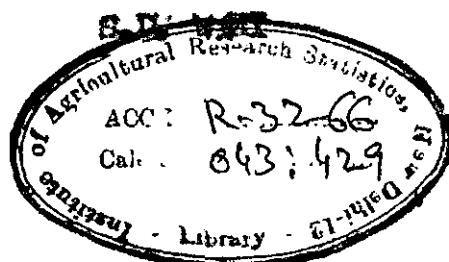


✓ 93

**SOME ASPECTS OF COMBINED SELECTION
IN
POULTRY**



Dissertation submitted in fulfilment of the requirements
for the award of Diploma in Agricultural Statistics
of the Indian Agricultural Statistics Research
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1979**

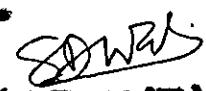
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(S.D. WAHI)

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CHAPTER
INTRODUCTION

The importance of poultry keeping for the production of meat and eggs to raise the present lower human nutritional standards of the country needs no emphasis. Eggs have become part of the diet even in most conservative homes and thus the requirement of eggs for the country has come to about three hundred million eggs per day even considering that only fifty per cent of our population is potential consumer of eggs. Hence it is imperative that we need to have good genetic stock of birds to achieve this production target of eggs.

It is an established fact that the native stocks are very poor egg producers and we have to mostly depend upon newly evolved strains with high production potentials. Thus evolving high yielding strains both for egg and meat production using once imported exotic stocks has become primary necessity for the poultry researchers of the country at this juncture. Question arises thus as to what type of stock and what methods of comparisons among different stocks be used to develop superior stocks.

As far as egg production is concerned, it is a low heritable trait and therefore different methods of selection and mating systems are adopted for its improvement. The

different methods of breeding used in poultry are inbreeding and hybridization, strain crossing and breed crossing. But the results of those breeding methods are useful only if we have some good procedure of selection. In investigations on breeding programmes, the chief aim is to study the possibility of improving the performance of the stock in respect of a given character and the alternative methods by means of which this can be achieved. The main tool the breeder has in his hands is selection.

Selection is an important tool for bringing about genetic improvements in quantitative traits of livestock. The expected genetic progress per generation is the product of heritability and selection differential. The heritability has a predictive role in expressing the reliability of phenotypic value as a guide to the breeding value. The primary effect of selection whether natural or artificial is to change the gene frequencies and the frequency of genotypes carrying certain gene combinations and thereby change the average of the population far from the original position. However, when the heritability of a character to be selected is low, the mean value of number of individuals of a particular relation often provides with a more reliable guide to the breeding value than the individuals own phenotypic value. The correlation between the breeding value and the phenotypic value can be increased by combining the informations

from one or more relatives of the individual for the given trait resulting in greater response to selection.

In poultry, the family classification is usually a hierarchical one, viz., each sire is mated to a number of dams and each dam produces one or more offspring. In such cases the relative merits of individual selection and family selection in breeding for traits of low heritability were first discussed by Lush (1947) and Lerner (1950). The general conclusion is that for traits of low heritability; selection of complete families of full-sibs or half-sibs without regard of individual's performance is more efficient than individual selection.

Selection of individuals on the basis of index with appropriate weights attached to its own performance and the average performance of families was developed by Osborne (1957). The various schemes of combined selection given by him are:

- (i) Selection on the basis of an index with optimum weights attached to the individual's performance and it's full-sib family average
- (ii) Selection the basis of an index with optimum weights attached to the individual's performance and it's half-sib family average.
- (iii) Selection on the basis of an index with optimum weights attached to full-sib family average, half-sib family average and the individual's performance.

However, the maximum efficiency was found to be achieved by selection on the basis of the index (iii) mentioned above.

A series of Coordinated Poultry Breeding Programmes for improvement of egg production are currently being undertaken in the country by various State Governments at the instance of Government of India. One such programme was initiated by Madhya Pradesh State Government at their Regional Poultry Farm at Bhopal with view of evolving a strain of poultry with high level of egg production by selecting birds on the basis of index combining the performance of the individual bird with average performance of the sires and dam families to which bird belongs.

Narain et al (1973) reported the results of the analysis of data collected during this programme. Subsequently Narain et al (1973b) reported that the average rate of lay which was about 46% in the production stock increased to about 53% by the third generation. However, the egg weight which was about 49 grams in the foundation stock decreased to about 45 grams by the third generation. The estimates of heritability of rate of lay decreased consistently from it's value of 0.129 in the foundation stock to 0.002 in the third generation. It appeared, therefore, that although the method of selection based on the index is effective in increasing the rate of lay, it resulted in correlated decline

in the average egg weight, which is also an economically important trait. Keeping in view, Faruqi et al (1977,79) developed a new index which takes into account the correlation between an auxiliary trait such as egg weight and rate of lay the trait under improvement. It was found that the efficiency of the new index is always greater if the phenotypic and genotypic correlation of the auxiliary characters and the characters under improvement are of opposite signs. The effect of inclusion of the auxiliary trait on the efficiency was however studied only for a few combination of genetic parameters involved. The effect of including more than one auxiliary trait as such egg weight and age at first egg or egg production was also investigated.

The main objective of the present study is therefore to construct an index which can take care of more than one auxiliary character of economic value besides the information on the trait under improvement for the averages of full-sib and half-sib families. In addition, the effect of including one, two or three auxiliary traits on the efficiency of the index has also been studied with the help of 1st generation data, obtained from State Poultry Farm at Bhoyal.

CHAPTER IV

SELECTION INDEX WITH ONE ADULTIVE TRAIT

2.1 Construction of Selection Index.

A selection index is used by the breeders to simultaneously select for several characters. The index is based on the phenotypic value of the characters to be improved. The phenotypic value of an individual measured as a deviation from the population mean is the sum of two parts, the deviation of its family mean from the population mean, E_p and the individual's from the family mean, E_w i.e. if P is the phenotypic value of the individual, we have

$$P = E_p + E_w$$

In case of hierarchical classification when the character under improvement is denoted by Y , the individuals phenotypic value can be written as follow:

$$Y_{ijk} = (Y_{ijk} - \bar{Y}_{ij.}) + (\bar{Y}_{ij.} - \bar{Y}_{i..}) + (\bar{Y}_{i..} - \bar{Y}_{...})$$

where Y_{ijk} denotes the k -th progeny of j -th dam and i -th sire. Then the deviation of phenotypic value of the individual from the population mean can be partitioned into three components, as given below:

- 1) the deviation of the individual from the mean of the individuals of j -th dam and the i -th sire (\bar{Y}_j)

- ii) $(\bar{Y}_{1j} - \bar{Y}_{1e})$, the deviation of the full-sib family average from the sire family (\bar{Y}_2) and
- iii) $(\bar{Y}_{1e} - \bar{Y}_{2e})$ the deviation of sire family average from the overall population mean (\bar{Y}_3).

The Osborne (1957) constructed the index by giving appropriate weights to all the three components

$$I = b_1 \bar{Y}_1 + b_2 \bar{Y}_2 + b_3 \bar{Y}_3$$

Now, we are interested in getting the optimum values of b_i 's which will maximise the correlation between the selection Index I and the genotypic value G_i of the character under improvement. The different information required to estimate b_i 's are a) the phenotypic and genotypic variances for every trait considered in the selection index b) the genotypic and phenotypic covariances between each pair of traits.

Now, on maximizing the correlation between genotypic and phenotypic index i.e. r_{GI} the normal equations for the above index are as follows:

$$b_1 V(\bar{Y}_1) + b_2 \text{Cov}(\bar{Y}_1, \bar{Y}_2) + b_3 \text{Cov}(\bar{Y}_1, \bar{Y}_3) = V_G(\bar{Y}_1)$$

$$b_1 \text{Cov}(\bar{Y}_2, \bar{Y}_1) + b_2 V(\bar{Y}_2) + b_3 \text{Cov}(\bar{Y}_2, \bar{Y}_3) = \text{Cov}_G(\bar{Y}_1, \bar{Y}_2)$$

$$b_1 \text{Cov}(\bar{Y}_3, \bar{Y}_1) + b_2 \text{Cov}(\bar{Y}_3, \bar{Y}_2) + b_3 V(\bar{Y}_3) = \text{Cov}_G(\bar{Y}_1, \bar{Y}_3)$$

where V and Cov denotes the phenotypic variances and covariances; V_G and Cov_G denotes the genotypic variance and covariance respectively.

The above equations are solved to get the estimates of b_1 's. The detail formulae of b_1 's are presented in the Chapter IV.

2.2 Selection Index with one Auxiliary Trait

The index I_1 , which was given by Narain et al (1977)

$$I_1 = b_1 P_X + b_2 P_Y + b_3 \bar{P}_Y + b_4 \bar{E}_Y$$

predicts the breeding value (G_y) of the individual for the character (y) under improvement (rate of lay) by combining in optimal manner its own performance (P_y) for y, its performance (P_X) for another correlated character X (egg weight), the average \bar{P}_Y of the phenotypic values of n paternal half-sibs for y and the average (\bar{E}_Y) of the phenotypic values of n full-sibs for y. The other index given by Narain et al (1977) was the modified version of the index given by Narutikorn et al (1973) and is given by

$$I_2 = b_1' P_X + b_2' P_Y + b_3' \bar{P}_Y + b_4' \bar{E}_Y + b_5' D_y$$

It predicts the G_y by combining in an optimal manner, in addition to the information included in I_1 , the information supplied by the average performance (D_y) of the individual's dam for y. The details estimates of b_i' 's in terms of genetic

parameters are presented in the Chapter IV.

2.3 Graphical Analysis of the Indices

In order to study the effect of including individuals performance for Y in the selection indices I_1 and I_2 for trait Y as well as on the accuracy of breeding values, the coefficients along with the efficiency of both the indices over individuals performance for Y were worked out numerically for different combinations of genetic parameters involved in the estimates of b_g 's.

Narsain et al (1977) worked out the efficiency of these two indices over individuals performances for y for the four combinations of r_g and r_p viz. (-0.5, 0.5); (0.2, -0.5); (0.5, 0.0) and (0.0, 0.0) for different values of h_y^2 . In the present study the relative efficiency of the indices I_1 and I_2 over direct selection of individual were studied for some more combinations of r_g and r_p . Since heritability of the individuals egg weight would generally be on the higher side, h_y^2 was taken as 0.5 whereas σ_x^2 / σ_y^2 was taken as one. The graphs for different values of r_g and r_p for different values of h_y^2 against the relative efficiency (E) of these indices over individuals selection were drawn and are presented in Fig. 1 to 6. The number of full-sibs and half-sibs considered were 3 and 20 respectively. The following conclusions were drawn from the

present graphical analysis:

- a) The efficiency of the indices was found to be high for larger values of r_g and r_p and with opposite signs.
- b) For r_p equals to zero the efficiency of both the indices was high and was increasing with rise in value of r_g .
- c) When r_g and r_p was of same sign the efficiency was increasing as the r_p was increasing and r_g was decreasing.
- d) Whenever r_g and r_p was of opposite sign the efficiency of these indices was found to be ~~remained~~ remained constant on the change of sign. So, in the present study e.g. the graph with $r_g = -0.2$ and $r_p = 0.2$ was drawn but the graph with $r_g = 0.2$ and $r_p = -0.2$ was not drawn.
- e) The inclusion of individual dams performance for the character y (D_y) has increased the efficiency of the Index I_2 over I_1 ⁱⁿ all the combinations of r_g , r_p and h_y^2 under study.
- f) For low values of r_p the efficiency of the indices was found to be high. For low values of heritabilities and

it was found decreasing as the h_y^2 was increasing. But with the large values of r_p , the efficiency of these indices was high for extreme values of h_y^2 and was low for middle range of heritability and as a result the curve assumes the typical 'U' shape.

The graphs for r_p equal to and greater than 0.6 was not drawn because of negative values of b_1 's in both the indices. The b_1 's was negative for some range of r_g and h_y^2 and was negative only in cases where r_g was of opposite sign to r_p or was zero. The all b_1 's was negative only due to the negative sign of the denominator in the estimates of b_1 's. The slight efforts were made to investigate the range of r_g and r_p for which the b_1 's was negative. The denominator (D) at the estimates of b_1 's in Index I_p after putting

$$n_{\max} = 4/h_y^2 \quad \text{and} \quad n_{\min} = 2/h_y^2$$

is given by

$$D = 4\sqrt{2h_y^2 - r_g^2 h_x^2 + 2r_g r_p h_x h_y - 2r_p^2}$$

which on simplification gives

$$D = \sqrt{(1-r_p^2)(2h_y^2) - h_y^2 (r_p - r_g \frac{h_x}{h_y})^2}$$

Now, D will be negative iff this inequality will hold true

$$(1-r_p^2)(2h_y^2) < h_y^2 (r_p - r_g \frac{h_x}{h_y})^2$$

which is equal to

$$2(1-r_p^2) - \frac{h_y^2}{h_y} \left[1 + \left(r_g \frac{h_x}{h_y} \right)^2 - 2r_p \left(r_g \frac{h_x}{h_y} \right) \right] < 0$$

Now on putting $r_g \frac{h_x}{h_y} = c$

$$\frac{2(1-r_p^2)}{h_y^2} < [1 + c^2 - 2r_p c]$$

$$< [1 \ 0] \begin{bmatrix} 1 - r_p \\ -r_p \ 1 \end{bmatrix} \begin{bmatrix} 1 \\ c \end{bmatrix}$$

p.d. matrix

which is equal to

$$2h_y^2 - (c^2 h_y^2 - \frac{c^2 h_y^2}{2}) = 2(r_p - c \frac{h_x^2}{2})^2 < 0$$

or

$$(2h_y^2)(1 - c^2 \frac{h_x^2}{2}) < 2(r_p - \frac{ch_x^2}{2})^2$$

or

$$r_p - c \frac{h_x^2}{2} > \sqrt{(1 - \frac{h_x^2}{2})(1 - r_g \frac{h_x^2}{2})}$$

or

$$r_p > r_g \frac{h_x h_y}{2} + \sqrt{(1 - \frac{h_x^2}{2})(1 - r_g \frac{h_x^2}{2})} \quad \text{--- (I)}$$

$\Rightarrow \Delta$ will be negative iff (I) will hold true.

