आई.ए.एस.आर.आई./पी.आर.--05/2020
I.A.S.R.I./P.R.-05/2020

संस्थान परियोजना कोड: AGENIASRISIL201700400090

हर कदम, हर डगर
किसानों का हमसफर
भारतीय कृषि अनुसंधान परिषद
Agrisearch with a human touch

PROJECT REPORT
परियोजना रिपोर्ट

Generalized Row-Column Designs for Crop and Animal Experiments

फसल और पशु प्रयोगों के लिए सामान्यीकृत पंक्ति–स्तम्भ अभिकल्पनायें

Project Team

Anindita Datta	अनिंदिता दत्ता
Mohd Harun (upto 11.09.2018)	मोहम्मद हारुन
Seema Jaggi	सीमा जग्गी
Cini Varghese	सिनी वरगीस
Arpan Bhowmik (w.e.f. 12.09.2018)	अर्पण भौमिक

परीक्षण अभिकल्पना प्रभाग
Division of Design of Experiments

भा.कृ.अनु.प.–भारतीय कृषि सांख्यिकी अनुसंधान संस्थान
लाइब्रेरी एवेन्यू, पूसा, नई दिल्ली–110012

ICAR-Indian Agricultural Statistics Research Institute
Library Avenue, PUSA, New Delhi – 110 012

(2020)

आमुख

जब परीक्षण इकाइयों में परिवर्तन के दो ऐसे श्रोत हों तो रो-कॉलम अभिकल्पना ऐसी प्रायोगिक स्थिति के लिए उपयोगी होते हैं। इन डिज़ाइनों का उपयोग क्षेत्र और पशु प्रयोगों में परिवर्तनशीलता को नियंत्रित करने के लिए किया जाता है। साहित्य में विकसित अधिकांश रो-कॉलम अभिकल्पना में प्रत्येक पंक्ति और स्तंभ के प्रतिच्छेदन के अनुरूप एक इकाई होती है। हालांकि, उदाहरणों के लिए जब सीमित प्रायोगिक संसाधनों के साथ उपचार की संख्या बड़ी है, तो सामान्यीकृत रो-कॉलम अभिकल्पनाओं का उपयोग किया जाता है जहां प्रत्येक पंक्ति-स्तंभ प्रतित्छेदन में एक से अधिक इकाई होती है। सामान्यीकृत रो-कॉलम अभिकल्पनाओं में p पंक्तियों और q कॉलम में वी ट्रीटमेंट की एक व्यवस्था है, जैसे कि प्रत्येक पंक्ति और स्तंभ (सेल) के प्रतिच्छेदन में एक से अधिक यूनिट होते हैं।

आदर्श स्थिति संभालने वाले विभिन्न पैरामीट्रिक संयोजनों के लिए साहित्य में सामान्यीकृत रो-कॉलम अभिकल्पना विकसित किए गए हैं। हालांकि, प्रयोग के दौरान आउटलेयर की उपस्थिति, डेटा में गुम टिप्पणियों, प्रयोगात्मक इकाइयों में एक व्यवस्थित प्रवृत्ति की उपस्थिति, उपचार के आदान-प्रदान आदि हो सकते हैं। इन गड़बड़ियों से प्रयोग में आजमाए गए उपचारों की तुलना में कम सटीक तुलना हो सकती है। ऐसी स्थितियों को दूर करने के लिए, इन गड़बड़ियों के खिलाफ असंवेदनशील या मजबूत होने वाले डिजाइनों की आवश्यकता होती है। इस अध्ययन में, दक्षता मानदंडों के अनुसार एक सेल के भीतर एक या एक से अधिक टिप्पणियों के लापता होने के खिलाफ जीआरसी डिजाइनों के विभिन्न वर्गों के प्रबलता की जांच की गई है। मजबूत सामान्यीकृत रो-कॉलम अभिकल्पना की एक सूची ने मापदंडों और डिजाइनों की दक्षता को तैयार किया है।

सामान्यीकृत रो-कॉलम अभिकल्पना में, चूंकि एक सेल में अधिक संख्या में इकाइयाँ होती हैं, इसलिए यह संभावना है कि एक प्रायोगिक इकाई पर लगाया गया उपचार एक ही सेल में पड़ोसी इकाई की प्रतिक्रिया को प्रभावित कर सकता है, यदि इकाइयों को गोलाकार प्रभाव देने के लिए रैखिक रूप से आसन्न रखा जाता है। इस अध्ययन में, इन स्थानिक प्रभावों के लिए संतुलित जीआरसी डिजाइनों की श्रृंखला विकसित की गई है। कुशल डिजाइनों की एक सूची तैयार की गई है। राष्ट्रीय कृषि अनुसंधान और शिक्षा प्रणाली (NARES) के तहत अंतिम उपयोगकर्ताओं को एक रेडीमेड समाधान प्रदान करने के लिए, एक SAS मैक्रो विकसित किया गया है जो डिजाइनों के लेआउट को उत्पन्न करता है।

WebGRC नामक एक वेब सॉल्यूशन को सामान्यीकृत रो-कॉलम अभिकल्पना की पीढ़ी के लिए विकसित किया गया है जो कि प्रयोगकर्ताओं के लिए अत्यधिक उपयोगी होगा। वेबपेज उपचार की दी गई संख्या के लिए यादृच्छिक लेआउट के साथ लेआउट योजनाओं को प्रदर्शित करता है। जीआरसी डिजाइनों का एक ऑनलाइन कैटलॉग भी तैयार किया गया है और सॉफ्टवेयर में शामिल किया गया है जिसमें उपयोगकर्ता सभी मापदंडों को देखकर डिजाइन का चयन कर सकता है और फिर यादृच्छिक लेआउट प्राप्त कर सकता है।

मेटिंग प्लान (आंशिक डायलेल क्रॉस, आंशिक ट्रायल समानांतर क्रॉस) के निर्माण के लिए सामान्यीकृत **रो-कॉलम अभिकल्पना** के एक आवेदन पर भी चर्चा की गई है। ब्रीडर्स आँकड़ों में आरामदायक ज्ञान के साथ छोटे और कुशल डायलेल और समानांतर क्रॉस प्लान प्राप्त कर सकते हैं।

सभी लेखक, निदेशक (का.), भा.कृ.अनु.प.–भा.कृ.सां.अ.सं. को उनके समर्थन एवं अनुसंधान कार्य को सफलतापूर्वक करने के लिए सभी आवश्यक सुविधाएं उपलब्ध कराने के लिए हार्दिक धन्यवाद अभिव्यक्त करते हैं। भा.कृ.अनु.प. –भा.कृ.सां.अ.सं., परीक्षण अभिकल्पना प्रभाग के अध्यक्ष (का.), वैज्ञानिक, तकनीकी एवं प्रभाग के अन्य कर्मचारियों के सहयोग का धन्यवाद सहित आभार व्यक्त करते है। हम सभी लेखक, भारतीय सांख्यिकी संस्थान, कोलकाता के सेवा निवृत प्राध्यापक विकास कुमार सिन्हा के प्रति भी उनसे उपयोगी चर्चा करने के लिए कृतज्ञ हैं। हम आन्तरिक निर्णायक का भी धन्यवाद व्यक्त करते हैं, जिनके सुझावों ने इस प्रतिवेदन की विषय वस्तु सुधारने एवं प्रस्तुतीकरण में सहायता की।

अनिंदिता दत्ता	मोहम्मद हारुन	सीमा जग्गी	सिनी वरगीस	अर्पण भौमिक
प्र. प.	स. प्र. प.	स. प्र. प.	स. प्र. प.	स. प्र. प.

PREFACE

When there is cross classified variation in the experimental unit then Row-Column (RC) designs are useful for such experimental situation. These designs are used to control variability in field and animal experiments. Most of the row-column designs developed in the literature have one unit corresponding to the intersection of each row and column. However, for the instances when the number of treatments is large with limited experimental resources, Generalized Row-Column (GRC) designs are used where there is more than one unit in each row-column intersection. GRC design is an arrangement of v treatments in p rows and q columns such that the intersection of each row and column (cell) consists of more than one unit.

GRC designs have been developed in the literature for different parametric combinations assuming ideal situation. However, there may be presence of outliers, missing observations in the data, presence of a systematic trend in the experimental units, exchange or interchange of treatments etc. during the experimentation. These disturbances may lead to less precise comparisons among treatments tried in the experiment. In order to overcome such situations, designs which was insensitive or robust against these disturbances are required. In this study, Robustness of different classes of GRC designs against missing of one or more observations within a cell as per the efficiency criteria has been investigated. A list of robust GRC designs has prepared giving the parameters and the efficiency of the designs.

In GRC designs, since there are more number of units in a cell, it is likely that the treatment applied to one experimental unit may affect the response of the neighbouring unit in the same cell if the units are placed linearly adjacent giving rise to spatial effects. In this study, series of GRC designs balanced for these spatial effects have been developed. A list of efficient designs has been prepared. For providing a readymade solution to the end users under National Agricultural Research and Education Systems (NARES), a SAS macro has been developed that generates the layout of the designs.

A web solution named *Web*GRC has been developed for the generation of GRC designs that would be highly useful to the experimenters. The webpage displays the layout plans along with the randomized layout for given number of treatments. An online catalogue of the GRC designs is also prepared and included in the software wherein the user can select the design by seeing all the parameters and then can get the randomized layout.

An application of GRC designs for construction of mating plan (partial diallel cross, partial triallel cross) has also been discussed. Breeders can obtain small and efficient diallel and triallel cross plans with comfortable knowledge in statistics.

The authors express their sincere thanks to the Director (A), ICAR-IASRI for his support and for providing all necessary facilities to carry out the research work successfully. The cooperation received from Head (A), Division of Design of Experiments, ICAR-IASRI, scientific, technical and other staff in the Division are thankfully acknowledged. The authors are also grateful to Professor Bikas K Sinha, Retired Professor, Indian Statistical Institute, Kolkata for some useful discussion. We also express our thanks to the internal referee whose suggestions helped in improving the contents and presentation of this report.

Anindita Datta	Mohd Harun	Seema Jaggi	Cini Varghese	Arpan Bhowmik
PI	Co-PI	Co-PI	Co-PI	Co-PI

CONTENTS

CHAPTER 1

INTRODUCTION AND BACKGROUND

1.1 Introduction

When the heterogeneity present in the experimental material is from two sources, then two-dimensional blocking or double blocking of the experimental units is recommended for control or reduction of experimental error. The two blocking systems are referred to generally as row blocking and column blocking and the resulting designs are termed as Row-Column (RC) designs. These designs are used to control variability in field and animal experiments. For example, in a greenhouse experiment on tobacco mosaic virus, the experimental unit is a single leaf. The plant and the position of the leaf on the plant may affect the number of lesions produced per leaf by rubbing the leaf with a solution, which contain the virus. Thus, here individual plant is one source of variability and represents rows and the position of the leaf from top to bottom on each plant represent columns (Youden, 1937). Another situation is in case of a laboratory trial to compare the percentage of protein in various grains, rows may be the different analysts and columns may be the occasions. Further, in an irrigation experiment in horticultural research, rows may be represented by channels and columns by the positions along the channels.

Latin square design is the simplest row-column design. In a Latin square design, v treatments are arranged in v rows and v columns in such a way that each treatment occurs once in each row and once in each column e.g. an animal experiment is conducted to compare the effects of four feeds eliminating the variation due to four breeds and four age groups of calves. Data is on growth rate of calves during a certain period. Here, rows represent age groups and columns represent different breeds. Following is the arrangement of a Latin square design for this situation with rows and columns complete.

Rows (Age Groups)	Columns (Breeds)			
	I	II	III	IV
I	1	2	3	4

II	2	3	4	1
III	3	4	1	2
IV	4	1	2	3

Latin squares have the restriction that the rows and columns must be equal in number to one another and to the number of treatments. In practical experiments Latin squares are very useful where the number of treatments is small. The available range of designs is generally restricted to sizes from about 4 × 4 to about 7 × 7. The upper end of the size range can be extended by using incomplete Latin squares of size $(n-1) \times n$ or size $n \times (n-1)$ obtained by deleting one complete row or one complete column from a Latin square of size $n \times n$ (Yates, 1936) whereas the lower end of the scale can be extended by using augmented Latin squares of size $(n + 1) \times n$ or of size $n \times (n + 1)$ obtained by repeating a complete row or a complete column of a Latin square of size $n \times n$ (Pearce, 1952). So when experimental units are in rectangular array then these designs are useful. Various types of row-column designs and their properties are discussed in Hinkelmann and Kempthorne (2005).

1.2 Genesis and Rationale of the Project

Most of the row-column designs developed in the literature have one unit corresponding to the intersection of each row and column. However, for the instances when the number of treatments is large with limited experimental resources, Generalized Row-Column (GRC) designs are used where there is more than one unit in each row-column intersection. GRC design is an arrangement of v treatments in p rows and q columns such that the intersection of each row and column consists of more than one unit. Following are some examples:

- To compare a number of dietary treatments on mice, different breeds and different age groups constitute the two sources of variability. The cages available with the experimenter have two partitions accommodating two mice of same parity, one in each partition. Hence, corresponding to each breed-age combination there are two mice, each receiving one distinct treatment.
- In an experiment to compare twelve pest control treatments on apple trees, four long replicate rows, each one tree wide, are used with twelve plots per row. Each row is

subdivided into four blocks or cells of three plots and the four adjacent blocks at any one position along the four rows formed a replicate column of twelve plots.

Some more experimental situations are described below along with designs appropriate for such situations.

Experimental Situation 1 (Bailey and Monod, 2001): An experiment was conducted on tobacco plants at Rothamsted Experimental Station to check whether a mechanism to inhibit tobacco mosaic virus had been carried over to following generations. Each treatment was a solution made from an extract of one of the offspring plants. The solution was rubbed onto several half-leaves of normal tobacco plants. The number of lesions per half leaf was measured and the logarithm of this number analyzed by ANOVA. There are eight plants and pair of half leaves at four heights. A row-column design which has less number of rows than columns is useful in such situations as the number of plants available for the experiment is typically more than the number of usable leaves and their positions per plant. The experimenter is interested to compare more than two treatments in c plants each with leaves at r heights, where typically $r < c$. Generally, the two half leaves of each of the rc leaves form the plots. So here leaf heights represent rows and the plants as columns and two plots in the intersection of each row and column. For such situations the following GRC designs is useful:

Heights	Plants							
	I	II	III	IV	V	VI	VII	VIII
I	5 6	6 7	7 8	8 1	1 2	2 3	3 4	4 5
II	2 8	3 1	4 2	5 3	6 4	7 5	8 6	1 7
III	1 4	2 5	3 6	4 7	5 8	6 1	7 2	8 3
IV	3 7	4 8	5 1	6 2	7 3	8 4	1 5	2 6

Experimental Situation 2 (Bailey, 1992)**:** Consider a food sensory experiment where 6 food items are to be compared. The experiment is conducted in 3 sessions. There are 6 panelists and each of them will taste 2 food items at each session. In this case, a GRC design with 3 rows, 6 columns with each row-column intersection having cell of size 2 can be used. Following is the arrangement of such a design:

Sessions	Panelists					
	I	II	III	IV	V	VI
I	1 4	2 6	2 5	3 5	6 3	4 1
II	2 3	1 5	4 6	6 1	4 5	3 2
III	6 5	4 3	3 1	2 4	1 2	6 5

Experimental Situation 3 (Edmondson, 1998)**:** An experiment was conducted to compare the colour intensities of apple sauce. The treatments consist of all combinations of 12 blends of apple sauce with 4 concentrations of cinnamon. Treatments could be stored for 4 different lengths of time. A GRC design was used in which rows, columns and symbols represented cinnamon concentrations, storage times and blend respectively as shown below:

Rows (Cinnamon Concentrations)	Columns (Storage Time)			
	I	II	III	IV
I	1 5 9	2 6 10	3 7 11	4 8 12
II	2 7 10	1 8 9	4 5 12	3 6 11
III	3 8 12	4 7 11	1 6 10	2 5 9
IV	4 6 11	3 5 12	2 8 9	1 7 10

This arrangement ensures that each of the 12 treatments occurred once and that both treatment factors were orthogonal to storage times. Part of the interaction between blends and concentrations was totally confounded with storage times.

In usual practice, these trials are conducted under controlled conditions and it is assumed that there are no disturbances that occur while conducting or measuring the observations. The presence of missing observations, outliers in the data, etc. are some of the disturbances that may occur during experimentation. These disturbances may lead to wrong interpretation of results or less precise comparisons among treatments tried in the experiment. In order to overcome such situations, designs which are insensitive or robust against missing observations/ outliers were required.

In case of a generalized row-column design there are more number of units in a plot and the treatment applied to one experimental unit in a plot may affect the response on neighbouring unit in the same plot. Experiments conducted in field may show neighbour effects (spatial indirect effects), like when treatments are varieties, neighbour effects may be due to differences in height, root vigour, or germination date, especially on small plots. Treatments such as fertilizer, irrigation, or pesticide may spread to adjacent unit of the same plots causing neighbour effects. Such experiments exhibit neighbour effects, because the effect of having no treatment as a neighbour is different from the neighbour effects of any treatment. Thus, neighbour effects resulting in competition or interference between neighbouring units may contribute to variability in experimental results and lead to substantial losses in efficiency. In order to compare the effects of treatments in this situation, it is important to ensure that no treatment is unduly disadvantaged by its neighbour. Neighbour balance is considered a desirable property for an experiment to possess in situations where neighbour effects from the treatments applied in adjacent experimental units are known to exist. Thus, neighbour-balanced designs or designs balanced for spatial indirect effects, wherein the allocation of treatments is such that every treatment occurs equally often with every other treatment as neighbour(s), are used for these situations. These designs permit the estimation of direct and neighbour effect(s) of treatments. So GRC designs balanced for spatial indirect effects were required to be developed.

A number of GRC designs are developed in the literature. For easy accessibility and quick reference of GRC designs by the experimenters, a web solution for cataloging and generation of GRC designs is to be developed. A number of web solutions have been developed by IASRI, viz., Design Resource Server, web generation of experimental designs balanced for indirect effects of treatments, online analysis of block designs / row-column designs, web service for Analysis of Augmented Designs, web solutions for PBIB designs, statistical package for factorial experiments. A web solution for GRC designs, on similar lines, would be helpful.

Keeping the above in view, the following objectives have been formulated:

Objectives

- To identify robust GRC designs in the presence of missing observation(s)/ outlier(s)
- To obtain methods for constructing GRC designs balanced for spatial/temporal indirect effects
- To develop a web solution for the cataloguing and generation of GRC designs

1.3 Critical Review of the Technology at National and International Levels

National

GRC designs are studied in the literature in different names such as Semi-Latin square in which there are *n* rows, *n* columns and intersection of each row and column contains a cell of k units , Trojan square Semi-latin rectangles, Generalized incomplete Trojan-type designs and Row-column designs with multiple units per cell. Some work related to GRC designs are given here. SahaRay (2001) studied designs with unequal row and column sizes. Chigbu (2003) obtained the best of the three optimal $(4 \times 4)/4$ semi-Latin squares by finding and comparing the variances of elementary contrasts of treatments for the squares. Parsad (2006) discussed a method of constructing semi-Latin square with $v = 2n$ treatments in n rows, n columns and cell size $k = 2$ by developing initial column. Varghese and Jaggi (2011) obtained generalized row-column designs with unequal cell sizes. Datta *et al*. (2014) obtained some methods of constructing row-column designs with multiple units per cell that are structurally incomplete. Datta *et al.* (2015) developed methods of constructing row-column designs with multiple units per cell with equal/ unequal cell sizes that are structurally complete, i.e. all the cells corresponding to the intersection of row and column receive at least two treatments.

There is some work done related to the study of robustness of RC designs in national level. Lal *et al.* (2003) investigated the robustness of Youden square and Latin square designs against the loss of any t ($\geq$ l) observations in a column/row and for the loss of any two observations in the design as per connectedness criterion. Bhar (2014) defined E-efficiency criterion and obtained lower bound of this criterion for the loss of any t observations in binary variance balanced block design.

Online generation of experimental designs provides an easy accessibility to the users. In this direction a lot of work has been done at IASRI. Taksande *et al*. (2012) developed software solution for the generation of partial diallel crosses. Many other open sources and commercial packages are also available for generation of readymade layouts of designs based on different situations [for example AgroPlotter (2002), Design resource server (2007), webPD (2015), etc.]

