

Soil Water Retention Characteristics of Black Soils of India and Pedotransfer Functions Using Different Approaches

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Abstract: Information on soil hydraulic properties is a prerequisite in irrigation management decisions and crop planning. Such information on soils of the black soil region (BSR) occupying 7.7×10^7 ha of India is sparse. Soil profile information for 49 representative sites (244 samples) was collected and used for analysis. Ten different functions were evaluated for their efficacy to describe soil water retention characteristics (SWRC) of the BSR soils. Campbell model fitted to measured SWRC data with relatively lower root mean square error (RMSE = $0.0214 \text{ m}^3 \cdot \text{m}^{-3}$), higher degree of agreement $d = 0.9653$, and lower absolute error on average (MAE = $0.0165 \text{ m}^3 \cdot \text{m}^{-3}$). The next best description was by van Genuchten (VG) function with RMSE ($0.0249 \text{ m}^3 \cdot \text{m}^{-3}$), $d(0.9489)$, and MAE ($0.0868 \text{ m}^3 \cdot \text{m}^{-3}$). Pedotransfer functions (PTF) were developed to predict field capacity (FC) and permanent wilting point (PWP) using nearest neighbor (kNN) algorithm and artificial neural networks (ANN). Four levels of input information used for point PTF development include (1) textural data (data on sand, silt, and clay fraction-SSC), (2) level 1+bulk density data (SSCBD), (3) level 2+organic matter (SSCBDOM), and (4) level 1 +organic matter (SSCOM). The RMSE of predictions by kNN PTFs ranged from 0.0346 to $0.0611 \text{ m}^3 \cdot \text{m}^{-3}$ with an average of $0.0483 \text{ m}^3 \cdot \text{m}^{-3}$. The ANN PTFs performed with an average RMSE of $0.0550 \text{ m}^3 \cdot \text{m}^{-3}$ and a range of 0.0367 to $0.0905 \text{ m}^3 \cdot \text{m}^{-3}$. Relatively better estimates of FC/PWP were obtained using SSCBD-based PTF. Accuracy of FC and PWP estimates obtained by using analytical functions was relatively greater than the estimates by kNN and ANN PTFs. Campbell and VG functions were relatively more accurate. The study demonstrated the efficacy of kNN technique vis-a-vis neural regression with the additional benefit of appending the development data as and when desired. The proposed PTFs could be useful in making irrigation management decisions for BSR soils of India. Identification of the most suitable SWRC function for the study soils will help in crop modeling/water balance studies of the region. DOI: 10.1061/(ASCE)IR.1943-4774.0000527. © 2013 American Society of Civil Engineers.

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Introduction

Any description of the hydraulic properties suitable for irrigation management must include saturated and near-saturated hydraulic conductivity and soil water retention characteristics (SWRC). Black soils in India occupy 7.7×10^7 ha and have great potential for agricultural production. Black soils also occupy sizeable areas in the developing world but are mostly underutilized because of limited understanding as regard to their behavior and management. Irrigation water and rainwater management decisions in these soils are often dictated by special problems associated with these soils compared with other soils, the constituents of available water capacity (difference between field capacity and permanent wilting point), and rooting depth (Ahmad and Mermut 1996). These soils are considered difficult to manage because of the shrink-swell nature leading to complex hydraulic behavior.

In the wake of global warming, inter alia, resilience of black soils and changes in strategies to adapt to the emerging/anticipated climate are also being debated. Management of these soils is also critical to meet the continuously growing food grain requirement of the country. Information on SWRC of black soils especially at regional scale in India is very sparse. Decisions on irrigation planning and hydrological management/simulations routinely suffer from broad approximations in the absence of adequate data and inevitable expenses/manpower involved in acquiring the same. Thus, SWRC data carry high value and it is obviously necessary to use available information to the fullest possible extent. Many

functions/models to describe SWRC are found in the literature. The experience shows that no single function can be termed generic, although the van Genuchten function has been the most widely adopted (Yichang et al. 2004).

Indirect estimation of soil hydraulic properties (e.g., SWRC) as an alternative to direct estimation is one of the most widely researched topics. Relating basic soil information empirically to properties of interest mostly using regression tools is the most favored method of developing rules known as pedotransfer functions (PTF). Most of the PTFs reported in the literature pertain to estimation of SWRC. The PTF could be built to predict a point of interest on the SWRC curve (point PTF) like field capacity (FC-soil water retained at -33 kPa) or permanent wilting point (PWP-soil water retained at $-1,500$ kPa) or to predict the full SWRC curve (parametric PTF).

In many applications, such as crop water requirement/management and irrigation scheduling, information on FC and PWP is adequate enough to facilitate analysis and decision-making. Therefore, point PTFs to estimate FC/PWP carry greater value than parametric PTFs to predict SWRC. Since soil water retention in different ranges of soil water potential is affected by different basic soil properties, point PTFs are expected to perform with better accuracy than parametric PTFs. However, parametric PTFs offer an advantage of continuous simulations (soil water retention at any level of potential can be predicted) and facilitating prediction of hydraulic conductivity at varied soil moisture levels.

A literature survey shows that neural regression technique is favored by researchers for developing PTFs. The advantage of using neural networks (nonparametric approach) to develop PTFs lies in the fact that they do not require a priori regression models, which relate input and output data (Schaap et al. 1998). An analog approach such as k-nearest neighbor (kNN) based on similarity functions is another alternative preferred by researchers (Lall and Sharma 1996; Rajagopalan and Lall 1999) when a priori information on the relationship is unknown.

Most of the reports relating to PTF development focus on estimation of soil hydraulic properties for different geographical areas or soil types, and they attempt to identify the most important basic soil properties to be used as input (Pachepsky and Rawls 1999). Calibrated PTFs have been evaluated for spatial validity, efficacy of different techniques (regression versus ANN models), and different input parameters. However, the authors did not come across reports comparing neural regression and k-nearest neighbor tools in Indian context. The reason for lack of such studies is perhaps the sparse availability of hydraulic properties data. Further, it is essential that the data used for calibrating PTFs should account for most of the variations that are likely to be encountered in the soilscape of the area they are meant to be used, and therefore a strong database is required for such studies. It was opined that using a small set of relevant data, if available, is better than using a large and general data set (Mayr and Jarvis 1999; Nemes et al. 2002). Very little information related to SWRC of BSR soils in India is known, and there certainly is an information gap on their hydraulic behavior and the suitability of a parametric functions to describe it. Because the data acquisition for these soils is at the initial stage, it is appropriate to opt for a PTF development technique such as kNN that provides flexibility for continuous refinement as more data are acquired. This study (1) evaluated 10 different functions to describe SWRC of BSR soils and (2) compared flexible/alternative pattern recognition algorithm namely kNN as used by Nemes et al. (2006a, b) with widely used neural regression to calibrate PTFs for prediction of FC and PWP.

