0	0	0	0	0	0	0
1	0	1	1	2	0	1	1	0	0	1	1	2
0	1	2	1	1	1	1	2	1	1	0	1	0
1	1	0	2	0	1	2	0	1	1	1	2	2
1	2	2	0	1	2	0	2	2	2	1	0	2
2	0	2	2	1	0	2	2	0	0	2	2	1
0	2	1	2	2	2	2	1	2	2	0	2	0
2	2	0	1	0	2	1	0	2	2	2	1	1
2	1	1	0	2	1	0	1	1	1	2	0	1

INDEPENDENT INTERACTIONS CONFOUNDED

$AB^2C^2, ABD^2, AB^2E, BF^2, ABG^2, AB^2H^2, BI^2$

I.S(2)-ALL EXCEPT $AK^2, AM, KM, BF^2, BI^2, BJ^2, FI^2, FJ^2, IJ^2, CE, CH^2, EH, DG^2, DL^2, GL^2$

KEY TO OTHER BLOCKS

$CJKM^2, DJKL, EKL^2, F, G, H, I$

29. DESIGN WITH N=13, P= 5, R=2
IDENTITY GROUP OF INTERACTIONS

$I=A^2C^2DEI^2=ABCDJ^2=B^2CDEK^2=A^2BDE^2L=ABC^2EM^2$

KEY BLOCK

A	B	C	D	E	F	G	H	I	J	K	L	M
0	0	0	0	0	0	0	0	0	0	0	0	0
1	0	1	1	2	0	1	1	1	0	1	1	2
0	1	2	1	1	1	1	2	0	1	0	1	0
1	1	0	2	0	1	2	0	1	1	1	2	2
1	2	2	0	1	2	0	2	1	2	1	0	2
2	0	2	2	1	0	2	2	2	0	2	2	1
0	2	1	2	2	2	2	1	0	2	0	2	0
2	2	0	1	0	2	1	0	2	2	2	1	1
2	1	1	0	2	1	0	1	2	1	2	0	1

INDEPENDENT INTERACTIONS CONFOUNDED

$AB^2C^2, ABD^2, AB^2E, BF^2, ABG^2, AB^2H^2$

$I.S(2)$-ALL EXCEPT $AK^2, AM, KM, AI^2, IM, IK^2, BF^2, BJ^2, FJ^2, CE, CH^2, EH, DG^2, DL^2, GL^2$

KEY TO OTHER BLOCKS

$CI^2JKM^2, DIJKL, EIKL^2, F, G, H$

30. DESIGN WITH N= 7, P= 1, R=3
IDENTITY GROUP OF INTERACTIONS

I=ABCDEFG

KEY BLOCK

A	B	C	D	E	F	G
0	0	0	0	0	0	0
1	0	0	1	0	2	2
0	1	0	2	2	2	2
1	1	0	0	2	1	1
1	2	0	2	1	0	0
0	0	1	1	2	0	2
1	0	1	2	2	2	1
0	1	1	0	1	2	1
1	1	1	1	1	1	0
1	2	1	0	0	0	2
1	0	2	0	1	2	0
0	1	2	1	0	2	0
1	1	2	2	0	1	2
1	2	2	1	2	0	1
2	0	0	2	0	1	1
0	2	0	1	1	1	1
2	2	0	0	1	2	2
2	1	0	1	2	0	0
0	0	2	2	1	0	1
2	0	2	1	1	1	2
0	2	2	0	2	1	2
2	2	2	2	2	2	0
2	1	2	0	0	0	1
2	0	1	0	2	1	0
0	2	1	2	0	1	0
2	2	1	1	0	2	1
2	1	1	2	1	0	2

INDEPENDENT INTERACTIONS CONFOUNDED

AB^2CD^2, BCE, ABF

I.S(2)-ALL

KEY TO OTHER BLOCKS

DG^2, EG^2, FG^2

31. DESIGN WITH N= 7, P= 2, R=3

IDENTITY GROUP OF INTERACTIONS

$I=ABCDF^2=B^2CDEG^2$

KEY BLOCK

A	B	C	D	E	F	G
0	0	0	0	0	0	0
1	0	0	1	0	2	1
0	1	0	2	1	0	2
1	1	0	0	1	2	0
1	2	0	2	2	2	2
0	0	1	1	1	2	0
1	0	1	2	1	1	1
0	1	1	0	2	2	2
1	1	1	1	2	1	0
1	2	1	0	0	1	2
1	0	2	0	2	0	1
0	1	2	1	0	1	2
1	1	2	2	0	0	0
1	2	2	1	1	0	2
2	0	0	2	0	1	2
0	2	0	1	2	0	1
2	2	0	0	2	1	0
2	1	0	1	1	1	1
0	0	2	2	2	1	0
2	0	2	1	2	2	2
0	2	2	0	1	1	1
2	2	2	2	1	2	0
2	1	2	0	0	2	1
2	0	1	0	1	0	2
0	2	1	2	0	2	1
2	2	1	1	0	0	0
2	1	1	2	2	0	1

INDEPENDENT INTERACTIONS CONFOUNDED

AB^2CD^2, BCE^2

I.S(2) ALL

KEY TO OTHER BLOCKS

DFG,EG

32. DESIGN WITH N= 8, P= 1, R=3

IDENTITY GROUP OF INTERACTIONS

$I=ABCD^{2}EFGH^{2}$

KEY BLOCK

A	B	C	D	E	F	G	H
0	0	0	0	0	0	0	0
1	0	0	1	1	2	0	0
0	1	0	2	1	1	1	2
1	1	0	0	2	0	1	2
1	2	0	2	0	1	2	1
0	0	1	0	0	2	2	2
1	0	1	1	1	1	2	2
0	1	1	2	1	0	0	1
1	1	1	0	2	2	0	1
1	2	1	2	0	0	1	0
1	0	2	1	1	0	1	1
0	1	2	2	1	2	2	0
1	1	2	0	2	1	2	0
1	2	2	2	0	2	0	2
2	0	0	2	2	1	0	0
0	2	0	1	2	2	2	1
2	2	0	0	1	0	2	1
2	1	0	1	0	2	1	2
0	0	2	0	0	1	1	1
2	0	2	2	2	2	1	1
0	2	2	1	2	0	0	2
2	2	2	0	1	1	0	2
2	1	2	1	0	0	2	0
2	0	1	2	2	0	2	2
0	2	1	1	2	1	1	0
2	2	1	0	1	2	1	0
2	1	1	1	0	1	0	1

INDEPENDENT INTERACTIONS CONFOUNDED

$AB^{2}D^{2}$, ABE^{2}, ~~AB^{2}~~, $AB^{2}CF$, $BC^{2}G^{2}$

I.S(2)-ALL

KEY TO OTHER BLOCKS

DH^{2}, EH, FH, GH

33. DESIGN WITH N= 8, P= 2, R=3

IDENTITY GROUP OF INTERACTIONS

$I=ABCDG^2=B^2CDEH^2$

KEY BLOCK

A	B	C	D	E	F	G	H
0	0	0	0	0	0	0	0
1	0	0	1	1	1	2	2
0	1	0	2	0	1	0	1
1	1	0	0	1	2	2	0
1	2	0	2	1	0	2	1
0	0	1	1	2	1	2	1
1	0	1	2	0	2	1	0
0	1	1	0	2	2	2	2
1	1	1	1	0	0	1	1
1	2	1	0	0	1	1	2
1	0	2	0	2	0	0	1
0	1	2	1	1	0	1	0
1	1	2	2	2	1	0	2
1	2	2	1	2	2	0	0
2	0	0	2	2	2	1	1
0	2	0	1	0	2	0	2
2	2	0	0	2	1	1	0
2	1	0	1	2	0	1	2
0	0	2	2	1	2	1	2
2	0	2	1	0	1	2	0
0	2	2	0	1	1	1	1
2	2	2	2	0	0	2	2
2	1	2	0	0	2	2	1
2	0	1	0	1	0	0	2
0	2	1	2	2	0	2	0
2	2	1	1	1	2	0	1
2	1	1	2	1	1	0	0

INDEPENDENT INTERACTIONS CONFOUNDED

$AB^2CD^2, AC^2E^2, ABCF^2$

I.S(2)-ALL

KEY TO OTHER BLOCKS

DGH, EH, F

34. DESIGN WITH N= 8, P= 3, R=3
IDENTITY GROUP OF INTERACTIONS

$I=A^2C^2DEF^2=A^2BDE^2G^2=ABC^2EH^2$

KEY BLOCK

A	B	C	D	E	F	G	H
0	0	0	0	0	0	0	0
1	0	0	1	2	2	1	0
0	1	0	2	0	2	0	1
1	1	0	0	2	1	1	1
1	2	0	2	2	0	1	2
0	0	1	1	2	2	2	1
1	0	1	2	1	1	0	1
0	1	1	0	2	1	2	2
1	1	1	1	1	0	0	2
1	2	1	0	1	2	0	0
1	0	2	0	0	0	2	2
0	1	2	1	1	0	1	0
1	1	2	2	0	2	2	0
1	2	2	1	0	1	2	1
2	0	0	2	1	1	2	0
0	2	0	1	0	1	0	2
2	2	0	0	1	2	2	2
2	1	0	1	1	0	2	1
0	0	2	2	1	1	1	2
2	0	2	1	2	2	0	2
0	2	2	0	1	2	1	1
2	2	2	2	2	0	0	1
2	1	2	0	2	1	0	0
2	0	1	0	0	0	1	1
0	2	1	2	2	0	2	0
2	2	1	1	0	1	1	0
2	1	1	2	0	2	1	2

INDEPENDENT INTERACTIONS CONFOUNDED

AB^2CD^2, ACE

I.S(2)-ALL

KEY TO OTHER BLOCKS

DFG, EFG^2H

35. DESIGN WITH N= 9, P= 1, R=3

IDENTITY GROUP OF INTERACTIONS

I=ABCDEFGHI

KEY BLOCK

A	B	C	D	E	F	G	H	I
0	0	0	0	0	0	0	0	0
1	0	0	2	1	0	1	1	0
0	1	0	2	2	2	2	2	1
1	1	0	1	0	2	0	0	1
1	2	0	0	2	1	2	2	2
0	0	1	1	0	2	2	1	2
1	0	1	0	1	2	0	2	2
0	1	1	0	2	1	1	0	0
1	1	1	2	0	1	2	1	0
1	2	1	1	2	0	1	0	1
1	0	2	1	1	1	2	0	1
0	1	2	1	2	0	0	1	2
1	1	2	0	0	0	1	2	2
1	2	2	2	2	2	0	1	0
2	0	0	1	2	0	2	2	0
0	2	0	1	1	1	1	1	2
2	2	0	2	0	1	0	0	2
2	1	0	0	1	2	1	1	1
0	0	2	2	0	1	1	2	1
2	0	2	0	2	1	0	1	1
0	2	2	0	1	2	2	0	0
2	2	2	1	0	2	1	2	0
2	1	2	2	1	0	2	0	2
2	0	1	2	2	2	1	0	2
0	2	1	2	1	0	0	2	1
2	2	1	0	0	0	2	1	1
2	1	1	1	1	1	0	2	0

INDEPENDENT INTERACTIONS CONFOUNDED

$ABC^2D, AB^2E^2, BCF, AB^2C^2G^2, AB^2CH^2$

I.S(2)-ALL

KEY TO OTHER BLOCKS

$DI^2, EI^2, FI^2, GI^2, HI^2$

36. DESIGN WITH N= 9, P= 2, R=3
IDENTITY GROUP OF INTERACTIONS

$I=ABCDH^2=B^2CDEI^2$

KEY BLOCK

A	B	C	D	E	F	G	H	I
0	0	0	0	0	0	0	0	0
1	0	0	1	1	0	1	2	2
0	1	0	2	0	2	1	0	1
1	1	0	0	1	2	2	2	0
1	2	0	2	1	1	0	2	1
0	0	1	1	2	1	0	2	1
1	0	1	2	0	1	1	1	0
0	1	1	0	2	0	1	2	2
1	1	1	1	0	0	2	1	1
1	2	1	0	0	2	0	1	2
1	0	2	0	2	2	1	0	1
0	1	2	1	1	1	1	1	0
1	1	2	2	2	1	2	0	2
1	2	2	1	2	0	0	0	0
2	0	0	2	2	0	2	1	1
0	2	0	1	0	1	2	0	2
2	2	0	0	2	1	1	1	0
2	1	0	1	2	2	0	1	2
0	0	2	2	1	2	0	1	2
2	0	2	1	0	2	2	2	0
0	2	2	0	1	0	2	1	1
2	2	2	2	0	0	1	2	2
2	1	2	0	0	1	0	2	1
2	0	1	0	1	1	2	0	2
0	2	1	2	2	2	2	2	0
2	2	1	1	1	2	1	0	1
2	1	1	2	1	0	0	0	0