International

Trojan squares were first discussed by Harshbarger and Davis (1952) but then it was named as Latinized Near Balanced Rectangular Lattices having $k = n-1$. Later, Darby and Gilbert (1958) discussed the general case for $k < n$ and introduced the name Trojan square designs where $k > 2$. However, all designs of the Latinized Rectangular Lattice type are now commonly described as Trojan squares for any $1 < k < n$. Williams (1986) generalized the notion and called semi-Latin squares as Latinized incomplete-block designs. Andersen and Hilton (1980) called semi-Latin squares as (1, 1, k) Latin rectangles. Preece and Freeman (1983) discussed the combinatorial properties of semi-Latin squares and related designs. Bailey (1988) discussed further construction for a range of semi-Latin and Trojan square designs. Bailey (1992) gave methods of constructing a range of semi-Latin and Trojan square designs, studied their efficiencies and showed that the Trojan squares are the optimal choice of semi-Latin squares for pair-wise comparisons of treatment means. These are particularly suitable for crop research experiments either in field or in the glasshouse. Trojan squares are normally the best choice of semi-Latin squares for crop research (Edmondson, 1998)). Bedford and Whitaker (2001) have given several methods of construction of semi-Latin squares. Dharmalingam (2002) gave an application of Trojan square designs and used it to obtain partial triallel crosses. Jaggi *et al.* (2010) defined generalized incomplete Trojan-Type designs to be a row-column design in which each cell, corresponding to the intersection of row and column, contains more than one treatment and the rows are incomplete. A method of constructing generalized incomplete Trojan-Type design was developed and some properties of this class of designs are discussed. The contrasts properties of the optimal semi-Latin squares with side six and block size two was investigated by Uto and Ekpenyong (2014) with a view to discriminating amongst them. Some reference of semi-Latin squares and Trojan squares can be found in Dean *et al.* (2015). It is seen in the literature that most of the work on designs with neighbour effects is concentrated under block design set up. There are few work related to neighbour effect under row-column set up. Jaggi *et al*. (2016) obtained another series of generalized incomplete Trojan-type designs for number of treatments v= sm+1.

It seems that some work related to study of robustness of RC designs in international level. Singh *et al.* (1987) studied robustness of designs eliminating heterogeneity in two directions to outliers. Varghese *et al.* (2002) showed that Williams square change-over designs are robust against missing of last α [$\leq$ v -1: v being the number of period in the design for v treatments] observations from an experimental unit.

There is some work done related to study of RC designs incorporating spatial indirect in international level. Freeman (1979) has given some row-column designs balanced for neighbours with and without border plots. Federer and Basford (1991) have given three methods of constructing balanced nearest neighbour row-column or competition effect designs. Chan and Eccleston (2003) have given an algorithm which generates neighbour balanced row–column Designs. However, the designs obtained are found to be only combinatorially balanced. Varghese *et al.*(2014) obtained row-column designs incorporating directional neighbour effects.

Sharma *et al*. (2013) developed web solution for generating partially balanced incomplete block designs. Jaggi *et al*. (2015) developed web-enabled software for generation of experimental designs balanced for indirect effects of treatments.

1.4 Scope of Present Study

Robustness of different classes of GRC designs against missing of one or more observations has been investigated and the efficiency of the residual designs have been reported and summarized. Neighbour Balanced Generalized Row-Column (NBP-GRC) designs have been defined. Methods of constructing series of NBGRC have been described. Construction of Generalized Row-Column design involves theoretical understanding and it may not be easy for the experimenters to understand. So, a readily available web solution named ***webGRC*** along with online catalogue has been developed. This would provide a readymade solution to the experimenters which will ultimately reduce the effort of the experimenter. Further SAS macros have also been developed which would help experimenters under NARES to get readymade layout plans. An application of GRC designs for construction of mating designs has been discussed.

CHAPTER 2

ROBUSTNESS OF GRC DESIGNS AGAINST MISSING OBSERVATION(S)

2.1 Introduction

The presence of missing observations, outliers in the data, etc. are some of the disturbances that may occur during experimentation. These disturbances may lead to less precise comparisons among treatments tried in the experiment. A lot of work has been done on robustness of designs in block set up or row-column set up.

A GRC design is robust against loss of observations, if the loss of efficiency of the residual design as compared to the original design is small. If C_d is the information matrix for estimating the treatment effects of GRC design d and C_{d*} is that of the residual design $d*$ after the observations are lost, then the efficiency E of the residual design relative to the original design is given by

$$E = \frac{\text{Harmonic mean of non-zero eigen values of } C_{d*}}{\text{Harmonic mean of non-zero eigen values of } C_d}$$

A GRC design is said to be robust if the efficiency of the resulting design after loss of information is more than 90%.

A list of robust GRC design has prepared giving the parameters and the efficiency of the designs. A SAS code (given in the Annexure I) has been written in PROC IML to calculate the harmonic mean of non-zero eigen-values of information matrix of original design and the residual design under the following three-way model for GRC design.

A GRC design is considered here with v treatments arranged in p rows, q columns and in each row-column intersection (i.e. cells) there are k units or plots resulting in total $n= pqk$ experimental units or observations. The following three-way classified model with treatments, rows and columns is considered:

$$Y_{l(ij)} = \mu + \tau_{l(ij)} + \alpha_i + \beta_j + e_{l(ij)}; \qquad ...(2.1)$$

$$i = 1,2,\ldots,p;\ j = 1,2,\ldots,q;\ l = 1,2,\ldots,k$$

where $Y_{l(ij)}$ is the response from the l^{th} unit corresponding to the intersection of i^{th} row and j^{th} column. μ is the general mean, $\tau_{l(ij)}$ is the effect of the treatment appearing in the l^{th} unit corresponding to the intersection of i^{th} row and j^{th} column, α_i is the i^{th} row effect and β_j is the j^{th} column effect. $e_{l(ij)}$ is the error term identically and independently distributed and following normal distribution with mean zero and constant variance.

2.2 Robustness of GRC Designs Against Missing Observation(s)

Here in this section, the robustness of different classes of GRC designs (Bailey, 1992; Jaggi *et al.*, 2010; Datta *et al.* 2012; Datta *et al.,* 2015) against missing of one or more observations within a cell as per the efficiency criteria, has been investigated. We consider a design be highly robust against missing observation(s) if the loss in efficiency of the residual design is not more than 5% and robust if the loss in efficiency of the residual design is between 5% to 10%.

Series I: Bailey (1992) defined semi-Latin square $(n \times n / k)$ as an arrangement of $v = nk$ treatments in n rows and n columns and intersection of each row and column containing k units each. These semi-Latin squares are constructed by superimposing k number of Latin squares of order n and symbols of each Latin square are represented by different symbols.

Example I.1: Following is a semi-Latin square for v = 10 treatments arranged in 5 rows, 5 columns and intersection of each row-column having 2 units:

Rows	Columns				
	I	II	III	IV	V
I	1 6	2 7	3 8	4 9	5 10
II	2 8	3 9	4 10	5 6	1 7
III	3 10	4 6	5 7	1 8	2 9
IV	4 7	5 8	1 9	2 10	3 6
V	5 9	1 10	2 6	3 7	4 8

Example I.2: Following is a semi-Latin square for v = 12 treatments arranged in 4 rows, 4 columns and intersection of each row-column having 3 units:

Rows	Columns			
	I	II	III	IV
I	1 5 9	2 8 11	3 6 12	4 7 10
II	2 6 10	1 7 12	4 5 11	3 8 9
III	3 7 11	4 6 9	1 8 10	2 5 12
IV	4 8 12	3 5 10	2 7 9	1 6 11

The robustness of this class of designs has been investigated against missing of some/ all observations of last column. Without loss of generality, the observations from units of last column are assumed to be missing as the columns can always be interchanged. Table 2.1 gives the parameters of the designs considered i.e., number of treatments ($v \leq 25$), number of rows (p), number of columns (q), replication (r), cell size (k) and the number of observation(s) missing with the unit/ cell number of the last column from which the observation(s) are missing along with the efficiency (E) of the residual design relative to the original design. The efficiency has been obtained by taking the ratio of harmonic means (HM) of information matrix $\mathbf{C}_d$ for treatment effects of original design with all observations to that of residual design $\mathbf{C}_{d*}$ with missing observations.

Table 2.1: Parameters and efficiency of the residual design for Series I

S. No	**v**	**p**	**q**	**r**	**k**	**No. of observations missing**	**Unit/ Cell No.**	**HM ($\mathbf{C}_d$)**	**HM ($\mathbf{C}_{d*}$)**	**E**
1	6	3	3	3	2	1	last unit in last cell	3.00	2.67	0.89
2	6	3	3	3	2	2	both units in last cell	3.00	2.31	0.77

3	6	3	3	3	2	2	any two units from different cells	3.00	2.33	0.78
4	6	3	3	3	2	3	any three units from different cells	3.00	2.07	0.69
5	6	3	3	3	2	4	last two units from different cells and last cell total	3.00	2.07	0.69
6	8	4	4	4	2	1	last unit in last cell	3.87	3.49	0.90
7	8	4	4	4	2	2	last cell total	3.87	3.31	0.86
8	8	4	4	4	2	2	any two observations from last units of last column	3.87	3.29	0.85
9	8	4	4	4	2	3	any three observations from last unit of last column	3.87	3.00	0.78
10	8	4	4	4	2	4	last two units from different cells and last cell total	3.87	2.80	0.72
11	8	4	4	4	2	5	last three units from different cells and last cell total	3.87	2.68	0.69
12	12	4	4	4	3	1	last unit in last cell	4.00	3.87	0.97
13	12	4	4	4	3	2	last any two units from last cell	4.00	3.74	0.94
14	12	4	4	4	3	3	last cell total	4.00	3.62	0.90
15	12	4	4	4	3	2	any two observations from last unit of last column	4.00	3.73	0.93

16	12	4	4	4	3	3	any three observations from last unit of last column	4.00	3.59	0.90
17	12	4	4	4	3	4	last unit from other cell and last cell total	4.00	3.46	0.86
18	12	4	4	4	3	6	last three units from different cells and last cell total	4.00	3.25	0.81
19	10	5	5	5	2	1	last unit in last cell	5.00	4.85	0.97
20	10	5	5	5	2	2	last cell total	5.00	4.70	0.94
21	10	5	5	5	2	2	any two observations from last unit of last column	5.00	4.68	0.94
22	10	5	5	5	2	3	any three observations from last unit of last column	5.00	4.49	0.90
23	10	5	5	5	2	4	any four observations from last unit of last column	5.00	4.28	0.86
24	10	5	5	5	2	5	last unit of each cell last column	5.00	4.06	0.81
25	10	5	5	5	2	6	last unit of last cell last column last cell total	5.00	3.94	0.79
26	15	5	5	5	3	1	last unit in last cell	5.00	4.91	0.98
27	15	5	5	5	3	2	any two observations from last cell	5.00	4.81	0.96
28	15	5	5	5	3	3	last cell total	5.00	4.72	0.94

29	15	5	5	5	3	2	any two observations from last unit of last column	5.00	4.81	0.96
30	15	5	5	5	3	3	any three observations from last unit of last column	5.00	4.70	0.94
31	15	5	5	5	3	4	any four observations from last unit of last column	5.00	4.58	0.92
32	15	5	5	5	3	5	last unit of last cell last column	5.00	4.46	0.89
33	15	5	5	5	3	7	last unit of last cell last column last cell total	5.00	4.46	0.89
34	14	7	7	7	2	1	last unit in last cell	7.00	6.90	0.99
35	14	7	7	7	2	2	last cell total	7.00	6.81	0.97
36	14	7	7	7	2	2	any two observations from last unit of last column	7.00	6.80	0.97
37	14	7	7	7	2	3	any three observations from last unit of last column	7.00	6.80	0.97
38	14	7	7	7	2	4	any four observations from last unit of last column	7.00	6.56	0.94
39	14	7	7	7	2	5	any five observations from last unit of last column	7.00	6.44	0.92
40	14	7	7	7	2	6	any six observations	7.00	6.30	0.90

							from last unit of last column			
41	14	7	7	7	2	7	last unit of last cell last column	7.00	6.17	0.88
42	14	7	7	7	2	8	last unit of last cell last column last cell total	7.00	6.17	0.88
43	21	7	7	7	3	1	last unit in last cell	7.00	6.94	0.99
44	21	7	7	7	3	2	any two observations from last cell	7.00	6.94	0.99
45	21	7	7	7	3	3	last cell total	7.00	6.82	0.97
46	21	7	7	7	3	2	any two observations from last unit of last column	7.00	6.87	0.98
47	21	7	7	7	3	3	any three observations from last unit of last column	7.00	6.81	0.97
48	21	7	7	7	3	4	any four observations from last unit of last column	7.00	6.73	0.96
49	21	7	7	7	3	5	any five observations from last unit of last column	7.00	6.65	0.95
50	21	7	7	7	3	6	any six observations from last unit of last column	7.00	6.58	0.94
51	21	7	7	7	3	7	last unit of last cell last column	7.00	6.50	0.93
52	21	7	7	7	3	9	last unit of last cell last column last cell total	7.00	6.39	0.91

The efficiency of the designs obtained above in Table 2.1 has been summarized in Table 2.2. It is seen that the efficiency of the resultant design is quite high for most of the designs.

Table 2.2: Summary of efficiency

S. No.	Efficiency	No. of Designs
1	< 0.70	3
2	0.70 - 0.80	5
3	0.80 - 0.85	2
4	0.85 - 0.90	9
5	0.90 - 0.95	17
6	≥ 0.95	16

Out of 52 designs investigated, 16 designs have efficiency more than and equal to 95% and are highly robust where as there are 17 designs that have efficiency between 0.90 - 0.95 and are thus robust. There is a decreasing trend in efficiency with increase in number of missing observations. In fact, the intensity or the consequences depends upon the size of the design. It is seen that smaller designs are more affected by the missing observations.

Series II: Jaggi *et al.* (2010) developed a series of generalized incomplete Trojan-type design for $v = sm$ ($s \geq 2$, m distinct group), cells of size k with $p = m$ rows and q columns.

Example II.1: Following is a generalized incomplete Trojan-type design for $v = 16$ treatments arranged in 8 rows, 2 columns and intersection of each row-column having $k = 4$ units:

Rows	Columns							
	I				II			
I	1	2	3	4	5	6	7	8
II	3	4	5	6	7	8	9	10
III	5	6	7	8	9	10	11	12

IV	7	8	9	10	11	12	13	14
V	9	10	11	12	13	14	15	16
VI	11	12	13	14	15	16	1	2
VII	13	14	15	16	1	2	3	4
VIII	15	16	1	2	3	4	5	6

The robustness of this class of designs has been investigated against missing of some/ all of observations pertaining to last column. Table 2.3 gives the parameters and efficiency of the residual design for this series of GRC designs.

Table 2.3: Parameters and efficiency of the residual design for Series II

S. No.	**v**	**p**	**q**	**r**	**k**	**No of Observations Missing**	**Cell/ Unit No**	**HM (C_d)**	**HM (C_{d*})**	**E**
1	16	8	2	4	4	1	last unit in last cell	3.60	3.51	0.97
2	16	8	2	4	4	2	any two unit from last cell	3.60	3.40	0.94
3	16	8	2	4	4	3	any three unit from last cell	3.60	3.32	0.92
4	16	8	2	4	4	4	total last cell	3.60	3.22	0.89
5	16	8	2	4	4	2	any two observation from last unit of last column	3.60	3.22	0.89
6	16	8	2	4	4	3	any three observation from last unit of last column	3.60	3.20	0.89
7	16	8	2	4	4	4	any four observation	3.60	2.99	0.83

							from last unit of last column			
8	16	8	2	4	4	5	any five observation from last unit of last column	3.60	2.99	0.83
9	16	8	2	4	4	6	any six observation from last unit of last column	3.60	2.60	0.72
10	16	8	2	4	4	7	any seven observation from last unit of last column	3.60	2.52	0.70
11	16	8	2	4	4	8	last unit of last cell last column	3.60	2.49	0.69
12	16	8	2	4	4	11	last unit of last cell last column last cell total	3.60	2.26	0.63
13	16	8	3	6	4	1	last	5.86	5.78	0.99
14	16	8	3	6	4	2	any two observation from last cell	5.86	5.70	0.97
15	16	8	3	6	4	3	any three observation from last cell	5.86	5.62	0.96
16	16	8	3	6	4	4	total last cell	5.86	5.54	0.94
17	16	8	3	6	4	2	any two observation from last unit of last column	5.86	5.54	0.94
18	16	8	3	6	4	3	any three observation	5.86	5.58	0.95

							from last unit of last column			
19	16	8	3	6	4	4	any four observation from last unit of last column	5.86	5.46	0.93
20	16	8	3	6	4	5	any five observation from last unit of last column	5.86	5.34	0.91
21	16	8	3	6	4	6	any six observation from last unit of last column	5.86	5.34	0.91
22	16	8	3	6	4	7	any seven observation from last unit of last column	5.86	5.08	0.87
23	16	8	3	6	4	8	last unit of last cell last column	5.86	4.95	0.84
24	16	8	3	6	4	11	last unit of last cell last column last cell total	5.86	4.73	0.81
25	6	6	2	4	2	1	last	3.57	3.28	0.92
26	6	6	2	4	2	2	total last cell	3.57	3.01	0.84
27	6	6	2	4	2	2	any two observation from last unit of last column	3.57	2.83	0.79
28	6	6	2	4	2	3	any three observation from last unit	3.57	2.13	0.60

							of last column			
29	6	6	2	4	2	4	any four observation from last unit of last column	3.57	2.19	0.61
30	6	6	2	4	2	5	any five observation from last unit of last column	3.57	2.00	0.56
31	6	6	2	4	2	6	last unit of last cell last column	3.57	1.58	0.44
32	6	6	2	4	2	7	last unit of last cell last column last cell total	3.57	1.41	0.40
33	6	7	2	6	3	1	last unit in last cell	5.83	5.63	0.97
34	6	7	2	6	3	2	any three units in last cell	5.83	5.63	0.97
35	6	7	2	6	3	3	all the units in last cell	5.83	5.25	0.90
36	6	7	2	6	3	2	any two observation from last unit of last column	5.83	5.38	0.92
37	6	7	2	6	3	3	any three observation from last unit of last column	5.83	5.38	0.92
38	6	7	2	6	3	4	any four observation from last unit	5.83	4.70	0.81

							of last column			
39	6	7	2	6	3	5	any five observation from last unit of last column	5.83	4.40	0.75
40	6	7	2	6	3	6	any six observation from last unit of last column	5.83	4.40	0.75
41	6	7	2	6	3	7	last unit of last cell last column	5.83	4.28	0.73
42	6	7	2	6	3	9	last unit of last cell last column last cell total	5.83	3.83	0.66

It is seen from Table 2.3 that out of 42 design, 7 designs have efficiency more than and equal to 0.95 and are highly robust where as there are 11 designs that have efficiency between 90% to 95% and are thus robust. Here also there is a decreasing trend in efficiency with increase in number of missing observations. Smaller designs, in terms of the total number of units, are more affected by the missing observations.

Series III: Datta *et al.*(2016) developed a series of GRC designs for $v = 2t + 1$ $(t > 1)$ and cells of size two with $p = t$ rows of size $2(2t+1)$, $q = (2t+1)$ columns of size $2t$, $r = 2t$ and $k = 2$ by developing the following initial columns mod $(2t + 1)$:

1	2t + 1
2	2t
3	2t – 1
.	.

.	.
.	.
t	2t - (t - 2)

Example III.1: For t = 3, v = 7 and the contents of the initial column are as follows:

1 7

2 6

3 5

Developing these columns mod 7 results in the following GRC design in three rows of size 14, 7 columns of size 6 with 2 units per cell and replication of each treatment being 6:

Rows	Columns						
	I	II	III	IV	V	VI	VII
I	1 7	2 1	3 2	4 3	5 4	6 5	7 6
II	2 6	3 7	4 1	5 2	6 3	7 4	1 5
III	3 5	4 6	5 7	6 1	7 2	1 3	2 4

The robustness of this class of designs has been investigated against missing of some/ all of observations pertaining to last column. Table 2.4 gives the parameters and efficiency of the residual design for this series of GRC designs.