Materials and Methods

Soil Characteristics

For the present study, soil information from seven republic states of the country—Madhya Pradesh, Maharashtra, Karnataka, Andhra Pradesh, Tamil Nadu, Gujarat, and Rajasthan—were collected (Fig. 1). The data of 45 soil profiles with 244 layer observations on physical, chemical, and hydraulic properties of the soils were used. All the locations are representative of the black soil region with similar physical, morphological, and chemical soil properties. Data are collected from various sources such as project reports, bulletins, and theses (Pal et al., unpublished report, 2003; Bhattacharyya et al. 2007; Ray et al. 2011; Balpande 1993; Vaidya 2001). The details about these soils and their representative characteristics could be accessed at <http://www.geosis-naip-nbsslup.org>. Data reported by Pal et al. (unpublished report, 2003) and analyzed by Patil et al. (2012) were included in this analysis. Thus, the appended database included information on 19 additional profiles (87 layers). The data were collected following an identical soil survey protocol through different research programs of the National Bureau of Soil Survey and Land Use Planning. The majority of these soils were developed in the alluvium of weathered Deccan basalt. The particle size distribution of the collected soil samples was determined by the international pipette method after removal of organic matter. Sand ($2,000 - 50 \mu\text{m}$), silt ($50 - 2 \mu\text{m}$), total clay ($<2 \mu\text{m}$), and fine clay ($<0.2 \mu\text{m}$) fractions were separated according to the procedure of Jackson (1973). A seven point SWRC (-33 , -100 , -300 , -500 , -800 , $-1,000$, and $-1,500$ kPa) was mapped using pressure plate apparatus. The BSR is characterized by shrink-swell behavior of soils (vertisols and their intergrades). Black soils are common in the semiarid tropics in India, although

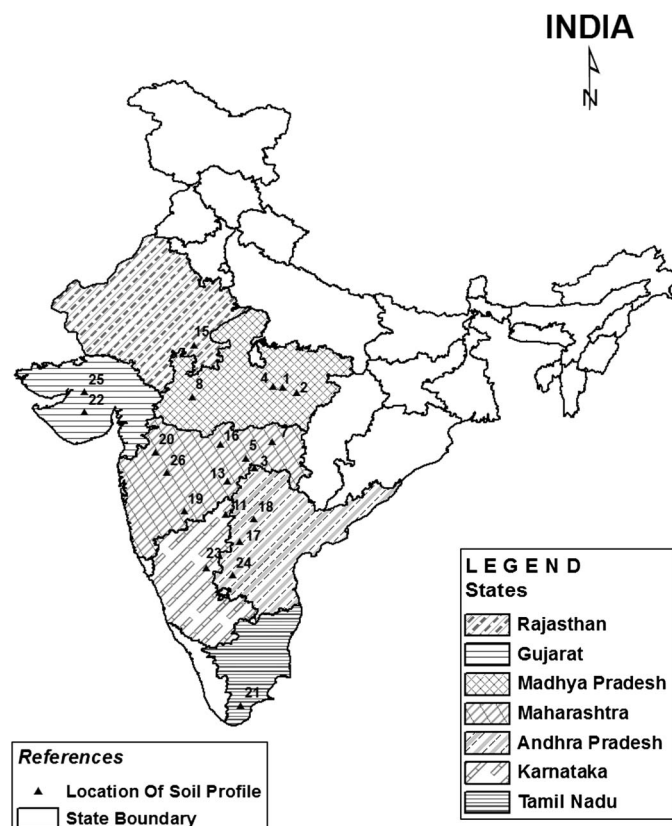


Fig. 1. Black soil region of India

their presence has been reported in the humid (mean annual rainfall of more than 1,600 mm) and arid (mean annual rainfall of less than 500 mm) bioclimates (Bhattacharyya et al. 1993, 2008). These soils are formed from basalts and other basic rocks (Pal et al., unpublished report, 2003) and are spatially associated with red soils, and thus form one of the major soil groups of India.

Salient features of the reference database used in the study are presented in Table 1. Soil water retention characteristics at six different tensions (−33, −100, −300, −500, −800, and −1,500 kPa) for 244 samples was used for analysis. A total of 1,464 paired data on soil water retained at varied suction pressure (−kPa) were used. Water retention ranged from 0.081 to 0.576 m³ · m^{−3}. The magnitude of the coefficient of variation at different points suggested that there was consistency in retention values. Mean standard deviation (SD) (measured data) was 0.04. This SD value was considered as the threshold value for judging suitability of evaluated SWRC functions.

Mathematical Model for Soil-Water Retention Characteristics

The SWRC data were fitted to the parametric relationship between water content θ , and the water potential of the soil, h as described by different researchers. A power law equation suggested by Brooks and Corey (1964) describes this relationship as

$$\theta = (h_b/h)^\lambda \quad \text{for } h < h_b \quad (1)$$

where θ = normalized water content such that

$$\theta = (\theta - \theta_r)/(\theta_s - \theta_r) \quad (2)$$

where θ = water content at pressure h ; θ_s = maximum water content; θ_r = residual water content; h_b = air entry pressure head; and λ = pore distribution index.

Another widely used function suggested by van Genuchten (1980) describes the relationship as

$$\theta = \theta_r + (\theta_s - \theta_r)/[1 + (\alpha * h)^n]^m \quad (3)$$

where $m = 1 - 1/n$. Here, α is related to the inverse of the air entry suction, $\alpha > 0$; and n is a measure of the pore-size distribution, ($n > 1$).

Campbell (1974) described water retention function as

$$\theta = \theta_s(h/h_b)^{-1/b} \quad \text{for } h < h_b \quad (4)$$

$$\theta = \theta_s \quad \text{for } h \geq h_b \quad (5)$$

Hutson and Cass (1987) modified the Campbell function, known as Cass-Hutson (CH) function, which shows

$$\theta = \theta_s(h/a)^{-1/b} \quad \text{for } \theta < \theta_i \quad (6)$$

and

$$\theta = \theta_s - \left[\theta_s h^2 \frac{(1 - \theta_i/\theta_s)}{a^2(\theta_i/\theta_s)} \right] \quad \text{for } \theta \geq \theta_i \quad (7)$$

here $\theta_i = 2b\theta_s/1 + 2b$ where a , b = empirical parameters; and h_b = air entry pressure. Six other functions—Drissen, exponential, Farrel and Larson, Libardi, Reichardt, and Nascimento (LRN), Simmons, and Power—evaluated in this study are shown in Table 2.

Pedotransfer Functions

A public-domain computer code “SWRC” (Dourado-Neto et al. 2000) was used for fitting water retention functions. The measured six-point soil water retention data of each sample was fitted to 10 parametric functions describing SWRC. Two techniques, neural regression and kNN, were used to build PTFs. Software developed by Nemes et al. (2008) to build PTFs for estimating FC and PWP from basic soil properties like textural distribution, bulk density, and organic matter in hierarchical order was used for building kNN PTFs. The software/tool combines kNN algorithm with the bootstrap data-subset selection technique to allow the development of model ensembles; that can be used to estimate the uncertainty of the final model. Basic soil properties like textural distribution, bulk density, and organic matter (in hierarchical order) were used for building kNN/PTFs. The software/tool combines kNN algorithm with the bootstrap data-subset selection technique. Four levels of input information were used to avoid possible bias towards one set of inputs, and dependencies between basic soil properties and FC/PWP were established:

- Input level 1 Textural data (data on sand, silt, and clay fraction-SSC),
- Input level 2 Level 1+bulk density data (SSCBD),
- Input level 3 Level 2+organic matter (SSCBDOM), and
- Input level 4 Level 1+organic matter (SSCOM).