Now, find the minimum of $r_{po} = r_g \frac{h_x h_y}{2} + \sqrt{(1 - \frac{h_x^2}{2})}$

$$\frac{x(1 - \frac{r_g h_x^2}{2})}{x(1 - \frac{r_g h_x^2}{2})}$$

--- (II)

which \Rightarrow

$$\frac{r_g h_x h_y}{2} + \sqrt{\left(1 - \frac{h_y^2}{2}\right) \left(1 - r_g^2 \frac{h_x^2}{2}\right)} < 0$$

which is equal to

$$\left(1 - \frac{h_y^2}{2}\right) \left(1 - r_g^2 \frac{h_x^2}{2}\right) < \left(1 - r_g h_x h_y\right)^2$$

which on simplification gives

$$(h_y + r_g h_x)^2 - 2 r_g h_x h_y > 0$$

or

$$(h_y + r_g h_x)^2 > 0$$

Now on differentiating (II) with reference to r_g .

$$\frac{\partial r_{po}}{\partial r_g} = \frac{h_x h_y}{2} + \sqrt{1 - \frac{h_y^2}{2}} \left(1 - r_g^2 \frac{h_x^2}{2}\right)^{-\frac{1}{2}} \left(-r_g \frac{h_x^2}{2}\right) = 0$$

or

$$h_x h_y = r_g^2 h_x^2 \frac{1 - h_y^2/2}{1 - r_g^2 h_x^2/2} = 0$$

which implies

$$h_y^2 = r_g^2 h_x^2 \quad \text{or} \quad r_g = \pm \frac{h_y}{h_x} \quad (\text{III})$$

Now, Max/Min r_{po} will be

$$\begin{aligned} & \pm \frac{h_y}{h_x} \frac{h_x h_y}{2} \sqrt{\left(1 - \frac{h_y^2}{2}\right) \left(1 - \frac{h_y^2}{2h_x^2}\right)} h_x^2 \\ & = \pm \frac{h_y^2}{2} + \left(1 - \frac{h_y^2}{2}\right) \end{aligned}$$

$$= 1 - \frac{h_y^2}{2} - \frac{h_y^2}{2}$$

Let us consider the positive sign, then $r_{po} = 1(\text{max.})$ and on considering negative sign,

$$r_{po} = 1-h_y^2$$

so, the Max/Min $r_{po} = 1, 1-h_y^2$ -(IV)

$$\text{and } r_g = \left(\frac{h_y}{h_x}, -\frac{h_y}{h_x} \right)$$

Now, on considering the min. value of

$$r_{po} = 1-h_y^2 \text{ and } r_g = -h_y/h_x$$

$$D = -8h_y^2 (1-h_y^2)$$

which proves that D will be negative for some values of r_g and r_p . Now on taking h_y^2 equal to zero or one to the D will become zero. But at optimum value of h_y^2 i.e. $h_y^2 = \frac{1}{2}$, D will become -2 . Hence on putting the optimum value of h_y^2 the following conditions will hold good i.e.

$$D_{(\min)} = 2; h_y^2 = \frac{1}{2}; r_g \geq \frac{-1}{\sqrt{2h_x}} \quad ; \quad r_p \geq 1 - h_y^2 .$$

Now, on taking the optimum value of h_y^2 , the $r_p = 0.5$ and D will become negative. Hence the D will always be positive for $r_p (1-h_y^2)$. The similar results can also be obtained for Index I_2 but due to the limited time it was not tried.

CHAPTER III

GEN INDEX WITH ADDITIVE AND DOMINANT AUXILIARY TRAITS

Three indices are constructed to predict the breeding value of y (G_y) of the individual under improvement by integrating the information supplied by Osborne's index or more correlated traits. Let us first consider two auxiliary varieties X_1 and X_2 and Index will be

$$= b_1 X_1 + b_2 X_2 + b_3 \bar{X}_1 + b_4 \bar{X}_2 + b_5 \bar{X}_3$$

so the b_i 's are estimated by maximizing the correlation between G_y and I_b . The normal equations obtained after dividing the correlation are as follow

$$1 = b_1 \sum X_1^2 + b_2 \sum X_1 X_2 + b_3 \sum X_1 \bar{X}_1 + b_4 \sum X_1 \bar{X}_2 + b_5 \sum X_1 \bar{X}_3$$

$$2 = b_1 \sum X_2 X_1 + b_2 \sum X_2^2 + b_3 \sum X_2 \bar{X}_1 + b_4 \sum X_2 \bar{X}_2 + b_5 \sum X_2 \bar{X}_3$$

$$3 = b_1 \sum X_1 \bar{X}_1 + b_2 \sum \bar{X}_1^2 + b_3 \sum \bar{X}_1 \bar{X}_2 + b_4 \sum \bar{X}_1 \bar{X}_3 + b_5 \sum \bar{X}_1 \bar{X}_3$$

$$\bar{X}_2 = b_1 \sum X_1 \bar{X}_2 + b_2 \sum X_2 \bar{X}_2 + b_3 \sum \bar{X}_1 \bar{X}_2 + b_4 \sum \bar{X}_2^2 + b_5 \sum \bar{X}_2 \bar{X}_3$$

$$\bar{X}_3 = b_1 \sum X_1 \bar{X}_3 + b_2 \sum X_2 \bar{X}_3 + b_3 \sum \bar{X}_1 \bar{X}_3 + b_4 \sum \bar{X}_2 \bar{X}_3 + b_5 \sum \bar{X}_3^2$$

Now, for estimating b_i 's, it is necessary to know heritropic and genotypic variances and covariances, the biometrical correlation coefficients between relatives or given values of heritabilities of the character under study, the genetic correlations (r_g) as well as

CHAPTER IX

SELECTION INDEX WITH TWO AND MORE AUXILIARY TRAITS

These indices are constructed to predict the breeding value of Y (G_y) of the individual under improvement by synthesizing the information supplied by Osborne's index by two or more correlated traits. Let us first consider the two auxiliary variates X_1 and X_2 and Index will be

$$I_{b_1} = b_1 X_1 + b_2 X_2 + b_3 \bar{X}_1 + b_4 \bar{X}_2 + b_5 \bar{X}_3$$

where the b_i 's are estimated by maximizing the correlation between G_y and I_{b_1} . The normal equations obtained after maximizing the correlation are as follow

$$\sum G X_1 = b_1 \sum X_1^2 + b_2 \sum X_1 X_2 + b_3 \sum X_1 \bar{X}_1 + b_4 \sum X_1 \bar{X}_2 + b_5 \sum X_1 \bar{X}_3$$

$$\sum G \bar{X}_2 = b_1 \sum X_1 \bar{X}_2 + b_2 \sum X_2^2 + b_3 \sum X_2 \bar{X}_1 + b_4 \sum X_2 \bar{X}_2 + b_5 \sum X_2 \bar{X}_3$$

$$\sum G \bar{X}_3 = b_1 \sum X_1 \bar{X}_3 + b_2 \sum X_2 \bar{X}_3 + b_3 \sum \bar{X}_1^2 + b_4 \sum \bar{X}_1 \bar{X}_2 + b_5 \sum \bar{X}_1 \bar{X}_3$$

$$\sum G \bar{Y}_2 = b_1 \sum X_1 \bar{Y}_2 + b_2 \sum X_2 \bar{Y}_2 + b_3 \sum \bar{X}_1 \bar{Y}_2 + b_4 \sum \bar{Y}_2^2 + b_5 \sum \bar{Y}_2 \bar{Y}_3$$

$$\sum G \bar{Y}_3 = b_1 \sum X_1 \bar{Y}_3 + b_2 \sum X_2 \bar{Y}_3 + b_3 \sum \bar{X}_1 \bar{Y}_3 + b_4 \sum \bar{Y}_2 \bar{Y}_3 + b_5 \sum \bar{Y}_2^2$$

Now, for estimating b_i 's, it is necessary to know the phenotypic and genotypic variances and covariances, or the biometrical correlation coefficients between relatives for given values of heritabilities of the character under study, the genetic correlations (r_g) as well as

phenotypic correlations (P). Zero correlations were presented by Varain et al (1977) and can be derived by using path coefficient analysis. While deriving these correlations it was assumed that the average of full-sibs (\bar{Y}_3) and the half-sibs (\bar{Y}_2) do not include the observation on the individual (Y_1) the matrix of Biometrical correlations used here is given in Table (A).

Now, the variance of family means are expressed in terms of variance of individual phenotypic value i.e.

$$\sigma_{\bar{Y}_2}^2 = \frac{\sigma^2_{Y_1}}{n}, \quad \sigma_{\bar{Y}_3}^2 = \frac{\sigma^2_{Y_1}}{n}$$

where

$$n = \frac{n}{1 + (n-1) \frac{h_y^2}{4}} \quad & n = \frac{n}{1 + (n-1) \frac{h_y^2}{2}}$$

where n is the number of half-sibs and
 n is the number of full-sibs.

Table A Matrix of Biometrical Correlation between relatives

	x_1	x_2	g_y	\bar{x}_1	\bar{x}_2	\bar{x}_3
x_1	1	$\rho_{12} + r_{1g} b_{x1} h_y \sqrt{n}$	$\rho_{1y} + r_{1g} b_{x1} h_y \sqrt{n}$	$\frac{1}{2} r_{1g} b_{x1} h_y \sqrt{n}$		
x_2		1	$r_{2g} b_{x2}$	$\rho_{2y} + r_{2g} b_{x2} h_y \sqrt{n}$	$\frac{1}{2} r_{2g} b_{x2} h_y \sqrt{n}$	
g_y			1	$b_y + h_y \sqrt{n}$	$\frac{1}{2} h_y \sqrt{n}$	
x_1				1	$\frac{1}{2} h_y^2 \sqrt{n}$	
\bar{x}_2					1	$\frac{1}{2} h_y^2 \sqrt{n}$
\bar{x}_3						1

Now, on using these correlations and phenotypic variances, the above normal equations for estimating the b_g 's in terms of genetic parameters can be written in matrix form and is presented in Table B.

The matrix of the above normal equation is symmetric.

Now, these equations have been solved for deriving the expressions for b_g 's by inverting the matrix and

Table II The moral evaluations of the selection index which has been found most effective

$$\begin{aligned}
 & \left[\frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} + \frac{\partial^2}{\partial z^2} \right] \psi(x, y, z) \\
 &= -\frac{E}{\hbar^2 c^2} \psi(x, y, z) \\
 & \left(\frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} + \frac{\partial^2}{\partial z^2} \right) \psi(x, y, z) \\
 &= -\frac{E}{\hbar^2 c^2} \psi(x, y, z) \\
 & \left(\frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} + \frac{\partial^2}{\partial z^2} \right) \psi(x, y, z) \\
 &= -\frac{E}{\hbar^2 c^2} \psi(x, y, z) \\
 & \left(\frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} + \frac{\partial^2}{\partial z^2} \right) \psi(x, y, z) \\
 &= -\frac{E}{\hbar^2 c^2} \psi(x, y, z)
 \end{aligned}$$

Multiplying the inverse with the vector on R.H.S. and are given below:

$$D_1 = \sigma_y b_y^2 + A \left[C_1 (1 - P_{2y}^2 - P_{2y} P_{1y}) + P_{12} (P_{2y}^2 - C_2^2) - P_{1y} \right] / D_4.$$

$$D_2 = \sigma_y b_y^2 + A \left[C_2 (1 - P_{1y}^2 - P_{1y} P_{2y}) + P_{12} (P_{1y}^2 - C_1^2) - P_{2y} \right] / D_4.$$

$$D_3 = b_y^2 + A \left[1 - P_{12}^2 + C_1 (P_{12} P_{2y} - P_{1y}) + C_2 (P_{12} P_{1y} - P_{2y}) \right] / D_4.$$

$$D_4 = 2B_1 b_y^2 \left[2Mm b_y^2 \right] / D_4.$$

$$D_5 = B_1 b_y^2 \left[8M - m b_y^2 \right] / D_4.$$

where,

$$C_1 = r_{1g} b_{x1} / b_y$$

$$C_2 = r_{2g} b_{x2} / b_y$$

$$A = (16m b_y^2 - 4m b_y^2)$$

$$B = \left[(1 - b_y^2)(1 - P_{12}^2 - P_{1y}^2 - P_{2y}^2 + 2P_{1y} P_{2y} P_{12} + b_y^2) \right. \\ \left. + C_1^2 (P_{2y}^2 - 1) + C_2^2 (P_{1y}^2 - 1) + 2C_1 (2P_{1y} - 2P_{12} P_{2y} + P_{2y}^2) + \right. \\ \left. 2C_2 (2P_{2y} - 2P_{12} P_{1y} + P_{1y}^2) + 2C_1 C_2 (P_{12} - P_{1y} P_{2y}) \right] /$$

$$D_4 = (16m b_y^4 - m b_y^4 + m b_y^6 - 4m b_y^6) (1 - P_{12}^2) - \\ (P_{2y}^2 + P_{1y}^2) (16m b_y^4) + 2P_{12} P_{1y} b_y (16m b_y^4) + \\ b_y^6 (16m b_y^2 + 4m) \left[C_1^2 (P_{2y}^2 - 1) + C_2^2 (P_{1y}^2 - 1) + 2C_1 (P_{1y} - P_{12} P_{2y}) \right. \\ \left. + 2C_2 (P_{2y} - P_{12} P_{1y}) + 2C_1 C_2 (P_{12} - P_{1y} P_{2y}) \right] / B$$