Table 2.4: Parameters and efficiency of the residual design for Series III

S. No.	**v**	**p**	**q**	**r**	**k**	**No. of observations missing**	**Unit/ Cell No.**	**HM (C_d)**	**HM (C_{d*})**	**E**
1	5	2	4	4	2	1	last unit in last cell	3.75	3.41	0.91

2	5	2	4	4	2	2	both units in last cell	3.75	3.08	0.82
3	5	2	4	4	2	2	any two units from different cells	3.75	3.07	0.82
4	5	2	4	4	2	3	any three units from different cells	3.75	2.79	0.74
5	7	3	7	6	2	1	last unit in last cell	5.83	5.63	0.96
6	7	3	7	6	2	2	both units in last cell	5.83	5.42	0.93
7	7	3	7	6	2	3	any three units from different cells	5.83	5.21	0.89
8	9	4	9	8	2	1	last unit in last cell	7.88	7.73	0.98
9	9	4	9	8	2	2	both units in last cell	7.88	7.58	0.96
10	9	4	9	8	2	4	any four observations from last unit of last column	7.88	7.23	0.92
11	9	4	9	8	2	5	any three observations from last unit and last cell total	7.88	7.09	0.90
12	11	5	11	10	2	1	last unit in last cell	9.90	9.78	0.99
13	11	5	11	10	2	2	both units in last cell	9.90	9.67	0.98
14	11	5	11	10	2	5	last unit of the cells	9.90	9.29	0.94
15	11	5	11	10	2	6	any four observations from last unit	9.90	9.18	0.93

							and last cell total			
16	13	6	13	12	2	1	last unit in last cell	11.92	11.82	0.99
17	13	6	13	12	2	2	both units in last cell	11.92	11.73	0.98
18	13	6	13	12	2	6	last units of the cells	11.92	11.31	0.95
19	13	6	13	12	2	7	any five observations from last unit and last cell total	11.92	11.22	0.94
20	15	7	15	14	2	1	last unit in last cell	13.93	13.85	0.99
21	15	7	15	14	2	2	both units in last cell	13.93	13.77	0.99
22	15	7	15	14	2	7	last units of the cells	13.93	13.34	0.96
23	15	7	15	14	2	8	any six observations from last unit and last cell total	13.93	13.27	0.95

It is seen from Table 2.4 that the efficiency of the resultant design is quite high for most of the designs. Out of 23 design, 11 designs have efficiency more than and equal to 0.95 and are highly robust where as there are 7 designs that have efficiency between 90% to 95% and are thus robust. Here also there is a decreasing trend in efficiency with increase in number of missing observations. Smaller designs, in terms of the total number of units, are more affected by the missing observations.

Series IV: Datta *et al.* (2016) developed GRC designs with parameters v (even), p = (v-1) rows of size v, q = $\frac{v}{2}$ columns of size 2(v-1), r = (v-1) and k = 2 by developing following initial columns mod v:

1	v
v	2
2	v-1
v-1	3
.	.
.	.
.	.
$v - (\frac{v}{2} - 2)$	$v - \frac{v}{2}$
$\frac{v}{2}$	$\frac{v}{2} + 1$

Example IV.1: For v = 8, following is a GRC design with cells containing 2 units in 7 rows of size 8 each and 4 columns of size 14 each:

Rows	Columns							
	I		II		III		IV	
I	1	8	2	1	3	2	4	3
II	8	2	1	3	2	4	3	5
III	2	7	3	8	4	1	5	2
IV	7	3	8	4	1	5	2	6
V	3	6	4	7	5	8	6	1
VI	6	4	7	5	8	6	1	7

VII	4	5	5	6	6	7	7	8

The efficiency of this class of design has been worked out against missing of some/ all of observations of last cell/ column. Table 2.5 contains the parameters ($v \leq 12$) and efficiency of the residual design for this series.

Table 2.5 Parameters and efficiency of the residual design for Series IV

S. No.	v	p	q	r	k	No. of observations missing	Unit/ Cell No.	HM (C_d)	HM (C_{d*})	E
1	6	5	3	5	2	1	last unit in last cell	4.41	4.06	0.92
2	6	5	3	5	2	2	both units in last cell	4.41	3.92	0.89
3	6	5	3	5	2	2	any two units from different cells	4.41	3.75	0.85
4	6	5	3	5	2	3	any three units from different cells	4.41	3.29	0.75
5	6	5	3	5	2	4	any four observations from last unit	4.41	2.53	0.57
6	6	5	3	5	2	5	last unit in each cell of last column	4.41	3.03	0.69
7	6	5	3	5	2	6	last unit in each cell of last column and total last cell	4.41	3.03	0.69
8	8	7	4	7	2	1	last unit in last cell	6.37	6.15	0.97
9	8	7	4	7	2	2	both units in last cell	6.37	6.04	0.95

10	8	7	4	7	2	2	any two units from different cells	6.37	5.98	0.94
11	8	7	4	7	2	3	any three units from different cells	6.37	5.73	0.90
12	8	7	4	7	2	4	any four observations from last unit of last column	6.37	5.28	0.83
13	8	7	4	7	2	5	any five observations from last unit of last column	6.37	5.25	0.83
14	8	7	4	7	2	6	any six observations from last unit of last column	6.37	4.83	0.76
15	8	7	4	7	2	7	last unit in each cell of last column	6.37	5.11	0.80
16	8	7	4	7	2	8	last unit in each cell of last column and total last cell	6.37	5.11	0.80
17	10	9	5	9	2	1	last unit in last cell	8.34	8.19	0.98
18	10	9	5	9	2	2	both units in last cell	8.34	8.09	0.97
19	10	9	5	9	2	2	any two units from different cells	8.34	8.06	0.97
20	10	9	5	9	2	3	any three units from different cells	8.34	7.89	0.95
21	10	9	5	9	2	4	any four observations	8.34	7.67	0.92

							from last unit of last column			
22	10	9	5	9	2	5	any five observations from last unit of last column	8.34	7.50	0.90
23	10	9	5	9	2	6	any six observations from last unit of last column	8.34	7.24	0.87
24	10	9	5	9	2	7	any seven observations from last unit of last column	8.34	7.21	0.86
25	10	9	5	9	2	8	any eight observations from last unit of last column	8.34	6.96	0.83
26	10	9	5	9	2	9	last unit in each cell of last column	8.34	7.11	0.85
27	10	9	5	9	2	10	last unit in each cell of last column and total last cell	8.34	7.11	0.85
28	12	11	6	11	2	1	last unit in last cell	10.32	10.21	0.99
29	12	11	6	11	2	2	total last cell	10.32	10.12	0.98
30	12	11	6	11	2	2	any two observations from last unit of last column	10.32	10.09	0.98
31	12	11	6	11	2	3	any three observations from last unit of last column	10.32	9.99	0.97

32	12	11	6	11	2	4	any four observations from last unit of last column	10.32	9.99	0.97
33	12	11	6	11	2	5	any five observations from last unit of last column	10.32	9.73	0.94
34	12	11	6	11	2	6	any six observations from last unit of last column	10.32	9.54	0.92
35	12	11	6	11	2	7	any seven observations from last unit of last column	10.32	9.49	0.92
36	12	11	6	11	2	8	any eight observations from last unit of last column	10.32	9.28	0.90
37	12	11	6	11	2	9	any nine observations from last unit of last column	10.32	9.27	0.90
38	12	11	6	11	2	10	any ten observations from last unit of last column	10.32	9.10	0.88
39	12	11	6	11	2	11	last unit in each cell of last column	10.32	9.10	0.88
40	12	11	6	11	2	12	last unit in each cell of last column and total last cell	10.32	9.10	0.88
41	14	13	7	13	2	1	last unit in last cell	12.31	12.31	1.00

42	14	13	7	13	2	2	both units in last cell	12.31	12.14	0.99
43	14	13	7	13	2	2	any two units from different cells	12.31	12.12	0.98
44	14	13	7	13	2	3	any three units from different cells	12.31	12.04	0.98
45	14	13	7	13	2	4	any four observations from last unit of last column	12.31	12.04	0.98
46	14	13	7	13	2	5	any five observations from last unit of last column	12.31	11.82	0.96
47	14	13	7	13	2	6	any six observations from last unit of last column	12.31	11.70	0.95
48	14	13	7	13	2	7	any seven observations from last unit of last column	12.31	11.60	0.94
49	14	13	7	13	2	8	any eight observations from last unit of last column	12.31	11.45	0.93
50	14	13	7	13	2	9	any nine observations from last unit of last column	12.31	11.41	0.93
51	14	13	7	13	2	10	any ten observations from last unit of last column	12.31	11.27	0.92
52	14	13	7	13	2	11	any eleven observations	12.31	11.26	0.91

							from last unit of last column			
53	14	13	7	13	2	12	any twelve observations from last unit of last column	12.31	11.13	0.90
54	14	13	7	13	2	13	last unit in each cell of last column	12.31	11.12	0.90
55	14	13	7	13	2	14	last unit in each cell of last column and total last cell	12.31	11.12	0.90

It is seen from the Table 2.5 that the efficiency of the resultant design is quite high for most of the designs. Out of 55 designs, 36 designs have efficiency more than 90% and are thus robust.

Series V: Datta *et al.* (2015) developed a method of constructing GRC designs with v (prime) treatments in p = 2 rows of size $\frac{kv(v-1)}{2}$, q = $\frac{v(v-1)}{2}$ columns of size 2k and each cell of size k.

Example V.1: Following is a GRC design with v = 5 treatments in 2 rows of size 20 each and 10 columns of size 4 each and cells containing 2 units:

Rows	Columns									
	I	II	III	IV	V	VI	VII	VIII	IX	X
I	1 2	2 3	3 4	4 5	5 1	1 3	2 4	3 5	4 1	5 2
II	2 3	3 4	4 5	5 1	1 2	3 5	4 1	5 2	1 3	2 4

Example V.2: For v = 5, a GRC design with cell size 3 is obtained in 2 rows of size 30 each and 10 columns of size 6 each as follows:

Rows	Columns									
	I	II	III	IV	V	VI	VII	VIII	IX	X
I	1 2 3	2 3 4	3 4 5	4 5 1	5 1 2	1 3 5	2 4 1	3 5 2	4 1 3	5 2 4
II	2 3 4	3 4 5	4 5 1	5 1 2	1 2 3	3 5 2	4 1 3	5 2 4	1 3 5	2 4 1

The robustness of this class of designs is investigated against missing of observations of last cell/ column. Table 2.6 lists the parameters and efficiency of the residual design for this series.

Table 2.6: Parameters and efficiency of the residual design for Series V

S. No.	v	p	q	r	k	No. of observations missing	Unit/ Cell No.	HM (C_d)	HM (C_{d^*})	E
1	5	2	10	8	2	1	last unit in last cell	6.25	5.90	0.94
2	5	2	10	8	2	2	both units in last cell	6.25	5.82	0.93
3	5	2	10	8	2	2	any two units from different cells	6.25	5.82	0.93
4	5	2	10	8	2	3	any three units from different cells	6.25	5.52	0.88
5	5	2	10	12	3	1	last unit in last cell	10.83	10.52	0.97
6	5	2	10	12	3	2	any two observations from last cell	10.83	10.21	0.94
7	5	2	10	12	3	3	total last cell	10.83	10.22	0.94
8	5	2	10	12	3	2	last column each cell last unit	10.83	10.33	0.95
9	5	2	10	12	3	4	last column each cell last	10.83	9.93	0.92

							unit and last cell total			
10	5	2	10	16	4	1	last unit in last cell	15.63	15.33	0.98
11	5	2	10	16	4	2	any two observations from last cell	15.63	15.12	0.97
12	5	2	10	16	4	3	any three observations from last cell	15.63	14.94	0.96
13	5	2	10	16	4	4	total last cell	15.63	14.80	0.95
14	5	2	10	16	4	2	last unit in each cell of last column	15.63	15.12	0.97
15	5	2	10	16	4	5	last unit in each cell of last column and last cell total	15.63	14.52	0.93
16	7	2	21	12	2	1	last unit in last cell	8.75	8.53	0.97
17	7	2	21	12	2	2	two observations from last cell	8.75	8.47	0.97
18	7	2	21	12	2	2	last column each cell last unit	8.75	8.47	0.97
19	7	2	21	12	2	3	last column each cell last unit and last cell total	8.75	8.28	0.95
20	7	2	21	18	3	1	last unit in last cell	15.17	14.96	0.99
21	7	2	21	18	3	2	any two observations from last cell	15.17	14.84	0.98

22	7	2	21	18	3	3	total last cell	15.17	14.76	0.97
23	7	2	21	18	3	2	last column each cell last unit	15.17	14.84	0.98
24	7	2	21	18	3	4	last column each cell last unit and last cell total	15.17	14.56	0.96
25	7	2	21	24	4	1	last unit in last cell	21.88	21.68	0.99
26	7	2	21	24	4	2	any two observations from last cell	21.88	21.54	0.98
27	7	2	21	24	4	3	any three observations from last cell	21.88	21.42	0.98
28	7	2	21	24	4	4	total last cell	21.88	21.32	0.97
29	7	2	21	24	4	2	last column each cell last unit	21.88	21.54	0.98
30	7	2	21	24	4	5	last column each cell last unit and last cell total	21.88	21.14	0.97
31	7	2	21	30	5	1	last unit in last cell	28.70	28.51	0.99
32	7	2	21	30	5	2	any two observations from last cell	28.70	28.37	0.99
33	7	2	21	30	5	3	any three observations from last cell	28.70	28.23	0.98
34	7	2	21	30	5	4	any four observations from last cell	28.70	28.10	0.98
35	7	2	21	30	5	5	total last cell	28.70	27.99	0.98

36	7	2	21	30	5	2	each cell in last unit of last column	28.70	28.36	0.99
37	7	2	21	30	5	6	last unit in each cell of last column and total last cell	28.70	27.81	0.97
38	11	2	55	20	2	1	last unit in last cell	13.75	13.62	0.99
39	11	2	55	20	2	2	total last cell	13.75	13.59	0.99
40	11	2	55	20	2	2	each cell in last unit of last column	13.75	13.59	0.99
41	11	2	55	20	2	3	each cell in last unit of last column and total last cell	13.75	13.48	0.98
42	11	2	55	30	3	1	last unit in last cell	23.83	23.71	1.00
43	11	2	55	30	3	2	any two observations from last cell	23.83	23.64	0.99
44	11	2	55	30	3	3	total last cell	23.83	23.59	0.99
45	11	2	55	30	3	2	last unit in each cell of last column	23.83	23.64	0.99
46	11	2	55	30	3	4	last unit in each cell of last column and last cell total	23.83	23.48	0.99
47	11	2	55	40	4	1	last unit in last cell	34.35	34.24	1.00

48	11	2	55	40	4	2	any two observations from last cell	34.35	34.16	0.99
49	11	2	55	40	4	3	any three observations from last cell	34.35	34.08	0.99
50	11	2	55	40	4	4	total last cell	34.35	34.02	0.99
51	11	2	55	40	4	2	last unit in each cell of last column	34.35	34.16	0.99
52	11	2	55	40	4	5	last unit in each cell of last column and last cell total	34.35	33.92	0.99
53	11	2	55	50	5	1	last unit in last cell	45.04	44.93	1.00
54	11	2	55	50	5	2	any two observations from last cell	45.04	44.85	1.00
55	11	2	55	50	5	3	any three observations from last cell	45.04	44.77	0.99
56	11	2	55	50	5	4	any four observations from last cell	45.04	44.69	0.99
57	11	2	55	50	5	5	total last cell	45.04	44.63	0.99
58	11	2	55	50	5	2	last unit in each cell of last column	45.04	44.85	1.00
59	11	2	55	50	5	6	last unit in each cell of last column and last cell total	45.04	44.53	0.99

It is seen from the Table 2.6 that the efficiency of the resultant design is quite high for most of the designs. Out of 59 designs, 51 designs have efficiency more than and equal to 0.95 and are highly robust where as there are 7 designs that have efficiency 0.90-0.95 and are thus robust. There are few designs with no loss of efficiency.

Series VI: Datta *et al.* (2015) developed this series of GRC design for unequal cell sizes. This design is developed by using a BIB design with parameters v*, b* (even), r*, k*, λ*. The resulting design have parameters v = v*, p = 2 rows of size $\frac{v^*b^*}{2}$, q = b* columns of size v*, r = b*, k_1 = k*, and k_2 = v*- k*.

Example VI.1: Consider a BIB design with parameters v* = 5, b* = 10, r* = 4, k* = 2, λ* = 1. The following is a GRC design with parameters v = 5, p = 2 of size 25 each and q = 10 columns of size 5, r = 10, k_1 = 2 and k_2 = 3.

Rows	Columns									
	I	II	III	IV	V	VI	VII	VIII	IX	X
I	1 2	1 3	1 4	1 5	2 3	3 4 5	2 4 5	2 3 5	2 3 4	1 4 5
II	1 3 5	1 3 4	1 2 5	1 2 4	1 2 3	2 4	2 5	3 4	3 5	4 5

The following Table 2.7 the parameter of the GRC designs developed by Series V along with number of observation missing and the cell number from which the observations are missing, harmonic mean of non-zero eigen values of information matrix of original design and the residual design under the three-way model and The efficiency (E) of the residual design relative to the original design.

Table 2.7: Parameters and efficiency of the residual design for Series VI

S. No.	v	p	q	r	k	No. of observation missing	Unit/ Cell No.	HM (C_d)	HM (C_{d^*})	E
1	5	2	10	10	2 3	1	last unit in last cell	8.50	8.29	0.98
2	5	2	10	10	3 3	2	last any two units from last cell	8.50	8.17	0.96
3	5	2	10	10	4 3	3	last cell total	8.50	7.93	0.93
4	5	2	10	10	5 3	2	last unit of each cell of last column	8.50	8.01	0.94
5	5	2	10	10	6 3	5	last unit of each cell of last column and last cell total	8.50	7.69	0.91
6	9	2	12	12	3 6	1	last unit	12.00	11.86	0.99
7	9	2	12	12	4 6	2	last any two units from last cell	12.00	11.72	0.98
8	9	2	12	12	5 6	3	last any three units from last cell	12.00	11.59	0.97
9	9	2	12	12	6 6	4	last any four units from last cell	12.00	11.47	0.96
10	9	2	12	12	7 6	5	last any five units from last cell	12.00	11.34	0.94
11	9	2	12	12	8 6	6	total last cell	12.00	11.21	0.93

12	9	2	12	12	9 6	2	last unit of each cell of last column	12.00	11.73	0.98
13	9	2	12	12	10 6	9	last unit of each cell of last column and last cell total	12.00	11.09	0.92
14	9	2	18	8	4 5	1	last unit	18.00	17.86	0.99
15	9	2	18	8	5 5	2	last any two units from last cell	18.00	17.73	0.99
16	9	2	18	8	6 5	3	any three units from last cell	18.00	17.60	0.98
17	9	2	18	8	7 5	4	last four units from last cell	18.00	17.47	0.97
18	9	2	18	8	8 5	5	total last cell	18.00	17.34	0.96
19	9	2	18	8	9 5	2	last unit of each cell of last column	18.00	17.73	0.99
20	9	2	18	8	10 5	9	last unit of each cell of last column and last cell total	18.00	17.35	0.96
21	10	2	30	30	3 7	1	last unit in last cell	29.76	29.64	1.00
22	10	2	30	30	4 7	2	last two units from last cell	29.76	29.52	0.99
23	10	2	30	30	5 7	3	last any three units from last cell	29.76	29.40	0.99

24	10	2	30	30	6 7	4	last any four units from last cell	29.76	29.29	0.98
25	10	2	30	30	7 7	5	last any five units from last cell	29.76	29.19	0.98
26	10	2	30	30	8 7	6	last any six units from last cell	29.76	29.09	0.98
27	10	2	30	30	9 7	7	last cell total	29.76	28.95	0.97
28	10	2	30	30	10 7	2	last unit of each cell of last column	29.76	29.53	0.99
29	10	2	30	30	11 7	8	last unit of each cell of last column and last cell total	29.76	28.84	0.97

It is seen from the Table 2.7 that the efficiency of the resultant design is quite high for most of the designs. Out of 28 designs, 23 design have efficiency more than and equal to 95% and are highly robust and 5 designs are robust.

Thus all the series of GRC designs investigated are found to be robust against loss of observations.