The technique is based on pattern recognition rather than on fitting equations to data. Application of the kNN means identifying and retrieving the most similar instances, on the basis of their input

Table 1. Statistical Summary of Soil Properties of Selected Soil Samples

Property	Mean	Standard deviation	Variance	Coefficient of variation	Minimum	Maximum
Sand (%)	6.41	9.68	93.68	1.51	0	48.5
Silt (%)	26.1	8.03	198.6	0.54	0.1	49
Clay (%)	49.63	11.22	637.13	0.51	0.6	86.1
Bulk density (Mg · m ^{−3})	1.38	0.16	0.02	0.11	1.04	1.8
Organic matter	0.5	0.23	0.05	0.45	0.04	1.55
Field capacity (m ³ · m ^{−3})	0.4077	0.0779	0.0061	0.1912	0.2140	0.5760
Permanent wilting point (m ³ · m ^{−3})	0.2278	0.0567	0.0032	0.2486	0.0810	0.3580
Soil water retention at suction pressure (−kPa)						
33	0.4077	0.0779	0.0061	0.1912	0.2140	0.5760
100	0.3452	0.0669	0.0045	0.1938	0.1570	0.5000
300	0.2835	0.0521	0.0027	0.1839	0.1300	0.3890
500	0.2649	0.0500	0.0025	0.1886	0.1190	0.3700
800	0.2533	0.0489	0.0024	0.1932	0.1160	0.3590
1,500	0.2278	0.0567	0.0032	0.2486	0.0810	0.3580

Table 2. Functions to Describe SWRC and Calibrated Parameters

Reference	Function	Calibrated parameters
Brooks-Corey (1964)	$\theta = (h_b/h)^\lambda$	h_b, λ_r
Campbell (1974)	$\theta = \theta_s (h/h_b)^{-1/b}$	θ_s, h_b, b
Driessen (1986)	$\theta = \theta_s h^{-\gamma \ln(h)}$	γ, θ_r
Exponential	$\theta = -1/\beta \ln(h/\alpha)$	α, β
Farrel and Larson (1972)	$\theta = \theta_r + (\theta_s - \theta_r)[1 - 1/\alpha \ln(h/h_e)]$	$\theta_r, \theta_s, \alpha$
Hutson-Cass (1987)	$\theta = \theta_s (h/a)^{-1/b}$	a, b
Libardi et al. (1979)	$\theta = \theta_s + (1/\beta) \ln(h/\alpha + 1)$	α, β, θ_s
Power	$\theta = (h/\alpha)^{-1/\beta}$	α, β
Simmons et al. (1979)	$\theta = \phi + (1/\beta) \ln(h/\alpha + 1)$	α, β, ϕ
Van Genuchten (1980)	$\Theta = \Theta_{r+}(\theta_s - \theta_r)/[1 + (\alpha * h)^n]^m$	$\alpha_{m,n}, \theta_s, \theta_r$

attributes, to the target object from a known set of stored instances. For developing ANN-based PTFs, software “neurointelligence” was used. The neural networks learn and generalize from experimental data even if they are noisy, imperfect, or nonlinear in nature. The underlying relationship between materials and properties is directly learned from the result of experiments, and subsequently, parameters can be fitted to the assumed function/relationship. From previous experience, feedforward neural network model with three hidden nodes was preferred (Patil et al. 2010). The data set was partitioned into training (178 samples) and test (53 samples) sets. Several (13) samples were discarded for inconsistency. On finding an appropriate network model, the PTF was calibrated. For network training, a Levenberg-Marquardt (L-M) algorithm was chosen because the data were limited (<250 samples).

Performance Evaluation

Efficacy of parametric functions was evaluated on the basis of (1) root mean square error (RMSE), (2) index of agreement (d), (3) maximum absolute error (ME), (iv) mean absolute error (MAE), and (5) correlation coefficient (R^2). The values of RMSE, d , ME, and MAE statistics were calculated using the following equations, respectively, where n represents the number of data used for modeling, M_i and E_i represent measured and computed value, respectively, whereas S_M and S_E represent sum of measured and computed values, respectively. The unit of errors is $\text{m}^3 \cdot \text{m}^{-3}$:

$$\text{Root mean square error} \quad \text{RMSE} = \sqrt{\frac{\sum_{i=1}^n (E_i - M_i)^2}{n}} \quad (8)$$

$$\text{Index of agreement} \quad d = 1 - \frac{\sum_{i=1}^n (E_i - M_i)^2}{\sum_{i=1}^n (|E_i - \bar{M}| + |M_i - \bar{M}|)^2} \quad (9)$$

$$\text{Maximum absolute error} \quad \text{ME} = \text{Max}|E_i - M_i| \quad (10)$$

$$\text{Mean absolute error} \quad \text{MAE} = \frac{\sum_{i=1}^n |E_i - M_i|}{n} \quad (11)$$

$$\text{Linear correlation coefficient} \quad r = \frac{1}{n-1} \frac{\sum_{i=1}^n (M_i - \bar{M})(E_i - \bar{E})}{S_M S_E} \quad (12)$$

The two tools of PTF development (kNN and ANN) were assessed by comparing estimations made by respective PTFs developed using the same data and input soil attributes. The same set of statistical indices was used for the comparison of measured and

estimated data. However, greater emphasis was placed on the RMSE statistic, which indicates the model's ability to predict away from the mean. Root mean square error imparts more weight to high values because it involves square of the difference between observed and predicted values. Ideally, the model should have the smallest MAE and smallest overall dispersion (RMSE).

Results and Discussion

The performance of 10 different models in describing SWRC of BSR soils indicated that six functions—Drissen, exponential, Farrel and Larson, LRN, Simmons, and Power—fitted poorly to the measured SWRC data. Mean RMSE and other indices showed that these functions described SWRC with a high magnitude of error (RMSE ranging from 0.1249 to 0.1545). These six functions were discarded after the assessment. Detailed evaluation of four functions that showed reasonable accuracy with lower mean RMSE (0.0214), lower mean absolute error (0.01645), and higher degree of agreement (0.9653) is presented in Table 3 and Figs. 2–6. The Campbell function emerged as the best performing function to describe SWRC of the soils. The difference between measured and computed values at -100 and $-1,500$ kPa was relatively lower when the Campbell function was used. The Brooks-Corey (BC) expression resulted in comparatively better fitting at the points -300 , -500 , and -800 kPa. However, the estimate errors were highest for the BC function at -33 kPa. It was apparent that the Campbell function performed relatively better if mean errors are considered. The van Genuchten (VG) function also fitted well with mean RMSE (0.0249), d (0.9489), ME (0.0868), MAE (0.0188), and R^2 (0.86). The worst fitting was observed in the modified Campbell function. Graphical representation (Figs. 7–10) highlighted that the Campbell function overestimated, whereas the BC and VG functions underestimated water retention in wet range. But the overestimates by the Campbell function were relatively few. It could also be inferred that Campbell function in general was better suited to describe SWRC irrespective of suction pressure, depth of soil granulometric composition, and geographic location of BSR soils. Most of the hydrological/irrigation related applications require soil water content in wet range and in this context, the Campbell function appears to be better. It is in this region that most of the flow is expected to occur. The function also has an advantage of parsimony (fewer number of parameters) and simplicity of expression. These findings could be interesting to analyze if data on more points on SWRC in saturation and dry range are acquired. The computation time and easier applicability in calculations such as hydraulic conductivity also favor the use of the Campbell function. The VG and BC functions underestimated retention in the same range. Because BC and VG models are derived on a similar

Table 3. Statistical Indexesto Judge Efficacy of Four Shortlisted Soil Water Retention Functions in Describing SWRCData of Soils of Black Soil Region of India