Now, the correlation between the index I_h and G_y i.e.
 $EC_y I_h$ in terms of b_i 's are given as follow

$$EC_y I_h = \sqrt{r_{g1} b_1 + r_{g2} b_2 + b_3 + \frac{1}{2} b_4 + \frac{1}{2} b_5}$$

and the relative efficiency of the index I_h over the individual performance of the trait y is given by

$$E_h = EC_y I_h / h_y .$$

Now, if we consider the case of K auxiliary variate besides the variate Y under improvement, then our index will be

$$I = b_1 X_1 + b_2 X_2 + \dots + b_K X_K + b_{K+1} \bar{Y}_1 + b_{K+2} \bar{Y}_2 + b_{K+3} \bar{Y}_3$$

The normal equations for estimating the b_i 's after maximising the correlation between G_y and I with reference to b_i 's are as follows

$$\sum X_1 G_y = b_1 \sum X_1^2 + b_2 \sum X_1 X_2 + \dots + b_K \sum X_1 X_K + b_{K+1} \sum X_1 \bar{Y}_1 + b_{K+2} \sum X_1 \bar{Y}_2 + b_{K+3} \sum X_1 \bar{Y}_3$$

$$\sum X_2 G_y = b_1 \sum X_1 X_2 + b_2 \sum X_2^2 + \dots + b_K \sum X_2 X_K + b_{K+1} \sum X_2 \bar{Y}_1 + b_{K+2} \sum X_2 \bar{Y}_2 + b_{K+3} \sum X_2 \bar{Y}_3$$

$$\vdots \quad \vdots \quad \vdots \quad \vdots \quad \vdots \quad \vdots \quad \vdots \quad \vdots$$

$$\sum X_K G_y = b_1 \sum X_1 X_K + b_2 \sum X_2 X_K + \dots + b_K \sum X_K^2 + b_{K+1} \sum X_K \bar{Y}_1 + b_{K+2} \sum X_K \bar{Y}_2 + b_{K+3} \sum X_K \bar{Y}_3$$

$$\sum Y_1 G_y = b_1 \sum X_1 \bar{Y}_1 + b_2 \sum X_2 \bar{Y}_1 + \dots + b_K \sum X_K \bar{Y}_1 + b_{K+1} \sum \bar{Y}_1^2 + b_{K+2} \sum \bar{Y}_1 \bar{Y}_2 + b_{K+3} \sum \bar{Y}_1 \bar{Y}_3$$

$$\sum Y_2 G_y = b_1 \sum X_1 \bar{Y}_2 + b_2 \sum X_2 \bar{Y}_2 + \dots + b_K \sum X_K \bar{Y}_2 + b_{K+1} \sum \bar{Y}_1 \bar{Y}_2 + b_{K+2} \sum \bar{Y}_2^2 + b_{K+3} \sum \bar{Y}_2 \bar{Y}_3$$

$$\sum Y_3 G_y = b_1 \sum X_1 \bar{Y}_3 + b_2 \sum X_2 \bar{Y}_3 + \dots + b_K \sum X_K \bar{Y}_3 + b_{K+1} \sum \bar{Y}_1 \bar{Y}_3 + b_{K+2} \sum \bar{Y}_2 \bar{Y}_3 + b_{K+3} \sum \bar{Y}_3^2$$

Now, by using the Biometrie correlation and phenotypic variances, we can write the normal equations in terms of genetic parameters as given in Table 'C'.

Now, for the sake of simplicity we can write the normal equations given in Table C by partitioning the matrix into two parts (i) due to the X_1 's the auxiliary variates and (ii) due to the X_2 's the character under improvement and are denoted as follow

$$\begin{bmatrix} z_1 \\ z_2 \end{bmatrix} = \begin{bmatrix} \sum_{xx} & \sum_{xy} \\ \sum_{yx} & \sum_{yy} \end{bmatrix} \begin{bmatrix} b_x \\ b_y \end{bmatrix}'$$

and the Index I as

$$I = [b_x \ b_y] [x \ x]'$$

Now, the b_x and b_y can be estimated by inverting the above matrix and multiplying the corresponding inverse by vector Z

$$\begin{bmatrix} b_x \\ b_y \end{bmatrix} = \begin{bmatrix} \sum_{xx} & \sum_{xy} \\ \sum_{yx} & \sum_{yy} \end{bmatrix}^{-1} \begin{bmatrix} z_1 \\ z_2 \end{bmatrix}'$$

where,

$$\sum_{xx}^{-1} = \sum_{xx}^{-1} + \sum_{xy}^{-1} \sum_{yy} (\sum_{yy} - \sum_{yx} \sum_{xy}^{-1} \sum_{yy})^{-1} \sum_{yx} \sum_{xx}^{-1}$$

$$\sum_{xy}^{-1} = -\sum_{xx}^{-1} \sum_{xy} (\sum_{yy} - \sum_{yx} \sum_{xx}^{-1} \sum_{xy})^{-1}$$

$$\sum_{yy}^{-1} = (\sum_{yy} - \sum_{yx} \sum_{xx}^{-1} \sum_{xy})^{-1}$$

$$\sum_{yx} = (\sum_{xy})^{-1}$$

The correlation between G_y and I can also be viewed as the multiple correlation coefficient between the genotype G_y and the mutually independent parts X_1, X_2, \dots, X_k and \bar{Y}_1, \bar{Y}_2 and \bar{Y}_3 .

$$R_{G_y I}^2 = \sum_{i=1}^k b_i \text{Cov}(X_i, G_y) + b_{k+1} \text{Cov}(G_y, \bar{Y}_1) + b_{k+2} \text{Cov}(G_y, \bar{Y}_2) + b_{k+3} \text{Cov}(G_y, \bar{Y}_3) - J/\sigma_A^2$$

On putting the values of covariances in terms of genetic parameters the above expressions will reduce to the form given below:

$$R_{G_y I}^2 = b_1 r_{gx_1} + b_2 r_{gx_2} + \dots + b_k r_{gx_k} + b_{k+1} + b_{k+2} + b_{k+3}$$

or in terms of matrix notation

$$R_{G_y I}^2 = [b_x \ b_y] [I \ r_{gx}] + [c]$$

where vector $c = [1, \bar{y}, \bar{J}]$

The relative efficiency of I over the individual performance of X is given as follow

$$E_X = R_{G_y I} / b_y$$

whereas, the relatives efficiency of index I over the index I^* can be given as follow

$$E_I = R_{G_y I} / R_{G_y I^*}$$

Table C: The normal equations of the selection index with k auxiliary traits in matrix notation

$r_{1g} h_{x1}^2 y \sigma_{x1}$	$\frac{2}{\sigma_{x1}^2}$	$\frac{r_{12} h_{x1} h_{x2}}{\sigma_y^2}$	\dots	$\frac{r_{1k} h_{xk} h_{x1}}{\sigma_y^2}$	$\frac{c_{1y} \sigma_{x1}}{\sigma_y}$	$\frac{t r_{1g} h_{x1} h_{x1}}{\sigma_y}$	$\frac{t r_{1g} h_{x1} h_{x1}}{\sigma_y}$	b_1
$r_{2g} h_{x2}^2 y \sigma_{x2}$	$\frac{0}{\sigma_y}$	$\frac{r_{12} h_{x1} h_{x2}}{\sigma_y^2}$	\dots	$\frac{r_{2k} h_{xk} h_{x2}}{\sigma_y^2}$	$\frac{c_{2y} \sigma_{x2}}{\sigma_y}$	$\frac{t r_{2g} h_{x2} h_{x2}}{\sigma_y}$	$\frac{t r_{2g} h_{x2} h_{x2}}{\sigma_y}$	b_2
\vdots	\vdots	\vdots	\vdots	\vdots	\vdots	\vdots	\vdots	\vdots
$r_{kg} h_{xk}^2 y \sigma_{xk}$	$\frac{0}{\sigma_y}$	$\frac{r_{12} h_{x1} h_{x2}}{\sigma_y^2}$	\dots	$\frac{r_{kg} h_{xk} h_{xk}}{\sigma_y^2}$	$\frac{c_{ky} \sigma_{xk}}{\sigma_y}$	$\frac{t r_{kg} h_{xk} h_{xk}}{\sigma_y}$	$\frac{t r_{kg} h_{xk} h_{xk}}{\sigma_y}$	b_k
h_y^2	$\frac{2}{\sigma_y^2}$	$\frac{0}{\sigma_y^2}$	\dots	$\frac{0}{\sigma_y^2}$	0	$\frac{t h_y^2}{\sigma_y}$	$\frac{t h_y^2}{\sigma_y}$	b_{k+1}
$t h_y^2$	$\frac{0}{\sigma_y^2}$	$\frac{c_{1y} \sigma_{x1}}{\sigma_y}$	\dots	$\frac{c_{ky} \sigma_{xk}}{\sigma_y}$	1	$\frac{t h_y^2}{\sigma_y}$	$\frac{t h_y^2}{\sigma_y}$	b_{k+2}
$\frac{t h_y^2}{\sigma_y^2}$	$\frac{t r_{1g} h_{x1} h_{x1}}{\sigma_y}$	$\frac{t r_{2g} h_{x2} h_{x2}}{\sigma_y}$	\dots	$\frac{t r_{kg} h_{xk} h_{xk}}{\sigma_y}$	$\frac{t h_y^2}{\sigma_y}$	$\frac{1/H}{\sigma_y}$	$\frac{t h_y^2}{\sigma_y}$	b_{k+3}
$(k+3)x1$	$\frac{0}{\sigma_y}$	$\frac{0}{\sigma_y}$	\dots	$\frac{0}{\sigma_y}$	$\frac{t h_y^2}{\sigma_y}$	$\frac{t h_y^2}{\sigma_y}$	$\frac{1/H}{\sigma_y}$	$(k+3)(k+3)$

CHAPTER - IV

EFFECT OF INCLUSION OF AUXILIARY TRAITS ON THE VIGILANCE OF THE INDEX

4.1 Nature and extent of data

The data collected under the I.A.R.S. Project entitled "statistical methodology for developing efficient selection procedure in poultry breeding" from regional Poultry Farm Bhopal during 1970 was used for this study.

The foundation stock at State Poultry Farm at Bhopal was originally derived from Regional Poultry Farm, Bombay. The mating were made in two batches of 50 sires. Within each batch a sire was mated to 12 dams and from each mating 7 to 8 batches were taken out. These batches were subsequently raised at 6 to 7 locations of which two were at the Poultry Farm at Bhopal. For propagating the next generation, however, birds raised at Bhopal were only considered. The records on age at first egg, average egg weight, total egg production upto 240 days of age and incidence of mortality were collected. The rate of lay was expressed as total egg production upto 240 days of age divided by 240 minus the age at first egg.

The use of early post-production records as a selection criterion for improving annual egg production was proposed by Denster and Lerner (1947). When they emphasised the selection on the basis of part egg production records, the genetic correlation between post and full year production was reported to be high (about 0.75). So in the present study the post-production records are used and are economic than the full year

records.

4.2 Adjustment of data for Non-Genetic Factors

Since the CGG production is a character largely affected by environmental differences it was deemed necessary to adjust the data for non-genetic factors such as the date of hatch, location and maternal effects. Since the data was not sufficient for correcting all the three factors i.e. location, batch and dam effect. Therefore the data were adjusted for location and dams for each sire separately. For this purpose, the method of fitting constants given by Harvey (1960) was used.

As the disproportionate number of offsprings exist for each dam and for each location. The model used for two-way classification assuming no interaction between dam's and location's effect is as follows

$$Y_{ijk} = u + a_i + b_j + \epsilon_{ijk}$$
$$i = 1, 2, \dots, d; j = 1, 2, \dots, k$$
$$k = 1, 2, \dots, n_{ij}$$

where

- Y_{ijk} = the k -th observation of j -th location & i -th dam.
u = the overall mean common to all individuals
 a_i = the effect of the i -th dam
 b_j = the effect of the j -th location
 ϵ_{ijk} = random error distributed normally around zero and a constant variance σ^2 .

and

In the present case the fixed effect model has been assumed and each of the terms on the right hand side of the equation (1) is population parameters of constants are to be estimated. Estimates of these constants are denoted by $\hat{\mu}$, \hat{a}_j , \hat{b}_j and $\hat{\sigma}^2_0$.