INDEPENDENT INTERACTIONS CONFOUNDED

$AB^2CD^2, AC^2E^2, BC^2F, ABG^2$

I.S(2)-ALL

KEY TO OTHER BLOCKS

DHI,EI,F,G

37. DESIGN WITH N= 9, P= 3, R=3

IDENTITY GROUP OF INTERACTIONS

$I=A^2C^2DEF^2=A^2BDE^2G^2=ABC^2EH^2$

KEY BLOCK

A	B	C	D	E	F	G	H	I
0	0	0	0	0	0	0	0	0
1	0	0	1	2	2	1	0	0
0	1	0	2	0	2	0	1	1
1	1	0	0	2	1	1	1	1
1	2	0	2	2	0	1	2	2
0	0	1	1	2	2	2	1	2
1	0	1	2	1	1	0	1	2
0	1	1	0	2	1	2	2	0
1	1	1	1	1	0	0	2	0
1	2	1	0	1	2	0	0	1
1	0	2	0	0	0	2	2	1
0	1	2	1	1	0	1	0	2
1	1	2	2	0	2	2	0	2
1	2	2	1	0	1	2	1	0
2	0	0	2	1	1	2	0	0
0	2	0	1	0	1	0	2	2
2	2	0	0	1	2	2	2	2
2	1	0	1	1	0	2	1	1
0	0	2	2	1	1	1	2	1
2	0	2	1	2	2	0	2	1
0	2	2	0	1	2	1	1	0
2	2	2	2	2	0	0	1	0
2	1	2	0	2	1	0	0	2
2	0	1	0	0	0	1	1	2
0	2	1	2	2	0	2	0	1
2	2	1	1	0	1	1	0	1
2	1	1	2	0	2	1	2	0

INDEPENDENT INTERACTIONS CONFOUNDED

AB^2CD^2, ACE, AB^2I^2

I.S(2)-ALL

KEY TO OTHER BLOCKS

DFG, EF^2H, I

38. DESIGN WITH N= 9, P= 4, R=3
IDENTITY GROUP OF INTERACTIONS

$I=ABCDF^2=B^2CDEG^2=A^2C^2DEH^2=ABC^2EI^2$

KEY BLOCK

A	B	C	D	E	F	G	H	I
0	0	0	0	0	0	0	0	0
1	0	0	2	1	0	0	2	2
0	1	0	1	1	2	1	2	2
1	1	0	0	2	2	1	1	1
1	2	0	1	0	1	2	0	0
0	0	1	0	0	1	1	2	2
1	0	1	2	1	1	1	1	1
0	1	1	1	1	0	2	1	1
1	1	1	0	2	0	2	0	0
1	2	1	1	0	2	0	2	2
1	0	2	2	1	2	2	0	0
0	1	2	1	1	1	0	0	0
1	1	2	0	2	1	0	2	2
1	2	2	1	0	0	1	1	1
2	0	0	1	2	0	0	1	1
0	2	0	2	2	1	2	1	1
2	2	0	0	1	1	2	2	2
2	1	0	2	0	2	1	0	0
0	0	2	0	0	2	2	1	1
2	0	2	1	2	2	2	2	2
0	2	2	2	2	0	1	2	2
2	2	2	0	1	0	1	0	0
2	1	2	2	0	1	0	1	1
2	0	1	1	2	1	1	0	0
0	2	1	2	2	2	0	0	0
2	2	1	0	1	2	0	1	1
2	1	1	2	0	0	2	2	2

INDEPENDENT INTERACTIONS CONFOUNDED

AB^2D, ABE^2

$I.S(2)$-ALL EXCEPT HI^2

KEY TO OTHER BLOCKS

DFGH,EGHI

39. DESIGN WITH N=10, P= 1, R=3
IDENTITY GROUP OF INTERACTIONS

I=ABCDEFGHIJ

KEY BLOCK

A	B	C	D	E	F	G	H	I	J
0	0	0	0	0	0	0	0	0	0
1	0	0	1	1	0	0	1	1	1
0	1	0	2	1	1	1	1	0	2
1	1	0	0	2	1	1	2	1	0
1	2	0	2	0	2	2	0	1	2
0	0	1	2	0	1	2	2	1	0
1	0	1	0	1	1	2	0	2	1
0	1	1	1	1	2	0	0	1	2
1	1	1	2	2	2	0	1	2	0
1	2	1	1	0	0	1	2	2	2
1	0	2	2	1	2	1	2	0	1
0	1	2	0	1	0	2	2	2	2
1	1	2	1	2	0	2	0	0	0
1	2	2	0	0	1	0	1	0	2
2	0	0	2	2	0	0	2	2	2
0	2	0	1	2	2	2	2	0	1
2	2	0	0	1	2	2	1	2	0
2	1	0	1	0	1	1	0	2	1
0	0	2	1	0	2	1	1	2	0
2	0	2	0	2	2	1	0	1	2
0	2	2	2	2	1	0	0	2	1
2	2	2	1	1	1	0	2	1	0
2	1	2	2	0	0	2	1	1	1
2	0	1	1	2	1	2	1	0	2
0	2	1	0	2	0	1	1	1	1
2	2	1	2	1	0	1	0	0	0
2	1	1	0	0	2	0	2	0	1

INDEPENDENT INTERACTIONS CONFOUNDED

$AB^2C^2D^2, ABE^2, BCF^2, BC^2G^2, ABC^2H^2, ACI^2$

I.S(2)-ALL

KEY TO OTHER BLOCKS

$DJ^2, EJ^2, FJ^2, GJ^2, HJ^2, IJ$

40. DESIGN WITH N=10, P= 2, R=3
IDENTITY GROUP OF INTERACTIONS

$I=ABCDI^2=B^2CDEJ^2$

KEY BLOCK

A	B	C	D	E	F	G	H	I	J
0	0	0	0	0	0	0	0	0	0
1	0	0	1	1	0	1	1	2	2
0	1	0	2	0	2	1	1	0	1
1	1	0	0	1	2	2	2	2	0
1	2	0	2	1	1	0	0	2	1
0	0	1	1	2	1	0	2	2	1
1	0	1	2	0	1	1	0	1	0
0	1	1	0	2	0	1	0	2	2
1	1	1	1	0	0	2	1	1	1
1	2	1	0	0	2	0	2	1	2
1	0	2	0	2	2	1	2	0	1
0	1	2	1	1	1	1	2	1	0
1	1	2	2	2	1	2	0	0	2
1	2	2	1	2	0	0	1	0	0
2	0	0	2	2	0	2	2	1	1
0	2	0	1	0	1	2	2	0	2
2	2	0	0	2	1	1	1	1	0
2	1	0	1	2	2	0	0	1	2
0	0	2	2	1	2	0	1	1	2
2	0	2	1	0	2	2	0	2	0
0	2	2	0	1	0	2	0	1	1
2	2	2	2	0	0	1	2	2	2
2	1	2	0	0	1	0	1	2	1
2	0	1	0	1	1	2	1	0	2
0	2	1	2	2	2	2	1	2	0
2	2	1	1	1	2	1	0	0	1
2	1	1	2	1	0	0	2	0	0

INDEPENDENT INTERACTIONS CONFOUNDED

$AB^2CD^2, AC^2E^2, BC^2F, ABG^2, ABC^2H^2$

I.S(2)-ALL

KEY TO OTHER BLOCKS

DIJ,EJ,F,G,H

41. DESIGN WITH N=10, P= 3, R=3

IDENTITY GROUP OF INTERACTIONS

$I=A^2C^2DEH^2=A^2BDE^2I^2=ABC^2EJ^2$

KEY BLOCK

A	B	C	D	E	F	G	H	I	J
0	0	0	0	0	0	0	0	0	0
1	0	0	1	2	1	1	2	1	0
0	1	0	2	0	2	2	2	0	1
1	1	0	0	2	0	0	1	1	1
1	2	0	2	2	2	2	0	1	2
0	0	1	1	2	0	2	2	2	1
1	0	1	2	1	1	0	1	0	1
0	1	1	0	2	2	1	1	2	2
1	1	1	1	1	0	2	0	0	2
1	2	1	0	1	2	1	2	0	0
1	0	2	0	0	1	2	0	2	2
0	1	2	1	1	2	0	0	1	0
1	1	2	2	0	0	1	2	2	0
1	2	2	1	0	2	0	1	2	1
2	0	0	2	1	2	2	1	2	0
0	2	0	1	0	1	1	1	0	2
2	2	0	0	1	0	0	2	2	2
2	1	0	1	1	1	1	0	2	1
0	0	2	2	1	0	1	1	1	2
2	0	2	1	2	2	0	2	0	2
0	2	2	0	1	1	2	2	1	1
2	2	2	2	2	0	1	0	0	1
2	1	2	0	2	1	2	1	0	0
2	0	1	0	0	2	1	0	1	1
0	2	1	2	2	1	0	0	2	0
2	2	1	1	0	0	2	1	1	0
2	1	1	2	0	1	0	2	1	2

INDEPENDENT INTERACTIONS CONFOUNDED

$AB^2CD^2, ACE, AB^2F^2, AB^2C^2G^2$

I.S(2)-ALL

KEY TO OTHER BLOCKS

DHI, EHI^2J

42. DESIGN WITH N=10, P= 4, R=3

IDENTITY GROUP OF INTERACTIONS

$I=ABCDG^2=B^2CDEH^2=A^2C^2DEI^2=ABC^2EJ^2$

KEY BLOCK

A	B	C	D	E	F	G	H	I	J
0	0	0	0	0	0	0	0	0	0
1	0	0	2	1	1	0	0	2	2
0	1	0	1	1	1	2	1	2	2
1	1	0	0	2	2	2	1	1	1
1	2	0	1	0	0	1	2	0	0
0	0	1	0	0	2	1	1	2	2
1	0	1	2	1	0	1	1	1	1
0	1	1	1	1	0	0	2	1	1
1	1	1	0	2	1	0	2	0	0
1	2	1	1	0	2	2	0	2	2
1	0	2	2	1	2	2	2	0	0
0	1	2	1	1	2	1	0	0	0
1	1	2	0	2	0	1	0	2	2
1	2	2	1	0	1	0	1	1	1
2	0	0	1	2	2	0	0	1	1
0	2	0	2	2	2	1	2	1	1
2	2	0	0	1	1	1	2	2	2
2	1	0	2	0	0	2	1	0	0
0	0	2	0	0	1	2	2	1	1
2	0	2	1	2	0	2	2	2	2
0	2	2	2	2	0	0	1	2	2
2	2	2	0	1	2	0	1	0	0
2	1	2	2	0	1	1	0	1	1
2	0	1	1	2	1	1	1	0	0
0	2	1	2	2	1	2	0	0	0
2	2	1	0	1	0	2	0	1	1
2	1	1	2	0	2	0	2	2	2

INDEPENDENT INTERACTIONS CONFOUNDED

AB^2D, ABE^2, ABC^2F^2

I.S(2)-ALL EXCEPT IJ^2

KEY TO OTHER BLOCKS

DGHI,EHIJ,F

43. DESIGN WITH N=10, P= 5, R=3
IDENTITY GROUP OF INTERACTIONS

$I=A^2BDE^2F^2=ABCDG^2=B^2CDEH^2=A^2C^2DEI^2=ABC^2EJ^2$

KEY BLOCK

A	B	C	D	E	F	G	H	I	J
0	0	0	0	0	0	0	0	0	0
1	0	0	2	1	0	0	0	2	2
0	1	0	1	1	1	2	1	2	2
1	1	0	0	2	1	2	1	1	1
1	2	0	1	0	2	1	2	0	0
0	0	1	0	0	0	1	1	2	2
1	0	1	2	1	0	1	1	1	1
0	1	1	1	1	1	0	2	1	1
1	1	1	0	2	1	0	2	0	0
1	2	1	1	0	2	2	0	2	2
1	0	2	2	1	0	2	2	0	0
0	1	2	1	1	1	1	0	0	0
1	1	2	0	2	1	1	0	2	2
1	2	2	1	0	2	0	1	1	1
2	0	0	1	2	0	0	0	1	1
0	2	0	2	2	2	1	2	1	1
2	2	0	0	1	2	1	2	2	2
2	1	0	2	0	1	2	1	0	0
0	0	2	0	0	0	2	2	1	1
2	0	2	1	2	0	2	2	2	2
0	2	2	2	2	2	0	1	2	2
2	2	2	0	1	2	0	1	0	0
2	1	2	2	0	1	1	0	1	1
2	0	1	1	2	0	1	1	0	0
0	2	1	2	2	2	2	0	0	0
2	2	1	0	1	2	2	0	1	1
2	1	1	2	0	1	0	2	2	2