CHAPTER 3

GRC DESIGNS BALANCED FOR SPATIAL INDIRECT EFFECTS

3.1 Introduction

In case of a GRC design, there are more number of units in a cell and the treatment applied to one experimental unit in a cell may affect the response on neighbouring units in the same cell. Treatments such as fertilizer, irrigation, or pesticide may spread to adjacent units causing neighbour effects. Such experiments exhibit spatial effects, because the effect of having no treatment as a neighbour is different from the neighbour effects of any treatment. Thus, spatial effects resulting in competition between neighbouring units may contribute to variability in experimental results and lead to substantial losses in efficiency. In order to compare the effects of treatments in this situation, designs balanced for spatial effects are considered where effects from the treatments applied in adjacent experimental units are known to exist. Thus, neighbour-balanced designs wherein the allocation of treatments is such that every treatment occurs equally often with every other treatment as neighbour(s), are used for these situations. These designs permit the estimation of direct and neighbour effect(s) of treatments.

It is seen in the literature that most of the work on designs with neighbour effects is concentrated under block design set up. There are a few work done related to study of neighbour balanced RC designs. Freeman (1979) has given some row-column designs balanced for neighbours with and without border plots. Federer and Basford (1991) have given three methods of constructing balanced nearest neighbour row-column or competition effect designs. Chan and Eccleston (2003) have given an algorithm which generates neighbour balanced row–column designs. Varghese *et al.* (2014) obtained row-column designs incorporating directional neighbour effects.

In this study, it is assumed that the effect of a treatment applied to a given unit in a cell is the sum of the direct effect due to the treatment applied to the unit, spatial effect from the treatment applied to the immediate left-neighbouring unit and spatial effect from the treatment applied to the immediate right-neighbouring unit within the cell. It is further assumed that the spatial effects from both the adjacent units are same. In this chapter, series of GRC designs balanced for these spatial effects have been developed. The general expression for the joint information matrix for estimating contrasts pertaining to direct effect and spatial effect has been derived. The efficiency factor of the designs has also been worked out. SAS codes have been written in PROC IML (given in ANNEXURE II) to calculate the information matrix (**C**-matrix) of treatment effects for a GRC designs balanced for these spatial effects, study the properties of the designs.

3.2 Model and Experimental Setup

We consider a GRC design with v treatments arranged in p rows, q columns and in each row-column intersection (i.e. cells) there are k units resulting in total $n= pqk$ experimental units or observations. In order to capture the spatial effect of treatments from neighbouring units, the following fixed effect model is considered:

$$y_{l(ij)} = \mu + \tau_{l[i,j]} + \delta_{(l-1)[i,j]} + \delta_{(l+1)[i,j]} + \alpha_i + \beta_j + e_{l(ij)} \qquad \ldots(3.2.1)$$

$i = 1,2,\ldots,p$; $j = 1,2,\ldots,q$; $l = 1,2,\ldots,k$

where $Y_{l(ij)}$ is the response from the l^{th} unit corresponding to the intersection of i^{th} row and j^{th} column. μ is the general mean, $\tau_{l[i,j]}$ is the effect of the treatment appearing in the l^{th} unit corresponding to the intersection of i^{th} row and j^{th} column, $\delta_{(l-1)[i,j]}$ is the neighbour effect due to the treatment applied in the adjacent left unit, $\delta_{(l+1)[i,j]}$ is the neighbour effect due to the treatment applied in the adjacent right unit, α_i is the i^{th} row effect and βj is the j^{th} column effect. $e_{l(ij)}$ is the error term identically and independently distributed and following normal distribution with mean zero and constant variance.

The above model can be written in matrix notation as follows:

$$\mathbf{Y} = \mu\mathbf{1} + \mathbf{\Delta}'\boldsymbol{\tau} + \mathbf{\Delta}_1'\boldsymbol{\delta} + \mathbf{D}_1'\boldsymbol{\alpha} + \mathbf{D}_2'\boldsymbol{\beta} + \mathbf{e} \qquad \ldots(3.2.2)$$

where **Y** is a n × 1 vector of observations, μ is the grand mean, **1** is the $n \times 1$ vector of ones, $\mathbf{\Delta}'$ is $n \times v$ incidence matrix of observations versus treatments, $\boldsymbol{\tau}$ is a $v \times 1$ vector of direct treatment effects, $\mathbf{\Delta}_1'$ is $n \times v$ incidence matrix of observations versus neighbouring treatments $\mathbf{D}_1'$ is $n \times p$ incidence matrix of observations versus rows, $\boldsymbol{\alpha}$ is $p \times 1$ vector of row effects, $\mathbf{D}_2'$ is $n \times q$ incidence matrix of observations versus columns, $\boldsymbol{\beta}$ is $q \times 1$ vector of column effects and **e** is $n \times 1$ vector of random errors with E(**e**)=**0** and D(**e**)=$\sigma^2\mathbf{I}_n$. Further, $\mathbf{\Delta}'\mathbf{1}_v = \mathbf{\Delta}_1'\mathbf{1}_v = \mathbf{D}_1'\mathbf{1}_p = \mathbf{D}_2'\mathbf{1}_q = \mathbf{1}_n$.

The design matrix $\mathbf{X}_{n\times(2v+p+q+1)}$ consisting of treatment effects, neighbour effects, row effects, column effects and mean can be partitioned into parameters of interest $\mathbf{X}_1$ and nuisance parameters $\mathbf{X}_2$.

$$\mathbf{X}_1 = \begin{pmatrix}\boldsymbol{\Delta}' & \boldsymbol{\Delta}_1'\end{pmatrix},\ \mathbf{X}_2 = \begin{pmatrix}\mathbf{1} & \mathbf{D}_1' & \mathbf{D}'\end{pmatrix}$$

with

$$\mathbf{X}_1'\mathbf{X}_1 = \begin{pmatrix}\boldsymbol{\Delta}\boldsymbol{\Delta}' & \boldsymbol{\Delta}\boldsymbol{\Delta}_1' \\ \boldsymbol{\Delta}_1\boldsymbol{\Delta}' & \boldsymbol{\Delta}_1\boldsymbol{\Delta}_1'\end{pmatrix} = \begin{pmatrix}\mathbf{R} & \mathbf{M} \\ \mathbf{M}' & \mathbf{G}\end{pmatrix}$$

$$\mathbf{X}_1'\mathbf{X}_2 = \begin{pmatrix}\boldsymbol{\Delta}\mathbf{1} & \boldsymbol{\Delta}\mathbf{D}_1' & \boldsymbol{\Delta}\mathbf{D}_2' \\ \boldsymbol{\Delta}_1\mathbf{1} & \boldsymbol{\Delta}_1\mathbf{D}_1' & \boldsymbol{\Delta}_1\mathbf{D}'\end{pmatrix} = \begin{pmatrix}\mathbf{r} & \mathbf{N}_1 & \mathbf{N}_2 \\ \mathbf{r}_1 & \mathbf{N}_3 & \mathbf{N}_4\end{pmatrix}$$

and

$$\mathbf{X}_2'\mathbf{X}_2 = \begin{pmatrix}\mathbf{1}'\mathbf{1} & \mathbf{1}'\mathbf{D}_1' & \mathbf{1}'\mathbf{D}_2' \\ \mathbf{D}_1\mathbf{1} & \mathbf{D}_1\mathbf{D}_1' & \mathbf{D}_1\mathbf{D}_2' \\ \mathbf{D}_2\mathbf{1} & \mathbf{D}_2\mathbf{D}_1' & \mathbf{D}_2\mathbf{D}_2'\end{pmatrix} = \begin{pmatrix}n & \mathbf{p}' & \mathbf{q}' \\ \mathbf{p} & \mathbf{K} & \mathbf{W} \\ \mathbf{q} & \mathbf{W}' & \mathbf{H}\end{pmatrix}$$

Here, $\mathbf{N}_1$ is an incidence matrix of order $v \times p$ of direct treatments vs. rows; $\mathbf{N}_2$ is an incidence matrix of order $v \times q$ of treatments vs. columns; $\mathbf{N}_3$ is an incidence matrix of order $v \times p$ of neighbour treatments vs. rows; $\mathbf{N}_4$ is an incidence matrix of order $v \times q$ of neighbour treatments vs. columns; $\mathbf{M}$ is an incidence matrix of order $v \times v$ of direct treatments vs. neighbour treatments; $\mathbf{W}$ is an incidence matrix of order $p \times q$ of rows vs. columns; $\mathbf{r} = (r_1, r_2, \ldots, r_v)'$ is the $v \times 1$ replication vector of direct treatments $\mathbf{r}_1 = (r_{11}, r_{12}, \ldots, r_{1v})$ is the $v \times 1$ replication vector of the treatments as neighbour with r_{1m} ($m = 1,2,\ldots,v$) being the number of times the m^{th} treatment appears as neighbour in the design; $\mathbf{R} = \mathrm{diag}(r_1, r_2, \ldots, r_v)$ is the diagonal matrix of replication of treatments; $\mathbf{G} = \mathrm{diag}(r_{11}, r_{12}, \ldots, r_{1v})$ is the diagonal matrix of replication of treatments as neighbour; $\mathbf{p} = (p_1, p_2, \ldots, p_p)$ is the $p \times 1$ vector of row sizes; $\mathbf{q} = (q_1, q_2, \ldots, q_q)$ is the $q \times 1$ vector of column sizes; $\mathbf{K} = \mathrm{diag}(k_1, k_2, \ldots, k_p)$ is the diagonal matrix of row sizes; $\mathbf{H} = \mathrm{diag}(h_1, h_2, \ldots, h_q)$ is the diagonal matrix of column sizes.

The joint information matrix for estimating all the effects (direct and neighbors) can be obtained as

$$\mathbf{C} = \mathbf{X}_1'\mathbf{X}_1 - \mathbf{X}_1'\mathbf{X}_2(\mathbf{X}_2'\mathbf{X}_2)^{-}\mathbf{X}_2'\mathbf{X}_1$$

where $(\mathbf{X}_2'\mathbf{X}_2)^{-}$ is the generalized inverse of $(\mathbf{X}_2'\mathbf{X}_2)$ and is obtained using the following result:

$$\mathbf{X}=\begin{pmatrix}\mathbf{A} & \mathbf{B}\\ \mathbf{B}' & \mathbf{D}\end{pmatrix} \text{ then } \mathbf{X}^{-}=\begin{pmatrix}\mathbf{A}^{-}+\mathbf{F}\mathbf{E}^{-}\mathbf{F}' & -\mathbf{F}\mathbf{E}^{-}\\ -\mathbf{E}^{-}\mathbf{F}' & \mathbf{E}^{-}\end{pmatrix}$$

where $\mathbf{F}=\mathbf{A}^{-}\mathbf{B}$ and $\mathbf{E}=\mathbf{D}-\mathbf{B}'\mathbf{A}^{-}\mathbf{B}$.

Here, $\mathbf{F}=\mathbf{K}^{-}\mathbf{W}$ and $\mathbf{E}=\mathbf{H}-\mathbf{W}'\mathbf{K}^{-}\mathbf{W}$, thus

$$(\mathbf{X}_2'\mathbf{X}_2)^{-}==\begin{pmatrix}0 & \mathbf{0}' & \mathbf{0}'\\ \mathbf{0} & \mathbf{K}^{-}+\mathbf{F}\mathbf{E}^{-}\mathbf{F}' & -\mathbf{F}\mathbf{E}^{-}\\ \mathbf{0} & -\mathbf{E}^{-}\mathbf{F}' & \mathbf{E}^{-}\end{pmatrix}$$

The joint information matrix for treatment and neighbour effects is

$$\mathbf{C}=\begin{pmatrix}\mathbf{C}_{11} & \mathbf{C}_{12}\\ \mathbf{C}_{21} & \mathbf{C}_{22}\end{pmatrix} \qquad \ldots(3.2.3)$$

where,

$$\mathbf{C}_{11}=\mathbf{R}-(\mathbf{N}_1\mathbf{K}^{-}\mathbf{N}_1'+\mathbf{N}_1\mathbf{F}\mathbf{E}^{-}\mathbf{F}'\mathbf{N}_1'-\mathbf{N}_2\mathbf{E}^{-}\mathbf{F}'\,\mathbf{N}_1'-\mathbf{N}_1\mathbf{F}\mathbf{E}^{-}\mathbf{N}_2'+\mathbf{N}_2\mathbf{E}^{-}\,\mathbf{N}_2')$$

$$\mathbf{C}_{12}=\mathbf{M}-(\mathbf{N}_1\mathbf{K}^{-}\mathbf{N}_3'+\mathbf{N}_1\mathbf{F}\mathbf{E}^{-}\mathbf{F}'\mathbf{N}_3'-\mathbf{N}_2\mathbf{E}^{-}\mathbf{F}'\,\mathbf{N}_3'-\mathbf{N}_1\mathbf{F}\mathrm{E}^{-}\mathbf{N}_4'+\mathbf{N}_2\mathbf{E}^{-}\,\mathbf{N}_4')$$

$$\mathbf{C}_{21}=\mathbf{M}-(\mathbf{N}_3\mathbf{K}^{-}\mathbf{N}_1'+\mathbf{N}_3\mathbf{F}\mathbf{E}^{-}\mathbf{F}'\mathbf{N}_1'-\mathbf{N}_3\mathbf{E}^{-}\mathbf{F}'\,\mathbf{N}_2'-\mathbf{N}_4\mathbf{F}\mathrm{E}^{-}\mathbf{N}_1'+\mathbf{N}_4\mathbf{E}^{-}\,\mathbf{N}_2')$$

$$\mathbf{C}_{22}=\mathbf{G}-(\mathbf{N}_3\mathbf{K}^{-}\mathbf{N}_3'+\mathbf{N}_3\mathbf{F}\mathbf{E}^{-}\mathbf{F}'\mathbf{N}_3'-\mathbf{N}_4\mathbf{E}^{-}\mathbf{F}'\,\mathbf{N}_3'-\mathbf{N}_3\mathbf{F}\mathbf{E}^{-}\mathbf{N}_4'+\mathbf{N}_4\mathbf{E}^{-}\,\mathbf{N}_4')$$

The $2v \times 2v$ matrix **C** is symmetric, non negative definite with zero row and column sums. From the above, the information matrices for estimating the direct effects and neighbour effects are obtained respectively as

$$\mathbf{C}_{\tau}=\mathbf{C}_{11}-\mathbf{C}_{12}\mathbf{C}_{22}^{-}\,\mathbf{C}_{21}$$

and $$\mathbf{C}_{\delta}=\mathbf{C}_{22}-\mathbf{C}_{12}\mathbf{C}_{11}^{-}\,\mathbf{C}_{21}$$

Definition 3.2.1: A GRC design with v treatments in p rows and q columns is said to be balanced for spatial effects from neighbouring units if within a cell every treatment has every other treatment appearing as neighbour a constant number of times (say λ times). These designs are called here as Neighbour Balanced GRC (NBGRC) designs. Further, a NBGRC design, permitting the estimation of direct and neighbour effects, is called variance balanced if the variance of any estimated elementary contrast among the direct effects is constant.

3.3 NBGRC Design Construction

Method 3.3.1: Consider *v* (prime) treatments. Develop the contents of i[th] (i=1,2,…,*v*) row (mod v) with cell size k = s (3 ≤ s ≤ v-1) as follows:

i i+1 … i+(s-1)	i i+2 … i+2(s-1)	…	i i+(v-1) … i+(v-1)(s-1)

The design so obtained is a NBGRC design balanced for spatial effects with parameter *v* (prime), $p = v$, $q = v$-1, k = s (3 ≤ s ≤ v-1), r = s (v-1) and λ = 2(s-1).

The structure of the various incidence matrices as per model (3.2.2) for this class of the designs obtained is as follows:

$$\mathbf{\Delta\Delta_1' = M} = 2(\mathrm{s}-1)[\mathbf{J}-\mathbf{I}]$$

$$\mathbf{\Delta D_1'} = \mathbf{N_1} = (v-\mathrm{s})\mathbf{I} + (\mathrm{s}-1)\,\mathbf{J}$$

$$\mathbf{\Delta D_2'} = \mathbf{N_2} = \mathrm{s}\,\mathbf{J}$$

$$\mathbf{\Delta}_1\mathbf{D_1'} = \mathbf{N}_3 = (v-2\mathrm{s}+2)\mathbf{I} + (2\mathrm{s}-3)\,\mathbf{J}$$

$$\mathbf{\Delta}_1\mathbf{D}_2' = \mathbf{N}_4 = 2(\mathrm{s}-1)\ \mathbf{J}$$

$$\mathbf{D_1}\mathbf{D}_2' = \mathbf{W} = s\ \mathbf{J}$$

$$\mathbf{\Delta\Delta' = R_\tau} = \mathrm{s}(v-1)\ \mathbf{I}$$

$$\mathbf{\Delta}_1\mathbf{\Delta}_1' = \mathbf{G} = [2(v-1)(\mathrm{s}-1) - 2(\mathrm{s}-2)]\mathbf{I} + 2(\mathrm{s}-2)\,\mathbf{J}$$

$$\mathbf{D}_1\mathbf{D_1'} = \mathbf{K} = \mathrm{s}(v-1)\mathbf{I}$$

$$\mathbf{D_2}\mathbf{D}_2' = \mathbf{H} = \mathrm{s}v\,\mathbf{I}$$

The components of $2v \times 2v$ joint information matrix for estimating the contrast pertaining to direct and neighbour effects as in (3.2.3) is obtained as below:

$$\mathbf{C_{11}} = \left[s(v\text{-}1) - \frac{(v\text{-}s)^2}{s(v\text{-}1)} \right] \mathbf{I} - \frac{2(v\text{-}s)(s\text{-}1)+v(s\text{-}1)^2}{s(v\text{-}1)} \mathbf{J}$$

$$\mathbf{C_{12}} = \mathbf{C_{21}} = -\left[2(s\text{-}1) + \frac{(v\text{-}s)(v\text{-}2s+2)}{s(v\text{-}1)} \right] \mathbf{I} + \left[\frac{(v\text{-}s)(2s\text{-}3)+(s\text{-}1)(v\text{-}2s+2)+v(s\text{-}1)(2s\text{-}3)}{s(v\text{-}1)} - 2(s\text{-}1) \right] \mathbf{J}$$

$$\mathbf{C_{22}} = \left[2(v\text{-}1)(s\text{-}1)\text{-}2(s\text{-}2) - \frac{(v\text{-}2s+2)^2}{s(v\text{-}1)} \right] \mathbf{I} - \left[\frac{2(v\text{-}2s+2)(2s\text{-}3)+v(2s\text{-}3)^2}{s(v\text{-}1)} - 2(s\text{-}2) \right] \mathbf{J}$$

The information matrix for estimating the contrast for direct treatment effects is obtained as below:

$$\mathbf{C_\tau} = \mathbf{C_{11}} - \mathbf{C_{12}} \mathbf{C_{22}}^{-} \mathbf{C_{21}}$$

$$= A\mathbf{I} - B\mathbf{J}$$

where,

$$A = \left((sa - \frac{f^2}{sa}) - \frac{(2abs+df)^2}{sa(2a^2bs\text{-}2acs\text{-}d^2)} \right)$$

$$B = \left(\frac{2fb+vb^2}{sa} - D \right)$$

$$D = \frac{1}{2a^2bs\text{-}2acs\text{-}d^2} \left((ef+bd+vbe\text{-}2sab) - \frac{(2de+ve^2\text{-}2sac)[(2sab+df)+v(ef+bd+vbe\text{-}2abs)]}{e(3v^2\text{-}4vs+2s)\text{-}d^2\text{-}2vd} \right) \times \left(\frac{ef+bd+vbe\text{-}2sab}{sa} \right)$$

a = (v-1), b = (s-1), c = (s-2), d = (v-2s+2), e = (2s-3) and f = (v-s).

Example 3.3.1.1: For v = 5 and s = 3, following is a NBGRC design with parameters v = 5, p = 5, q = 4, k = 3, r = 12 and λ = 6:

	Columns			
Rows	1 2 3	1 3 5	1 4 2	1 5 4
	2 3 4	2 4 1	2 5 3	2 1 5
	3 4 5	3 5 2	3 1 4	3 2 1
	4 5 1	4 1 3	4 2 5	4 3 2

	5 1 2	5 2 4	5 3 1	5 4 3

For this design,

$$\mathbf{C_{11}} = 11.66\,\mathbf{I} - 2.33\,\mathbf{J}$$

$$\mathbf{C_{12}} = \mathbf{C_{21}} = -4.16\,\mathbf{I} + 0.83\,\mathbf{J}$$

$$\mathbf{C_{22}} = 13.92\,\mathbf{I} - 2.25\,\mathbf{J}$$

The information matrix for estimating direct treatment contrast is

$\mathbf{C}_\tau$ = 10.42 **I** - 2.08 **J**

Similarly, the information matrix for estimating neighbour treatment contrast is

$\mathbf{C}_\delta$ = 12.43 **I** – 1.95**J**.