Applying the least square principle the normal equations for two-way classification are as follow

$$\bar{y}_{..} = \bar{n}_{..} \hat{\mu} + \sum_j n_{1j} \hat{a}_j + \sum_j n_{-j} \hat{b}_j = Y_{..}$$

$$\bar{a}_j = \bar{n}_{1j} \hat{\mu} + n_{1j} \hat{a}_j + \sum_l n_{1lj} \hat{b}_j = Y_{1j}$$

$$\bar{b}_j = \bar{n}_{-j} \hat{\mu} + \sum_l n_{-lj} \hat{a}_j + n_{-j} \hat{b}_j = Y_{-j}$$

It should be noted that the sum of coefficients of the \hat{a}_j in the $\hat{\mu}$ equation equals to the sum of coefficients for \hat{b}_j and the coefficients for $\hat{\mu}$. In addition, the sum of the coefficients for the \hat{b}_j in an a_j equation equals the coefficients for the \hat{a}_j and the total of the right hand terms for the \hat{a}_j equation and the b_j equations equals the grand total of y_{1jk} . To, in order to solve the direct equations, it is necessary to impose certain restrictions on a_j 's and b_j 's.

A common restriction on these equations is to be imposed to solve them. The generally use restriction is to take $\sum \hat{a}_l = 0 = \sum \hat{b}_j$ and to make necessary subtractions before inversion. The constants obtained from the direct solution of the reduced matrix was in desired form, in most cases.

is for example, if we impose the restriction $\hat{b}_g = \hat{b}_d = 0$, $\hat{\mu}$ are estimated as $\hat{\mu} + \hat{a}_g + \hat{b}_g$ instead of a_g the \hat{a}_g and \hat{b}_g are estimated as $\hat{a}_g - \hat{a}_d$ and $\hat{b}_g - \hat{b}_d$ instead of \hat{a}_g and \hat{b}_d .

The reduced set of least square equations was solved directly to obtain estimates of constants a_g, b_g and $\hat{\mu}$. The \hat{a}_g and \hat{b}_g constants was obtained from

$$\hat{a}_g = -\sum \hat{a}_j$$

$$\hat{b}_g = -\sum \hat{b}_j$$

The total reduction in sum of squares is

$$R(\mu, a_g, b_g) = \hat{\mu} Y_{..} + \sum_{i=1}^{d-1} \hat{a}_i (Y_{i..} - Y_{d..}) + \sum_{j=1}^{g-1} \hat{b}_j (Y_{..j} - Y_{..g})$$

and the error sum of squares is equal to

$$\sum_i \sum_j \sum_k Y_{ijk}^2 - R(\mu, a_g, b_g)$$

Then the least square S.S. for testing significance of differences among M (days) class can most easily be computed from

$$B(\mu, a_g, b_g) = R(\mu, b_g)$$

$$\text{where, } B(\mu, b_g) = \frac{Y_{..g}^2}{n_{..g}}$$

is "between" uncorrected S.S. for L (locations). Similarly, for B classification

$$B(\mu, a_g) = R(\mu, a_g)$$

$$\text{where, } M(p, a_1) = \frac{1}{2} \sum_{i=1}^2 \frac{x_i}{n_{i*}}$$

to "between" uncorrected S.S. for a classification,
so, the final Analysis of Variance Table for testing
the ram's and location effects is as follow

ANOVA FOR TESTED DAIS AND LOCATION EFFECTS

source	d.f.	S.S.	S.(D.F.)	F
Daia	G-1	A	$k_1(\bar{a}_1^2 + \sigma_e^2)$	N/S
Locations	L-1	B	$k_2(\bar{a}_2^2 + \sigma_e^2)$	R/E
error	(L-G)(G-1)	E	σ_e^2	
Total	L-1			

$$\text{where } k_1 = \frac{1}{G-1} \left[a_{..} - \sum_j \frac{a_{1j}^2}{n_{1j}} \right]$$

$$k_2 = \frac{1}{G-1} \left[a_{..} - \sum_j \frac{a_{2j}^2}{n_{2j}} \right]$$

The significance of F indicates whether the particular effect is influencing the character under study or not.

The data on the performance of foundation stock in regard to egg production upto 240 days of age, age at first egg and egg weight already on punched cards were processed on the computer at the I.I.S.R.I. The data included only those daughters which were reared at Ibro-al.

Since the egg production does not take into account the differences in ages of the birds when they lay the eggs,

it was thought desirable to include the rate of lay under present study besides the other three characters. Using two characters; egg production upto 240 days as well as age at first egg, the rate of lay for a bird was expressed as

$$\text{Rate of Lay} = \frac{\text{Egg production upto 240 days}}{240 - \text{Age at first egg.}}$$

As it is a well known fact that locations and dams constitute an important source of environmental variation in the economic traits of poultry. These two effects were tested for both the sets of sires for each of the four characters under study and for each sire separately by using above mentioned Harvey's technique. It was found that the location and dam effect was affecting the age at first egg (X_1); egg production upto 240 days (X_2); egg weight (X_3) and the rate of lay (X_4) considerably in most of the cases.

4.3 Estimation of Genetic Parameters by Half-sib Analysis

The adjusted data were used for estimation of heritability, genotypic and phenotypic correlations for different traits under study.

Each of the 6-sires is mated to a set of dams which constitutes a random sample of female population and each dam produces a large number of offsprings. This method requires only the measurement on the offspring for each sire.

Let there be d_1 daughters per sire with " = $\sum_{i=1}^S d_i$ " as the total number of daughters. The random effect model used for the analysis is as follow

$$Y_{ij} = \mu + a_i + e_{ij}$$

where

Y_{ij} is the adjusted observation for j -th daughter of the sire i -th sire.

a_i is the effect of the i -th sire randomly distributed with σ_a^2 and e_{ij} is the residual error with variance σ_e^2 .

Half-add ANOVA

source	d.f.	N.s. S.e.	(N.s. S.e.)
Between sires	S-1	1	$\frac{\sigma_a^2}{\sigma_e^2} + k \frac{\sigma_s^2}{\sigma_e^2}$
Progeny within sires	N-S	S	$\frac{\sigma_s^2}{\sigma_e^2}$
Total	S-1		

where,

$$k = \frac{1}{S} \left[\sum a_{..} - \frac{\sum a_{..}^2}{d_{..}} \right]$$

The total phenotypic variance observed in the trait under study were separated into different components assignable to different sources as outlined in Table.

The genetic model is given as follow

$$\hat{\sigma}_g^2 = \text{Cov}_{Hg} = \frac{1}{4} V_A + \frac{1}{15} V_{AA} + \frac{1}{64} V_{AAA} + \dots$$

$$\hat{\sigma}_e^2 = \sigma_T^2 - \text{Cov}_{Hg} = \frac{2}{3} V_A + V_D + 15/16 V_{AA} + \frac{3}{8} V_{AD} + \dots$$

$$.. + V_{DD} + 63/64 V_{AAA} + \dots$$

The interaction parts of $\hat{\sigma}_g^2$ was ignored, since only small fraction of it contributes to the covariance and so it's effect on resemblance between half-sibs is very small (Falconer, 1960). Therefore,

$$\hat{\sigma}_g^2 = \frac{1}{4} V_A \text{ and}$$

$$\hat{h}^2 = 4 \hat{\sigma}_g^2 / \hat{\sigma}_g^2 + \hat{\sigma}_e^2$$

The S.E. of heritability estimates was computed by Swinegar et al. (1964) and quoted by Dekar (1964), assuming normality of the distribution of intraclass correlation t is approximately given by

$$\text{S.E.}(\hat{h}^2) \approx 4 \sqrt{\frac{2(k-1)(1-t)^2 [14(k-1)t]}{k^2 (N-k)(s-1)}}^2$$

The correlation between breeding values (additive genetic effects) is genetic correlation (Falconer, 1960). The genetic correlation r_g is, therefore, the ratio of the additive genetic covariance of the trait X and Y to the geometric mean of the additive genetic variance of these traits, i.e.

$$r_g(X,Y) = \frac{\widehat{\text{Cov}}_A(X,Y)}{\sqrt{\widehat{V}_A(X) \widehat{V}_A(Y)}}$$

The procedure outlined by Becker (1975) was followed for the estimation of r_g . The statistical and genetical models are same as in the case of heritability estimates. The Analysis of Covariance is prepared for two traits under study is

ANOVA For Half-sib

Source	D.F.	F.C.P.	E.P.C.P.)
Within sires	s-1	A	$\text{Cov}_e(X,Y) + k \text{Cov}_A(X,Y)$
Between progeny Within sires	R-s	B	$\text{Cov}_e(X,Y)$
Total	N-1		

$\text{Cov}_e(X,Y) = \text{NCP}_e = \text{Error component of Covariance between } X \text{ and } Y$

$\text{Cov}_A(X,Y) = \frac{\text{NCP}_A - \text{NCP}_e}{k} = \text{Sire component.}$

The genetic composition of the NCP's are

$$\text{Cov}_A(XY) = \frac{1}{4} \text{Cov}_A + \frac{1}{16} \text{Cov}_{AA} + \frac{1}{64} \text{Cov}_{AAA} + \dots$$

Thus, the sire component of the Cov is mainly additive genetic covariance and interactions can be ignored

$$r_g(X,Y) = \frac{\text{Cov}_A(X,Y)}{\sqrt{\widehat{V}_A(X) \widehat{V}_A(Y)}}$$

since, estimates of genetic correlation is subjected to varying amounts of sampling error, the reliability of r_g

is limited without an estimate of S.E. Robertson (1959) have given the following formulae for S.E. (r_g).

$$\text{S.E. } [r_g(X,Y)] = \frac{\sum_{i=1}^n r_g}{\sqrt{n}} \quad \sqrt{\frac{\text{S.E.}(h_x^2) \text{ S.E.}(h_y^2)}{h_x^2 h_y^2}}$$

$$\hat{r}_p(X,Y) = \frac{\widehat{\text{Cov}}_A(X,Y) + \widehat{\text{Cov}}_o(X,Y)}{\sqrt{[\widehat{v}_A(X) + \widehat{v}_B(X)] [\widehat{v}_A(Y) + \widehat{v}_B(Y)]}}$$

$$\text{S.E. } (r_p) = \sqrt{1 - r_p^2} / \sqrt{n-1}$$

Following the above method the adjusted data were analysed to estimate heritabilities, correlations for all the four traits under investigation along with their standard error for both the set's of sires separately. The results of the analysis are presented in Tables 1 and 2. The estimates of heritabilities for X_1 , X_2 and X_3 are slightly higher in Set I as compared to set II. The estimate of h^2 for rate of lay is 0.261 and 0.179 in set I and II respectively. The estimate of heritability was found to be slightly on the higher side when compared with the estimate of heritability by Narain et al (1973). The increase in heritability may be due to the more accurate correction of the data for location and dams effect for the each sire separately.

The rate of lay is found to be negatively correlated both genotypically and phenotypically with age at first

egg and egg weight, whereas positively correlated with egg production upto 240 days of age in both the sets of data. These estimates of heritabilities, genotypic and phenotypic correlations were used for working out the coefficient of different selection indices used.

4.4 Different Selection Indices used

1) The Index given by Osborne (1954) is used to predict the breeding value (q_y) of the individual for the character Y under improvement. This selection procedure combines in an efficient manner individual's own performance (p_y), the average of the phenotypic values of n paternal half-sibs (\bar{Y}_y) and average of the phenotypic values of n full-sibs for Y (\bar{F}_y). The index is given as follows

$$I_y = b_1 p_y + b_2 \bar{Y}_y + b_3 \bar{F}_y -$$

where the b_i 's are obtained by maximizing the correlation between q_y and the phenotypic Index I_y .

These are given below:

$$\begin{aligned} b_1 &= h_y^2 \left[16 - 41 h_y^2 - 11 h_y^2 \right] / D_1 \\ b_2 &= 2h_y^2 \left[21 - 11 h_y^2 - 3h_y^2 + 11 h_y^2 \right] / D_1 \\ b_3 &= h_y^2 \left[8 - 11 h_y^2 - 8h_y^2 + 11 h_y^2 \right] / D_1 \\ D_1 &= \left[16 + h_y^6 - 11 h_y^4 - 4h_y^4 - 11 h_y^2 \right] \end{aligned}$$

where

$$\bar{H} = \frac{H}{1+(n-1) h_y^2 / 4}$$

$$H = \frac{m}{1 + (n-1) \frac{h_y^2}{2}}$$

ii) One of the indices given by Morein et al (1977) is a modification over Osborne's index. It predicts the C_y by combining in an optimal manner, in addition to the information included in I_1 , the information supplied by a trait correlated with the trait under improvement. This index is given by

$$I_2 = b_1 P_x + b_2 P_y + b_3 \bar{H}_y + b_4 \bar{D}_y .$$

where the b_i 's are different from the above b_i 's and are similarly obtained by maximizing the correlation between C_y and I_2 . These are as follow

$$b_1 = \sigma_y h_y^2 (C - P) \text{ or } (\sigma_x^2 - D_2)$$

$$b_2 = h_y^2 (1 - C/P) C/D_2$$

$$b_3 = 2\bar{H} h_y^2 (2 - M_y^2) / D_2$$

$$b_4 = \bar{H} h_y^2 (8M_y^2) / D_2$$

where

where

$$D_2 = 16MB \frac{h_y^6}{y} - 16B \frac{h_y^4}{y} - 4M \frac{h_y^4}{y} - 16 \frac{h_y^4}{y}$$

$$+ Ch_y^2 (C-2P) (MB \frac{h_y^2}{y} - BM - E)$$

$$- P^2 (16-MB \frac{h_y^2}{y})$$

$$\pi = (16-4Ch_y^2 - MB \frac{h_y^2}{y})$$

$$T = (1-h_y^2) (1-P^2) - h_y^2 (C-P)^2$$

$$C = r_g b_x / b_y$$

σ_x = phenotypic standard deviation for X

σ_Y = phenotypic standard deviation for Y

P and r_g are phenotypic and genotypic correlations respectively.