INDEPENDENT INTERACTIONS CONFOUNDED

AB^2D, ABE^2

I.S(2)-ALL EXCEPT BF^2, IJ^2

KEY TO OTHER BLOCKS

$DFGHI, EF^2HIJ$

44. DESIGN WITH N=11, P= 1, R=3

IDENTITY GROUP OF INTERACTIONS

$I=ABCDE^2F^2GHIJK$

KEY BLOCK

A	B	C	D	E	F	G	H	I	J	K
0	0	0	0	0	0	0	0	0	0	0
1	0	0	0	0	1	1	1	1	1	2
0	1	0	1	1	1	0	2	2	0	2
1	1	0	1	1	2	1	0	0	1	1
1	2	0	2	2	0	1	2	2	1	0
0	0	1	1	2	0	1	0	1	2	2
1	0	1	1	2	1	2	1	2	0	1
0	1	1	2	0	1	1	2	0	2	1
1	1	1	2	0	2	2	0	1	0	0
1	2	1	0	1	0	2	2	0	0	2
1	0	2	2	1	1	0	1	0	2	0
0	1	2	0	2	1	2	2	1	1	0
1	1	2	0	2	2	0	0	2	2	2
1	2	2	1	0	0	0	2	1	2	1
2	0	0	0	0	2	2	2	2	2	1
0	2	0	2	2	2	0	1	1	0	1
2	2	0	2	2	1	2	0	0	2	2
2	1	0	1	1	0	2	1	1	2	0
0	0	2	2	1	0	2	0	2	1	1
2	0	2	2	1	2	1	2	1	0	2
0	2	2	1	0	2	2	1	0	1	2
2	2	2	1	0	1	1	0	2	0	0
2	1	2	0	2	0	1	1	0	0	1
2	0	1	1	2	2	0	2	0	1	0
0	2	1	0	1	2	1	1	2	2	0
2	2	1	0	1	1	0	0	1	1	1
2	1	1	2	0	0	0	1	2	1	2

INDEPENDENT INTERACTIONS CONFOUNDED

$BCD^2, BC^2E^2, ABF^2, ACG^2, AB^2H^2, AB^2CI^2, AC^2J^2$

IS(2)-ALL

KEY TO OTHER BLOCKS

$DK^2, EK, FK, GK^2, HK^2, IK^2, JK^2$

45. DESIGN WITH N=11, P= 2, R=3

IDENTITY GROUP OF INTERACTIONS

$I=ABCDJ^2=B^2CDEK^2$

KEY BLOCK

A	B	C	D	E	F	G	H	I	J	K
0	0	0	0	0	0	0	0	0	0	0
1	0	0	1	1	0	1	1	0	2	2
0	1	0	2	0	2	1	1	2	0	1
1	1	0	0	1	2	2	2	2	2	0
1	2	0	2	1	1	0	0	1	2	1
0	0	1	1	2	1	0	2	2	2	1
1	0	1	2	0	1	1	0	2	1	0
0	1	1	0	2	0	1	0	1	2	2
1	1	1	1	0	0	2	1	1	1	1
1	2	1	0	0	2	0	2	0	1	2
1	0	2	0	2	2	1	2	1	0	1
0	1	2	1	1	1	1	2	0	1	0
1	1	2	2	2	1	2	0	0	0	2
1	2	2	1	2	0	0	1	2	0	0
2	0	0	2	2	0	2	2	0	1	1
0	2	0	1	0	1	2	2	1	0	2
2	2	0	0	2	1	1	1	1	1	0
2	1	0	1	2	2	0	0	2	1	2
0	0	2	2	1	2	0	1	1	1	2
2	0	2	1	0	2	2	0	1	2	0
0	2	2	0	1	0	2	0	2	1	1
2	2	2	2	0	0	1	2	2	2	2
2	1	2	0	0	1	0	1	0	2	1
2	0	1	0	1	1	2	1	2	0	2
0	2	1	2	2	2	2	1	0	2	0
2	2	1	1	1	2	1	0	0	0	1
2	1	1	2	1	0	0	2	1	0	0

INDEPENDENT INTERACTIONS CONFOUNDED

$AB^2CD^2, AC^2E^2, BC^2F, ABG^2, ABC^2H^2, BCI$

I.S(2)-ALL

KEY TO OTHER BLOCKS

DJK, EK, F, G, H, I

46. DESIGN WITH N=11, P= 3, R=3

IDENTITY GROUP OF INTERACTIONS

$I=A^2C^2DEI^2=A^2BDE^2J^2=ABC^2EK^2$

KEY BLOCK

A	B	C	D	E	F	G	H	I	J	K
0	0	0	0	0	0	0	0	0	0	0
1	0	0	1	2	1	1	0	2	1	0
0	1	0	2	0	2	2	1	2	0	1
1	1	0	0	2	0	0	1	1	1	1
1	2	0	2	2	2	2	2	0	1	2
0	0	1	1	2	0	2	2	2	2	1
1	0	1	2	1	1	0	2	1	0	1
0	1	1	0	2	2	1	0	1	2	2
1	1	1	1	1	0	2	0	0	0	2
1	2	1	0	1	2	1	1	2	0	0
1	0	2	0	0	1	2	1	0	2	2
0	1	2	1	1	2	0	2	0	1	0
1	1	2	2	0	0	1	2	2	2	0
1	2	2	1	0	2	0	0	1	2	1
2	0	0	2	1	2	2	0	1	2	0
0	2	0	1	0	1	1	2	1	0	2
2	2	0	0	1	0	0	2	2	2	2
2	1	0	1	1	1	1	1	0	2	1
0	0	2	2	1	0	1	1	1	1	2
2	0	2	1	2	2	0	1	2	0	2
0	2	2	0	1	1	2	0	2	1	1
2	2	2	2	2	0	1	0	0	0	1
2	1	2	0	2	1	2	2	1	0	0
2	0	1	0	0	2	1	2	0	1	1
0	2	1	2	2	1	0	1	0	2	0
2	2	1	1	0	0	2	1	1	1	0
2	1	1	2	0	1	0	0	2	1	2

INDEPENDENT INTERACTIONS CONFOUNDED

$AB^2CD^2, ACE, AB^2F^2, AB^2C^2G^2, BC^2H^2$

I.S(2)-ALL

KEY TO OTHER BLOCKS

DIJ, EIJ^2K, F, G, H

47. DESIGN WITH N=11, P=4, R=3

IDENTITY GROUP OF INTERACTIONS

$I=ABCDH^2=B^2CDEI^2=A^2C^2DEJ^2=ABC^2EK^2$

KEY BLOCK

A	B	C	D	E	F	G	H	I	J	K
0	0	0	0	0	0	0	0	0	0	0
1	0	0	2	1	1	1	0	0	2	2
0	1	0	1	1	1	0	2	1	2	2
1	1	0	0	2	2	1	2	1	1	1
1	2	0	1	0	0	1	1	2	0	0
0	0	1	0	0	2	1	1	1	2	2
1	0	1	2	1	0	2	1	1	1	1
0	1	1	1	1	0	1	0	2	1	1
1	1	1	0	2	1	2	0	2	0	0
1	2	1	1	0	2	2	2	0	2	2
1	0	2	2	1	2	0	2	2	0	0
0	1	2	1	1	2	2	1	0	0	0
1	1	2	0	2	0	0	1	0	2	2
1	2	2	1	0	1	0	0	1	1	1
2	0	0	1	2	2	2	0	0	1	1
0	2	0	2	2	2	0	1	2	1	1
2	2	0	0	1	1	2	1	2	2	2
2	1	0	2	0	0	2	2	1	0	0
0	0	2	0	0	1	2	2	2	1	1
2	0	2	1	2	0	1	2	2	2	2
0	2	2	2	2	0	2	0	1	2	2
2	2	2	0	1	2	1	0	1	0	0
2	1	2	2	0	1	1	1	0	1	1
2	0	1	1	2	1	0	1	1	0	0
0	2	1	2	2	1	1	2	0	0	0
2	2	1	0	1	0	0	2	0	1	1
2	1	1	2	0	2	0	0	2	2	2

INDEPENDENT INTERACTIONS CONFOUNDED

$AB^2D, ABE^2, ABC^2F, ACG^2$

I.S(2)-ALL EXCEPT JK^2

KEY TO OTHER BLOCKS

DHIJ,EIJK,F,G

48. DESIGN WITH N=11, P= 5, R=3

IDENTITY GROUP OF INTERACTIONS

$I=A^2BDE^2G^2=ABCDH^2=B^2CDEI^2=A^2C^2DEJ^2=ABC^2EK^2$

KEY BLOCK

A	B	C	D	E	F	G	H	I	J	K
0	0	0	0	0	0	0	0	0	0	0
1	0	0	2	1	1	0	0	0	2	2
0	1	0	1	1	0	1	2	1	2	2
1	1	0	0	2	1	1	2	1	1	1
1	2	0	1	0	1	2	1	2	0	0
0	0	1	0	0	1	0	1	1	2	2
1	0	1	2	1	2	0	1	1	1	1
0	1	1	1	1	1	1	0	2	1	1
1	1	1	0	2	2	1	0	2	0	0
1	2	1	1	0	2	2	2	0	2	2
1	0	2	2	1	0	0	2	2	0	0
0	1	2	1	1	2	1	1	0	0	0
1	1	2	0	2	0	1	1	0	2	2
1	2	2	1	0	0	2	0	1	1	1
2	0	0	1	2	2	0	0	0	1	1
0	2	0	2	2	0	2	1	2	1	1
2	2	0	0	1	2	2	1	2	2	2
2	1	0	2	0	2	1	2	1	0	0
0	0	2	0	0	2	0	2	2	1	1
2	0	2	1	2	1	0	2	2	2	2
0	2	2	2	2	2	2	0	1	2	2
2	2	2	0	1	1	2	0	1	0	0
2	1	2	2	0	1	1	1	0	1	1
2	0	1	1	2	0	0	1	1	0	0
0	2	1	2	2	1	2	2	0	0	0
2	2	1	0	1	0	2	2	0	1	1
2	1	1	2	0	0	1	0	2	2	2

INDEPENDENT INTERACTIONS CONFOUNDED

AB^2D, ABE^2, ACF^2

I.S(2)-ALL EXCEPT BG^2, JK^2

KEY TO OTHER BLOCKS

$DGHIJ, EG^2IJK, F$

49. DESIGN WITH N=11, P= 6, R=3

IDENTITY GROUP OF INTERACTIONS

$I=A^2BCD^2EF^2=A^2BDE^2G^2=ABCDH^2=B^2CDEI^2=A^2C^2DEJ^2=ABC^2EK^2$

KEY BLOCK

A	B	C	D	E	F	G	H	I	J	K
0	0	0	0	0	0	0	0	0	0	0
1	0	0	2	1	1	0	0	0	2	2
0	1	0	1	1	1	1	2	1	2	2
1	1	0	0	2	2	1	2	1	1	1
1	2	0	1	0	0	2	1	2	0	0
0	0	1	0	0	1	0	1	1	2	2
1	0	1	2	1	2	0	1	1	1	1
0	1	1	1	1	2	1	0	2	1	2
1	1	1	0	2	0	1	0	2	0	0
1	2	1	1	0	1	2	2	0	2	2
1	0	2	2	1	0	0	2	2	0	0
0	1	2	1	1	0	1	1	0	0	0
1	1	2	0	2	1	1	1	0	2	2
1	2	2	1	0	2	2	0	1	1	1
2	0	0	1	2	2	0	0	0	1	1
0	2	0	2	2	2	2	1	2	1	1
2	2	0	0	1	1	2	1	2	2	2
2	1	0	2	0	0	1	2	1	0	0
0	0	2	0	0	2	0	2	2	1	1
2	0	2	1	2	1	0	2	2	2	2
0	2	2	2	2	1	2	0	1	2	2
2	2	2	0	1	0	2	0	1	0	0
2	1	2	2	0	2	1	1	0	1	1
2	0	1	1	2	0	0	1	1	0	0
0	2	1	2	2	0	2	2	0	0	0
2	2	1	0	1	2	2	2	0	1	1
2	1	1	2	0	1	1	0	2	2	2

INDEPENDENT INTERACTIONS CONFOUNDED

AB^2D, ABE^2

I.S(2)-ALL EXCEPT BG^2, FJ, FK, JK^2

KEY TO OTHER BLOCKS

DF^2GHIT, EFG^21JK

50. DESIGN WITH N=12, P= 1, R=3
IDENTITY GROUP OF INTERACTIONS

I=ABCDEFGHIJKL

KEY BLOCK

A	B	C	D	E	F	G	H	I	J	K	L
0	0	0	0	0	0	0	0	0	0	0	0
1	0	0	1	1	0	0	1	1	1	2	1
0	1	0	1	2	2	1	0	0	1	2	2
1	1	0	2	0	2	1	1	1	2	1	0
1	2	0	0	2	1	2	1	1	0	0	2
0	0	1	0	0	2	2	1	2	1	1	2
1	0	1	1	1	2	2	2	0	2	0	0
0	1	1	1	2	1	0	1	2	2	0	1
1	1	1	2	0	1	0	2	0	0	2	2
1	2	1	0	2	0	1	2	0	1	1	1
1	0	2	1	1	1	1	0	2	0	1	2
0	1	2	1	2	0	2	2	1	0	1	0
1	1	2	2	0	0	2	0	2	1	0	1
1	2	2	0	2	2	0	0	2	2	2	0
2	0	0	2	2	0	0	2	2	2	1	2
0	2	0	2	1	1	2	0	0	2	1	1
2	2	0	1	0	1	2	2	2	1	2	0
2	1	0	0	1	2	1	2	2	0	0	1
0	0	2	0	0	1	1	2	1	2	2	1
2	0	2	2	2	1	1	1	0	1	0	0
0	2	2	2	1	2	0	2	1	1	0	2
2	2	2	1	0	2	0	1	0	0	1	1
2	1	2	0	1	0	2	1	0	2	2	2
2	0	1	2	2	2	2	0	1	0	2	1
0	2	1	2	1	0	1	1	2	0	2	0
2	2	1	1	0	0	1	0	1	2	0	2
2	1	1	0	1	1	0	0	1	1	1	0