	Suction pressure (-kPa)						
Function/index	33	100	300	500	800	1,500	Mean
			Campbell				
Root mean square error	0.0343	0.018	0.02	0.0177	0.0176	0.0209	0.0214
Index of agreement, d	0.9503	0.9803	0.9639	0.9696	0.9681	0.9596	0.9653
Maximum absolute error	0.1115	0.0599	0.0816	0.0623	0.0578	0.0548	0.0713
Mean absolute error	0.0242	0.0145	0.0159	0.0145	0.0115	0.0181	0.0165
R^2	0.8451	0.9272	0.8781	0.8851	0.9091	0.9126	0.8929
			Modified Campbell				
Root mean square error	0.0959	0.0691	0.0557	0.0515	0.0478	0.0475	0.0613
Index of agreement, d	0.6209	0.706	0.7032	0.7267	0.759	0.779	0.7158
Maximum absolute error	0.3999	0.2782	0.2689	0.2275	0.1927	0.1505	0.2530
Mean absolute error	0.0645	0.0478	0.0366	0.0358	0.0348	0.0342	0.0423
R^2	0.0294	0.0429	0.0354	0.0681	0.0885	0.0746	0.0565
			BC				
Root mean square error	0.0531	0.0362	0.0207	0.0181	0.0157	0.0293	0.0289
Index of agreement, d	0.8523	0.9094	0.9595	0.9683	0.9756	0.9264	0.9319
Maximum absolute error	0.1877	0.1257	0.0857	0.0813	0.0784	0.937	0.2493
Mean absolute error	0.0352	0.0255	0.0149	0.0137	0.0103	0.218	0.0529
R^2	0.6882	0.7972	0.8502	0.8839	0.9077	0.7729	0.8167
			VG				
Root mean square error	0.0499	0.0283	0.0197	0.0156	0.0132	0.0228	0.0249
Index of agreement, d	0.87	0.9466	0.9653	0.9768	0.9822	0.9524	0.9489
Maximum absolute error	0.1858	0.1138	0.068	0.0479	0.0418	0.0634	0.0868
Mean absolute error	0.0366	0.0198	0.0153	0.0128	0.0091	0.0189	0.0188
R^2	0.7343	0.8363	0.8851	0.9148	0.9359	0.8567	0.8605

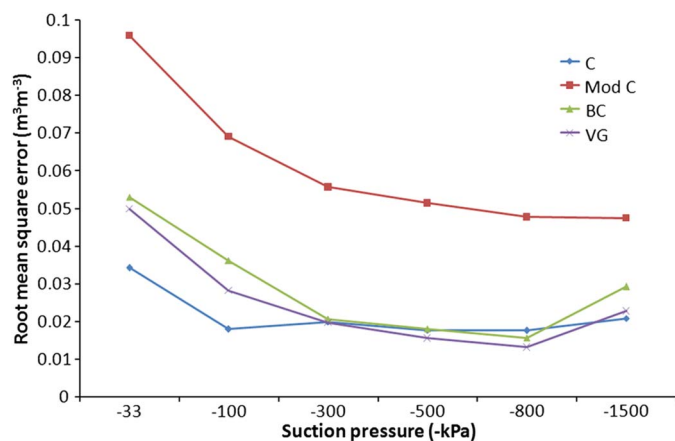


Fig. 2. Performance of different SWRC functions in BSR soils as indicated by root mean square error

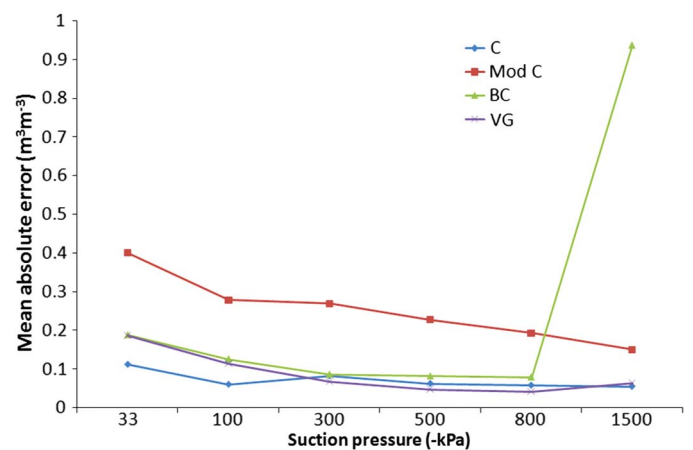


Fig. 3. Performance of different SWRC functions in BSR soils as indicated by mean absolute error

basis, the commonality could be attributed to better representation of capillarity by these functions.

The data were analyzed by subsets defined according to the textures and three major textural classes according to the U.S. Department of Agriculture (USDA) classification. The classes consisted of clay (158 samples), silt loam (44 samples), and silty clay (33 samples). Nine samples belonging to other classes were not included for analysis. The SWRC of clay soils (Table 4) were described well by all the four functions with reasonable accuracy, as the RMSE was well below the 0.04 criteria. The Campbell function was relatively better suited than any other function because of the lower RMSE (0.0199), higher degree of agreement (0.9647), and lower ME (0.0850) and MAE (0.0150). In silt loam soils,

all the four functions failed to achieve the RMSE < 0.04 (Table 5). Other indices also divulged that there were considerable errors in fitting functions to the measured SWRC data. The VG function came closest in meeting the criteria of RMSE 0.04 with relatively lower RMSE (0.0464). The SWRC of silty clay soils (Table 6) were best described by the VG function (RMSE 0.0161) followed by Campbell (RMSE 0.0171), modified Campbell, and the BC functions. All other indices also support these observations. The VG model historically has been the most widely adopted (Yichang et al. 2004) and was also confirmed by these findings. Previous findings (Patil et al. 2012) espoused the use of any of these four functions to describe SWRC of BSR soils. But with more data, it is inferred that the choice could be narrowed down to two functions—Campbell

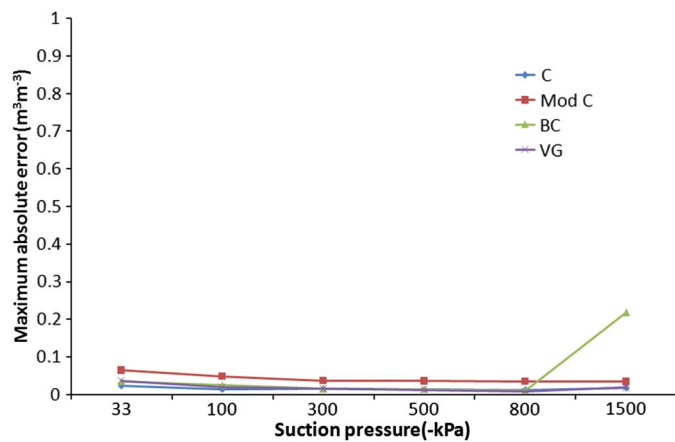


Fig. 4. Performance of different SWRC functions in BSR soils as indicated by maximum absolute error

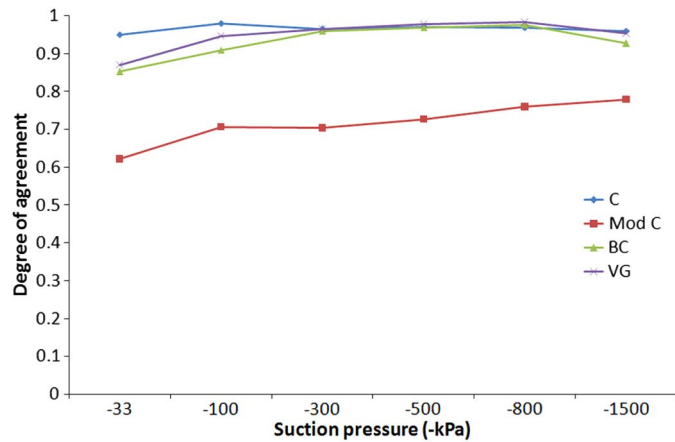


Fig. 5. Performance of different SWRC functions in BSR soils as indicated by degree of agreement

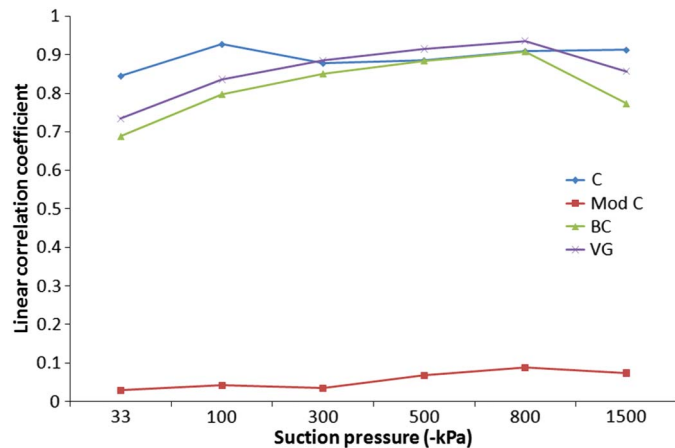


Fig. 6. Performance of different SWRC functions in BSR soils as indicated by linear correlation coefficient

and VG. The relatively superior performance of the VG function in silty loam soils and acceptable accuracy in clay soils indicated that it has applicability across the textures in BSR soils. Although soil hydraulic functions of the VG model are comparatively difficult to calculate and do not lead to rapid numerical solution, it could be