Example 3.3.1.2: For v = 5 and s = 4, following is a NBGRC design with parameters v = 5, p = 5, q = 4, k = 4, r = 16 and λ = 4:

	Columns			
	1 2 3 4	1 3 5 2	1 4 2 5	1 5 4 3
	2 3 4 5	2 4 1 3	2 5 3 1	2 1 5 4
Rows	3 4 5 1	3 5 2 4	3 1 4 2	3 2 1 5
	4 5 1 2	4 1 3 5	4 2 5 3	4 3 2 1
	5 1 2 3	5 2 4 1	5 3 1 4	5 4 3 2

Here,

$$\mathbf{C_{11}} = 15.93\,\mathbf{I} - 3.18\,\mathbf{J}$$

$$\mathbf{C_{12}} = \mathbf{C_{21}} = -5.94\,\mathbf{I} + 1.19\,\mathbf{J}$$

$$\mathbf{C_{22}} = 19.94\,\mathbf{I} - 3.19\,\mathbf{J}$$

The information matrix for estimating direct treatment contrast is

$\mathbf{C}_\tau$ = 14.17 **I** - 2.38 **J**.

Similarly, the information matrix for estimating neighbour treatment contrast is

$$\mathbf{C}_{\boldsymbol{\delta}} = 17.73\,\mathbf{I} - 2.75\mathbf{J}.$$

Thus, we see that the developed series of NBGRC design is variance balanced for estimating the contrast pertaining to direct treatments and also pertaining to neighbour effects.

Method 3.3.2: Consider a Balanced Incomplete Block (BIB) design with parameters (v^*, b^*, r^*, k^*, *and* λ^*). Let $v^* = 4t+3 = x^n$, where x is a prime and n (≥ 1) is a positive integer. Consider the odd powers of the primitive number of GF(x^n) as set 1 and the even powers of the primitive number of GF(x^n) as set 2. The block contents of set 1 comprises the 1st column of resulting GRC design and set 2 comprises the 2nd column of resulting GRC design. The parameters of the developed design are $v = v^*$, $p = v^*$, $q = 2$, $k = k^*$, r = r* and λ_i (i = 1,2,…, $\frac{v-1}{2}$). Thus, a GRC design with neighbour effects obtained through initial blocks of a BIB design is always a partially balanced design for estimating elementary direct treatment contrasts following a varying circular association scheme.

Example 3.3.2.1: Consider a BIB design with parameters (7,7,3,3,1). Following is a GRC design with neighbour effects with parameters $v = 7$, $p = 7$, $q = 2$, r = 6, $k = 3$, $\lambda_1 = 2$, $\lambda_2 = 1$ and $\lambda_3 = 1$:

	Columns	
Rows	1 2 4	3 6 5
	2 3 5	4 7 6
	3 4 6	5 1 7
	4 5 7	6 2 1
	5 6 1	7 3 2
	6 7 2	1 4 3
	7 1 3	2 5 4

The information matrix for estimating direct treatment contrasts is given by

$$\mathbf{C}_\tau = \begin{bmatrix} 4.54 & -0.52 & -0.89 & -0.85 & -0.85 & -0.89 & -0.52 \\ -0.52 & 4.54 & -0.52 & -0.89 & -0.85 & -0.85 & -0.89 \\ -0.89 & -0.52 & 4.54 & -0.52 & -0.89 & -0.85 & -0.85 \\ -0.85 & -0.89 & -0.52 & 4.54 & -0.52 & -0.89 & -0.85 \\ -0.85 & -0.85 & -0.89 & -0.52 & 4.54 & -0.52 & -0.89 \\ -0.89 & -0.85 & -0.85 & -0.89 & -0.52 & 4.54 & -0.52 \\ -0.52 & -0.89 & -0.85 & -0.85 & -0.89 & -0.52 & 4.54 \end{bmatrix}$$

The information matrix for estimating neighbour treatment contrasts is given by

$$\mathbf{C}_\delta = \begin{bmatrix} 5.39 & -0.89 & -0.47 & -0.67 & -0.67 & -0.47 & -0.89 \\ -0.89 & 5.39 & -0.89 & -0.47 & -0.67 & -0.67 & -0.47 \\ -0.47 & -0.89 & 5.39 & -0.89 & -0.47 & -0.67 & -0.67 \\ -0.67 & -0.47 & -0.89 & 5.39 & -0.89 & -0.47 & -0.67 \\ -0.67 & -0.67 & -0.47 & -0.89 & 5.39 & -0.89 & -0.47 \\ -0.47 & -0.67 & -0.67 & -0.47 & -0.89 & 5.39 & -0.89 \\ -0.89 & -0.47 & -0.67 & -0.67 & -0.47 & -0.89 & 5.39 \end{bmatrix}$$

It can be seen that treatment number 1 has treatment 2 and 7 as first associates (these treatments appear as neighbour twice), treatment 3 and 6 as second associates (these treatments appear as neighbour once) and remaining 4 and 5 as third associates (these treatments appear as neighbour once).

A SAS code (given in ANNEXURE II) has been written in PROC IML to calculate the information matrix (**C**-matrix) of treatment effects and neighbour effects and to study the properties of the designs under the three-way model with spatial effects.

3.4 Analysis

Consider the NBGRC design given in Example 3.3.1.1. The layout along with hypothetical data (within parenthesis) is as given below.

1 (27.84)	2 (23.20)	3 (34.03)	1 (21.27)	3 (14.18)	5 (16.07)	1 (40.22)	4 (15.68)	2 (67.03)	1 (65.74)	5 (17.53)	4 (55.52
2 (46.41)	3 (37.13)	4 (23.51)	2 (45.70)	4 (21.38)	1 (27.42)	2 (95.63)	5 (30.60)	3 (70.13)	2 (42.19)	1 (50.63)	5 (19.13)
3 (43.57)	4 (25.47)	5 (26.93)	3 (47.48)	5 (18.99)	2 (64.75)	3 (26.81)	1 (40.22)	4 (18.53)	3 (67.03)	2 (83.79)	1 (54.84)
4	5	1	4	1	3	4	2	5	4	3	2

(42.39)	(29.01)	(50.20)	(12.47)	(31.99)	(18.05)	(39.90)	(85.31)	(35.70)	(45.72)	(78.20)	(90.23)
5	1	2	5	2	4	5	3	1	5	4	3
(14.34)	(53.79)	(31.64)	(23.91)	(74.71)	(26.72)	(35.06)	(87.66)	(46.41)	(51.80)	(75.70)	(83.79)

The data was analysed as per the model defined in 3.2.1 and using SAS 9.3 (The code for analysis is given in ANNEXURE III). The Analysis of Variance is shown in Table 3.1.

Table 3.1: Analysis of Variance of NBGRC design for $v = 5$

Sources of Variation	DF	Sum of Squares	Mean Squares	F-Value	Pr > F
Rows	4	1180.51	295.13	10.06	<.0001
Columns	3	7658.68	2552.89	87.03	<.0001
Treatments	4	8013.43	2003.36	68.30	<.0001
Neighbours	4	9027.17	2256.79	76.94	<.0001
Error	44	1290.66	29.33		
Total	59	30535.15			

It is seen that all the effects including neighbour effects are significant. This shows that neighbour effects has an important role and must be incorporated in the model for better precision.

3.5 Efficiency of NBGRC Designs

The canonical efficiency of the NBGRC designs is obtained as follows:

$$E = \frac{H}{r}, \quad H = \left(\frac{1}{v\text{-}1} \sum_{i=1}^{v-1} \theta_i^{-1} \right)^{-1},$$

where θ_i are the eigen-values of **C**- matrix (obtained for direct treatment effects and neighbour treatment effects). Here, r is the number of replications of the treatments and is assumed to be same for the developed design and the orthogonal design to which it is compared.

The parameters of NBGRC designs obtained using Method 3.3.1.1 described above have been listed in Table 6.1. The list contains number of treatments ($v \leq 13$), cell sizes (k), number of rows (p), number of columns (q) and replications (r). The canonical efficiency of the developed designs for direct treatment effects and neighbour treatment effects are also reported in the Table 3.2.

Table 3.2: Parameters and efficiency factor of NBGRC designs

S. No.	***v***	***k***	***p***	***q***	**λ**	**Efficiency Factor (direct treatment effects)**	**Efficiency Factor (neighbour treatment effects)**
1	5	3	5	4	4	0.86	0.45
2	5	4	5	4	6	0.88	0.45
3	7	3	7	6	4	0.89	0.53
4	7	4	7	6	6	0.82	0.49
5	7	5	7	6	8	0.94	0.50
6	7	6	7	6	10	0.95	0.49
7	11	3	11	10	4	0.89	0.62
8	11	4	11	10	6	0.94	0.63
9	11	5	11	10	8	0.94	0.61
10	11	6	11	10	10	0.94	0.59
11	11	7	11	10	12	0.95	0.57
12	11	8	11	10	14	0.96	0.57
13	11	9	11	10	16	0.96	0.58
14	11	10	11	10	18	0.97	0.58

It is seen that the efficiency of direct treatment effects of NBGRC designs constructed is more as compared to neighbour treatment effects. The efficiency factor increases with increase in cell size for a given number of treatments.

3.6 SAS Macro for Generation of Neighbour Balanced GRC Designs

A SAS macro (given in ANNEXURE IV) has been developed to generate NBGRC designs for parameter v (prime), $p = v$, $q = v$-1, k = s (3 ≤ s ≤ v-1), r = s(v-1) and λ=2(s-1). Here, user need to enter the number of treatment as ***v*** **(**prime**)** and the number of units per cell as ***k*** **(**≥**2)**. If user run the macro after entering any prime number as the value of v and also as the value of k, then the SAS Macro will generate a particular NBGRC designs corresponding to the value of v and k under the heading Neighbour Balanced Generalized Row Column (GRC) Design. Once user run the macro, every time the SAS macro would also generate a word file containing the output. User can then save the word file.

3.7 Discussion

Two series of GRC designs balanced for spatial effects have been developed. One series is variance balanced for estimating the contrasts pertaining to direct treatment effects and also for estimating the contrasts pertaining to neighbour treatment effects. The second series is partially balanced for estimating elementary treatment contrasts for direct and neighbour treatment effects following circular association scheme. Further, the efficiency of the NBGRC designs have been worked out and are found to be quite high for estimating the direct treatment effects.

CHAPTER 4

WEB GENERATION OF GENERALIZED ROW-COLUMN DESIGNS (*web*GRC)

4.1 Introduction

A large number of experimental designs under different situations have been developed in the literature. For ready referencing and potential use of these designs, online software for generation of randomized layout of these designs is highly desirable. Online generation of designs are very much useful for the experimenters in providing a readymade solution. A large number of GRC designs are developed in the literature, construction of which involves a fair amount of theoretical understanding. Hence, for easy accessibility and quick reference of these designs by the experimenters, compilation and presentation of these designs at one platform is desirable. The rapid advancements on the internet technology have resulted in development of online software and hence expanding the horizon further. In this study, a web solution for generation of GRC designs has been developed which will help the experimenters for an easy accessibility and quick reference of these designs like the one developed by Taksande et al. (2012) with respect to partial diallel crosses, Sharma et al. (2013) for generating partially balanced incomplete block designs and Jaggi et al. (2015) for generating web-enabled software for generation of experimental designs balanced for indirect effects of treatments, was required. Many other open sources and commercial packages are also available for generation of readymade layouts of designs based on different situations [for example AgroPlotter (2002), Design-Expert Software (Version 7.0), webPD (2015), webFMC (2016) etc.].

The software *web*GRC generates both structurally complete and structurally incomplete GRC designs for different parametric combinations. Online catalogues for quick references of end users have also been developed for specific parametric combinations and integrated with *web*GRC.

4.2 Architecture of *web*GRC

The web solution for generation of GRC designs has been developed using client–server architecture along with an online catalogue of the designs within a permissible range. There are three main components i.e. user interface management, input data management and statistical engine for generation of GRC designs. At client side any communication to software from users is handled by user interface and input data handling is done by data management module. Statistical engine which hold the several procedures required for generation is implemented at server side. User interface has been separated from the statistical engine to free software developers from interface problem. Hyper Text Markup Language (HTML) and Cascading Style Sheets (CCS) have been used to develop the user interface management. ASP.NET has been used to develop input data management component. Web generation engine has been consructed using C# language. This engine contains the Dynamic Link Libraries (DLL) for generation and randomization of designs. Web generation of GRC Design has been developed for web platform and programming has been done with the ASP.NET and C# programming language. C# provides a complete set of tools for creation of rapid and powerful graphical user interface (GUI) based web applications. Microsoft Visual Studio 2010 integrated development environment has been used as a platform for development of the software. Fig. 4.1 shows the architecture of the software.

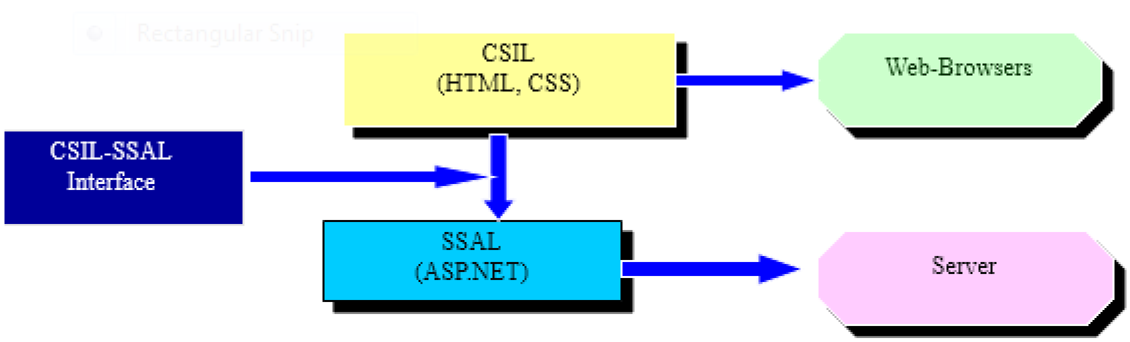

Fig. 4.1: Architecture for web generation of GRC design

4.3 *web*GRC Design

Software design of *web*GRC consists of three major modules namely (i) generation of Generalized Row Column designs, (ii) catalogue of Generalized Row- Column designs and (iii)about Generalized Row- Column designs. The hierarchical structure chart for the design of the software *web*GRC is shown in **Fig. 4.2**.

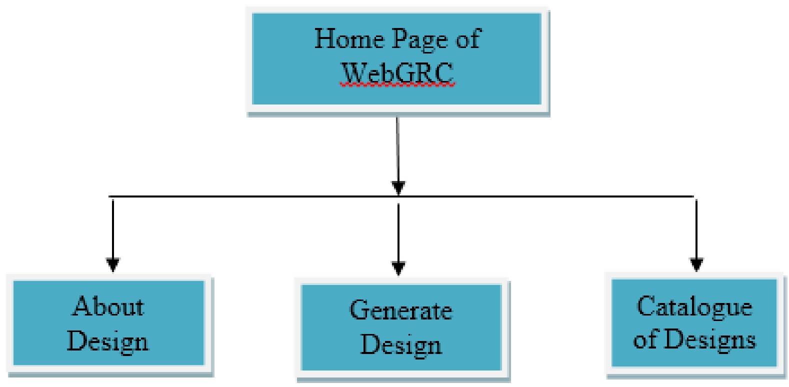

Fig. 4.2: Different module of webGRC

4.4 *web*GRC: Description

WebGRC generates design and randomized layout for various classes of GRC designs. It generates GRC Design for odd number of treatments (Datta *et al.*, 2016), GRC designs for even number of treatments (Datta *et al.*, 2016,; Parsad, 2006), GRC designs for prime number of treatments (Datta *et al.*, 2015). It also generates different series of structurally incomplete GRC designs developed by Datta *et al.*(2014).The webpage displays the layout plans along with the randomized layout for given number of treatments. It also displays various parameters of the generated designs viz. number of treatments, numbers of rows, number of columns and number of unit per cell. The output can be saved by the end user in excel sheet. To provide an idea about GRC designs a section named About Design has been created in the software which will provide the information about GRC designs along with example. The home page of the software is shown in Fig. 4.3.

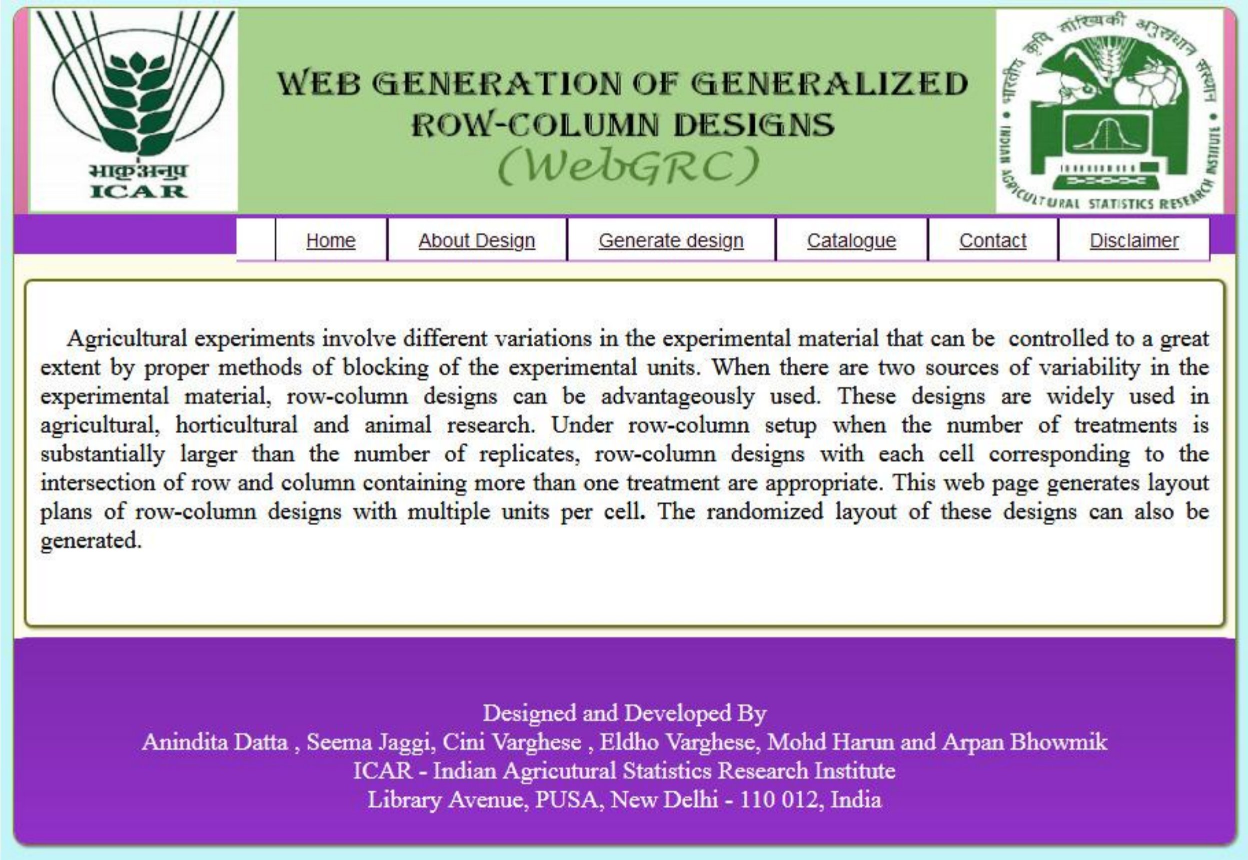

Fig. 4.3: Home page of *web*GRC

*web*GRC also consists of online catalogue for different series of GRC designs within a permissible range of parametric combinations. User can also generate designs from this online catalogue. To provide an idea about GRC designs a section named About Design has been created in the software which will provide the information GRC designs along with example.