INDEPENDENT INTERACTIONS CONFOUNDED

ABD^2, AB^2E^2, BCF, BC^2G^2, ACH^2, AC^2I^2, $ABCJ^2$, ABC^2K

I.S(2)-ALL

KEY TO OTHER BLOCKS

DL^2, EL^2, FL^2, GL^2, HL^2, IL^2, JL^2, KL^2

51. DESIGN WITH N=12, P= 2, R=3

IDENTITY GROUP OF INTERACTIONS

$I=ABCDK^2=B^2CDEL^2$

KEY BLOCK

A	B	C	D	E	F	G	H	I	J	K	L
0	0	0	0	0	0	0	0	0	0	0	0
1	0	0	1	1	0	1	1	0	1	2	2
0	1	0	2	0	2	1	1	2	2	0	1
1	1	0	0	1	2	2	2	2	0	2	0
1	2	0	2	1	1	0	0	1	2	2	1
0	0	1	1	2	1	0	2	2	0	2	1
1	0	1	2	0	1	1	0	2	1	1	0
0	1	1	0	2	0	1	0	1	2	2	2
1	1	1	1	0	0	2	1	1	0	1	1
1	2	1	0	0	2	0	2	0	2	1	2
1	0	2	0	2	2	1	2	1	1	0	1
0	1	2	1	1	1	1	2	0	2	1	0
1	1	2	2	2	1	2	0	0	0	0	2
1	2	2	1	2	0	0	1	2	2	0	0
2	0	0	2	2	0	2	2	0	2	1	1
0	2	0	1	0	1	2	2	1	1	0	2
2	2	0	0	2	1	1	1	1	0	1	0
2	1	0	1	2	2	0	0	2	1	1	2
0	0	2	2	1	2	0	1	1	0	1	2
2	0	2	1	0	2	2	0	1	2	2	0
0	2	2	0	1	0	2	0	2	1	1	1
2	2	2	2	0	0	1	2	2	0	2	2
2	1	2	0	0	1	0	1	0	1	2	1
2	0	1	0	1	1	2	1	2	2	0	2
0	2	1	2	2	2	2	1	0	1	2	0
2	2	1	1	1	2	1	0	0	0	0	1
2	1	1	2	1	0	0	2	1	1	0	0

INDEPENDENT INTERACTIONS CONFOUNDED

$AB^2CD^2, AC^2E^2, BC^2F, ABG^2, ABC^2H^2, BCI, AB^2J^2$

I.S(2)-ALL

KEY TO OTHER BLOCKS

DKL,EL,F,G,H,I,J

52. DESIGN WITH N=12, P= 3, R=3

IDENTITY GROUP OF INTERACTIONS

$I=A^2C^2DEJ^2=A^2BDE^2K^2=ABC^2EL^2$

KEY BLOCK

A	B	C	D	E	F	G	H	I	J	K	L
0	0	0	0	0	0	0	0	0	0	0	0
1	0	0	1	2	1	1	0	2	2	1	0
0	1	0	2	0	2	2	1	2	2	0	1
1	1	0	0	2	0	0	1	1	1	1	1
1	2	0	2	2	2	2	2	0	0	1	2
0	0	1	1	2	0	2	2	1	2	2	1
1	0	1	2	1	1	0	2	0	1	0	1
0	1	1	0	2	2	1	0	0	1	2	2
1	1	1	1	1	0	2	0	2	0	0	2
1	2	1	0	1	2	1	1	1	2	0	0
1	0	2	0	0	1	2	1	1	0	2	2
0	1	2	1	1	2	0	2	1	0	1	0
1	1	2	2	0	0	1	2	0	2	2	0
1	2	2	1	0	2	0	0	2	1	2	1
2	0	0	2	1	2	2	0	1	1	2	0
0	2	0	1	0	1	1	2	1	1	0	2
2	2	0	0	1	0	0	2	2	2	2	2
2	1	0	1	1	1	1	1	0	0	2	1
0	0	2	2	1	0	1	1	2	1	1	2
2	0	2	1	2	2	0	1	0	2	0	2
0	2	2	0	1	1	2	0	0	2	1	1
2	2	2	2	2	0	1	0	1	0	0	1
2	1	2	0	2	1	2	2	2	1	0	0
2	0	1	0	0	2	1	2	2	0	1	1
0	2	1	2	2	1	0	1	2	0	2	0
2	2	1	1	0	0	2	1	0	1	1	0
2	1	1	2	0	1	0	0	1	2	1	2

INDEPENDENT INTERACTIONS CONFOUNDED

$AB^2CD^2, ACE, AB^2F^2, AB^2C^2G^2, BC^2H^2, ABC^2I$

I.S(2)-ALL

KEY TO OTHER BLOCKS

DJK, EJK^2L, F, G, H, I

53. DESIGN WITH N=12, P= 4, R=3
IDENTITY GROUP OF INTERACTIONS

$I = ABCDI^2 = B^2CDEJ^2 = A^2C^2DEK^2 = ABC^2EL^2$

KEY BLOCK

A	B	C	D	E	F	G	H	I	J	K	L
0	0	0	0	0	0	0	0	0	0	0	0
1	0	0	2	1	1	1	1	0	0	2	2
0	1	0	1	1	1	0	0	2	1	2	2
1	1	0	0	2	2	1	1	2	1	1	1
1	2	0	1	0	0	1	1	1	2	0	0
0	0	1	0	0	2	1	2	1	1	2	2
1	0	1	2	1	0	2	0	1	1	1	1
0	1	1	1	1	0	1	2	0	2	1	1
1	1	1	0	2	1	2	0	0	2	0	0
1	2	1	1	0	2	2	0	2	0	2	2
1	0	2	2	1	2	0	2	2	2	0	0
0	1	2	1	1	2	2	1	1	0	0	0
1	1	2	0	2	0	0	2	1	0	2	2
1	2	2	1	0	1	0	2	0	1	1	1
2	0	0	1	2	2	2	2	0	0	1	1
0	2	0	2	2	2	0	0	1	2	1	1
2	2	0	0	1	1	2	2	1	2	2	2
2	1	0	2	0	0	2	2	2	1	0	0
0	0	2	0	0	1	2	1	2	2	1	1
2	0	2	1	2	0	1	0	2	2	2	2
0	2	2	2	2	0	2	1	0	1	2	2
2	2	2	0	1	2	1	0	0	1	0	0
2	1	2	2	0	1	1	0	1	0	1	1
2	0	1	1	2	1	0	1	1	1	0	0
0	2	1	2	2	1	1	2	2	0	0	0
2	2	1	0	1	0	0	1	2	0	1	1
2	1	1	2	0	2	0	1	0	2	2	2

INDEPENDENT INTERACTIONS CONFOUNDED

$AB^2D, ABE^2, ABC^2F^2, ACG^2, AC^2H^2$

I.S(2)-ALL EXCEPT KL^2

KEY TO OTHER BLOCKS

DIJK, EJKL, F, G, H

54. DESIGN WITH N=12, P= 5, R=3

IDENTITY GROUP OF INTERACTIONS

$I=A^2BDE^2H^2=ABCDI^2=B^2CDEJ^2=A^2C^2DEK^2=ABC^2EL^2$

KEY BLOCK

A	B	C	D	E	F	G	H	I	J	K	L
0	0	0	0	0	0	0	0	0	0	0	0
1	0	0	2	1	1	1	0	0	0	2	2
0	1	0	1	1	0	0	1	2	1	2	2
1	1	0	0	2	1	1	1	2	1	1	1
1	2	0	1	0	1	1	2	1	2	0	0
0	0	1	0	0	1	2	0	1	1	2	2
1	0	1	2	1	2	0	0	1	1	1	1
0	1	1	1	1	1	2	1	0	2	1	1
1	1	1	0	2	2	0	1	0	2	0	0
1	2	1	1	0	2	0	2	2	0	2	2
1	0	2	2	1	0	2	0	2	2	0	0
0	1	2	1	1	2	1	1	1	0	0	0
1	1	2	0	2	0	2	1	1	0	2	2
1	2	2	1	0	0	2	2	0	1	1	1
2	0	0	1	2	2	2	0	0	0	1	1
0	2	0	2	2	0	0	2	1	2	1	1
2	2	0	0	1	2	2	2	1	2	2	2
2	1	0	2	0	2	2	1	2	1	0	0
0	0	2	0	0	2	1	0	2	2	1	1
2	0	2	1	2	1	0	0	2	2	2	2
0	2	2	2	2	2	1	2	0	1	2	2
2	2	2	0	1	1	0	2	0	1	0	0
2	1	2	2	0	1	0	1	1	0	1	1
2	0	1	1	2	0	1	0	1	1	0	0
0	2	1	2	2	1	2	2	2	0	0	0
2	2	1	0	1	0	1	2	2	0	1	1
2	1	1	2	0	0	1	1	0	2	2	2

INDEPENDENT INTERACTIONS CONFOUNDED

$AB^2D, ABE^2, ACF^2, AC^2G^2$

I.S(2)-ALL EXCEPT BH^2, KL^2

KEY TO OTHER BLOCKS

$DHIJK, EH^2JKL, F, G$

55. DESIGN WITH N=12, P= 6, R=3

IDENTITY GROUP OF INTERACTIONS

$$I=A^2BCD^2EG^2=A^2BDE^2H^2=ABCDI^2=B^2CDEJ^2=A^2C^2DEK^2=ABC^2EL^2$$

KEY BLOCK

A	B	C	D	E	F	G	H	I	J	K	L
0	0	0	0	0	0	0	0	0	0	0	0
1	0	0	2	1	1	1	0	0	0	2	2
0	1	0	1	1	0	1	1	2	1	2	2
1	1	0	0	2	1	2	1	2	1	1	1
1	2	0	1	0	1	0	2	1	2	0	0
0	0	1	0	0	1	1	0	1	1	2	2
1	0	1	2	1	2	2	0	1	1	1	1
0	1	1	1	1	1	2	1	0	2	1	1
1	1	1	0	2	2	0	1	0	2	0	0
1	2	1	1	0	2	1	2	2	0	2	2
1	0	2	2	1	0	0	0	2	2	0	0
0	1	2	1	1	2	0	1	1	0	0	0
1	1	2	0	2	0	1	1	1	0	2	2
1	2	2	1	0	0	2	2	0	1	1	1
2	0	0	1	2	2	2	0	0	0	1	1
0	2	0	2	2	0	2	2	1	2	1	1
2	2	0	0	1	2	1	2	1	2	2	2
2	1	0	2	0	2	0	1	2	1	0	0
0	0	2	0	0	2	2	0	2	2	1	1
2	0	2	1	2	1	1	0	2	2	2	2
0	2	2	2	2	2	1	2	0	1	2	2
2	2	2	0	1	1	0	2	0	1	0	0
2	1	2	2	0	1	2	1	1	0	1	1
2	0	1	1	2	0	0	0	1	1	0	0
0	2	1	2	2	1	0	2	2	0	0	0
2	2	1	0	1	0	2	2	2	0	1	1
2	1	1	2	0	0	1	1	0	2	2	2

INDEPENDENT INTERACTIONS CONFOUNDED

AB^2D, ABE^2, ACF^2

I.S(2)-ALL EXCEPT BH^2, GK[redacted], ~~GL^2~~, KL^2, GL

KEY TO OTHER BLOCKS

DG^2HIJK, EGH^2JKL, F

56. DESIGN WITH N=13, P= 1, R=3

IDENTITY GROUP OF INTERACTIONS

$I=ABCDEF^2GHIJK^2LM$

KEY BLOCK

A	B	C	D	E	F	G	H	I	J	K	L	M
0	0	0	0	0	0	0	0	0	0	0	0	0
1	0	0	1	1	1	1	0	0	1	1	1	2
0	1	0	1	2	0	0	1	1	1	1	2	1
1	1	0	2	0	1	1	1	1	2	2	0	0
1	2	0	0	2	1	1	2	2	0	0	2	1
0	0	1	0	0	1	2	1	2	1	2	1	1
1	0	1	1	1	2	0	1	2	2	0	2	0
0	1	1	1	2	1	2	2	0	2	0	0	2
1	1	1	2	0	2	0	2	0	0	1	1	1
1	2	1	0	2	2	0	0	1	1	2	0	2
1	0	2	1	1	0	2	2	1	0	2	0	1
0	1	2	1	2	2	1	0	2	0	2	1	0
1	1	2	2	0	0	2	0	2	1	0	2	2
1	2	2	0	2	0	2	1	0	2	1	1	0
2	0	0	2	2	2	2	0	0	2	2	2	1
0	2	0	2	1	0	0	2	2	2	2	1	2
2	2	0	1	0	2	2	2	2	1	1	0	0
2	1	0	0	1	2	2	1	1	0	0	1	2
0	0	2	0	0	2	1	2	1	2	1	2	2
2	0	2	2	2	1	0	2	1	1	0	1	0
0	2	2	2	1	2	1	1	0	1	0	0	1
2	2	2	1	0	1	0	1	0	0	2	2	2
2	1	2	0	1	1	0	0	2	2	1	0	1
2	0	1	2	2	0	1	1	2	0	1	0	2
0	2	1	2	1	1	2	0	1	0	1	2	0
2	2	1	1	0	0	1	0	1	2	0	1	1
2	1	1	0	1	0	1	2	0	1	2	2	0