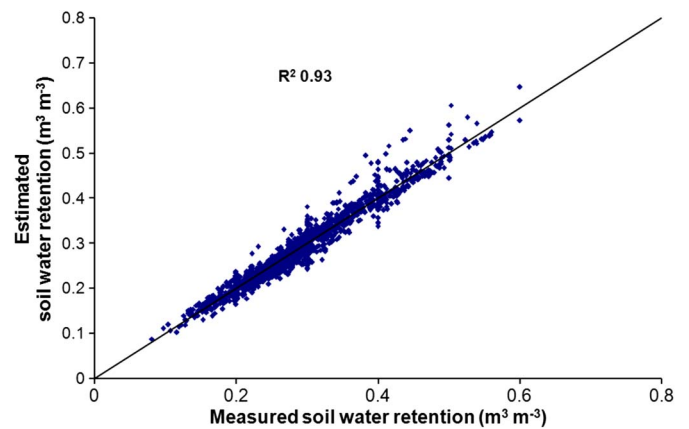


Fig. 7. Measured and estimated soil water retention described by Campbell function (best fitting)

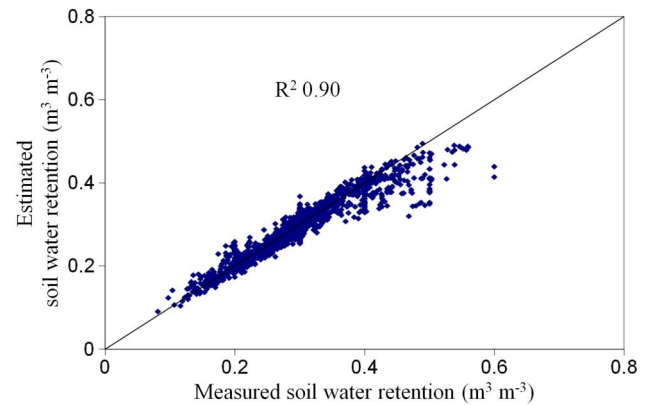


Fig. 8. Measured and estimated soil water retention described by VG function

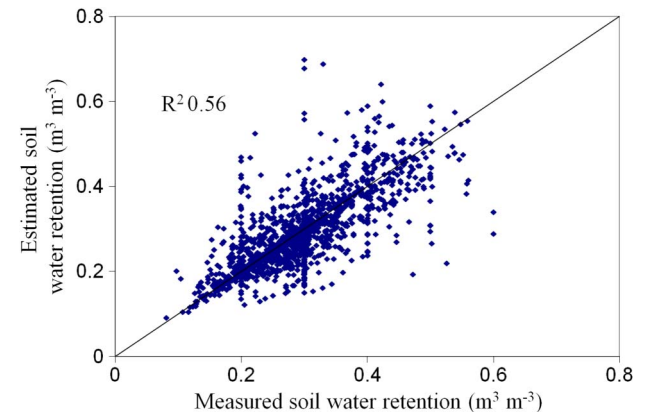


Fig. 9. Measured and estimated soil water retention described by modified Campbell function (poorest fitting)

considered equally important for usage. It appears that the VG function was better suited to describe SWRC (Patil and Rajput 2009) of seasonally impounded clay soils of the Jabalpur district, India (vertisols and their intergrades), which are in conformity with the present findings. In fine textured soils, the BC function

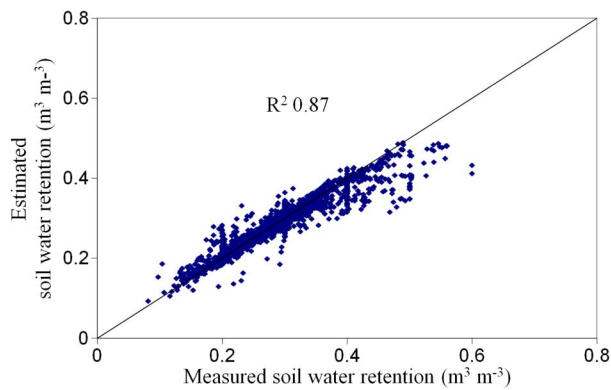


Fig. 10. Measured and estimated soil water retention described by BC function

is expected to falter near the saturation range, and it is apparent from Fig. 3 that the description was not good in the saturation range.

This study included soils with variations in soil genesis, climate, and topography. It is possible that, in some parts, the VG function could be more suitable than the Campbell function, and vice versa. With current knowledge and with available data, it could be concluded that either the VG or Campbell function could be used.

Pedotransfer Functions

The performance of PTFs developed using kNN and neural networks could be judged from the statistical indices (Table 7). The kNN PTFs were generally superior (RMSE 0.0346 to 0.0611) to neural PTFs (RMSE 0.0367 to 0.0905) irrespective of input used in prediction of FC/PWP (Figs. 11–15). Other indices

(d, ME, MAE, R^2) also confirmed the superiority of kNN PTF. Incremental inclusion of bulk density after texture (SSC) as a predictor variable improved accuracy of PTFs. The improvement was especially significant in predicting FC (RMSE reduced from 0.0905 to 0.0634) using neural PTF. Further inclusion of variable organic matter (OM) either in addition to SSCBD or at the expense of bulk density (BD) did not improve the accuracy of PTFs against the expectations. In fact, kNN PTFs for FC lost some accuracy after the inclusion of OM. This is in contrast to the reports by Patil et al. (2012) wherein inclusion of OM improved prediction of available water capacity (AWC), whereas BD was reported to decrease accuracy of predictions. Bulk density of black soils (vertisols) changes with change in soil water content and, therefore, SWRC of these soils differ from other soils. It is therefore expected that influence of BD will be reflected in estimates of soil water retention. Further, Patil et al. (2012) opined that their data were insufficient to capture the underlying relationship between SWRC and BD for developing PTFs. Because this study is based on more data in addition to the data they used, the relationship was noted and importance of BD in prediction of soil water contents was elaborated. Further, they have also reported improvement in accuracy of neural PTFs with inclusion of OM as a predictor variable, which was confirmed to a certain extent, as the current results showed that the best predictions were obtained at maximum input level (Fig. 12) essentially containing information on OM. Vereecken et al. (1989) opined that the most accurate PTFs for estimating the SWRC were obtained when textural properties, bulk density, soil organic matter, and soil moisture content were used. This study pertained to only two points (soil moisture constants) on SWRC and, inclusion of soil moisture constants was ruled out. But including all other variables, the authors observed no advantage. Perhaps, the OM content in these soils is too low to influence FC/PWP to a perceptible extent. Thus, BD was the most influential variable in predicting

Table 4. Statistical Indexes to Judge Efficacy of Different Soil Water Retention Functions in Describing SWRC Data of Soils of Black Soil Region of India (Clay Soils, 158 Samples)