4.5 Generation of Generalized Row- Column Designs through *web*GRC

In order to provide readymade layout to the end users, *web*GRC generates Generalized Row-Column designs (structurally complete and structurally incomplete) given in Fig 4.4. The generation of structurally complete Generalized Row- Column Designs through *web*GRC has been illustrated by Fig. 4.5. The generation of structurally incomplete GRC through *web*GRC has been illustrated by Fig. 4.6. Various web forms have been designed and developed for generation of these designs.

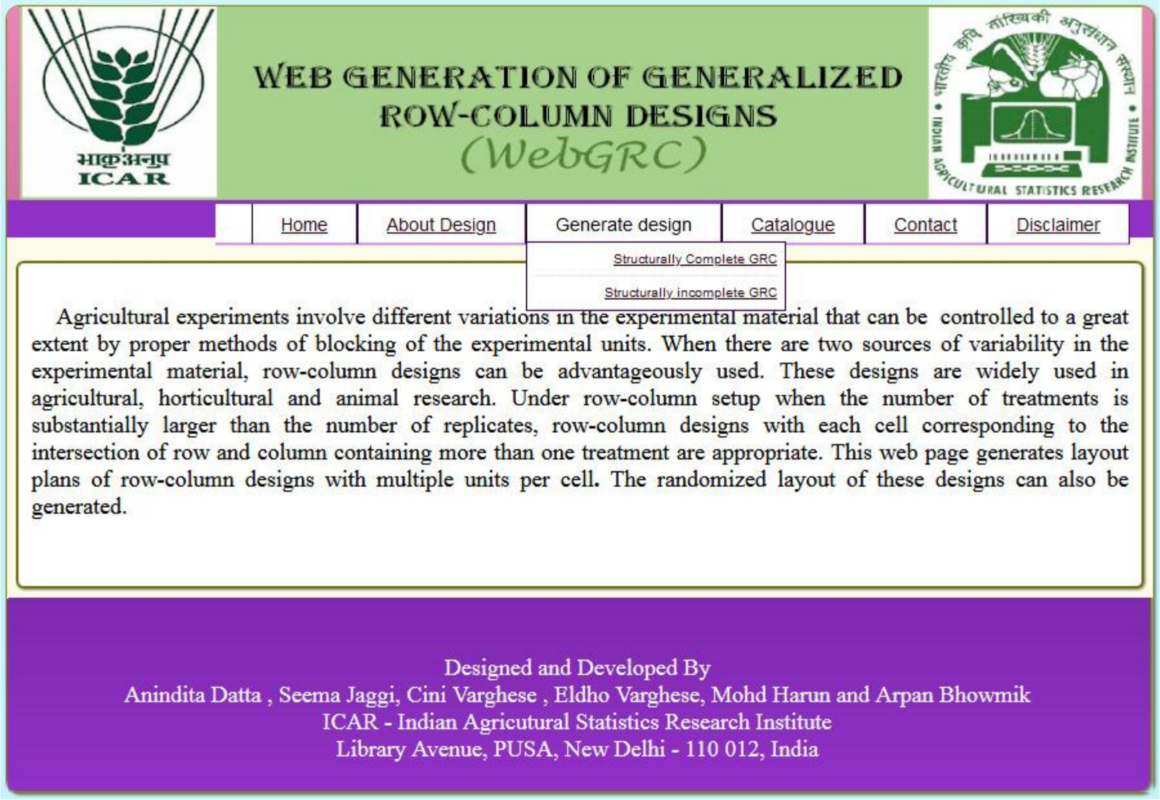

Fig. 4.4: Menu page of *web*GRC

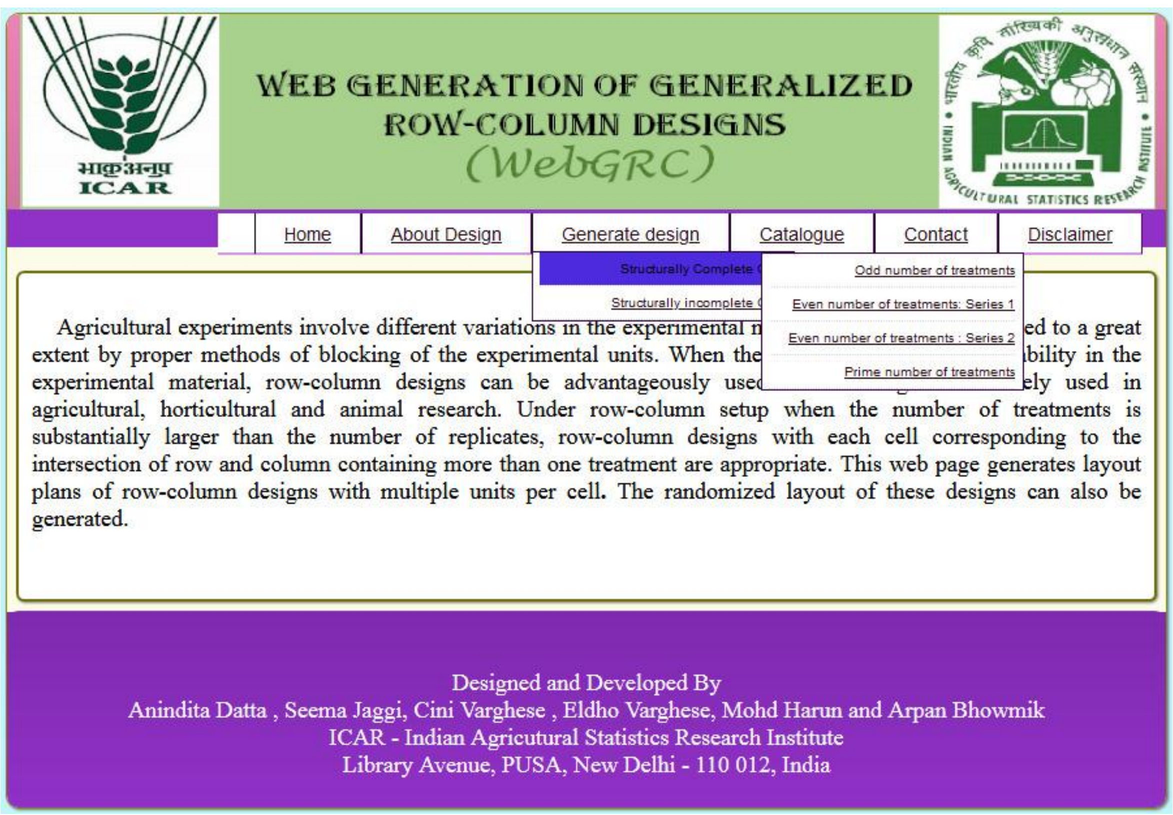

Fig. 4.5: Menu page of structurally complete *web*GRC

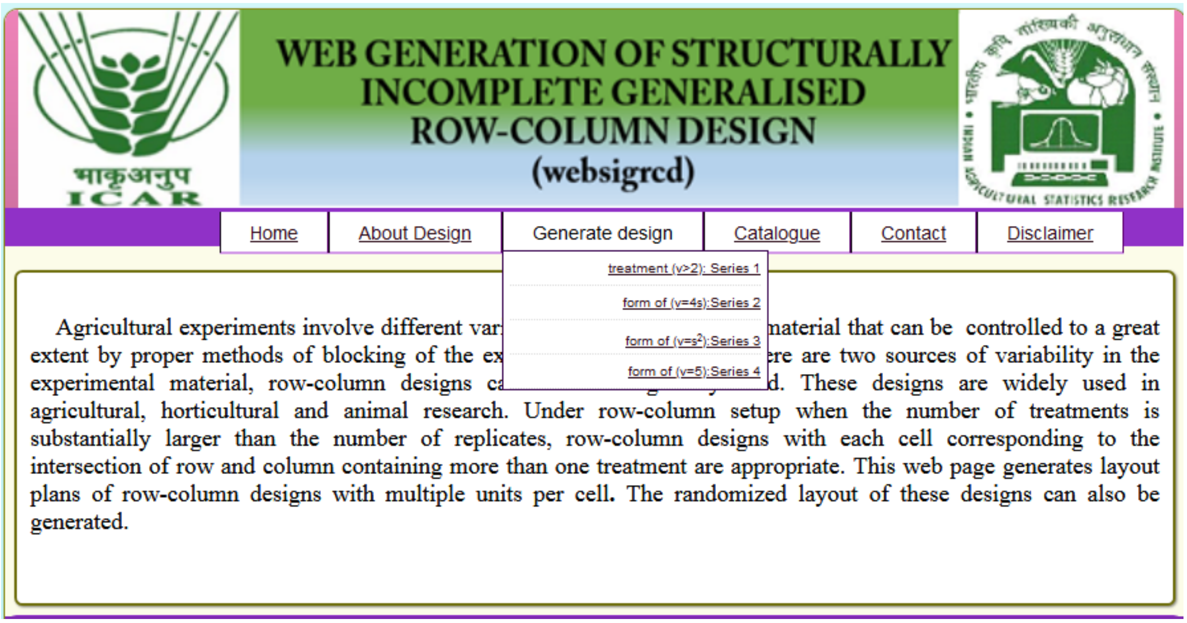

Fig. 4.6: Menu page of structurally incomplete *web*GRC

4.5.1 Generation of Structurally Complete GRC Designs through *web*GRC

To generate Structurally Complete GRC designs through *web*GRC, the following steps needs to be followed by the users:

In order to generate the design, user has to follow the following steps:

- Click on 'Generate Design' as shown in Fig. 4.5.
- Select 'prime number of treatments' under 'Generate Design'.
- Enter the number of treatments (v) = 7 (say) and enter the value of k = 3 (say)as shown in Fig. 4.6.
- Click on 'Generate Design' and the generated design along with parameters v = 7, p = 7, q = 21, and k =2) will be displayed as shown in Fig. 4.7.
- Click on 'Generate Randomized Layout' to get a randomized layout of the design as shown in Fig. 4.7.
- Output can be exported to MS-Excel spread sheet for further use.

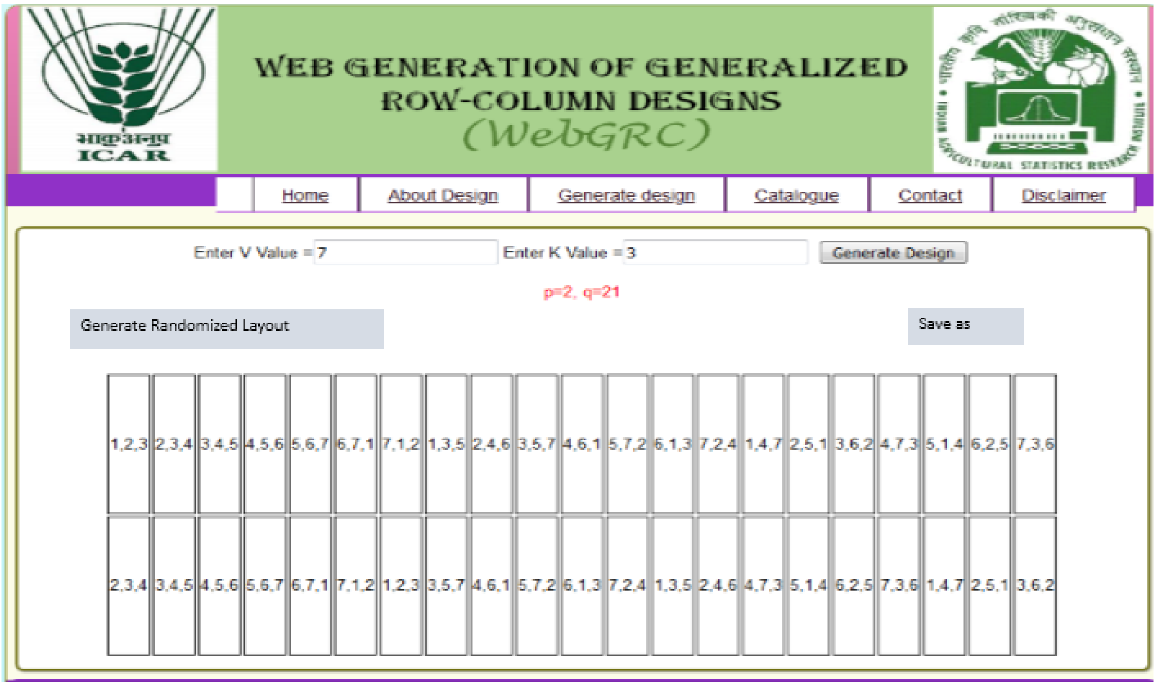

Fig. 4.7: Generation of GRC design for prime number of treatment (v=7)

Similarly for even number of treatments the design for v = 8 along with its randomized layout are shown in Fig. 4.8 and Fig. 4.9 respectively. Output can be exported to MS-Excel spread sheet for further use as shown in Fig. 4.10.

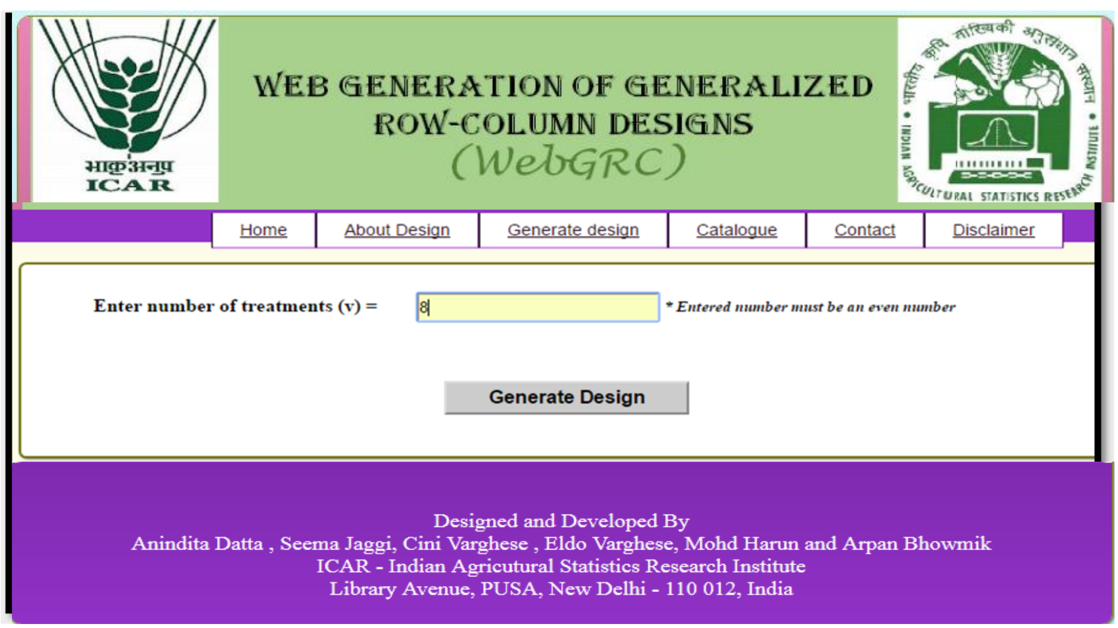

Fig. 4.8: Generation of GRC design for v=8

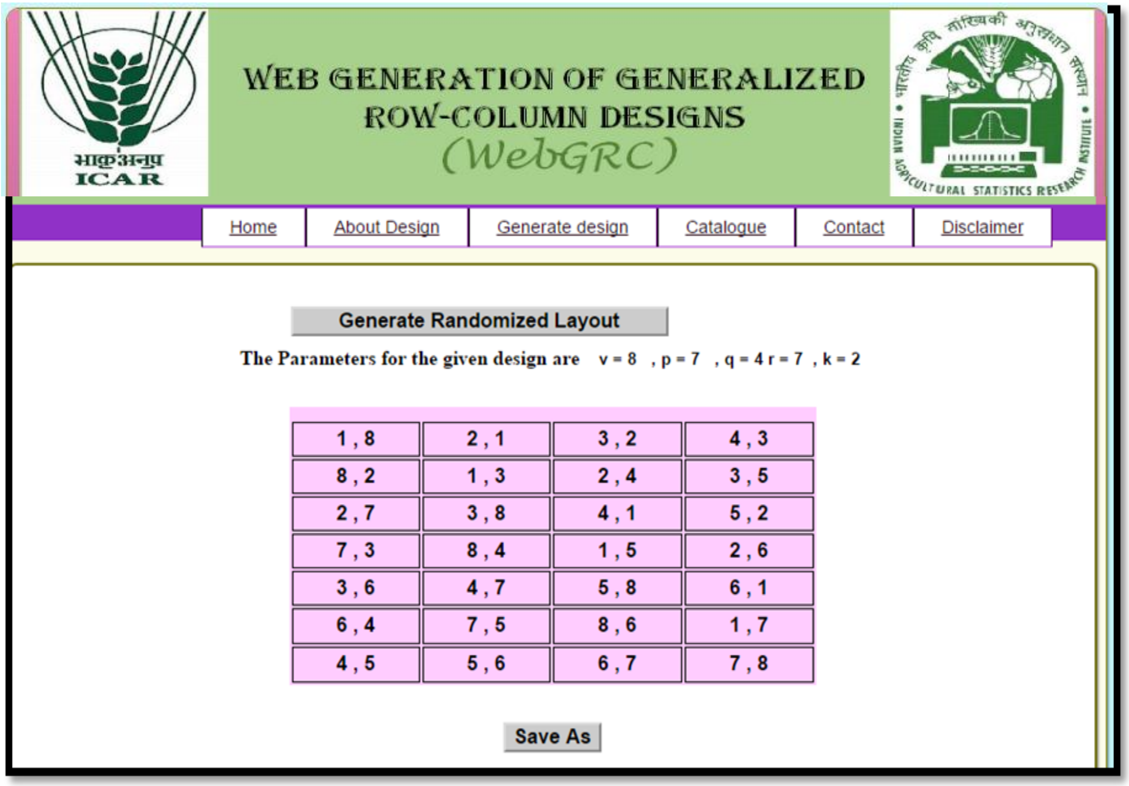

Fig. 4.9: Randomized layout of design for v = 8

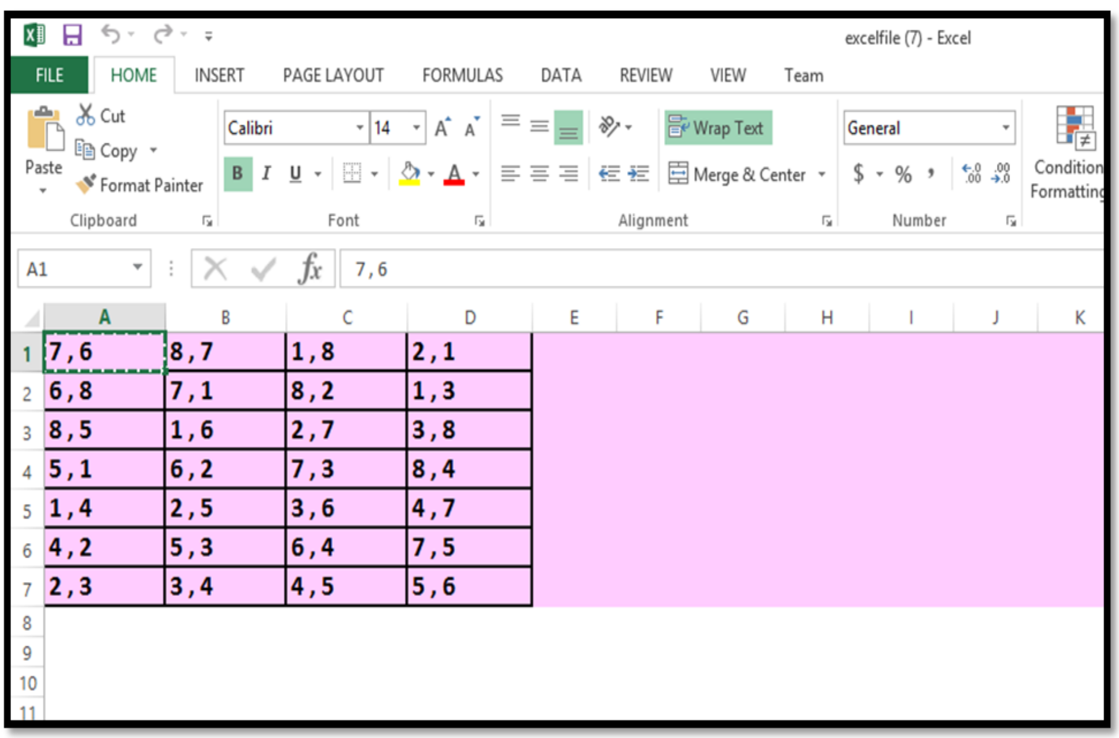

Fig. 4.10: Saving in excel

4.5.2. Generation of Structurally Complete GRC Designs through *web*GRC

To generate structurally incomplete GRC designs through *web*FMC, the following steps needs to be followed by the users:

i) Go to **Generate Design**.

ii) Select **Structurally incomplete GRC** under **Generate Design**.

iii) There are 4 series (developed by Datta *et al*., 2014) under the link

iv) After entering the value of the parameter, click **Generate Design** button and the generated layout along with different parameters will be displayed. User can save the output in MS-Excel spread sheet for further use.