INDEPENDENT INTERACTIONS CONFOUNDED

$ABD^2, AB^2E^2, ACF^2, AC^2G^2, BCH^2, BC^2I^2, ABCJ^2, ABC^2K^2, AB^2CL^2$

I.S(2)-ALL

KEY TO OTHER BLOCKS

$DM^2, EM^2, FM^2, GM^2, HM^2, IM^2, JM^2, KM^2, LM^2$

57. DESIGN WITH N=13, P= 2, R=3

IDENTITY GROUP OF INTERACTIONS

$I=ABCDL^2=B^2CDEM^2$

KEY BLOCK

A	B	C	D	E	F	G	H	I	J	K	L	M
0	0	0	0	0	0	0	0	0	0	0	0	0
1	0	0	1	1	0	1	1	0	1	1	2	2
0	1	0	2	0	2	1	1	2	2	1	0	1
1	1	0	0	1	2	2	2	2	0	2	2	0
1	2	0	2	1	1	0	0	1	2	0	2	1
0	0	1	1	2	1	0	2	2	0	1	2	1
1	0	1	2	0	1	1	0	2	1	2	1	0
0	1	1	0	2	0	1	0	1	2	2	2	2
1	1	1	1	0	0	2	1	1	0	0	1	1
1	2	1	0	0	2	0	2	0	2	1	1	2
1	0	2	0	2	2	1	2	1	1	0	0	1
0	1	2	1	1	1	1	2	0	2	0	1	0
1	1	2	2	2	1	2	0	0	0	1	0	2
1	2	2	1	2	0	0	1	2	2	2	0	0
2	0	0	2	2	0	2	2	0	2	2	1	1
0	2	0	1	0	1	2	2	1	1	2	0	2
2	2	0	0	2	1	1	1	1	0	1	1	0
2	1	0	1	2	2	0	0	2	1	0	1	2
0	0	2	2	1	2	0	1	1	0	2	1	2
2	0	2	1	0	2	2	0	1	2	1	2	0
0	2	2	0	1	0	2	0	2	1	1	1	1
2	2	2	2	0	0	1	2	2	0	0	2	2
2	1	2	0	0	1	0	1	0	1	2	2	1
2	0	1	0	1	1	2	1	2	2	0	0	2
0	2	1	2	2	2	2	1	0	1	0	2	0
2	2	1	1	1	2	1	0	0	0	2	0	1
2	1	1	2	1	0	0	2	1	1	1	0	0

INDEPENDENT INTERACTIONS CONFOUNDED

$AB^2CD^2, AC^2E^2, B^2CF^2, ABG^2, ABC^2H^2BCI, AB^2J^2, ABCK^2$

I.S(2)-ALL

KEY TO OTHER BLOCKS

DLM, EM, F, G, H, I, J, K

58. DESIGN WITH N=13, P= 3, R=3

IDENTITY GROUP OF INTERACTIONS

$I=A^2C^2DEK^2=A^2BDE^2L^2=ABC^2EM^2$

KEY BLOCK

A	B	C	D	E	F	G	H	I	J	K	L	M
0	0	0	0	0	0	0	0	0	0	0	0	0
1	0	0	1	2	1	1	0	1	1	2	1	0
0	1	0	2	0	2	2	1	1	1	2	0	1
1	1	0	0	2	0	0	1	2	2	1	1	1
1	2	0	2	2	2	2	2	0	0	0	1	2
0	0	1	1	2	0	2	2	2	0	2	2	1
1	0	1	2	1	1	0	2	0	1	1	0	1
0	1	1	0	2	2	1	0	0	1	1	2	2
1	1	1	1	1	0	2	0	1	2	0	0	2
1	2	1	0	1	2	1	1	2	0	2	0	0
1	0	2	0	0	1	2	1	2	1	0	2	2
0	1	2	1	1	2	0	2	2	1	0	1	0
1	1	2	2	0	0	1	2	0	2	2	2	0
1	2	2	1	0	2	0	0	1	0	1	2	1
2	0	0	2	1	2	2	0	2	2	1	2	0
0	2	0	1	0	1	1	2	2	2	1	0	2
2	2	0	0	1	0	0	2	1	1	2	2	2
2	1	0	1	1	1	1	1	0	0	0	2	1
0	0	2	2	1	0	1	1	1	0	1	1	2
2	0	2	1	2	2	0	1	0	2	2	0	2
0	2	2	0	1	1	2	0	0	2	2	1	1
2	2	2	2	2	0	1	0	2	1	0	0	1
2	1	2	0	2	1	2	2	1	0	1	0	0
2	0	1	0	0	2	1	2	1	2	0	1	1
0	2	1	2	2	1	0	1	1	2	0	2	0
2	2	1	1	0	0	2	1	0	1	1	1	0
2	1	1	2	0	1	0	0	2	0	2	1	2

INDEPENDENT INTERACTIONS CONFOUNDED

$AB^2CD^2, ACE, AB^2F^2, AB^2C^2G^2, BC^2H^2, ABC^2I^2, ABJ^2$

I.S(2)-ALL

KEY TO OTHER BLOCKS

$DKL, EKL^2M, F, G, H, I, J$

59. DESIGN WITH N=13, P= 4, R=3

IDENTITY GROUP OF INTERACTIONS

$I=ABCDJ^2=B^2CDEK^2=A^2C^2DEL^2=ABC^2EM^2$

KEY BLOCK

A	B	C	D	E	F	G	H	I	J	K	L	M
0	0	0	0	0	0	0	0	0	0	0	0	0
1	0	0	2	1	1	1	1	1	0	0	2	2
0	1	0	1	1	1	0	0	2	2	1	2	2
1	1	0	0	2	2	1	1	0	2	1	1	1
1	2	0	1	0	0	1	1	2	1	2	0	0
0	0	1	0	0	2	1	2	1	1	1	2	2
1	0	1	2	1	0	2	0	2	1	1	1	1
0	1	1	1	1	0	1	2	0	0	2	1	1
1	1	1	0	2	1	2	0	1	0	2	0	0
1	2	1	1	0	2	2	0	0	2	0	2	2
1	0	2	2	1	2	0	2	0	2	2	0	0
0	1	2	1	1	2	2	1	1	1	0	0	0
1	1	2	0	2	0	0	2	2	1	0	2	2
1	2	2	1	0	1	0	2	1	0	1	1	1
2	0	0	1	2	2	2	2	2	0	0	1	1
0	2	0	2	2	2	0	0	1	1	2	1	1
2	2	0	0	1	1	2	2	0	1	2	2	2
2	1	0	2	0	0	2	2	1	2	1	0	0
0	0	2	0	0	1	2	1	2	2	2	1	1
2	0	2	1	2	0	1	0	1	2	2	2	2
0	2	2	2	2	0	2	1	0	0	1	2	2
2	2	2	0	1	2	1	0	2	0	1	0	0
2	1	2	2	0	1	1	0	0	1	0	1	1
2	0	1	1	2	1	0	1	0	1	1	0	0
0	2	1	2	2	1	1	2	2	2	0	0	0
2	2	1	0	1	0	0	1	1	2	0	1	1
2	1	1	2	0	2	0	1	2	0	2	2	2

INDEPENDENT INTERACTIONS CONFOUNDED

$AB^2D, ABE^2, ABC^2F^2, ACG^2, AC^2H^2, AB^2CI^2$

I, S(2) ALL EXCEPT LM^2

KEY TO OTHER BLOCKS

DJKL, EKLM, F, G, H, I

60. DESIGN WITH N=13, P= 5, R=3

IDENTITY GROUP OF INTERACTIONS

$I=A^2BDE^2I^2=ABCDJ^2=B^2CDEK^2=A^2C^2DEL=AB^2CEM^2$

KEY BLOCK

A	B	C	D	E	F	G	H	I	J	K	L	M
0	0	0	0	0	0	0	0	0	0	0	0	0
1	0	0	2	1	1	1	1	0	0	0	2	2
0	1	0	1	1	0	0	1	1	2	1	2	2
1	1	0	0	2	1	1	2	1	2	1	1	1
1	2	0	1	0	1	1	0	2	1	2	0	0
0	0	1	0	0	1	2	2	0	1	1	2	2
1	0	1	2	1	2	0	0	0	1	1	1	1
0	1	1	1	1	1	2	0	1	0	2	1	1
1	1	1	0	2	2	0	1	1	0	2	0	0
1	2	1	1	0	2	0	2	2	2	0	2	2
1	0	2	2	1	0	2	2	0	2	2	0	0
0	1	2	1	1	2	1	2	1	1	0	0	0
1	1	2	0	2	0	2	0	1	1	0	2	2
1	2	2	1	0	0	2	1	2	0	1	1	1
2	0	0	1	2	2	2	2	0	0	0	1	1
0	2	0	2	2	0	0	2	2	1	2	1	1
2	2	0	0	1	2	2	1	2	1	2	2	2
2	1	0	2	0	2	2	0	1	2	1	0	0
0	0	2	0	0	2	1	1	0	2	2	1	1
2	0	2	1	2	1	0	0	0	2	2	2	2
0	2	2	2	2	2	1	0	2	0	1	2	2
2	2	2	0	1	1	0	2	2	0	1	0	0
2	1	2	2	0	1	0	1	1	1	0	1	1
2	0	1	1	2	0	1	1	0	1	1	0	0
0	2	1	2	2	1	2	1	2	2	0	0	0
2	2	1	0	1	0	1	0	2	2	0	1	1
2	1	1	2	0	0	1	2	1	0	2	2	2

INDEPENDENT INTERACTIONS CONFOUNDED

$AB^2D, ABE^2, ACF^2, AC^2G, ABC^2H^2$

I.S(2)-ALL EXCEPT BI^2, LM^2

KEY TO OTHER BLOCKS

$DIJKL, EI^2KLM, F, G, H$

61. DESIGN WITH N=13, P= 6, R=3

IDENTITY GROUP OF INTERACTIONS

$I=A^2BCD^2EH^2=A^2BDE^2I^2=ABCDJ^2=B^2CDEK^2=A^2C^2DEL^2=ABC^2EM^2$

KEY BLOCK

A	B	C	D	E	F	G	H	I	J	K	L	M
0	0	0	0	0	0	0	0	0	0	0	0	0
1	0	0	2	1	1	1	1	0	0	0	2	2
0	1	0	1	1	0	0	1	1	2	1	2	2
1	1	0	0	2	1	1	2	1	2	1	1	1
1	2	0	1	0	1	1	0	2	1	2	0	0
0	0	1	0	0	1	2	1	0	1	1	2	2
1	0	1	2	1	2	0	2	0	1	1	1	1
0	1	1	1	1	1	2	2	1	0	2	1	1
1	1	1	0	2	2	0	0	1	0	2	0	0
1	2	1	1	0	2	0	1	2	2	0	2	2
1	0	2	2	1	0	2	0	0	2	2	0	0
0	1	2	1	1	2	1	0	1	1	0	0	0
1	1	2	0	2	0	2	1	1	1	0	2	2
1	2	2	1	0	0	2	2	2	0	1	1	1
2	0	0	1	2	2	2	2	0	0	0	1	1
0	2	0	2	2	0	0	2	2	1	2	1	1
2	2	0	0	1	2	2	1	2	1	2	2	2
2	1	0	2	0	2	2	0	1	2	1	0	0
0	0	2	0	0	2	1	2	0	2	2	1	1
2	0	2	1	2	1	0	1	0	2	2	2	2
0	2	2	2	2	2	1	1	2	0	1	2	2
2	2	2	0	1	1	0	0	2	0	1	0	0
2	1	2	2	0	1	0	2	1	1	0	1	1
2	0	1	1	2	0	1	0	0	1	1	0	0
0	2	1	2	2	1	2	0	2	2	0	0	0
2	2	1	0	1	0	1	2	2	2	0	1	1
2	1	1	2	0	0	1	1	1	0	2	2	2