	Suction pressure (-kPa)						
Function/index	33	100	300	500	800	1,500	Mean
			Campbell				
Root mean square error	0.0338	0.0174	0.0170	0.0160	0.0150	0.0204	0.0199
Index of agreement, <i>d</i>	0.9417	0.9774	0.9696	0.9698	0.9731	0.9568	0.9647
Maximum absolute error	0.1794	0.0900	0.0771	0.0516	0.0641	0.0479	0.0850
Mean absolute error	0.0218	0.0135	0.0144	0.0134	0.0091	0.0179	0.0150
<i>R</i> ²	0.7970	0.9146	0.9068	0.8906	0.9195	0.9053	0.8889
			Modified Campbell				
Root mean square error	0.0351	0.0176	0.0164	0.0174	0.0185	0.0236	0.0214
Index of agreement, <i>d</i>	0.9415	0.9772	0.9709	0.9631	0.9576	0.9405	0.9585
Maximum absolute error	0.1869	0.0925	0.0764	0.0566	0.0674	0.0541	0.0890
Mean absolute error	0.0209	0.0135	0.0136	0.0132	0.0119	0.0202	0.0156
<i>R</i> ²	0.7972	0.9129	0.8956	0.8660	0.8913	0.8969	0.8766
			BC				
Root mean square error	0.0517	0.0332	0.0153	0.0140	0.0109	0.0269	0.0253
Index of agreement, <i>d</i>	0.8461	0.9087	0.9744	0.9772	0.9859	0.9265	0.9365
Maximum absolute error	0.1861	0.1100	0.0755	0.0487	0.0602	0.0937	0.0957
Mean absolute error	0.0349	0.0222	0.0118	0.0115	0.0073	0.0198	0.0179
<i>R</i> ²	0.6663	0.7782	0.9055	0.9205	0.9481	0.7636	0.8303
			VG				
Root mean square error	0.0269	0.0498	0.0167	0.0141	0.0112	0.0229	0.0236
Index of agreement, <i>d</i>	0.9417	0.8553	0.9707	0.9770	0.9848	0.9441	0.9456
Maximum absolute error	0.0936	0.1878	0.0782	0.0517	0.0631	0.0634	0.0896
Mean absolute error	0.0179	0.0346	0.0134	0.0117	0.0072	0.0191	0.0173
<i>R</i> ²	0.6914	0.8031	0.9128	0.9211	0.9457	0.8308	0.8508

Table 5. Statistical Indexes to Judge Efficacy of Different Soil Water Retention Functions in Describing SWRC Data of Soils of Black Soil Region of India (Silt Loam, 44 Samples)

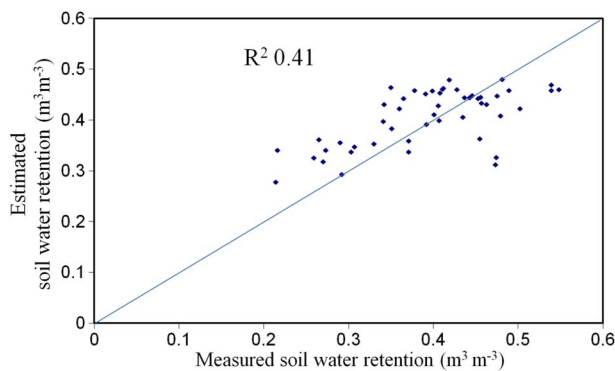
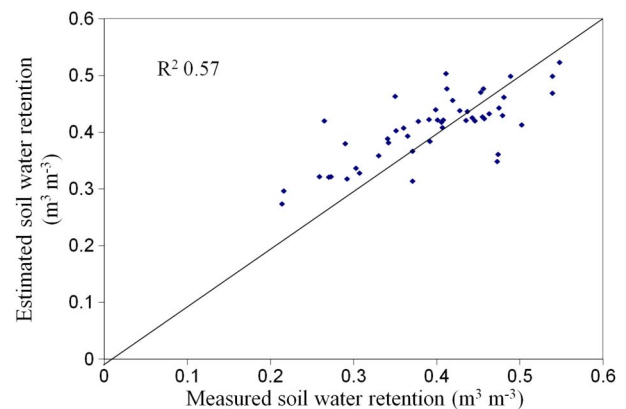
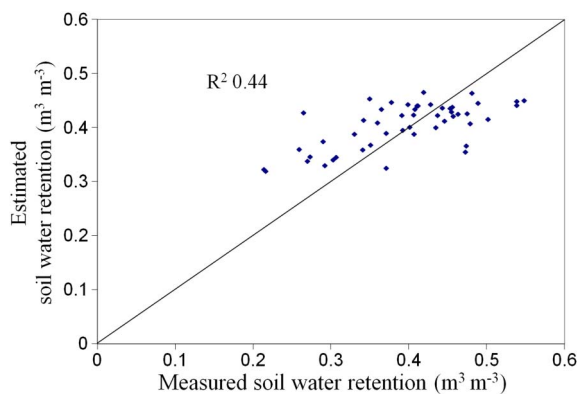
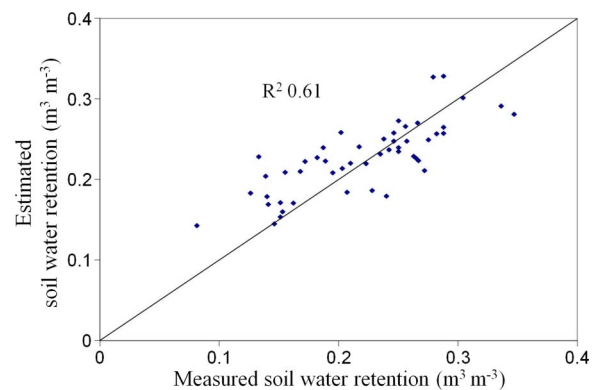
	Suction pressure (-kPa)						
Function/index	33	100	300	500	800	1,500	Mean
			Campbell				
Root mean square error	0.0673	0.0548	0.0543	0.0475	0.0492	0.0561	0.0549
Index of agreement, d	0.7672	0.7673	0.6824	0.7922	0.7779	0.7019	0.7482
Maximum absolute error	0.1673	0.1856	0.1434	0.2023	0.1273	0.1559	0.1636
Mean absolute error	0.0546	0.0394	0.0403	0.0345	0.0377	0.0432	0.0416
R^2	0.3407	0.4259	0.2579	0.4494	0.5654	0.3345	0.3956
			Modified Campbell				
Root mean square error	0.0713	0.0529	0.0505	0.0459	0.0472	0.0503	0.0530
Index of agreement, d	0.7603	0.7770	0.6866	0.7867	0.7772	0.7416	0.7549
Maximum absolute error	0.1666	0.1929	0.1516	0.2137	0.1390	0.1675	0.1719
Mean absolute error	0.0565	0.0387	0.0355	0.0318	0.0377	0.0381	0.0397
R^2	0.3351	0.4378	0.2601	0.4238	0.5703	0.4529	0.4133
			BC				
Root mean square error	0.0863	0.0655	0.0436	0.0295	0.0240	0.0396	0.0481
Index of agreement, d	0.6246	0.6845	0.7206	0.8868	0.9170	0.7936	0.7712
Maximum absolute error	0.2134	0.1743	0.1169	0.1291	0.0655	0.0839	0.1305
Mean absolute error	0.0653	0.0559	0.0307	0.0212	0.0188	0.0306	0.0371
R^2	0.2852	0.4548	0.3036	0.6330	0.7277	0.4176	0.4703
			VG				
Root mean square error	0.0839	0.0581	0.0433	0.0303	0.0277	0.0351	0.0464
Index of agreement, d	0.6340	0.7314	0.7471	0.8849	0.8962	0.8274	0.7868
Maximum absolute error	0.2044	0.1458	0.1230	0.1256	0.0623	0.0767	0.1230
Mean absolute error	0.0630	0.0485	0.0325	0.0226	0.0222	0.0294	0.0364
Root mean square error	0.2879	0.4697	0.3392	0.6292	0.7046	0.5169	0.4912