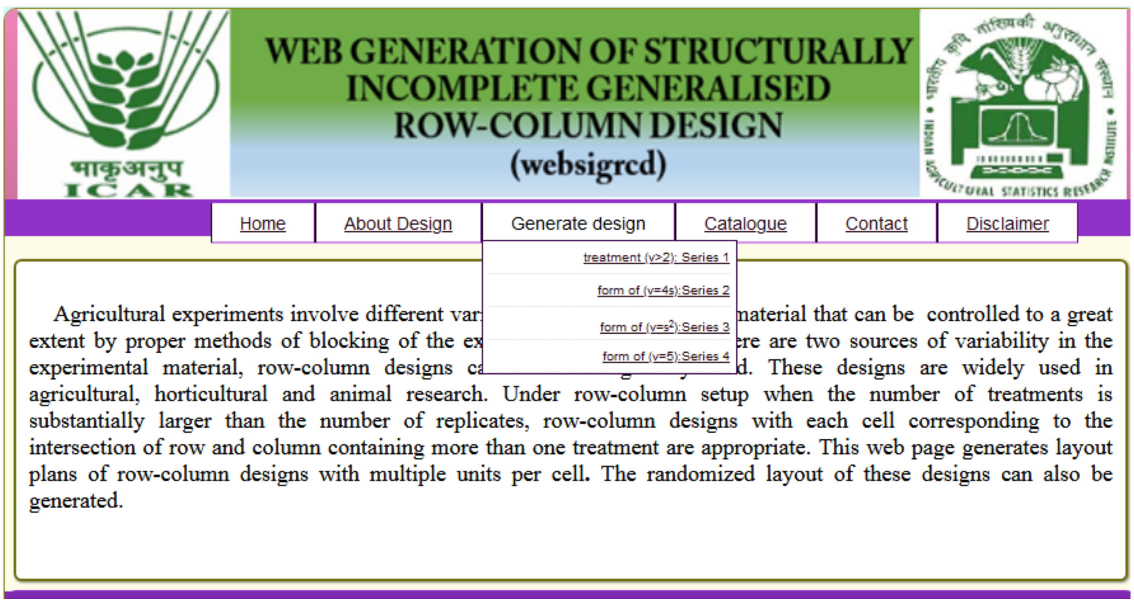

Fig. 4.11: Series of designs under Structurally incomplete GRC

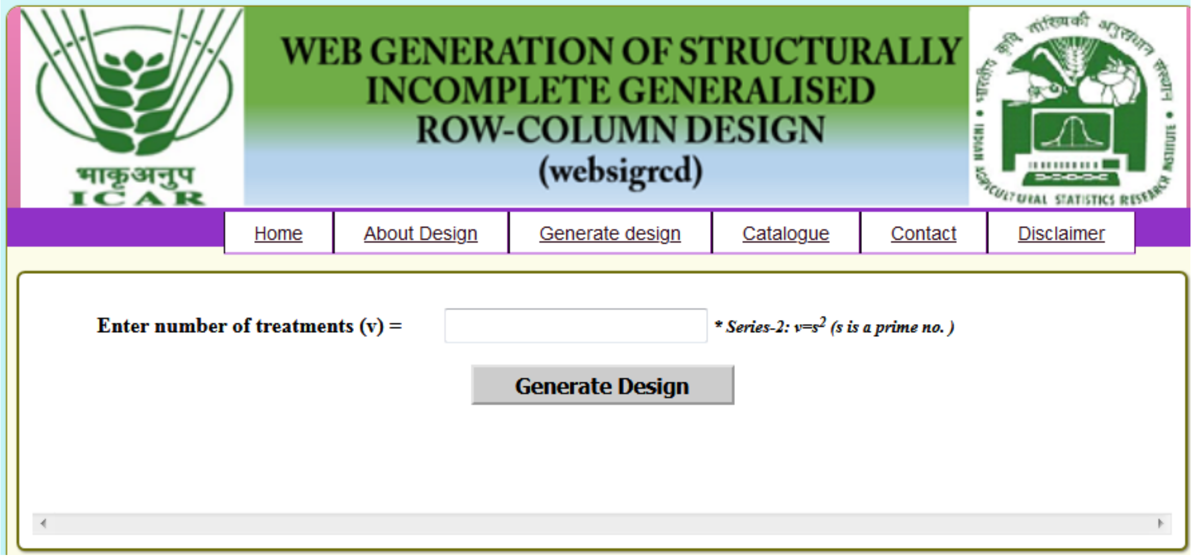

Fig. 4.12: Generation of structurally incomplete GRC for v= s^2 (s is a prime number)

4.6 About Designs

To provide an idea about generalized row- column designs and to guide the users about the online generation of such designs, a section under the option **About Design** has been created and linked with the software. If user wants to have an idea about structurally complete GRC designs, **Structurally Complete GRC** under **About Design** need to be clicked (**Fig. 4.12**), whereas **Structurally Incomplete GRC** option (**Fig. 4.12**) will give an idea about asymmetric factorial designs with minimum level changes.

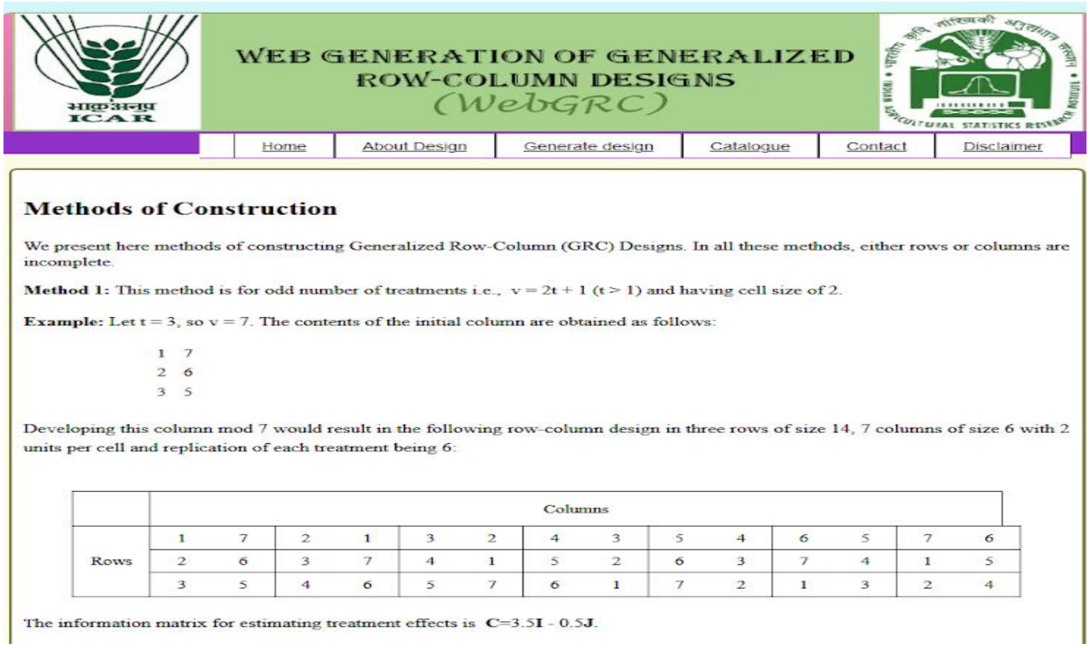

Fig. 4.11: About design for Structurally Complete GRC Designs

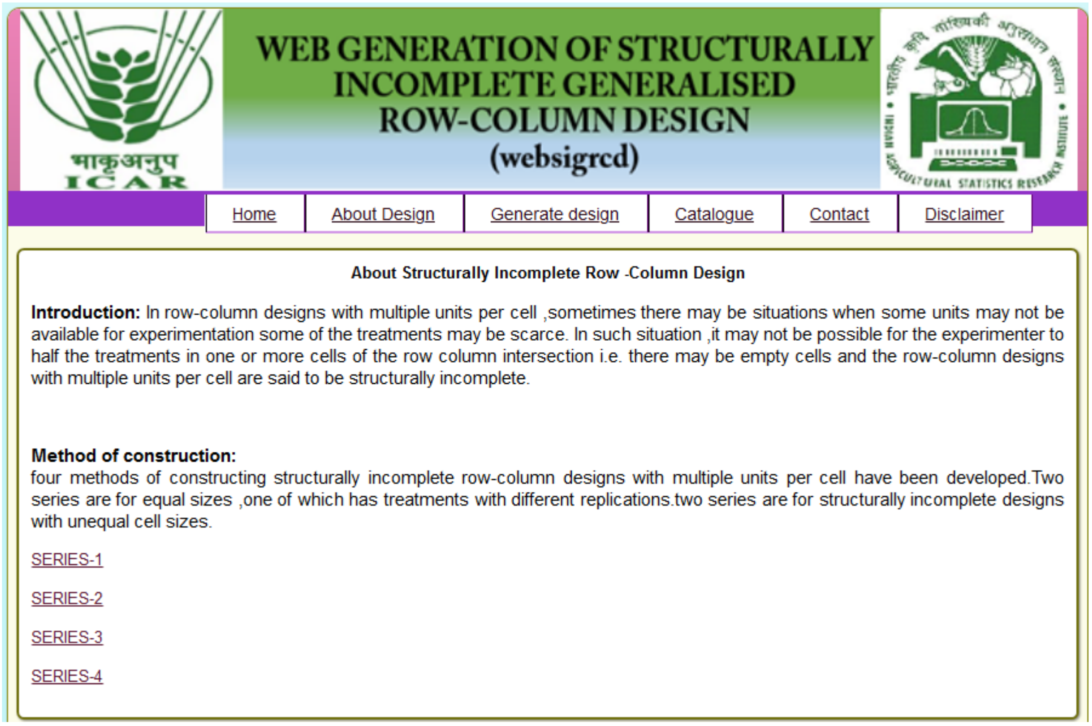

Fig. 4.12: About design for Structurally Incomplete GRC Designs

4.7 Online Catalogue

Online catalogue for both structurally complete and structurally incomplete GRC designs with a specific set of parametric combinations has also been developed and integrated with *web*GRC. User can also generate designs from these catalogues (**Fig. 4.13** and **Fig. 4.14**).

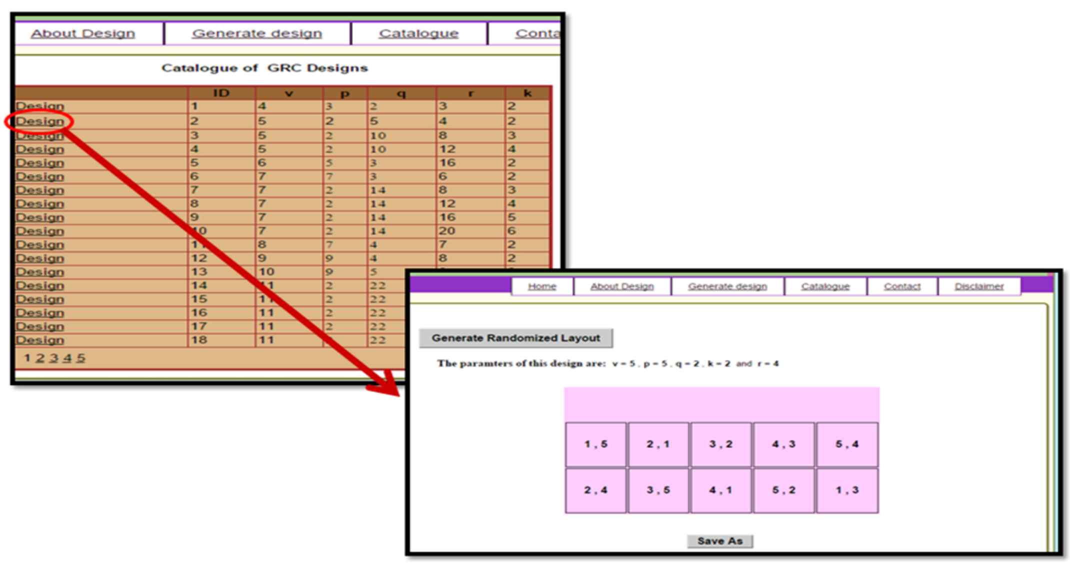

Fig. 4.12: Catalogue and generation of structurally complete GRC Designs

	ID	v	m	n	r_i	k_j	Efficiency Factor
Design	1	4	3	3	3	2	0.667
Design	2	5	10	3	15	2 , 3 , 4	0.804
Design	3	6	10	3	10	3	0.800
Design	4	6	5	4	10	4	0.900
Design	5	6	20	3	24	2 , 3 , 4	0.800
Design	6	7	35	3	31	2 , 3 , 4	0.809
Design	7	7	6	7	11 , 6	2	0.517
Design	8	7	6	7	16 , 12	3	0.740
Design	9	8	3	3	3	4 , 2	0.609
Design	10	8	70	3	35	2 , 3 , 4	0.737
Design	11	9	8	9	15 , 8	2	0.510
Design	12	9	4	4	4	3	0.750
Design	13	11	10	11	19 , 10	2	0.507
Design	14	12	3	3	3	6 , 3	0.709
Design	15	13	12	13	23 , 12	2	0.505
Design	16	15	14	15	27 , 14	2	0.503
Design	17	16	3	3	3	4 , 8	0.769
Design	18	16	5	5	5	4	0.800
Design	19	17	16	17	31 , 16	2	0.503
Design	20	19	18	19	35 , 18	2	0.502
Design	21	20	3	3	3	10 , 5	0.809

Fig. 4.13: Online catalogue of structurally incomplete GRC Designs

4.8 Discussion

*web*GRC is a web based software for generation of a generalized row- column designs. This software is menu driven and user-friendly. It will help the researchers for getting a readymade solution with respect to experiments involving hard-to-change factors and hence will be of immense use to various research experiments in the field of agriculture. Online catalogue will serve as a readymade reference to the available design options for easy selection from user point of view. Researchers can learn more about these designs and their construction methods through about designs menu.

CHAPTER 5

MATING PLANS FOR BREEDING TRIALS USING GENERALIZED ROW-COLUMN DESIGNS

5.1 Introduction

The breeding experiments comprise of two types of designs namely, mating designs and environmental designs. Mating design is a procedure of producing the progenies, while environmental design is subjecting these progenies to the environmental conditions in a systematic manner. Diallel , Partial diallel , Triallel , Partial triallel and Double crosses are some examples of mating designs. A judicious choice of a mating design is essential to attain the breeder's goal. Diallel cross is a set of all possible mating between several genotypes which may be clones, homozygous lines *etc*. These crosses are frequently used in plant breeding trials for estimating genetic components of total variance of a quantitative character. These are also used in estimating general and specific combining abilities of inbred lines involved in the crosses. With exclusion of reciprocal crosses and parental inbred, there are $\frac{n(n-1)}{2}$ possible diallel crosses among a set of n lines that increases rapidly with increase in n. With limited facilities available for testing, a diallel cross may only be possible for a relatively small number of inbred lines. It may be desirable to have a large number of inbred lines but raise only a sample of all possible crosses among them giving rise to Partial diallel cross (PDC). A good amount of literature is available which deals with different aspect of diallel and partial diallel crosses [for example Hinkelmann (1965), Choi *et al.* (2004), Hsu and Ting (2005), Srivastava *et al.* (2013), Harun *et al.* (2016a, 2016b and 2019) etc.] The set of all possible three-way hybrids based on n lines will constitute triallel crosses and there would be $N_T = \frac{n(n-1)(n-2)}{2}$ distinct triallel crosses resulting in distinct three-way hybrids. As the number of lines (n) involved increases, the number of crosses also increases manifold and becomes unmanageably large for the investigator to handle. An answer to this situation lies in taking sample of triallel crosses rather than conducting the experiment with Complete Triallel Crosses (CTC). This leads to the adaption of Partial Triallel Crosses (PTC).

Let there be n lines denoted by $1,2,\ldots,n$. A three-way cross is represented by $(i \times j) \times k$, where the offspring of the cross $i \times j$ is crossed with k and hence i and j are half-parents whereas k is a full-parent for $i \neq j \neq k = 1,2,\ldots,n.$ On the lines of Hinkelmann (1965), a set of mating is said to be a PTC if each line occurs exactly r_H times as half-parent and r_F times as full-parent. Further, each $(i \times j) \times k$ (including the structural symmetricity) either do not occur or occurs exactly once. Since each line is equally often represented as half-parent, it follows that $r_H = 2r_F$ and further, a PTC plan has to be connected.

There are many crops like maize and corn where three-way crosses are commonly used to develop commercial hybrids. Weatherspoon (1970) recommended the use of three-way crosses as they are more uniform, high yielding and stable than the single cross hybrids. A series of PTC plans using Trojan square design, Generalized incomplete trojan type designs and Mutually orthogonal Latin squares have been obtained by Dharmalingam (2002), Varghese and Jaggi (2011) and Sharma *et al.* (2012). In literature, there are mating plans which are developed using block/ row-column designs.

In this chapter, method of constructing PDC plans and PTC plans have been discussed based on GRC designs. The characterization properties of such plans have also been investigated.

5.2 Model and Experimental Setup

The statistical model underlying the analysis of variance of diallel crosses is given by

$$y_{ij} = \mu + g_i + g_j + e_{ij}, \quad i < j = 1,2,\ldots,n \qquad \ldots 5.2.1$$

with restriction $\sum g_i = 0$ for $i = 1, 2, \ldots,(n-1)$. y_{ij} is the response of crosses, μ is the overall mean, g_i, g_j is the g.c.a. effect of i^{th} and j^{th} line and e_{ij} is the error term with mean zero and variance σ^2.

The statistical model underlying the analysis of variance of triallel crosses is given by

$$y_{(ij)k} = \mu + h_i + h_j + g_k + e_{(ij)k} \qquad \ldots 5.2.2$$

($i, j, k = 1,2, \ldots$, n, $i \neq j \neq k$) where $y_{(ij)k}$ stands for the response of triple cross $(i\times j)k$, μ is the overall mean, h is the g.c.a effect of half parents and g is the g.c.a. effect of full parents and $e_{(ij)k}$ are considered to be independent random variables with mean zero and variance σ^2.

5.3 Method of Construction of PDC plans using Generalized Row- Column Designs

Consider a Latin square of order s and another orthogonal Latin square of the same order. Renumber the s treatments of the second Latin square by $s+1$, $s+2,\ldots,2s$. Superimpose the second

Latin square on the first Latin square. This results in a GRC design (Bailey, 1988) with parameter *v=2s (s>2), p=s, q=s* and *k*= 2. A PDC plan can be obtained by making all possible distinct 2-way crosses within each cell of the GRC design. The parameters of the developed PDC plan will be *n* (no of lines/genotypes) = *v*, *N* (no of crosses)= s^2 and *f* (degree of fractionation) = *s/(2s-1)* which is the ratio of crosses in the given plan to Complete Diallel Crosses (CDC) for the same no of lines. In terms of the statistical characterization properties the developed PDC plan is partially balanced following a group divisible association scheme which is described below.

The *v=2s* lines are arranged in two rows of size s each as shown below.

1	2	...	*s*
s+1	*s+2*	...	*2s*

The lines in the other row are first associates to each other and the lines in the first row are second associates.