INDEPENDENT INTERACTIONS CONFOUNDED

$AB^2D, ABE^2, ACF^2, AC^2G^2$

I.S(2)-ALL EXCEPT BI^2, HL^2, HM^2, LM^2

KEY TO OTHER BLOCKS

DH^2IJKL, EHI^2KLM, F, G

62. DESIGN WITH N=14, P= 1, R=3

IDENTITY GROUP OF INTERACTIONS

I=ABCDEFGHIJKLMN

KEY BLOCK

A	B	C	D	E	F	G	H	I	J	K	L	M	N
0	0	0	0	0	0	0	0	0	0	0	0	0	0
1	0	0	1	1	1	1	0	0	1	1	1	2	2
0	1	0	1	2	0	0	1	1	1	1	2	1	1
1	1	0	2	0	1	1	1	1	2	2	0	0	0
1	2	0	0	2	1	1	2	2	0	0	2	1	1
0	0	1	0	0	1	2	1	2	1	2	1	1	0
1	0	1	1	1	2	0	1	2	2	0	2	0	2
0	1	1	1	2	1	2	2	0	2	0	0	2	1
1	1	1	2	0	2	0	2	0	0	1	1	1	0
1	2	1	0	2	2	0	0	1	1	2	0	2	1
1	0	2	1	1	0	2	2	1	0	2	0	1	2
0	1	2	1	2	2	1	0	2	0	2	1	0	1
1	1	2	2	0	0	2	0	2	1	0	2	2	0
1	2	2	0	2	0	2	1	0	2	1	1	0	1
2	0	0	2	2	2	2	0	0	2	2	2	1	1
0	2	0	2	1	0	0	2	2	2	2	1	2	2
2	2	0	1	0	2	2	2	2	1	1	0	0	0
2	1	0	0	1	2	2	1	1	0	0	1	2	2
0	0	2	0	0	2	1	2	1	2	1	2	2	0
2	0	2	2	2	1	0	2	1	1	0	1	0	1
0	2	2	2	1	2	1	1	0	1	0	0	1	2
2	2	2	1	0	1	0	1	0	0	2	2	2	0
2	1	2	0	1	1	0	0	2	2	1	0	1	2
2	0	1	2	2	0	1	1	2	0	1	0	2	1
0	2	1	2	1	1	2	0	1	0	1	2	0	2
2	2	1	1	0	0	1	0	1	2	0	1	1	0
2	1	1	0	1	0	1	2	0	1	2	2	0	2

INDEPENDENT INTERACTIONS CONFOUNDED

$ABD^2, AB^2E^2, ACF^2, AC^2G^2, BCH^2, BC^2I^2, ABCJ^2, ABC^2K^2, AB^2CL^2, AB^2C^2M$

I.S(2) ALL EXCEPT EN

KEY TO OTHER BLOCKS

$DN^2, EN^2, FN^2, GN^2, HN^2, IN^2, JN^2, KN^2, LN^2, MN^2$

63. DESIGN WITH N=14, P= 2, R=3

IDENTITY GROUP OF INTERACTIONS

$I=ABCDM^2=B^2CDEN^2$

KEY BLOCK

A	B	C	D	E	F	G	H	I	J	K	L	M	N
0	0	0	0	0	0	0	0	0	0	0	0	0	0
1	0	0	1	1	0	1	1	0	1	1	1	2	2
0	1	0	2	0	2	1	1	2	2	1	2	0	1
1	1	0	0	1	2	2	2	2	0	2	0	2	0
1	2	0	2	1	1	0	0	1	2	0	2	2	1
0	0	1	1	2	1	0	2	2	0	1	1	2	1
1	0	1	2	0	1	1	0	2	1	2	2	1	0
0	1	1	0	2	0	1	0	1	2	2	0	2	2
1	1	1	1	0	0	2	1	1	0	0	1	1	1
1	2	1	0	0	2	0	2	0	2	1	0	1	2
1	0	2	0	2	2	1	2	1	1	0	0	0	1
0	1	2	1	1	1	1	2	0	2	0	1	1	0
1	1	2	2	2	1	2	0	0	0	1	2	0	2
1	2	2	1	2	0	0	1	2	2	2	1	0	0
2	0	0	2	2	0	2	2	0	2	2	2	1	1
0	2	0	1	0	1	2	2	1	1	2	1	0	2
2	2	0	0	2	1	1	1	1	0	1	0	1	0
2	1	0	1	2	2	0	0	2	1	0	1	1	2
0	0	2	2	1	2	0	1	1	0	2	2	1	2
2	0	2	1	0	2	2	0	1	2	1	1	2	0
0	2	2	0	1	0	2	0	2	1	1	0	1	1
2	2	2	2	0	0	1	2	2	0	0	2	2	2
2	1	2	0	0	1	0	1	0	1	2	0	2	1
2	0	1	0	1	1	2	1	2	2	0	0	0	2
0	2	1	2	2	2	2	1	0	1	0	2	2	0
2	2	1	1	1	2	1	0	0	0	2	1	0	1
2	1	1	2	1	0	0	2	1	1	1	2	0	0

INDEPENDENT INTERACTIONS CONFOUNDED

$AB^2CD^2, AC^2E^2, B^2CF^2, ABG^2, ABC^2HBCI, AB^2J^2, ABCK^2, AB^2CL^2$

I.S(2)-ALL EXCEPT DL^2

KEY TO OTHER BLOCKS

DMN, EN, F, G, H, I, J, K, L

64. DESIGN WITH N=14, P= 3, R=3

IDENTITY GROUP OF INTERACTIONS

$I=A^2C^2DEL^2=A^2BDE^2M^2=ABC^2EN^2$

KEY BLOCK

A	B	C	D	E	F	G	H	I	J	K	L	M	N
0	0	0	0	0	0	0	0	0	0	0	0	0	0
1	0	0	1	2	1	1	0	1	1	1	2	1	0
0	1	0	2	0	2	2	1	1	1	1	2	0	1
1	1	0	0	2	0	0	1	2	2	2	1	1	1
1	2	0	2	2	2	2	2	0	0	0	0	1	2
0	0	1	1	2	0	2	2	2	0	1	2	2	1
1	0	1	2	1	1	0	2	0	1	2	1	0	1
0	1	1	0	2	2	1	0	0	1	2	1	2	2
1	1	1	1	1	0	2	0	1	2	0	0	0	2
1	2	1	0	1	2	1	1	2	0	1	2	0	0
1	0	2	0	0	1	2	1	2	1	0	0	2	2
0	1	2	1	1	2	0	2	2	1	0	0	1	0
1	1	2	2	0	0	1	2	0	2	1	2	2	0
1	2	2	1	0	2	0	0	1	0	2	1	2	1
2	0	0	2	1	2	2	0	2	2	2	1	2	0
0	2	0	1	0	1	1	2	2	2	2	1	0	2
2	2	0	0	1	0	0	2	1	1	1	2	2	2
2	1	0	1	1	1	1	1	0	0	0	0	2	1
0	0	2	2	1	0	1	1	1	0	2	1	1	2
2	0	2	1	2	2	0	1	0	2	1	2	0	2
0	2	2	0	1	1	2	0	0	2	1	2	1	1
2	2	2	2	2	0	1	0	2	1	0	0	0	1
2	1	2	0	2	1	2	2	1	0	2	1	0	0
2	0	1	0	0	2	1	2	1	2	0	0	1	1
0	2	1	2	2	1	0	1	1	2	0	0	2	0
2	2	1	1	0	0	2	1	0	1	2	1	1	0
2	1	1	2	0	1	0	0	2	0	1	2	1	2

INDEPENDENT INTERACTIONS CONFOUNDED

$AB^2CD^2, ACF^2, AB^2F^2, AB^2C^2G^2, BC^2HABC^2I^2, ABJ^2, ABCK^2$

I.S(2)-ALL EXCEPT KL^2

KEY TO OTHER BLOCKS

DLM, ELM^2N, F, G, H, I, J, K

65. DESIGN WITH N=14, P= 4, R=3
IDENTITY GROUP OF INTERACTIONS

$I=ABCDK^2=B^2CDEL^2=A^2C^2DEM^2=ABC^2EN^2$

KEY BLOCK

A	B	C	D	E	F	G	H	I	J	K	L	M	N
0	0	0	0	0	0	0	0	0	0	0	0	0	0
1	0	0	2	1	1	1	1	1	1	0	0	2	2
0	1	0	1	1	1	0	0	2	2	2	1	2	2
1	1	0	0	2	2	1	1	0	0	2	1	1	1
1	2	0	1	0	0	1	1	2	2	1	2	0	0
0	0	1	0	0	2	1	2	1	2	1	1	2	2
1	0	1	2	1	0	2	0	2	0	1	1	1	1
0	1	1	1	1	0	1	2	0	1	0	2	1	1
1	1	1	0	2	1	2	0	1	2	0	2	0	0
1	2	1	1	0	2	2	0	0	1	2	0	2	2
1	0	2	2	1	2	0	2	0	2	2	2	0	0
0	1	2	1	1	2	2	1	1	0	1	0	0	0
1	1	2	0	2	0	0	2	2	1	1	0	2	2
1	2	2	1	0	1	0	2	1	0	0	1	1	1
2	0	0	1	2	2	2	2	2	2	0	0	1	1
0	2	0	2	2	2	0	0	1	1	1	2	1	1
2	2	0	0	1	1	2	2	0	0	1	2	2	2
2	1	0	2	0	0	2	2	1	1	2	1	0	0
0	0	2	0	0	1	2	1	2	1	2	2	1	1
2	0	2	1	2	0	1	0	1	0	2	2	2	2
0	2	2	2	2	0	2	1	0	2	0	1	2	2
2	2	2	0	1	2	1	0	2	1	0	1	0	0
2	1	2	2	0	1	1	0	0	2	1	0	1	1
2	0	1	1	2	1	0	1	0	1	1	1	0	0
0	2	1	2	2	1	1	2	2	0	2	0	0	0
2	2	1	0	1	0	0	1	1	2	2	0	1	1
2	1	1	2	0	2	0	1	2	0	0	2	2	2

INDEPENDENT INTERACTIONS CONFOUNDED

$AB^2D, ABE^2, ABC^2F^2, ACG^2, AC^2H^2, AB^2CI^2, AB^2C^2J^2$

I.S(2)-ALL EXCEPT MN^2

KEY TO OTHER BLOCKS

DKLM,ELMN,F,G,H,I,J

66. DESIGN WITH N=14, P= 5, R=3

IDENTITY GROUP OF INTERACTIONS

$I=A^2BDE^2J^2=ABCDK^2=B^2CDEL^2=A^2C^2DEM^2=ABC^2EN^2$

KEY BLOCK

A	B	C	D	E	F	G	H	I	J	K	L	M	N
0	0	0	0	0	0	0	0	0	0	0	0	0	0
1	0	0	2	1	1	1	1	1	0	0	0	2	2
0	1	0	1	1	0	0	1	2	1	2	1	2	2
1	1	0	0	2	1	1	2	0	1	2	1	1	1
1	2	0	1	0	1	1	0	2	2	1	2	0	0
0	0	1	0	0	1	2	2	2	0	1	1	2	2
1	0	1	2	1	2	0	0	0	0	1	1	1	1
0	1	1	1	1	1	2	0	1	1	0	2	1	1
1	1	1	0	2	2	0	1	2	1	0	2	0	0
1	2	1	1	0	2	0	2	1	2	2	0	2	2
1	0	2	2	1	0	2	2	2	0	2	2	0	0
0	1	2	1	1	2	1	2	0	1	1	0	0	0
1	1	2	0	2	0	2	0	1	1	1	0	2	2
1	2	2	1	0	0	2	1	0	2	0	1	1	1
2	0	0	1	2	2	2	2	2	0	0	0	1	1
0	2	0	2	2	0	0	2	1	2	1	2	1	1
2	2	0	0	1	2	2	1	0	2	1	2	2	2
2	1	0	2	0	2	2	0	1	1	2	1	0	0
0	0	2	0	0	2	1	1	1	0	2	2	1	1
2	0	2	1	2	1	0	0	0	0	2	2	2	2
0	2	2	2	2	2	1	0	2	2	0	1	2	2
2	2	2	0	1	1	0	2	1	2	0	1	0	0
2	1	2	2	0	1	0	1	2	1	1	0	1	1
2	0	1	1	2	0	1	1	1	0	1	1	0	0
0	2	1	2	2	1	2	1	0	2	2	0	0	0
2	2	1	0	1	0	1	0	2	2	2	0	1	1
2	1	1	2	0	0	1	2	0	1	0	2	2	2