Table 6. Statistical Indexes to Judge Efficacy of Different Soil Water Retention Functions in Describing SWRC Data of Soils of Black Soil Region of India (Silty Clay, 33 Samples)

	Suction pressure (-kPa)						
Function/index	33	100	300	500	800	1,500	Mean
			Campbell				
Root mean square error	0.0307	0.0150	0.0126	0.0145	0.0159	0.0139	0.0171
Index of agreement, d	0.9330	0.9813	0.9737	0.9659	0.9536	0.9649	0.9621
Maximum absolute error	0.0728	0.0379	0.0339	0.0325	0.0405	0.0253	0.0405
Mean absolute error	0.0231	0.0119	0.0102	0.0120	0.0123	0.0116	0.0135
R^2	0.8529	0.9613	0.9206	0.8923	0.9078	0.9091	0.9073
			Modified Campbell				
Root mean square error	0.0252	0.0161	0.0128	0.0153	0.0181	0.0187	0.0177
Index of agreement, d	0.9584	0.9788	0.9716	0.9596	0.9344	0.9280	0.9551
Maximum absolute error	0.0765	0.0420	0.0279	0.0366	0.0507	0.0391	0.0455
Mean absolute error	0.0172	0.0128	0.0108	0.0121	0.0127	0.0145	0.0134
R^2	0.9049	0.9541	0.9124	0.8968	0.8767	0.8245	0.8949
			BC				
Root mean square error	0.0253	0.0264	0.0114	0.0111	0.0106	0.0232	0.0180
Index of agreement, d	0.9363	0.9354	0.9780	0.9805	0.9805	0.9144	0.9542
Maximum absolute error	0.0910	0.0754	0.0288	0.0223	0.0250	0.0502	0.0488
Mean absolute error	0.0157	0.0182	0.0095	0.0098	0.0084	0.0187	0.0134
R^2	0.8418	0.9200	0.9365	0.9280	0.9347	0.8473	0.9013
			VG				
Root mean square error	0.0238	0.0181	0.0136	0.0114	0.0114	0.0185	0.0161
Index of agreement, d	0.9461	0.9729	0.9715	0.9801	0.9762	0.9357	0.9638
Maximum absolute error	0.0843	0.0578	0.0389	0.0214	0.0296	0.0430	0.0458
Mean absolute error	0.0161	0.0129	0.0103	0.0096	0.0090	0.0155	0.0122
R^2	0.8522	0.9344	0.9317	0.9265	0.9421	0.8541	0.9068

Table 7. Statistical Indexesto Judge Efficacy of Point Pedotransfer Functions to Estimate Field Capacity and Permanent Wilting Point

Input/index	Neural pedotransfer functions				K-nearest pedotransfer functions			
	SSC	SSC BD	SSC BDOM	SSC OM	SSC	SSC BD	SSC BDOM	SSC OM
Pedotransfer functions to estimate field capacity								
Root mean square error	0.0905	0.0634	0.0632	0.0693	0.0591	0.055	0.06	0.0611
Index of agreement, d	0.4827	0.7694	0.7191	0.7148	0.7962	0.8354	0.7894	0.7811
Maximum absolute error	0.2297	0.1606	0.1624	0.1983	0.1497	0.1551	0.1589	0.1661
Mean absolute error	0.0723	0.0510	0.0520	0.0528	0.0465	0.0436	0.0483	0.049
R^2	0.0270	0.4164	0.4400	0.3148	0.4872	0.574	0.4912	0.4562
Pedotransfer functions to estimate permanent wilting point								
Root mean square error	0.0413	0.0373	0.0367	0.0400	0.0417	0.0346	0.0346	0.0407
Index of agreement, d	0.8000	0.8553	0.8590	0.8253	0.8094	0.8847	0.88	0.8164
Maximum absolute error	0.1090	0.0955	0.0899	0.0978	0.101	0.0819	0.0991	0.1122
Mean absolute error	0.0324	0.0300	0.0299	0.0331	0.0347	0.0285	0.0274	0.0339
R^2	0.5305	0.6152	0.6211	0.5599	0.4994	0.6723	0.6728	0.5309

**Fig. 11.** One-to-one comparison of observed and predicted field capacity using neural PTF with input of texture and BD**Fig. 13.** One-to-one comparison of observed and predicted field capacity using kNN PTF with input of texture and BD**Fig. 12.** One-to-one comparison of observed and predicted field capacity using neural PTF with maximum input (texture, BD, OM)**Fig. 14.** One-to-one comparison of observed and predicted permanent wilting point using neural PTF with input of texture and BD

FC/PWP. It was inferred that input SSCBD was adequate enough to obtain acceptable FC estimates and, therefore, kNN PTF using SSCBD as an input was recommended. The results could be useful to irrigation managers as often the decisions are based on broad approximations of texture. Textural composition and bulk density data are relatively easy to measure and/or available. Thus accuracy of decisions could be improved.

In general, PWP estimates were obtained with better accuracy than estimates of FC. At a minimum input level (SSC), kNN as well as neural PTFs were on par. At other input levels, the difference in prediction as displayed by the statistical indices was indicative of

par performance of both the types of PTFs. Compared with PTFs to predict FC, inclusion of OM did improve prediction of PWP, but the magnitude of improvement was marginal. Though better predictions of FC/PWP were obtained at the maximum input level (SSCBDOM) in both the types of PTFs, the error improvement was again marginal, and it is reasonable to conclude that PTFs using SSCBD input (Figs. 11, 13, 14, and 15) would serve the purpose of utility, and maximum input could be used only when such data are available. Comparative evaluation of FC and PWP estimates by ANN PTFs, kNN PTFs, and SWRC functions revealed that the

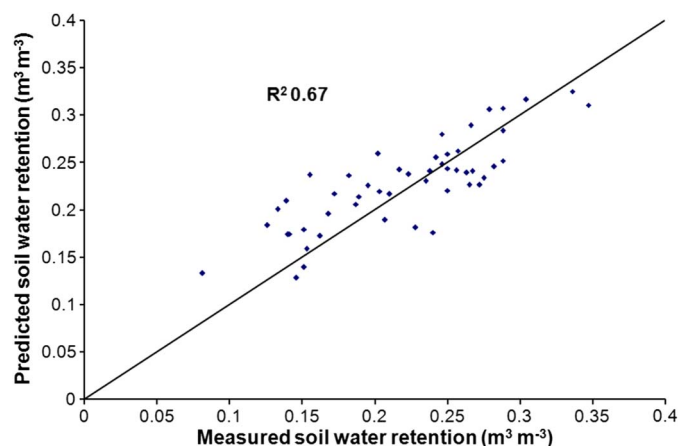


Fig. 15. One-to-one comparison of observed and predicted permanent wilting point using kNN PTF with input of texture and BD

continuous functions performed relatively better (Figs. 16–19) because of a lower magnitude of errors (Tables 3 and 7) and better agreement between measured and estimated data. Campbell and VG functions in general outperformed other functions and approaches. It is evident that analytic functions are preferable for accurate estimates, but the function must be chosen after evaluation, as not all the four functions evaluated in this paper exhibited a comparative edge over ANN and kNN estimates. The modified Campbell function was the poorest performer among all the approaches. Neural networks are expected to perform better with greater than three variables, and it is also logical to expect improved modeling if the number of variables known to influence the dependent property are increased. However, the results outlined that parsimony is also important, as fewer number of variables also provided reasonable estimates. The earlier findings (Patil et al. 2012) were also confirmed that, as a tool, kNN performed better (Figs. 11–15) than neural networks with additional advantage of simplicity in use, and it is also possible to append the development