The correlation between q_y and I_2 is given by

$$r_{q_y I_2} = \sqrt{r_g b_y + D_2 + \frac{1}{2} b_3 + \frac{1}{2} b_y \frac{1}{T}}$$

(iii) The second Index given in Karun et al (1977) includes another auxiliary character in the modified version of the Osborne's index given in Karubiran et al (1973). This index predicts the genotypic value of Y (q_y) by combining in an efficient manner the information supplied by the average performance of dams (E_y) in addition to the informations used in I_2 . This index is given by

$$I_3 = b_1 P_x + b_2 P_y + b_3 \bar{E}_y + b_4 \bar{E}_y + b_5 - D_y$$

The b_i 's are all different from the above indices and are obtained in similar fashion. These are

$$b_1 = \sigma_y^2 b_y^2 (C - \rho) (S - 4b_y^2) / (\sigma_x^2 D_3)$$

$$b_2 = b_y^2 (1 - \rho^2) (S - 4b_y^2) / D_3$$

$$b_3 = 2b_y^2 (2 - 4b_y^2) T / D_3$$

$$b_4 = 4b_y^2 (8 - 16b_y^2 - 4b_y^4) T / D_3$$

$$b_5 = 4b_y^2 (2 + 2b_y^2) T / D_3$$

$$\begin{aligned} D_3 = & 16481 b_y^6 + 161 b_y^6 - 121 b_y^4 - 811 b_y^4 - 4b_y^4 - 11b_y^4 \\ & + (b_y^4 (C - 2\rho) (11b_y^2 + 4b_y^2 - 10 - 4)) \\ & - \rho^2 (16 - 11b_y^4 - 4b_y^4) \end{aligned}$$

The correlation between \bar{Q}_y and I_3 is then given by

$$R_{Q_y I_3} = [r_g b_1 + b_2 + \frac{1}{4} b_3 + \frac{1}{2} b_4 + \frac{1}{2} b_5]^{\frac{1}{2}}$$

iv) The index with more than one auxiliary traits besides all the other traits in Osborne's index (I_4) were studied in the present investigation. This index is given by

$$I_4 = b_1 P X_1 + \frac{1}{2} b_2 P X_2 + b_3 P_y + b_4 \bar{P}_y + b_5 \bar{P}_y$$

In addition to this another index with three auxiliary traits has been used in the present analysis and is given by

$$I_5 = b_1 P_{X_1} + b_2 P_{X_2} + b_3 P_{X_3} + b_4 P_y + b_5 \bar{P}_y + b_6 \bar{\bar{P}}_y .$$

The b_i 's of these indices are already presented in Chapter III in great detail. The correlation between G_y and the I_4 and I_5 are given by

$$R_{G_y} I_4 = [r_{g1} b_1 + r_{g2} b_2 + r_{g3} b_3 + r_{g4} b_4 + r_{g5} b_5]^{\frac{1}{2}}$$

and

$$R_{G_y} I_5 = [r_{g1} b_1 + r_{g2} b_2 + r_{g3} b_3 + b_4 + b_5 + b_6]^{\frac{1}{2}}$$

Now, for comparing the relative efficiency of any of the index say I_4 over the Osborne's Index (I_4') can be given by the formula

$$E_{I_4} = R_{G_y} I_4 / R_{G_y} I_4'$$

4.5 As in the present study the data with three auxiliary traits i.e. age at first egg (X_1); egg production upto 240 days (X_2) and the egg weight (X_3) were available, all the seven possible combinations of these traits along with Osborne's index for rate of lay (Y) was studied numerically. So, in all the following eight indices were studied:

a) $b_1 P_{X_1} + b_2 \bar{P}_y + b_3 \bar{\bar{P}}_y$

- b) $b_1 PX_1 + b_2 PY + b_3 \bar{H}_y + b_4 \bar{P}_y$
- c) $b_1 PX_2 + b_2 PY + b_3 \bar{H}_y + b_4 \bar{P}_y$
- d) $b_1 PX_3 + b_2 PY + b_3 \bar{H}_y + b_4 \bar{P}_y$
- e) $b_1 PX_1 + b_2 PX_2 + b_3 PY + b_4 \bar{H}_y + b_5 \bar{P}_y$
- f) $b_1 PX_1 + b_2 PX_3 + b_3 PY + b_4 \bar{H}_y + b_5 \bar{P}_y$
- g) $b_1 PX_2 + b_2 PX_3 + b_3 PY + b_4 \bar{H}_y + b_5 \bar{P}_y$
- h) $b_1 PX_1 + b_2 PX_2 + b_3 PX_3 + b_4 PY + b_5 \bar{H}_y + b_6 \bar{P}_y$

Besides this, the above eight indices were also studied after the inclusion of individual's dam performance (D_y). The increase in efficiency of these indices with auxiliary traits will depend on the number of auxiliary traits, the heritabilities of the traits and on the size and signs of genotypic as well as phenotypic correlations between the pairs of traits.

4.6 Regression Coefficients and Relative efficiency of Selection Indices

It is clear from the previous sections that the coefficients of the indices are functions of $h_x^2 + h_y^2 + r_{xy}$, r_p , $\sigma_x^2 + \sigma_y^2$, \square and n . In order to study the effect of including the different auxiliary traits in addition to the individual performance for Y (rate of lay); full-sib family average for y and half-sib family average for Y, with and without the average performance of individual's dam, the regression coefficient t of indices with different

combinations of auxiliary traits i.e. age at first egg, egg production upto 240 days and egg weight were worked out along with their relative efficiencies over the Osborne's index for both the sets of data and are presented in Tables 3 and 4. These tables show that the regression coefficient for the egg weight (X_3) is constant. For all the indices in each set of data the coefficient for the other two auxiliary traits vary slightly with the change in combinations of auxiliary traits in the index and the same is true for the rate of lay, the half-sib family average, full-sib family average and for individuals dam's performance.

The inclusion of average performance of individual's dam (D_y) had slightly increased the efficiency in all eight indices under study for both the sets of data. The inclusion of the auxiliary traits in the selection index had lowered the efficiency over the Osborne's Index in almost all the cases in both sets of data. Tables 3b and 4b show that in set II the efficiency of Index with all the three auxiliary traits and with two auxiliary traits i.e. egg production upto 240 days and the age at first egg is more than one. Whereas in the rest of the cases of Set II the efficiency is very close to one. Tables 3a and 4a show that in set I the efficiency of the index with one auxiliary trait i.e. age at first egg was very close to one and the same is true for the index with egg weight as auxiliary trait. Whereas in most of the cases

of set I the efficiency is considerably lower than one. The differences in the results of two sets are due to differing values of the parametric estimates. The relative efficiency of all the indices worked out for Set II is uniformly greater than the corresponding indices of Set I and this difference can be attributed to the differences in estimates of heritability which are 0.261 for Set I and 0.179 for Set II. The decrease in efficiency of modified Osborne's Index (with one and more auxiliary traits) over the Osborne's index is due to the same signs of genotypic and phenotypic correlations between the auxiliary traits and the trait under improvement.

In spite of lot of variation in the results of set I and II the inclusion of auxiliary traits having genotypic and phenotypic correlation with opposite signs will increase the efficiency of these modified Osborne's index. The other traits correlated with rate of lay satisfying the above condition can be tried for getting rapid genetic improvement in the rate of lay by using the modified Osborne's index i.e. with one or more auxiliary traits.

The analysis of data collected under the IASRI Project entitled "Statistical methodology for developing efficient selection procedure in poultry breeding" from Regional Poultry Farm at Hopal during the laying period 1970 was conducted with a view to study the efficiency of different selection indices with one and more auxiliary traits (modified Osborne's index) over the Osborne's index. The data was first adjusted for dams and locations effects for each sire separately. The adjusted data was finally used to work out the various genetic parameters such as heritabilities, genotypic and phenotypic correlations. Using these genetic parameters the coefficients of the selection indices along with their efficiency over the Osborne's index was worked out.

The regression coefficient of the egg weight was found constant for all the indices in each sets of the data, but it was varying slightly for the other two auxiliary traits i.e. age at first egg and egg production upto 240 days. The regression coefficients for the character under improvement i.e. rate of lay (Y); half sib-family average for Y, full-sib family average for Y and the individual dam performance for Y was also found changing with change in the combinations of auxiliary traits in the index.

The relative efficiency of modified Osborne's index for rate of lay with various combinations of three auxiliary traits i.e. age at first egg, egg production upto 240 days and the egg weight over the Osborne's index was found less than one in majority of the cases. But the efficiency of the index with all the three auxiliary traits and with two auxiliary traits i.e. age at first egg and egg production upto 240 days was found more than one in one of the two sets of data under investigation. The all selection indices with individuals dams performance was having slightly greater efficiency as compared to the indices without the dam's performance.

Besides, this the graphical analysis of selection index with one auxiliary trait (modified Osborne's index) and with and without inclusion of individuals dams performance was also conducted and the relative efficiency of the c indices over the individuals selection was calculated for various combinations of genotypic (r_g) and phenotypic (r_p) correlations. The graphs for different combination of r_g and r_p was drawn and the efficiency was found considerably higher than one in all the cases under study.

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Table 1 Genotypic and Phenotypic Correlations Estimated by Half-sib Method for Set-I

x_1	x_2	x_3	x_4
0.502 ± 0.109	-0.799 ± 0.064	0.383 ± 0.127	-0.276 ± 0.158
$\sum 16,1838$			
-0.263 ± 0.029	0.503 ± 0.109	-0.547 ± 0.102	0.738 ± 0.080
	$\sum 16,1833$		
0.080 ± 0.024	-0.117 ± 0.029	0.833 ± 0.158	0.620 ± 0.101
		$\sum 45, 1485$	
-0.0058 ± 0.025	0.746 ± 0.006	-0.103 ± 0.029	0.31 ± 0.070
			$\sum 46,1838$

x_1 = age at first egg; x_2 = EGG production upto 240 days; $-x_3$ = egg weight x_4 = ratio of lay

* Diagonals are heritability estimates; above diagonals are genotypic correlation (r_g) and below diagonals are phenotypic correlations (r_p).

** In parameters the first is the number of sires and second is the total no. of observations.

Table 26. Mean total wind parameters during the winter months at different stations

	\bar{X}_1	\bar{X}_2	\bar{X}_3	\bar{X}_4	\bar{X}_5	\bar{X}_6	\bar{X}_7	\bar{X}_8	\bar{X}_9	\bar{X}_{10}	\bar{X}_{11}	\bar{X}_{12}
\bar{X}_1	0.425 \pm 0.079	0.405 \pm 0.039	0.480 \pm 0.039	0.425 \pm 0.079	0.396 \pm 0.063	0.324 \pm 0.063	0.396 \pm 0.063	0.304 \pm 0.138	0.650 \pm 0.038	0.650 \pm 0.038	0.687 \pm 0.038	0.687 \pm 0.038
\bar{X}_2	-0.014 \pm 0.136	-0.033 \pm 0.139	-0.033 \pm 0.139	-0.014 \pm 0.136	-0.033 \pm 0.139	-0.033 \pm 0.139	-0.033 \pm 0.139	-0.004 \pm 0.138	0.650 \pm 0.038	0.650 \pm 0.038	0.687 \pm 0.038	0.687 \pm 0.038
\bar{X}_3	0.017 \pm 0.021	-0.038 \pm 0.021	-0.038 \pm 0.021	0.017 \pm 0.021	-0.038 \pm 0.021	-0.038 \pm 0.021	-0.038 \pm 0.021	0.004 \pm 0.138	0.650 \pm 0.038	0.650 \pm 0.038	0.687 \pm 0.038	0.687 \pm 0.038
\bar{X}_4	0.019 \pm 0.021	-0.038 \pm 0.021	-0.038 \pm 0.021	0.019 \pm 0.021	-0.038 \pm 0.021	-0.038 \pm 0.021	-0.038 \pm 0.021	0.004 \pm 0.138	0.650 \pm 0.038	0.650 \pm 0.038	0.687 \pm 0.038	0.687 \pm 0.038
\bar{X}_5	0.019 \pm 0.021	-0.038 \pm 0.021	-0.038 \pm 0.021	0.019 \pm 0.021	-0.038 \pm 0.021	-0.038 \pm 0.021	-0.038 \pm 0.021	0.004 \pm 0.138	0.650 \pm 0.038	0.650 \pm 0.038	0.687 \pm 0.038	0.687 \pm 0.038
\bar{X}_6	0.019 \pm 0.021	-0.038 \pm 0.021	-0.038 \pm 0.021	0.019 \pm 0.021	-0.038 \pm 0.021	-0.038 \pm 0.021	-0.038 \pm 0.021	0.004 \pm 0.138	0.650 \pm 0.038	0.650 \pm 0.038	0.687 \pm 0.038	0.687 \pm 0.038
\bar{X}_7	0.019 \pm 0.021	-0.038 \pm 0.021	-0.038 \pm 0.021	0.019 \pm 0.021	-0.038 \pm 0.021	-0.038 \pm 0.021	-0.038 \pm 0.021	0.004 \pm 0.138	0.650 \pm 0.038	0.650 \pm 0.038	0.687 \pm 0.038	0.687 \pm 0.038
\bar{X}_8	0.019 \pm 0.021	-0.038 \pm 0.021	-0.038 \pm 0.021	0.019 \pm 0.021	-0.038 \pm 0.021	-0.038 \pm 0.021	-0.038 \pm 0.021	0.004 \pm 0.138	0.650 \pm 0.038	0.650 \pm 0.038	0.687 \pm 0.038	0.687 \pm 0.038
\bar{X}_9	0.019 \pm 0.021	-0.038 \pm 0.021	-0.038 \pm 0.021	0.019 \pm 0.021	-0.038 \pm 0.021	-0.038 \pm 0.021	-0.038 \pm 0.021	0.004 \pm 0.138	0.650 \pm 0.038	0.650 \pm 0.038	0.687 \pm 0.038	0.687 \pm 0.038
\bar{X}_{10}	0.019 \pm 0.021	-0.038 \pm 0.021	-0.038 \pm 0.021	0.019 \pm 0.021	-0.038 \pm 0.021	-0.038 \pm 0.021	-0.038 \pm 0.021	0.004 \pm 0.138	0.650 \pm 0.038	0.650 \pm 0.038	0.687 \pm 0.038	0.687 \pm 0.038
\bar{X}_{11}	0.019 \pm 0.021	-0.038 \pm 0.021	-0.038 \pm 0.021	0.019 \pm 0.021	-0.038 \pm 0.021	-0.038 \pm 0.021	-0.038 \pm 0.021	0.004 \pm 0.138	0.650 \pm 0.038	0.650 \pm 0.038	0.687 \pm 0.038	0.687 \pm 0.038
\bar{X}_{12}	0.019 \pm 0.021	-0.038 \pm 0.021	-0.038 \pm 0.021	0.019 \pm 0.021	-0.038 \pm 0.021	-0.038 \pm 0.021	-0.038 \pm 0.021	0.004 \pm 0.138	0.650 \pm 0.038	0.650 \pm 0.038	0.687 \pm 0.038	0.687 \pm 0.038