INDEPENDENT INTERACTIONS CONFOUNDED

$AB^2D, ABE^2, ACF^2; AC^2G^2, ABC^2H^2, AB^2C^2I^2$

I.S(2)-ALL EXCEPT, BJ^2, MN^2

KEY TO OTHER BLOCKS

$DJKLM, EJ^2LMN, F, G, H, I$

67. DESIGN WITH N=14, P= 6, R=3

IDENTITY GROUP OF INTERACTIONS

$I=A^{2}BCD^{2}EI^{2}=A^{2}BDE^{2}J^{2}=ABCDK^{2}=B^{2}CDEL^{2}=A^{2}C^{2}DEM^{2}=ABC^{2}EN^{2}$

KEY BLOCK

A	B	C	D	E	F	G	H	I	J	K	L	M	N
0	0	0	0	0	0	0	0	0	0	0	0	0	0
1	0	0	2	1	1	1	1	1	0	0	0	2	2
0	1	0	1	1	0	0	1	1	1	2	1	2	2
1	1	0	0	2	1	1	2	2	1	2	1	1	1
1	2	0	1	0	1	1	0	0	2	1	2	0	0
0	0	1	0	0	1	2	2	1	0	1	1	2	2
1	0	1	2	1	2	0	0	2	0	1	1	1	1
0	1	1	1	1	1	2	0	2	1	0	2	1	1
1	1	1	0	2	2	0	1	0	1	0	2	0	0
1	2	1	1	0	2	0	2	1	2	2	0	2	2
1	0	2	2	1	0	2	2	0	0	2	2	0	0
0	1	2	1	1	2	1	2	0	1	1	0	0	0
1	1	2	0	2	0	2	0	1	1	1	0	2	2
1	2	2	1	0	0	2	1	2	2	0	1	1	1
2	0	0	1	2	2	2	2	2	0	0	0	1	1
0	2	0	2	2	0	0	2	2	2	1	2	1	1
2	2	0	0	1	2	2	1	1	2	1	2	2	2
2	1	0	2	0	2	2	0	0	1	2	1	0	0
0	0	2	0	0	2	1	1	2	0	2	2	1	1
2	0	2	1	2	1	0	0	1	0	2	2	2	2
0	2	2	2	2	2	1	0	1	2	0	1	2	2
2	2	2	0	1	1	0	2	0	2	0	1	0	0
2	1	2	2	0	1	0	1	2	1	1	0	1	1
2	0	1	1	2	0	1	1	0	0	1	1	0	0
0	2	1	2	2	1	2	1	0	2	2	0	0	0
2	2	1	0	1	0	1	0	2	2	2	0	1	1
2	1	1	2	0	0	1	2	1	1	0	2	2	2

INDEPENDENT INTERACTIONS CONFOUNDED

$AB^{2}D, ABE^{2}, ACF^{2}, AC^{2}G^{2}, ABC^{2}H^{2}$

I.S(2)-ALL EXCEPT $BJ^{2}, IM^{2}, IN^{2}, MN^{2}$

KEY TO OTHER BLOCKS

$DI^{2}JKLM, EIJ^{2}LMN, F, G, H$

68. DESIGN WITH N=14, P= 7, R=3

IDENTITY GROUP OF INTERACTIONS

$I=ABFGH^2=ABCDI^2=B^2CDEJ^2=A^2C^2DEK^2=A^2BDE^2L^2=ABC^2EM^2=A^2BCD^2EN^2$

KEY BLOCK

A	B	C	D	E	F	G	H	I	J	K	L	M	N
0	0	0	0	0	0	0	0	0	0	0	0	0	0
1	0	0	1	1	0	0	1	2	2	1	2	2	2
0	1	0	1	0	2	1	1	2	0	1	2	1	0
1	1	0	2	1	2	1	2	1	2	2	1	0	2
1	2	0	0	1	1	2	0	0	2	0	0	1	2
0	0	1	2	2	2	2	1	0	2	0	0	1	1
1	0	1	0	0	2	2	2	2	1	1	2	0	0
0	1	1	0	2	1	0	2	2	2	1	2	2	1
1	1	1	1	0	1	0	0	1	1	2	1	1	0
1	2	1	2	0	0	1	1	0	1	0	0	2	0
1	0	2	2	2	1	1	0	2	0	1	2	1	1
0	1	2	2	1	0	2	0	2	1	1	2	0	2
1	1	2	0	2	0	2	1	1	0	2	1	2	1
1	2	2	1	2	2	0	2	0	0	0	0	0	1
2	0	0	2	2	0	0	2	1	1	2	1	1	1
0	2	0	2	0	1	2	2	1	0	2	1	2	0
2	2	0	1	2	1	2	1	2	1	1	2	0	1
2	1	0	0	2	2	1	0	0	1	0	0	2	1
0	0	2	1	1	1	1	2	0	1	0	0	2	2
2	0	2	0	0	1	1	1	1	2	2	1	0	0
0	2	2	0	1	2	0	1	1	1	2	1	1	2
2	2	2	2	0	2	0	0	2	2	1	2	2	0
2	1	2	1	0	0	2	2	0	2	0	0	1	0
2	0	1	1	1	2	2	0	1	0	2	1	2	2
0	2	1	1	2	0	1	0	1	2	2	1	0	1
2	2	1	0	1	0	1	2	2	0	1	2	1	2
2	1	1	2	1	1	0	1	0	0	0	0	0	2

INDEPENDENT INTERACTIONS CONFOUNDED

$ABC^2D^2, AC^2E^2, BCF, BC^2G^2$

I.S(2)-ALL EXCEPT IK, IL^2, KL, EN

KEY TO OTHER BLOCKS

$DIJKLN^2, EJKL^2MN, FH^2, G$

69. DESIGN WITH N=15, P= 1, R=3

IDENTITY GROUP OF INTERACTIONS

I=ABCDEFGHIJKLMNO

KEY BLOCK

A	B	C	D	E	F	G	H	I	J	K	L	M	N	O
0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
1	0	0	1	1	1	1	0	0	1	1	1	1	1	2
0	1	0	1	0	2	0	1	1	1	1	2	2	0	0
1	1	0	2	1	0	1	1	1	2	2	0	0	1	2
1	2	0	0	1	2	1	2	2	0	0	2	2	1	2
0	0	1	0	1	0	2	1	2	1	2	1	2	0	2
1	0	1	1	2	1	0	1	2	2	0	2	0	1	1
0	1	1	1	1	2	2	2	0	2	0	0	1	0	2
1	1	1	2	2	0	0	2	0	0	1	1	2	1	1
1	2	1	0	2	2	0	0	1	1	2	0	1	1	1
1	0	2	1	0	1	2	2	1	0	2	0	2	1	0
0	1	2	1	2	2	1	0	2	0	2	1	0	0	1
1	1	2	2	0	0	2	0	2	1	0	2	1	1	0
1	2	2	0	0	2	2	1	0	2	1	1	0	1	0
2	0	0	2	2	2	2	0	0	2	2	2	2	2	1
0	2	0	2	0	1	0	2	2	2	2	1	1	0	0
2	2	0	1	2	0	2	2	2	1	1	0	0	2	1
2	1	0	0	2	1	2	1	1	0	0	1	1	2	1
0	0	2	0	2	0	1	2	1	2	1	2	1	0	1
2	0	2	2	1	2	0	2	1	1	0	1	0	2	2
0	2	2	2	2	1	1	1	0	1	0	0	2	0	1
2	2	2	1	1	0	0	1	0	0	2	2	1	2	2
2	1	2	0	1	1	0	0	2	2	1	0	2	2	2
2	0	1	2	0	2	1	1	2	0	1	0	1	2	0
0	2	1	2	1	1	2	0	1	0	1	2	0	0	2
2	2	1	1	0	0	1	0	1	2	0	1	2	2	0
2	1	1	0	0	1	1	2	0	1	2	2	0	2	0

INDEPENDENT INTERACTIONS CONFOUNDED

ABD^2, ACE^2, AB^2F^2, AC^2G^2, BCH^2, BC^2I^2, $ABCJ^2$, ABC^2K^2,

AB^2CL^2, $AB^2C^2M^2$, AN^2

I.S(2)-ALL EXCEPT AN^2, EO

KEY TO OTHER BLOCKS

DO^2, EO^2, FO^2, GO^2, HO^2, IO^2, JO^2, KO^2, LO^2, MO^2, NO^2

70. DESIGN WITH N=15, P= 2, R=3

IDENTITY GROUP OF INTERACTIONS

$I=ABCDN^2=B^2CDEO^2$

KEY BLOCK

A	B	C	D	E	F	G	H	I	J	K	L	M	N	O
0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
1	0	0	1	1	0	1	1	0	1	1	1	2	2	2
0	1	0	2	0	2	1	1	2	2	1	2	0	0	1
1	1	0	0	1	2	2	2	2	0	2	0	2	2	0
1	2	0	2	1	1	0	0	1	2	0	2	2	2	1
0	0	1	1	2	1	0	2	2	0	1	1	2	2	1
1	0	1	2	0	1	1	0	2	1	2	2	1	1	0
0	1	1	0	2	0	1	0	1	2	2	0	2	2	2
1	1	1	1	0	0	2	1	1	0	0	1	1	1	1
1	2	1	0	0	2	0	2	0	2	1	0	1	1	2
1	0	2	0	2	2	1	2	1	1	0	0	0	0	1
0	1	2	1	1	1	1	2	0	2	0	1	1	1	0
1	1	2	2	2	1	2	0	0	0	1	2	0	0	2
1	2	2	1	2	0	0	1	2	2	2	1	0	0	0
2	0	0	2	2	0	2	2	0	2	2	2	1	1	1
0	2	0	1	0	1	2	2	1	1	2	1	0	0	2
2	2	0	0	2	1	1	1	1	0	1	0	1	1	0
2	1	0	1	2	2	0	0	2	1	0	1	1	1	2
0	0	2	2	1	2	0	1	1	0	2	2	1	1	2
2	0	2	1	0	2	2	0	1	2	1	1	2	2	0
0	2	2	0	1	0	2	0	2	1	1	0	1	1	1
2	2	2	2	0	0	1	2	2	0	0	2	2	2	2
2	1	2	0	0	1	0	1	0	1	2	0	2	2	1
2	0	1	0	1	1	2	1	2	2	0	0	0	0	2
0	2	1	2	2	2	2	1	0	1	0	2	2	2	0
2	2	1	1	1	2	1	0	0	0	2	1	0	0	1
2	1	1	2	1	0	0	2	1	1	1	2	0	0	0

INDEPENDENT INTERACTIONS CONFOUNDED

$AB^2CD^2, AC^2E^2, BC^2F, ABG^2, ABC^2H^2, BCI, AB^2J^2, ABCK^2, AB^2CL^2, ACM$

I.S(2)-ALL EXCEPT DL^2 , MN^2

KEY TO OTHER BLOCKS

DNO, EO, F, G, H, I, J, K, L, M

71. DESIGN WITH N=15, P= 3, R=3

IDENTITY GROUP OF INTERACTIONS

$I=A^2C^2DEM^2=A^2BDE^2N^2=ABC^2EO^2$

KEY BLOCK

A	B	C	D	E	F	G	H	I	J	K	L	M	N	O
0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
1	0	0	1	2	1	1	0	1	1	2	1	2	1	0
0	1	0	2	0	2	2	1	1	1	2	0	2	0	1
1	1	0	0	2	0	0	1	2	2	1	1	1	1	1
1	2	0	2	2	2	2	2	0	0	0	1	0	1	2
0	0	1	1	2	0	2	2	2	0	2	2	2	2	1
1	0	1	2	1	1	0	2	0	1	1	0	1	0	1
0	1	1	0	2	2	1	0	0	1	1	2	1	2	2
1	1	1	1	1	0	2	0	1	2	0	0	0	0	2
1	2	1	0	1	2	1	1	2	0	2	0	2	0	0
1	0	2	0	0	1	2	1	2	1	0	2	0	2	2
0	1	2	1	1	2	0	2	2	1	0	1	0	1	0
1	1	2	2	0	0	1	2	0	2	2	2	2	2	0
1	2	2	1	0	2	0	0	1	0	1	2	1	2	1
2	0	0	2	1	2	2	0	2	2	1	2	1	2	0
0	2	0	1	0	1	1	2	2	2	1	0	1	0	2
2	2	0	0	1	0	0	2	1	1	2	2	2	2	2
2	1	0	1	1	1	1	1	0	0	0	2	0	2	1
0	0	2	2	1	0	1	1	1	0	1	1	1	1	2
2	0	2	1	2	2	0	1	0	2	2	0	2	0	2
0	2	2	0	1	1	2	0	0	2	2	1	2	1	1
2	2	2	2	2	0	1	0	2	1	0	0	0	0	1
2	1	2	0	2	1	2	2	1	0	1	0	1	0	0
2	0	1	0	0	2	1	2	1	2	0	1	0	1	1
0	2	1	2	2	1	0	1	1	2	0	2	0	2	0
2	2	1	1	0	0	2	1	0	1	1	1	1	1	0
2	1	1	2	0	1	0	0	2	0	2	1	2	1	2