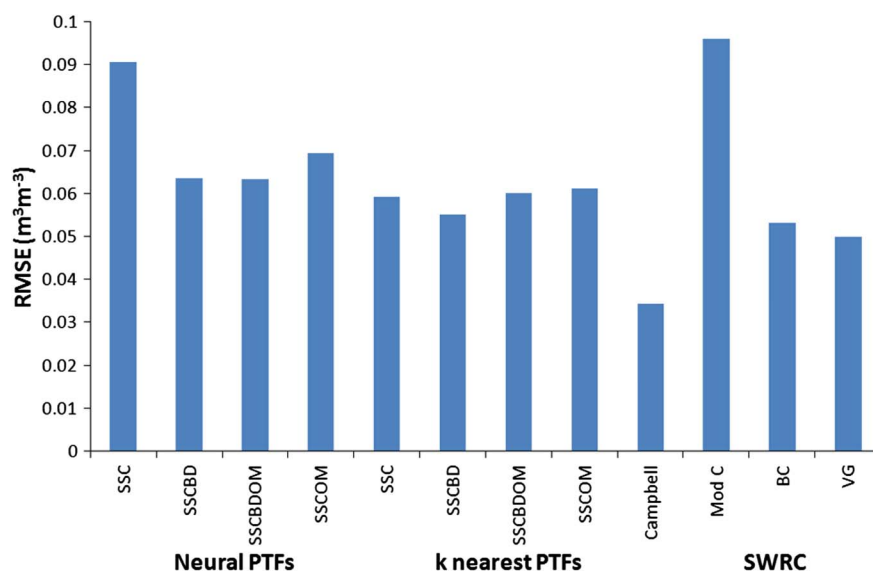


Fig. 16. Performance of ANN, kNN, and analytic functions in estimating FC as indicated by RMSE

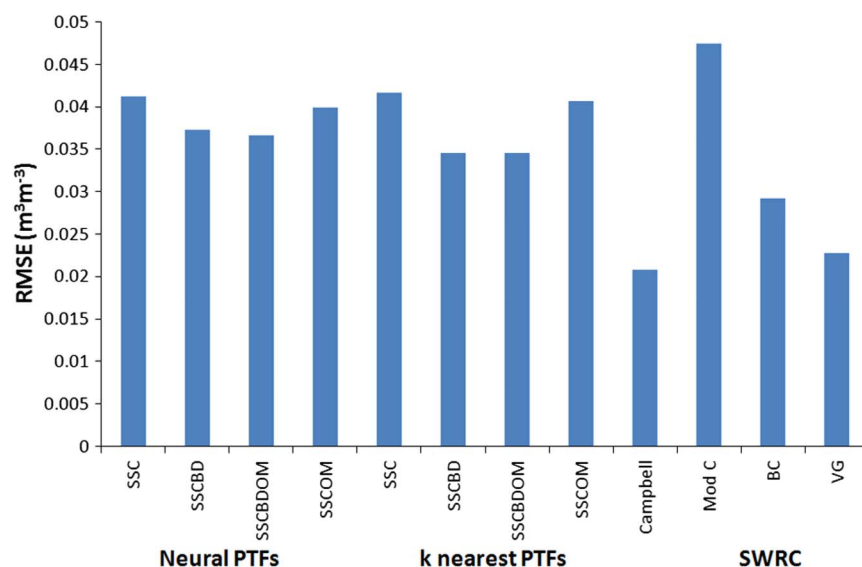


Fig. 17. Performance of ANN, kNN, and analytic functions in estimating PWP as indicated by RMSE

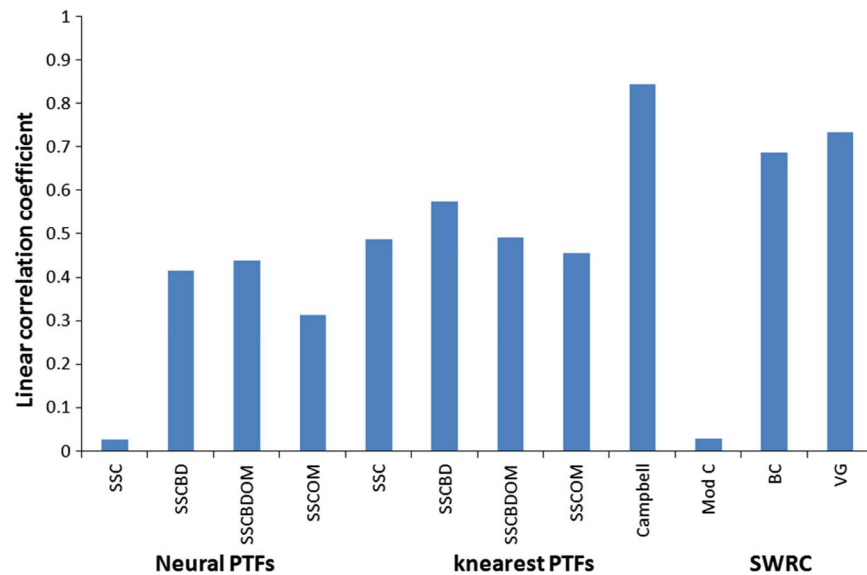


Fig. 18. Performance of ANN, kNN, and analytic functions in estimating FC as indicated by linear correlation coefficient

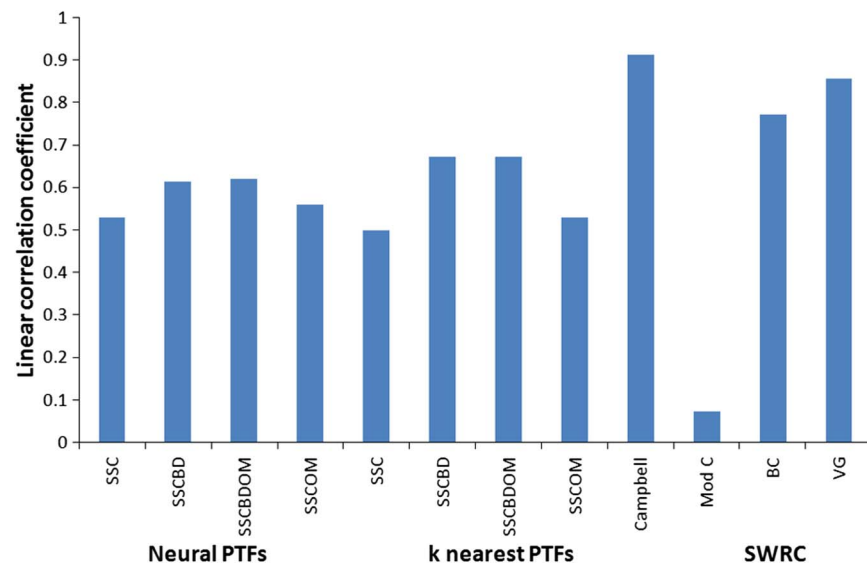


Fig. 19. Performance of ANN, kNN, and analytic functions in estimating PWP as indicated by linear correlation coefficient

data set. Therefore, it could be a tool of choice. The authors have also opined that, with acquisition of more SWRC data on vertisols, the PTFs need refinement for better predictions. Since ANN does not provide flexibility of appending data, PTFs would need to be redeveloped each time the data are added. In contrast, kNN PTFs could be refined without reprocessing. With proven acceptable accuracy and less computing time, kNN PTFs will be important for applications.

Conclusion

After evaluation, two of the 10 functions to describe SWRC of the black soil region of India were recommended—Campbell and Van Genuchten. Neural regression and kNN techniques of PTF development were evaluated. PTFs using textural information and bulk density as inputs were recommended to predict field capacity and permanent wilting point. Superior ability of kNN PTFs in predicting FC/PWP of BSR soils was noted. The study demonstrated

that the kNN technique can be as competitive as widely used neural regression with the additional benefit of appending the development data as and when desired. The proposed PTFs are expected to be useful in managing the BSR soils of India.

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