Table 3a: Regression Coefficients and Relative efficiency for of the index over Osborne's Index for Set I (1121 Dams)

x_1	x_2	x_3	x_4	\bar{x}_2	\bar{Y}_3	b_y	E
			0.1745	0.4423	0.1808	0.0738	1.0000
	-0.0013	0.4689	0.3163	0.1293	0.0523	0.9066	
0.0022	-0.0012	0.0722	0.2906	0.1188	0.0485	0.7452	
0.0023	-	0.0731	0.4141	0.1693	0.0691	0.8169	
-0.0009	-	-	0.1726	0.4298	0.1757	0.0717	0.9893
-0.0005	0.0020	-	0.0844	0.4104	0.1678	0.0635	0.8602
-0.0007	-	-0.0013	0.1672	0.3074	0.1257	0.0513	0.8979
-0.0004	0.0020	-0.0013	0.0806	0.2886	0.1180	0.04816	0.7561

x_1 = age at first egg; x_2 = Egg production upto 240 days + x_3 = egg weight

\bar{Y}_3 = the average of the phenotypic values of full-sibs for y .

b_y = average performance of individual's dam for y

E = efficiency of the index over osborne's index.

y_1 = the individual's own performance for rates of lay.

\bar{Y}_2 = the average of the phenotypic values of half-sibs, for y .

Table 3b: Regression coefficients and relative efficiency of the indices over the Osborne's Index for 1971 (5th Panel)

x_1	x_2	x_3	y_1	\bar{y}_2	\bar{y}_3	\bar{y}	σ
.	.	.	0.1347	0.5040	0.1819	0.0612	1.0000
-0.0002	0.1338	0.4984	0.1799	0.0605	0.9752		
0.0007	-	0.1021	0.4985	0.1799	0.0605	0.9527	
0.0007	-0.0002	0.1016	0.4930	0.1799	0.0598	0.9483	
-0.0010	-	--	0.1331	0.4781	0.1726	0.0580	0.9811
-0.0016	-0.0012	--	0.1863	0.4717	0.1703	0.0573	0.0458
-0.0010	-	-0.0002	0.1323	0.4727	0.1706	0.0573	0.9764
-0.0016	-0.0012	-0.0002	0.1859	0.4663	0.1683	0.0558	1.0420

Table 4a: Regression coefficients and jointive efficiencies of the Indices over the
Dobrovols Index for Set I.
(Without DWD)

x_1	x_2	x_3	\bar{Y}_1	\bar{Y}_2	\bar{Y}_3	π
			0.1812	0.4193	0.2076	1.000
	-0.0013	0.1734	0.2966	0.1469	0.9075	
0.0024	-	0.0757	0.3916	0.1939	0.8331	
0.0023	-0.0013	0.0739	0.2719	0.1346	0.7327	
-0.0009	-	0.1788	0.4070	0.2015	0.9890	
-0.0005	0.0021	-	0.0874	0.3880	0.1921	0.8482
-0.0008	-	-0.0013	0.1716	0.2881	0.1427	0.8988
-0.0004	0.0020	-0.0013	0.0926	0.27001	0.1337	0.7452

Table 4b: Regression Coefficients and Relative efficiency of the indices over the
Dobrotin's index for Set (II) (without D₂₂)

x_1	x_2	x_3	y_1	\bar{y}_2	\bar{y}_3	E
			0.1391	0.4846	0.2038	1.000
	-0.0002		0.1381	0.4791	0.2015	0.9951
0.0007	-		0.1054	0.4791	0.2015	0.9487
0.0007	-0.0002		0.1049	0.4737	0.1992	0.9442
-0.0010	-	-	0.1372	0.4589	0.1930	0.9809
-0.0016	-0.0012	-	0.1979	0.4528	0.1905	1.0515
-0.0010	-	-0.0002	0.1963	0.4536	0.19078	0.9761
-0.0017	-0.0013	-0.0002	0.1916	0.4473	0.1881	1.0477