INDEPENDENT INTERACTIONS CONFOUNDED

$AB^2CD^2, ACE, AB^2F^2, AB^2C^2G^2, BC^2H^2ABC^2I^2, ABJ^2, ABCK, AC^2L^2$

I.S(2)-ALL EXCEPT KM^2, LN^2

KEY TO OTHER BLOCKS

$DMN, EMN^2O, F, G, H, I, J, K, L$

72. DESIGN WITH N=15, P= 4, R=3
IDENTITY GROUP OF INTERACTIONS

$I=ABCDL^2=B^2CDEM^2=A^2C^2DEN^2=ABC^2EO^2$

KEY BLOCK

A	B	C	D	E	F	G	H	I	J	K	L	M	N	O
0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
1	0	0	2	1	1	1	1	1	1	0	0	0	2	2
0	1	0	1	1	1	0	0	2	2	1	2	1	2	2
1	1	0	0	2	2	1	1	0	0	1	2	1	1	1
1	2	0	1	0	0	1	1	2	2	2	1	2	0	0
0	0	1	0	0	2	1	2	1	2	1	1	1	2	2
1	0	1	2	1	0	2	0	2	0	1	1	1	1	1
0	1	1	1	1	0	1	2	0	1	2	0	2	1	1
1	1	1	0	2	1	2	0	1	2	2	0	2	0	0
1	2	1	1	0	2	2	0	0	1	0	2	0	2	2
1	0	2	2	1	2	0	2	0	2	2	2	2	0	0
0	1	2	1	1	2	2	1	1	0	0	1	0	0	0
1	1	2	0	2	0	0	2	2	1	0	1	0	2	2
1	2	2	1	0	1	0	2	1	0	1	0	1	1	1
2	0	0	1	2	2	2	2	2	2	0	0	0	1	1
0	2	0	2	2	2	0	0	1	1	2	1	2	1	1
2	2	0	0	1	1	2	2	0	0	2	1	2	2	2
2	1	0	2	0	0	2	2	1	1	1	2	1	0	0
0	0	2	0	0	1	2	1	2	1	2	2	2	1	1
2	0	2	1	2	0	1	0	1	0	2	2	2	2	2
0	2	2	2	2	0	2	1	0	2	1	0	1	2	2
2	2	2	0	1	2	1	0	2	1	1	0	1	0	0
2	1	2	2	0	1	1	0	0	2	0	1	0	1	1
2	0	1	1	2	1	0	1	0	1	1	1	1	0	0
0	2	1	2	2	1	1	2	2	0	0	2	0	0	0
2	2	1	0	1	0	0	1	1	2	0	2	0	1	1
2	1	1	2	0	2	0	1	2	0	2	0	2	2	2

INDEPENDENT INTERACTIONS CONFOUNDED

$AB^2D, ABE^2, ABC^2F^2, ACG^2, AC^2H^2, AB^2CI^2, AB^2C^2J^2, BCK^2$

I.S(2)-ALL EXCEPT KM^2, NO^2

KEY TO OTHER BLOCKS

DLMN, EMNO, F, G, H, I, J, K

73. DESIGN WITH N=15, P= 5, R=3

IDENTITY GROUP OF INTERACTIONS

$I = A^2BDE^2K^2 = ABCDL^2 = B^2CDEM^2 = A^2C^2DEN^2 = ABC^2EO^2$

KEY BLOCK

A	B	C	D	E	F	G	H	I	J	K	L	M	N	O
0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
1	0	0	2	1	1	1	1	1	1	0	0	0	2	2
0	1	0	1	1	1	0	0	2	2	1	2	1	2	2
1	1	0	0	2	2	1	1	0	0	1	2	1	1	1
1	2	0	1	0	0	1	1	2	2	2	1	2	0	0
0	0	1	0	0	2	1	2	1	2	0	1	1	2	2
1	0	1	2	1	0	2	0	2	0	0	1	1	1	1
0	1	1	1	1	0	1	2	0	1	1	0	2	1	1
1	1	1	0	2	1	2	0	1	2	1	0	2	0	0
1	2	1	1	0	2	2	0	0	1	2	2	0	2	2
1	0	2	2	1	2	0	2	0	2	0	2	2	0	0
0	1	2	1	1	2	2	1	1	0	1	1	0	0	0
1	1	2	0	2	0	0	2	2	1	1	1	0	2	2
1	2	2	1	0	1	0	2	1	0	2	0	1	1	1
2	0	0	1	2	2	2	2	2	2	0	0	0	1	1
0	2	0	2	2	2	0	0	1	1	2	1	2	1	1
2	2	0	0	1	1	2	2	0	0	2	1	2	2	2
2	1	0	2	0	0	2	2	1	1	1	2	1	0	0
0	0	2	0	0	1	2	1	2	1	0	2	2	1	1
2	0	2	1	2	0	1	0	1	0	0	2	2	2	2
0	2	2	2	2	0	2	1	0	2	2	0	1	2	2
2	2	2	0	1	2	1	0	2	1	2	0	1	0	0
2	1	2	2	0	1	1	0	0	2	1	1	0	1	1
2	0	1	1	2	1	0	1	0	1	0	1	1	0	0
0	2	1	2	2	1	1	2	2	0	2	2	0	0	0
2	2	1	0	1	0	0	1	1	2	2	2	0	1	1
2	1	1	2	0	2	0	1	2	0	1	0	2	2	2

INDEPENDENT INTERACTIONS CONFOUNDED

$AB^2D, ABE^2, ABC^2F^2, ACG^2, AC^2H^2 AB^2CI^2, AB^2C^2J^2$

I.S(2)-ALL EXCEPT BK^2, NO^2

KEY TO OTHER BLOCKS

DLMN, EMNO, F, G, H, I, J

74. DESIGN WITH N=15, P= 6, R=3

IDENTITY GROUP OF INTERACTIONS

$I=A^2BCD^2EJ^2=A^2BDE^2K^2=ABCDL^2=B^2CDEM^2=A^2C^2DEN^2=ABC^2EO^2$

KEY BLOCK

A	B	C	D	E	F	G	H	I	J	K	L	M	N	O
0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
1	0	0	2	1	1	1	1	1	1	0	0	0	2	2
0	1	0	1	1	0	0	1	2	1	1	2	1	2	2
1	1	0	0	2	1	1	2	0	2	1	2	1	1	1
1	2	0	1	0	1	1	0	2	0	2	1	2	0	0
0	0	1	0	0	1	2	2	1	1	0	1	1	2	2
1	0	1	2	1	2	0	0	2	2	0	1	1	1	1
0	1	1	1	1	1	2	0	0	2	1	0	2	1	1
1	1	1	0	2	2	0	1	1	0	1	0	2	0	0
1	2	1	1	0	2	0	2	0	1	2	2	0	2	2
1	0	2	2	1	0	2	2	0	0	0	2	2	0	0
0	1	2	1	1	2	1	2	1	0	1	1	0	0	0
1	1	2	0	2	0	2	0	2	1	1	1	0	2	2
1	2	2	1	0	0	2	1	1	2	2	0	1	1	1
2	0	0	1	2	2	2	2	2	2	0	0	0	1	1
0	2	0	2	2	0	0	2	1	2	2	1	2	1	1
2	2	0	0	1	2	2	1	0	1	2	1	2	2	2
2	1	0	2	0	2	2	0	1	0	1	2	1	0	0
0	0	2	0	0	2	1	1	2	2	0	2	2	1	1
2	0	2	1	2	1	0	0	1	1	0	2	2	2	2
0	2	2	2	2	2	1	0	0	1	2	0	1	2	2
2	2	2	0	1	1	0	2	2	0	2	0	1	0	0
2	1	2	2	0	1	0	1	0	2	1	1	0	1	1
2	0	1	1	2	0	1	1	0	0	0	1	1	0	0
0	2	1	2	2	1	2	1	2	0	2	2	0	0	0
2	2	1	0	1	0	1	0	1	2	2	2	0	1	1
2	1	1	2	0	0	1	2	2	1	1	0	2	2	2

INDEPENDENT INTERACTIONS CONFOUNDED

$AB^2D, ABE^2, ACF^2, AC^2G^2, ABC^2H, AB^2CI^2$

I.S(2)-ALL EXCEPT JN, JO, NO^2, BK^2

KEY TO OTHER BLOCKS

$DJ^2KLMN, EJK^2MNO, F, G, H, I$

75. DESIGN WITH N=15, P= 7, R=3

IDENTITY GROUP OF INTERACTIONS

$I=ABFGI^2=ABCDJ^2=B^2CDEK^2=A^2C^2DEL^2=A^2BDE^2M^2=ABC^2EN^2=A^2BCD^2EO^2$

KEY BLOCK

A	B	C	D	E	F	G	H	I	J	K	L	M	N	O
0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
1	0	0	1	1	0	0	1	1	2	2	1	2	2	2
0	1	0	1	0	2	1	2	1	2	0	1	2	1	0
1	1	0	2	1	2	1	0	2	1	2	2	1	0	2
1	2	0	0	1	1	2	2	0	0	2	0	0	1	2
0	0	1	2	2	2	2	1	1	0	2	0	0	1	1
1	0	1	0	0	2	2	2	2	2	1	1	2	0	0
0	1	1	0	2	1	0	0	2	2	2	1	2	2	1
1	1	1	1	0	1	0	1	0	1	1	2	1	1	0
1	2	1	2	0	0	1	0	1	0	1	0	0	2	0
1	0	2	2	2	1	1	0	0	2	0	1	2	1	1
0	1	2	2	1	0	2	1	0	2	1	1	2	0	2
1	1	2	0	2	0	2	2	1	1	0	2	1	2	1
1	2	2	1	2	2	0	1	2	0	0	0	0	0	1
2	0	0	2	2	0	0	2	2	1	1	2	1	1	1
0	2	0	2	0	1	2	1	2	1	0	2	1	2	0
2	2	0	1	2	1	2	0	1	2	1	1	2	0	1
2	1	0	0	2	2	1	1	0	0	1	0	0	2	1
0	0	2	1	1	1	1	2	2	0	1	0	0	2	2
2	0	2	0	0	1	1	1	1	1	2	2	1	0	0
0	2	2	0	1	2	0	0	1	1	1	2	1	1	2
2	2	2	2	0	2	0	2	0	2	2	1	2	2	0
2	1	2	1	0	0	2	0	2	0	2	0	0	1	0
2	0	1	1	1	2	2	0	0	1	0	2	1	2	2
0	2	1	1	2	0	1	2	0	1	2	2	1	0	1
2	2	1	0	1	0	1	1	2	2	0	1	2	1	2
2	1	1	2	1	1	0	2	1	0	0	0	0	0	2

INDEPENDENT INTERACTIONS CONFOUNDED

$ABC^2D, AC^2E^2, BCF, BC^2G^2, AB^2CH^2$

I.S(2)-ALL EXCEPT EO, LM, JL, JM^2

KEY TO OTHER BLOCKS

$DJKLMO^2, EKLM^2NO, FI, G, H$

76. DESIGN WITH N=15, P= 8, R=3

IDENTITY GROUP OF INTERACTIONS

$I = ACF^2GH^2 = ABFGI^2 = ABCDJ^2 = B^2CDEK^2 = A^2C^2EL^2 = A^2BDE^2M^2 = ABC^2EN^2$

$= A^2BCD^2EO^2$

KEY BLOCK

A	B	C	D	E	F	G	H	I	J	K	L	M	N	O
0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
1	0	0	1	1	0	0	1	1	2	2	1	2	2	2
0	1	0	1	0	2	1	2	1	2	0	1	2	1	0
1	1	0	2	1	2	1	0	2	1	2	2	1	0	2
1	2	0	0	1	1	2	2	0	0	2	0	0	1	2
0	0	1	2	2	2	2	1	1	0	2	0	0	1	1
1	0	1	0	0	2	2	2	2	2	1	1	2	0	0
0	1	1	0	2	1	0	0	2	2	2	1	2	2	1
1	1	1	1	0	1	0	1	0	1	1	2	1	1	0
1	2	1	2	0	0	1	0	1	0	1	0	0	2	0
1	0	2	2	2	1	1	0	0	2	0	1	2	1	1
0	1	2	2	1	0	2	1	0	2	1	1	2	0	2
1	1	2	0	2	0	2	2	1	1	0	2	1	2	1
1	2	2	1	2	2	0	1	2	0	0	0	0	0	1
2	0	0	2	2	0	0	2	2	1	1	2	1	1	1
0	2	0	2	0	1	2	1	2	1	0	2	1	2	0
2	2	0	1	2	1	2	0	1	2	1	1	2	0	1
2	1	0	0	2	2	1	1	0	0	1	0	0	2	1
0	0	2	1	1	1	1	2	2	0	1	0	0	2	2
2	0	2	0	0	1	1	1	1	1	2	2	1	0	0
0	2	2	0	1	2	0	0	1	1	1	2	1	1	2
2	2	2	2	0	2	0	2	0	2	2	1	2	2	0
2	1	2	1	0	0	2	0	2	0	2	0	0	1	0
2	0	1	1	1	2	2	0	0	1	0	2	1	2	2
0	2	1	1	2	0	1	2	0	1	2	2	1	0	1
2	2	1	0	1	0	1	1	2	2	0	1	2	1	2
2	1	1	2	1	1	0	2	1	0	0	0	0	0	2

INDEPENDENT INTERACTIONS CONFOUNDED

ABC^2D^2, AC^2E^2BCF, BC^2G^2

I.S(2)-ALL EXCEPT EO, LM, JM^2, JL

KEY TO OTHER BLOCKS

$DJKLMO^2$, $EKLM^2NO$, FIH^2, GH

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