$$\gamma_p = 0.0$$

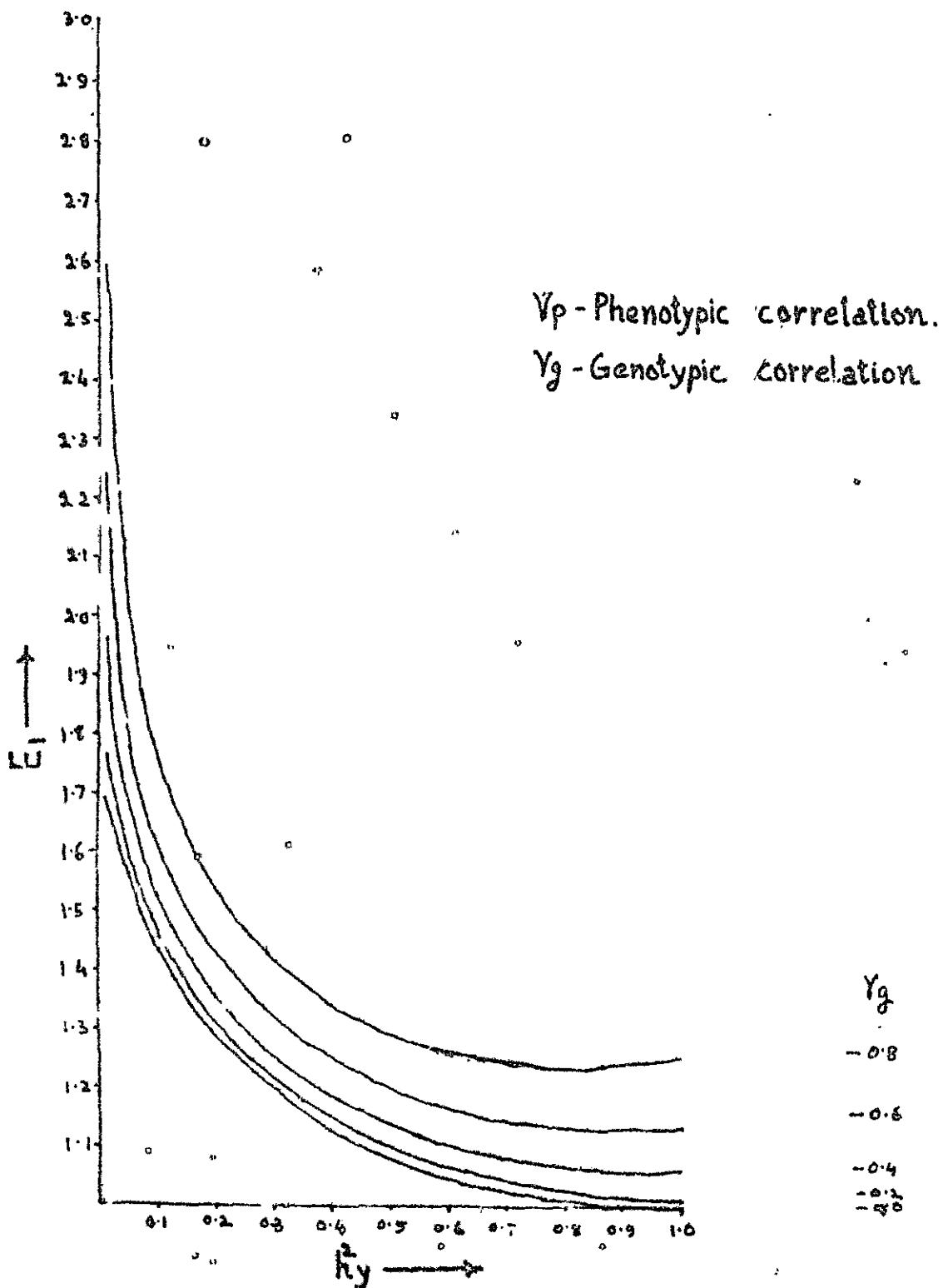


Fig. 1a: Efficiency of Selection Index I_1

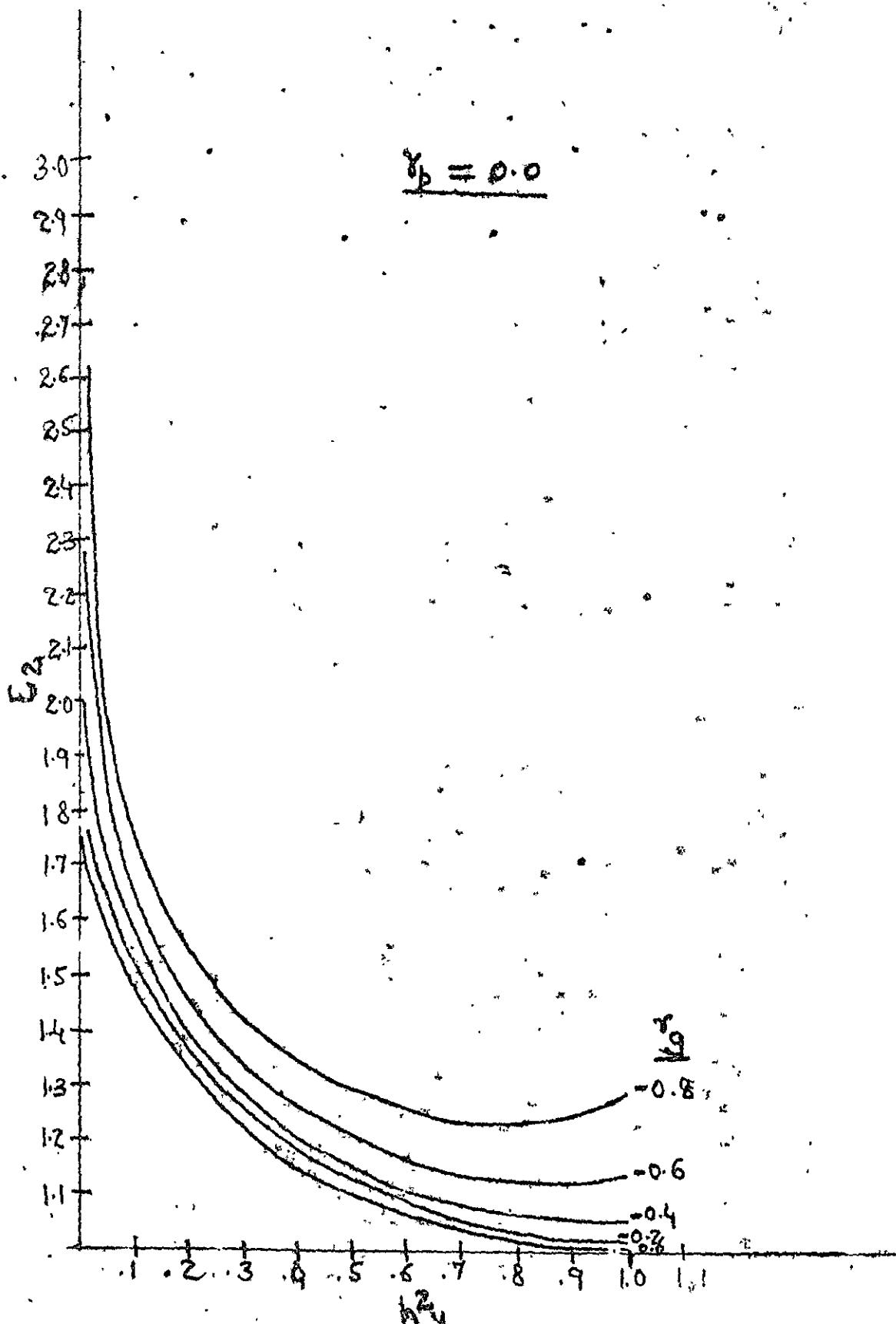


FIG. 6b; EFFICIENCY OF SELECTION INDEX I_2

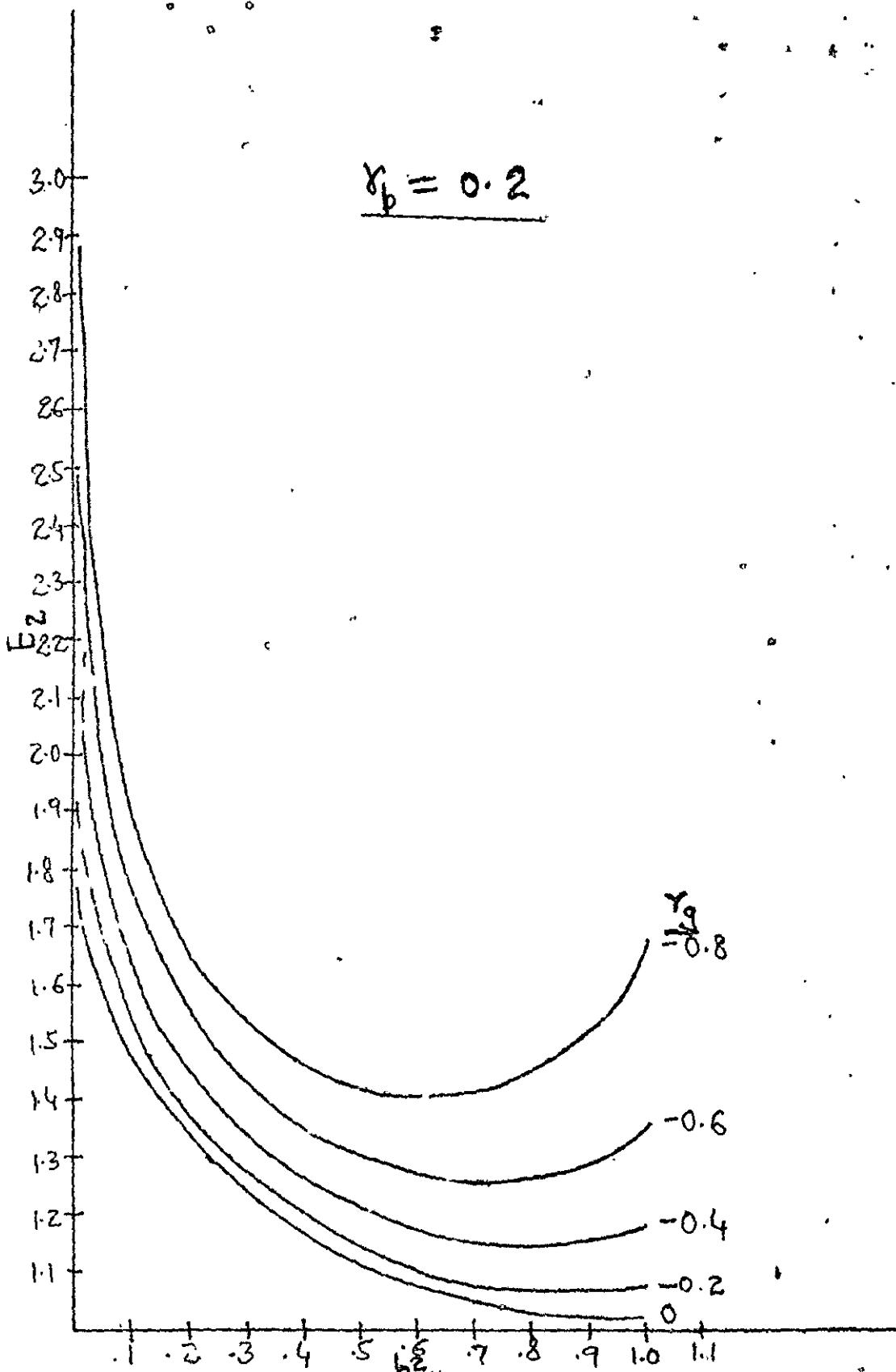


FIG 2b: EFFICIENCY OF SELECTION INDEX I_2

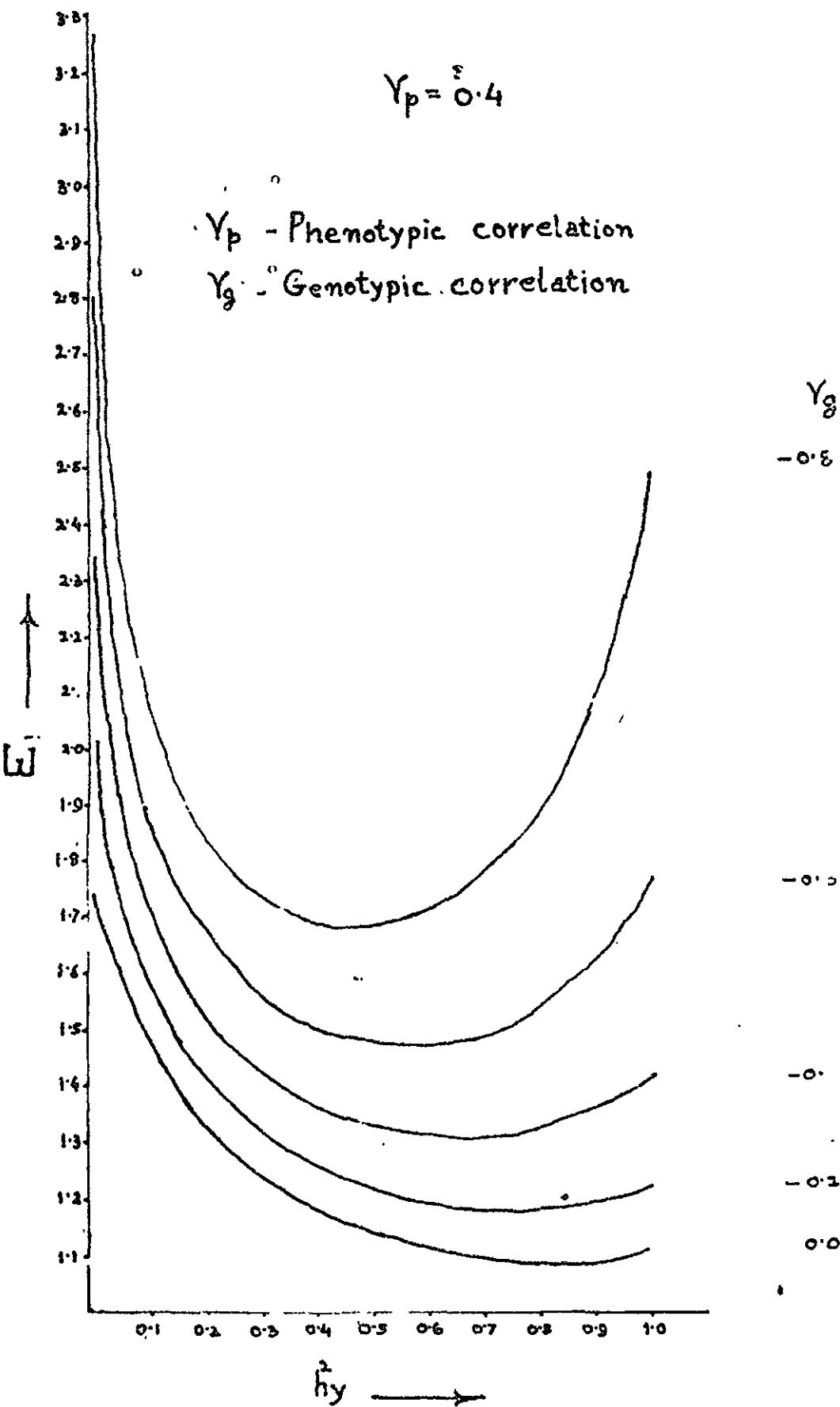


Fig. 3a: Efficiency of Selection Index I_1

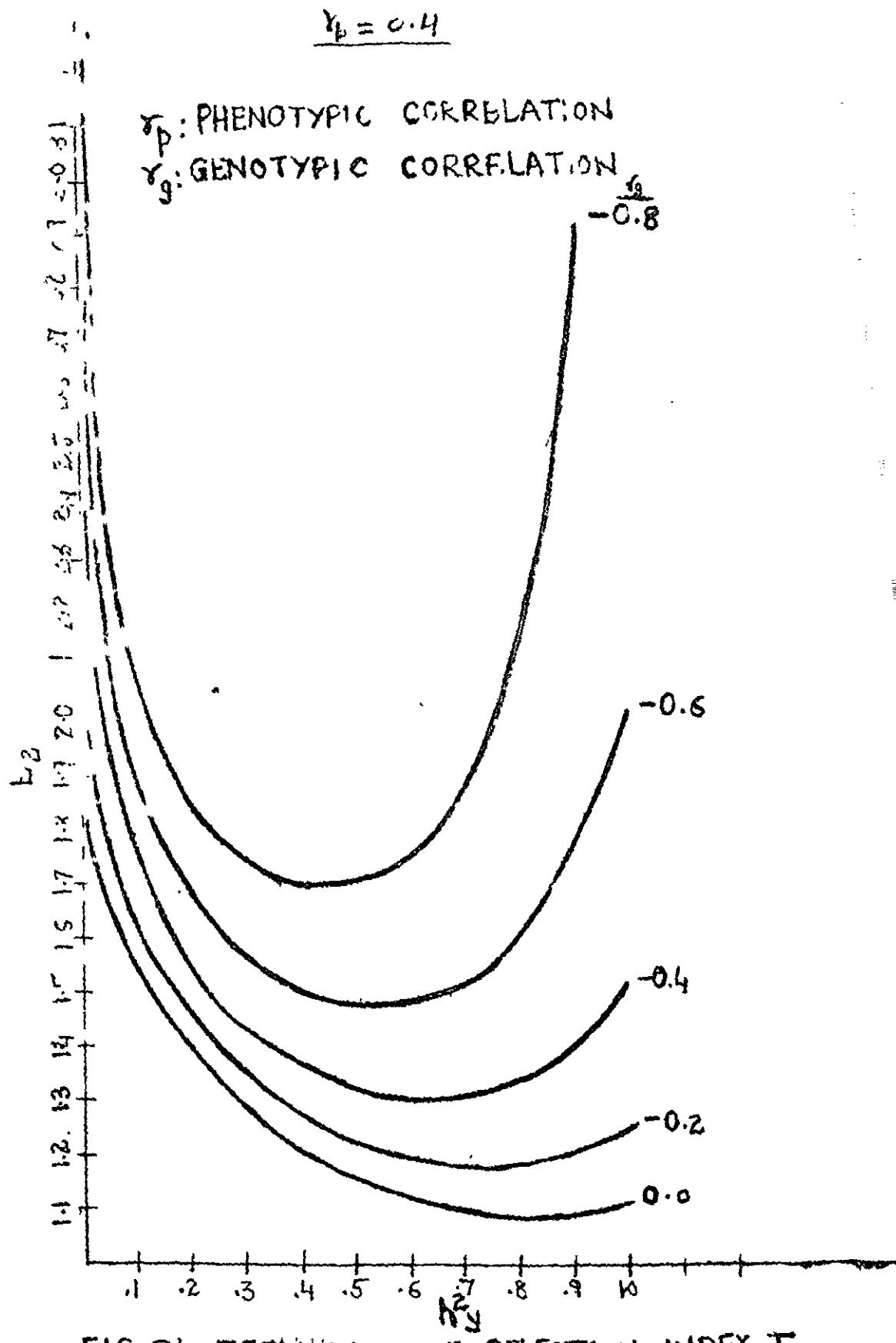


FIG 3b: EFFICIENCY OF SELECTION INDEX I_2

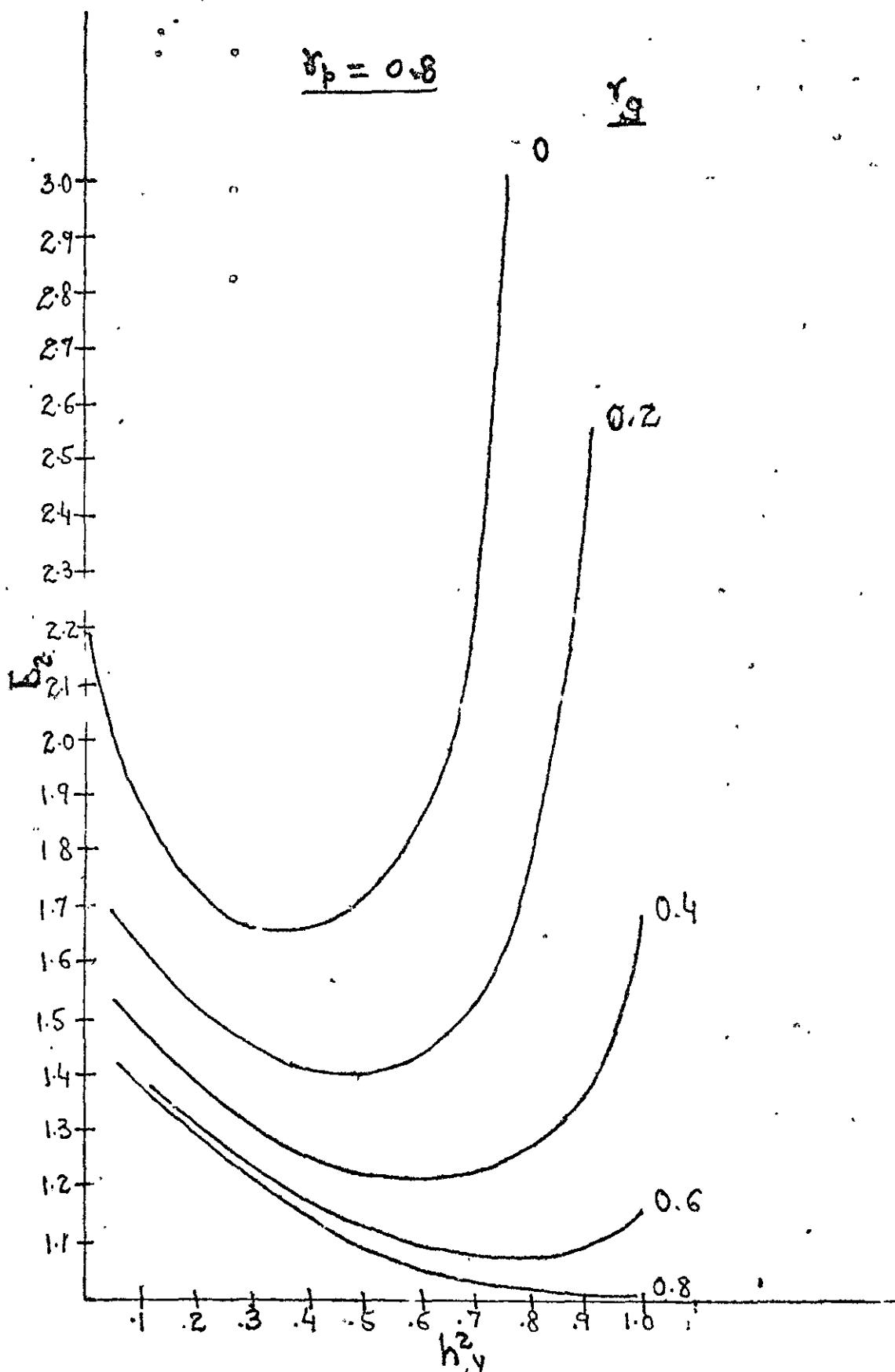


FIG 4b: EFFICIENCY OF SELECTION INDEX I_2 .

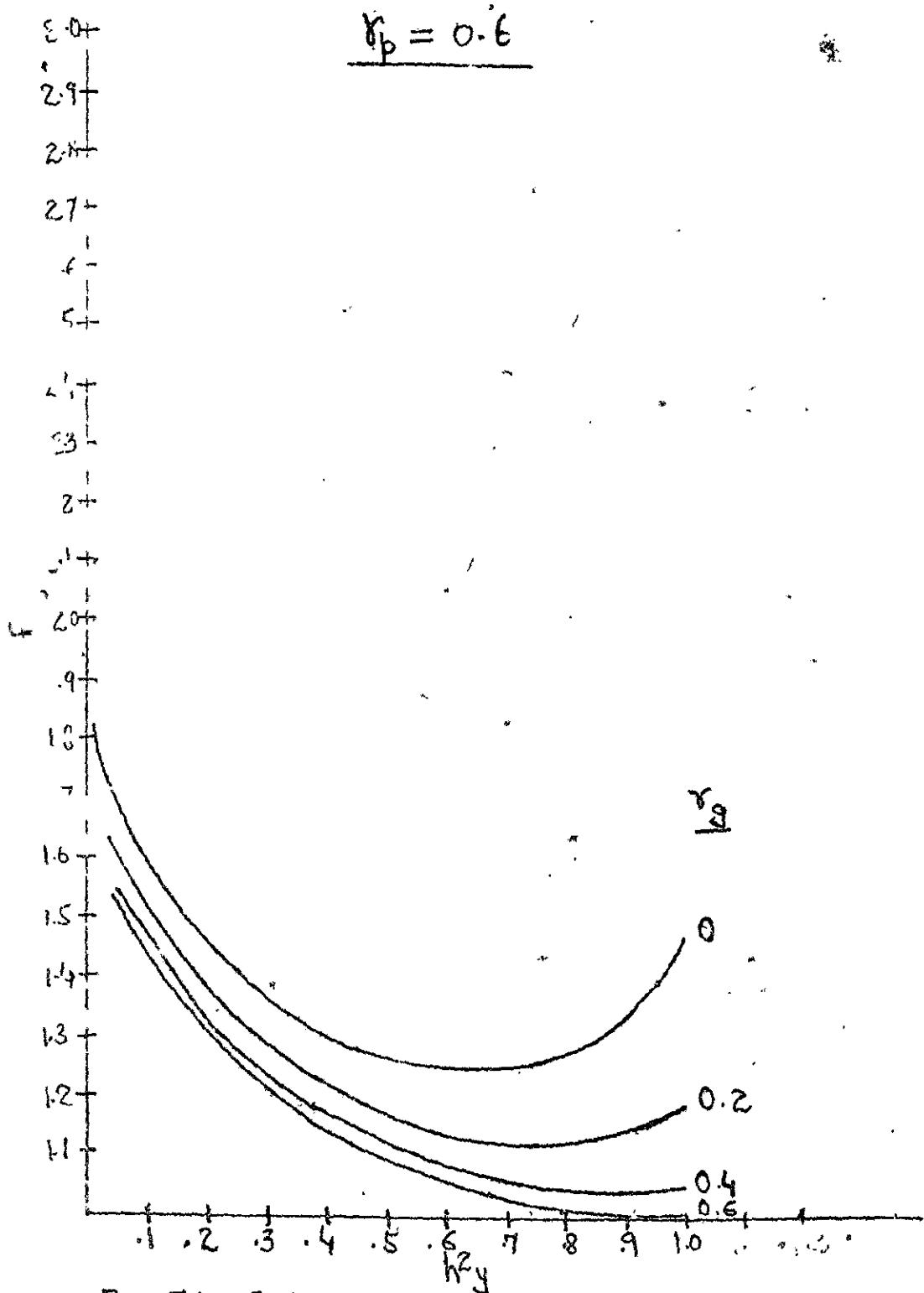


FIG 5 b: EFFICIENCY OF SELECTION INDEX I_2 .

$$\beta_p = 0.4$$

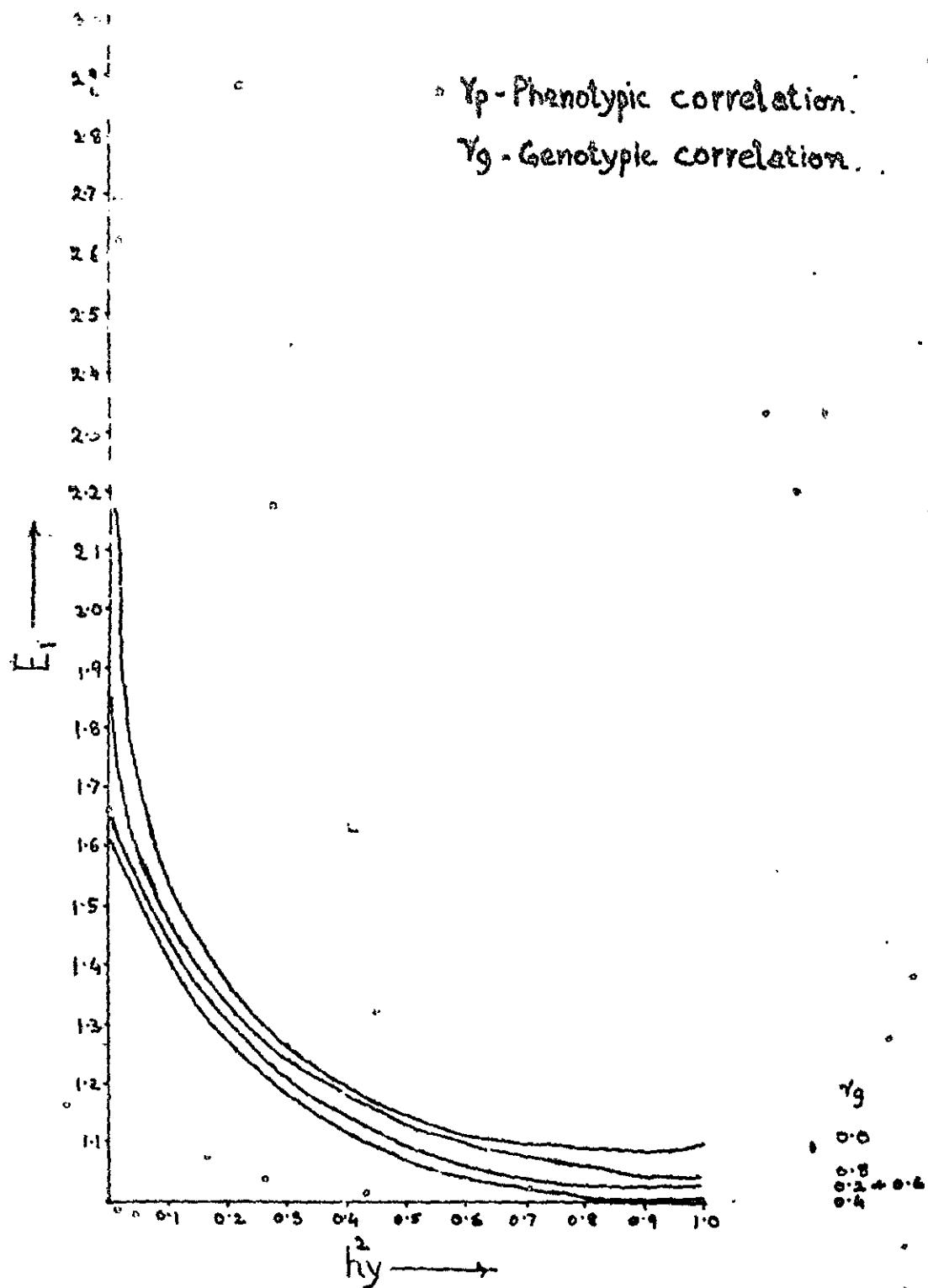


Fig. 6a: Efficiency of Selection Index I_1



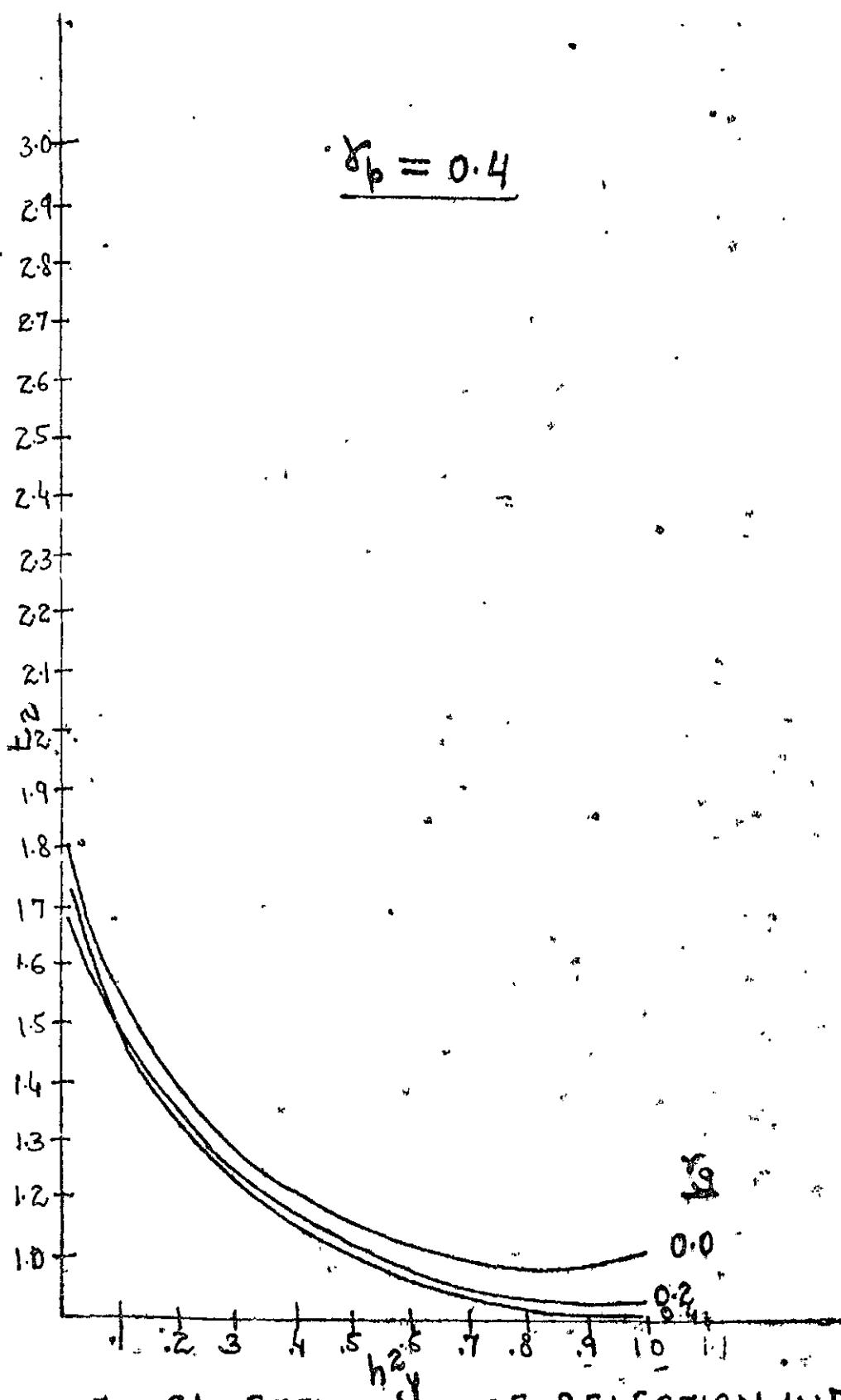


Fig. 6b: EFFICIENCY OF SELECTION INDEX I_2