The information matrix for PDC plan is

$$\boldsymbol{C}= a_0\, \boldsymbol{I}_v + a_1\, \boldsymbol{A}_v + a_2\, \boldsymbol{B}_v \qquad ...5.3.1$$

where,

$a_0 = \frac{(v-k)}{2}, a_1 = -1, a_2 = 0$, here *k= 2*

$A_v = \{a_{ij}\}$ = 1, if i and *j* are first associates

= 0, otherwise

$B_v = \{b_{ij}\}$ = 1, if i and *j* are second associate

=0, otherwise

Example 5.3.1. Let s=5, following is a GRC design with parameters *v* = 10, *p* = 5, *q* = 5 and *k* = 2

Columns	Rows				
	I	II	III	IV	V
I	1 6	2 7	3 8	4 9	5 10
II	2 8	3 9	4 10	5 6	1 7
III	3 10	4 6	5 7	1 8	2 9
IV	4 7	5 8	1 9	2 10	3 6
V	5 9	1 10	2 6	3 7	4 8

Now, considering each treatment as a line in the breeding programme, the following crosses are obtained by making crosses within each cell:

1 × 6	2 × 7	3 × 8	4 × 9	5 × 10
2 × 8	3 × 9	4 × 10	5 × 6	1 × 7
3 × 10	4 × 6	5 × 7	1 × 8	2 × 9
4 × 7	5 × 8	1 × 9	2 × 10	3 × 6
5 × 9	1 × 10	2 × 6	3 × 7	4 × 8

The parameters of this PDC plan are n = 10, N = 25 and f = 5/9. The information matrix for gca using PDC plan is

$$\boldsymbol{C} = 4\boldsymbol{I}_v - 1\boldsymbol{A}_v + 0\boldsymbol{B}_v$$

The 10 lines are arranged as given below:

1	2	3	4	5
6	7	8	9	10

The various associates of the lines based on the crosses involved are as follows:

Treatments	1st Associate	2nd Associate
1	6 7 8 9 10	2 3 4 5
2	6 7 8 9 10	1 3 4 5
3	6 7 8 9 10	1 2 4 5
4	6 7 8 9 10	1 2 3 5
5	6 7 8 9 10	1 2 3 4
6	1 2 3 4 5	7 8 9 10
7	1 2 3 4 5	6 8 9 10
8	1 2 3 4 5	6 7 9 10
9	1 2 3 4 5	6 7 8 10
10	1 2 3 4 5	6 7 8 9

5.4 Method of Construction of PTC Plans using Generalized Row- Column Designs

PTC plans can be obtained from the cell contents of appropriate GRC designs with cells of size 3. The treatments in the design are to be considered as the lines and then possible distinct 3-way crosses in a systematic order are to be made. If the condition of structural symmetry of PTC is not met,

For v (prime) treatments, consider a set of 2 mutually orthogonal Latin square (MOLS) juxtaposing one after other horizontally giving rise to an array (A) of dimension $v \times 2v$. The cell contents of the first row of the repeat the crosses by changing the role of full-parents and half-parents in circular manner. GRC design is formed by taking the first k ($3 \leq k \leq v\text{-}1$) rows of the above array (A). Similarly, cell contents of the second row are obtained by taking the k consecutive rows starting from the 2nd row of the array (A).The resulting design is a GRC design (Datta *et al.*, 2015) with v treatments in p = 2, q = 2v and each cell of size k. A PTC plan can be obtained by making all possible distinct 3-way crosses within each cell of first or second row in a systematic order. In order to meet the condition of a structural symmetry of PTC, distinct crosses of the form $(i \times j) \times k$, $(i \times k) \times j$ and $(j \times k) \times i$ $(i \neq j \neq k = 1,2,\ldots,v)$ are taken in a cell. Degree of fractionation (f) for the developed plans is $12/(v\text{-}1)(v\text{-}2)$.

Example 5.4.1. To illustrate the method of construction, GRC design with parameters v = 7, p = 2, q = 14 and k = 3 is given below:

1 2 3	2 3 4	3 4 5	4 5 6	5 6 7	6 7 1	7 1 2	1 3 5	2 4 6	3 5 7	4 6 1	5 7 2	6 1 3	7 2 4
2 3 4	3 4 5	4 5 6	5 6 7	6 7 1	7 1 2	1 2 3	3 5 7	4 6 1	5 7 2	6 1 3	7 2 4	1 3 5	2 4 6

Consider the treatments as lines. Form all distinct three-way crosses using each cell contents in a particular order, *i.e.*, by considering two lines as half-parents and third one as full parent. There are 52 three-way crosses, each of the form $(i \times j) \times k$, $(i \times k) \times j$ and $(j \times k) \times i$.

(1×2)×3 (1×3) ×2 (2×3) ×1	(6×7)×1 (6×1) ×7 (1×7) ×6	(4×6)×1 (4×1) ×6 (1×6) ×4
(2×3)×4 (2×4) ×3 (3×4) ×2	(7×1)×2 (7×2) ×1 (1×2) ×7	(5×7)×2 (5×2) ×7 (2×7) ×5

(3×4)×5 (3×5) ×4 (4×5) ×3	(1×3)×5 (1×5) ×3 (3×5) ×1	(6×1)×3 (6×3) ×1 (1×3) ×6
(4×5)×6 (4×6) ×5 (5×6) ×4	(2×4)×6 (2×6) ×4 (4×6) ×2	(7×2)×4 (7×4) ×2 (2×4) ×7
(5×6)×7 (5×7) ×6 (6×7) ×5	(3×5)×7 (3×7) ×5 (5×7) ×3	

Thus, altogether, there are 52 crosses in the final PTC plan and this PTC plan satisfies the structural symmetric property. A CTC plan for 7 lines requires 105 three-way crosses. The degree of fractionation for the above plan is f = 12/30 = 2/5.

5.5. Discussion

It can be deduced from the results that through the suggested methods, breeders can obtain small and efficient diallel and triallel cross plans with comfortable knowledge in statistics. The plans obtained here using these designs yield smaller degree of fractionation thereby reducing the resources and reduce the heterogeneity present in the experimental field, simultaneously. As the lines are being selected using diallel or triallel plans, uniformity, yield and stability of the selected ones are also ensured.

सार

खेत एवं पशुओं से संबन्धित परीक्षणों में जहाँ परीक्षण इकाइयों में परिवर्तन के दो ऐसे श्रोत हों जो परिणामी चर को प्रभावित करने की क्षमता रखते हों तो इस स्थिति में रो-कॉलम अभिकल्पनाओं का प्रयोग किया जाता है। पठन सामग्री में अभी तक उपलब्ध लगभग सभी रो-कॉलम अभिकल्पनाओं में रो-कॉलम प्रतित्छेदन पर केवल एक ही इकाई होती है। ऐसी स्थिति में जहाँ ट्रीटमेंट की संख्या अधिक हो और परीक्षण संसाधनों की कमी हो तो रो-कॉलम प्रतित्छेदन में एक से अधिक इकाइयां होने पर जनरलाईज्ड रो-कॉलम (GRC) अभिकल्पनाओं का प्रयोग किया जाता है। अभी तक उपलब्ध अभिकल्पनाओं से ट्रीटमेंटों के सभी संभव युग्मों की तुलनाओं का अध्ययन किया जाता है। प्रेक्षणों की अनुप्लब्धताए ; ऑउतलायरों का पाया जाना आदि कुछ ऐसी बातें है जो परीक्षण के दौरान सामने आ सकती हैं। इनके कारण ट्रीटमेंटों की परस्पर तुलनाओं के आकलन की शुद्धता में कमी आ सकती है। एक या अधिक अनुपलब्ध प्रेक्षणों वाली जनरलाईज्ड रो-कॉलम अभिकल्पनाओं के विभिन्न वर्गो की प्रबलता की भी जाँच की गयी है। यह देखने में आया है कि अधिकांश अभिकल्पनाओं से अधिकतम उच्च स्तर (>90) की दक्षता पायी गयी है तथा अभिकल्पनयें प्रबल हैं। साथ ही यह भी देखा गया है कि अनुपलब्ध प्रेक्षणों की संख्या के बढ़ने के साथ साथ परीक्षण की दक्षता में गिरावट का ट्रेंड आ जाता है। स्थानिक प्रभावों के लिए संतुलित जीआरसी डिजाइनों की श्रृंखला विकसित की गई है। प्रत्यक्ष प्रभाव और स्थानिक प्रभाव से संबंधित विरोधाभासों के आकलन के लिए सूचना मैट्रिसेस प्राप्त किया गया है। विकसित किए गए अभिकल्पनाओं यह सुनिश्चित करते हैं कि एक सेल के भीतर हर उपचार में पड़ोसी के रूप में दिखाई देने वाले हर दूसरे उपचार में कई बार होता है। इसके अलावा, प्रयोगकर्ताओं को एक रेडीमेड समाधान देने के लिए एक SAS मैक्रोस विकसित किया गया है जो डिजाइनों के लेआउट (layout) को उत्पन्न करता है। जनरलाईज्ड रो-कॉलम अभिकल्पनाओं की उपलब्धता को आसान बनाने के लिए WebGRC के नाम से एक वेब सोल्युशन (Web solution) विकसित किया गया है जिससे इन अभिकल्पनाओं के यादृछिक लेआउट (lay out) ऑनलाइन प्राप्त किए जा सकते हैं। इन डिजाइनों का उपयोग आंशिक रूप से डायलेल क्रॉस (PDC) या आंशिक त्रिकोणीय क्रॉस (PTC) योजनाओं जैसे कि प्रजनन कार्यक्रम में व्यक्तिगत पैतृक लाइनों के रूप में विचार करके और प्रत्येक सेल के बीच लाइनों के बीच क्रॉस बनाकर किया जा सकता है। यहां, जीआरसी डिजाइनों के विभिन्न वर्गों का उपयोग करके PDC और PTC योजनाओं को प्राप्त करने के तरीकों का वर्णन किया गया है। इन डिज़ाइनों का उपयोग करके प्राप्त की गई योजनाओं से छोटे अंशों का विभाजन होगा, जिससे संसाधनों में कमी आएगी और प्रायोगिक क्षेत्र में मौजूद विषमता को कम किया जा सकेगा।

ABSTRACT

In field and animal experiments, where there are two sources of variation in experimental units that may influence the response variable, row-column designs are used. Most of the row-column designs developed in the literature have only one unit corresponding to the intersection of row and column i.e. in a single cell. However, for the instances when the number of treatments is large with limited experimental resources, Generalized Row-Column (GRC) designs are used where there is more than one unit in each row-column intersection. The presence of missing observations, outliers in the data, etc. are some of the disturbances that may occur during experimentation. These disturbances may lead to less precise comparisons among treatments. Robustness of different classes of GRC designs against missing of one or more observations has been investigated. It is found that the efficiency is quite high (more than 90%) for most of the designs and the designs are robust and there is a decreasing trend in efficiency with increase in number of missing observations. Series of GRC designs balanced for spatial effects have been developed. The information matrices for estimating the contrasts pertaining to direct effect and spatial effect have been derived. The designs developed ensure that within a cell every treatment has every other treatment appearing as neighbour a constant number of times. Further, in order to give a readymade solution to the experimenters, a SAS macro has been developed that generates the layout of the designs. For easy accessibility of GRC designs, a web solution named WebGRC has been developed that provides the online generation of randomized layout of these designs along with an online catalogue within a permissible range.These designs can be advantageously used for obtaining mating plans like Partial diallel cross (PDC) or Partial triallel cross (PTC) plans by considering treatments in the design as individual parental lines in the breeding programme and by making crosses between lines within each cell. Here, methods of obtaining PDC and PTC plans using different classes of GRC designs have been described. The plans obtained using these designs will yield smaller degree of fractionation thereby reducing the resources and reduce the heterogeneity present in the experimental field, simultaneously.

SUMMARY

Row-column design is used when there are two cross classified sources of variation in experimental units that influence the response variable. These designs are used to control variability in field and animal experiments. Most of the row-column designs developed in the literature have one unit corresponding to the intersection of row and column. However, there may be instances when the number of treatments is substantially large with limited number of replicates. A more general class of row-column designs is required where there is more than one unit in each row-column intersection. These designs may be called as Generalized Row-Column (GRC) designs. GRC design is an arrangement of v treatments in p rows and q columns such that the intersection of each row and column consist of more than one unit.

In this study, Robustness of different classes of GRC designs against missing of one or more observations within a cell as per the efficiency criteria has been investigated. A list of robust GRC designs has prepared giving the parameters and the efficiency of the designs. A design is considered to be highly robust against missing observation(s) if the loss in efficiency of the residual design is not more than 5% and robust if the loss in efficiency of the residual design is between 5% to 10%. The efficiency of the GRC designs in the absence of one or more observations has been studied and the efficiency is found to be quite high for most of the designs and thus the designs are robust. There is a decreasing trend in efficiency with increase in number of missing observations. It is further seen that smaller designs are more affected by the missing observations.

In GRC designs, since there are more number of units in a cell, it is likely that the treatment applied to one experimental unit may affect the response of the neighbouring unit in the same cell if the units are placed linearly adjacent giving rise to spatial effects. The study in presence of spatial effects from neighbouring units requires construction of an environment or an arrangement in which the neighbouring units have to appear in a predetermined pattern. Here, series of GRC designs balanced for these spatial effects have been developed. The information matrices for estimating the contrasts pertaining to direct effect and spatial effect have been derived. The designs developed ensure that within a cell every treatment has every other treatment appearing as neighbour a constant number of times. A list of efficient designs has been prepared. It is seen that

the efficiency of direct treatment effects of these designs constructed is more as compared to neighbour treatment effects. The efficiency factor increases with increase in cell size for a given number of treatments. Further, in order to give a readymade solution to the experimenters, a SAS macro has been developed that generates the layout of the designs.

A web solution named *Web*GRC has been developed for the generation of GRC designs that would be highly useful to the experimenters. The webpage displays the layout plans along with the randomized layout for given number of treatments. The parameters of the design so generated are also displayed. An online catalogue of the GRC designs is also prepared and included in the software wherein the user can select the design by seeing all the parameters and then can get the randomized layout. The details regarding the method of obtaining these designs are also included. This software will provide freely available solution for the researchers and students working in this area.

Mating plan is a systematic procedure of producing the progenies. Diallel and triallel crosses are some examples of mating plans. GRC designs can be advantageously used for obtaining mating plans like Partial diallel cross (PDC) or Partial triallel cross (PTC) plans by considering treatments in the design as individual parental lines in the breeding programme and by making crosses between lines within each cell. Here, methods of obtaining PDC and PTC plans using different classes of GRC designs have been described. Breeders can obtain small and efficient diallel and triallel cross plans with comfortable knowledge in statistics. The plans obtained here using these designs yield smaller degree of fractionation thereby reducing the resources and reduce the heterogeneity present in the experimental field, simultaneously. As the lines are being selected using diallel or triallel plans, uniformity, yield and stability of the selected ones are also ensured. SAS code has been developed to obtain the information matrix for the PDC and PTC plans.

REFERENCES

AgroPlotter. https://agrosoftsystems.com/plotter_launch.html.

Andersen, L. D. and Hilton, A. J. W. (1980). Generalized Latin rectangles I: Constructions and decomposition. *Discrete Mathematics*, **31**, 125-152.

Bailey, R. A. (1988). Semi Latin squares. *Journal of Statistical Planning and Inference*, **18**, 299-312.

Bailey, R. A. (1992). Efficient semi-Latin squares. *Statistica Sinica*, **2**, 413-437.

Bailey, R. A. and Monod, H. (2001). Efficient semi-Latin rectangles: Designs for plant disease experiments. *Scandanavian Journal of Statistics*, **28**, 257-270.

Bedford, D. and Whitaker, R. M. (2001). A new construction for efficient semi-Latin squares. *Journal of Statistical Planning and Inference*,**98**, 287-292.

Bhar, L. (2014). Robustness of variance balanced block designs, *Sankhya* B, **76**(2), 305-361.

Chan, B.S.P. and Eccleston, J. A. (2003). On the construction of nearest–neighbour balanced row–column designs, *Australian & New Zealand Journal of Statistics*, **45(1)**, 97 - 106.

Choi K. C., Gupta S. and Kageyama S. (2004). Designs for diallel crosses for test versus control comparisons.*Utilitas Mathematica*, **65**: 167-180.

Chigbu, P. E. (2003). The "best" of the three optimal $(4 \times 4)/4$ semi-Latin squares. *Sankhya: The Indian Journal of Statistics*, **65(3**), 641-648.

Darby, L. A. and Gilbert, N. (1958). The Trojan Square. *Euphytica*, **7**, 183-188.

Datta, A., Jaggi, S., Varghese, C. and Varghese, E. (2014). Structurally incomplete row-column designs with multiple units per cell. *Statistics and Applications*, **12(1&2)**, 71-79.

Datta, A., Jaggi, S., Varghese, C. and Varghese, E. (2015). Some series of row-column designs with multiple units per cell. *Calcutta Statistical Association Bulletin*, **67,** (265-266), 89-99.

Dean, A., Morris, M., Stufken, J. and Bingham, D. (2015). Handbook of Design and Analysis of Experiments. Edited book. CRC Press, Taylor and Francis Group.

Design Resources Server. http://drs.icar.gov.in2007.

Dharmalingam, M. (2002). Construction of partial triallel crosses based on Trojan square design. *Journal of Applied Statistics*, **29(5)**, 675-702.

Edmondson, R. N. (1998). Trojan square and incomplete Trojan square design for crop research. *Journal of Agricultural Science*,**131**, 135-142.

Federer, W.T. and Basford, K.E. (1991). Competing effects designs and models for two-dimensional field arrangements. *Biometrics*, 1461-1472.

Freeman G. H. (1979). Some two-dimensional designs balanced for nearest neighbours. *Journal of Royal Statistical Society*, **41(B),** 88-95.

Harshbarger, B. and Davis, L. L. (1952). Latinized rectangular lattices. *Biometrics*, **8**, 73-84.

Harun, M., Varghese, C., Jaggi, S., Datta, A. and Varghese, E. (2019). Efficient and Cost Effective Partial Three-Way Cross Designs for Breeding Experiments with Scarce Resources. *Journal of the Indian Society of Agricultural Statistics*, **73**(1): 71-78.

Harun, M., Varghese, C., Varghese, E. and Jaggi, S. (2016 a). Three-way cross designs for animal breeding experiments. *The Indian Journal of Animal Sciences*, **86**(6): 710–714.

Harun, M., Varghese, C., Varghese, E. and Jaggi, S. (2016 b). Triallel cross designs for comparing a set of test lines with a control line. *The Indian Journal of Genetics and Plant Breeding*. **76**(1): 84-87.

Hinkelmann K. (1965). Partial triallel crosses. *Sankhya*, **27**: 173-196.

Hinkelmann, K. and Kempthorne, O. (2005). *Design and Analysis of Experiments. Advanced Experimental Design*. Vol. 2, John Wiley and Sons Inc. New Jersy.

Hsu Y. F. and Ting C. P. (2005). A-optimal and efficient diallel cross experiments for comparing test treatments with a control. *Statistics and probability letters*, **71**(1): 99-110.

Jaggi, S., Varghese, C., Varghese, E. and Sharma, V. K. (2010). Generalized incomplete Trojan-type designs. *Statistics and Probability Letters*, **80,** 706-710.

Jaggi, S., Varghese, C., Varghese, E., Sharma, A. (2015). Web generation of experimental designs balanced for indirect effects of treatments (WEB-DBIE). *Computers and Electronics in Agriculture*,**111**, 62–68.

Jaggi, Seema, Varghese, Cini, and Varghese Eldho (2016): A series of generalized incomplete Trojan-type designs. *Journal of Combinatorics, Information and System Sciences: American Journal*, **40**(1-4), 53-60.

Lal, K., Gupta, V. K. and Bhar, L. (2003). Robustness of Youden square designs against missing data. *Journal of Indian Society of Agricultural Statistics*, **56(2)**, 107-116.

Parsad, R. (2006). A note on semi-Latin squares. *Journal of Indian Society of Agricultural Statistics*, **60**(**2**), 131-133.

Pearce, S. C. (1952). Some new designs for Latin square type. *Journal of Royal Statistical Society*, **B14**, 101-106.

Preece, D. A. and Freeman, G. H. (1983). Semi-Latin squares and related designs. *Journal of Royal Statistical Society*, **B45**, 267-277.

SahaRay, R. (2001). Further study on optimal row-column designs with unequal row and column sizes. *Statistics and Applications*, **3**, 165-175.

Sharma, A., Varghese, C., Jaggi, S. (2013). A web solution for partially balanced incomplete block experimental designs. *Computers and Electronics in Agriculture*,**99**, 132–134.

Singh, G., Gupta, S. and Singh, M. (1987). Robustness of Row- column designs. *Statistics and Probability Letters*, **5**, 421-424.

Srivastava S., Varghese C., Jaggi S. and Varghese E. (2013). Diallel cross designs for test versus control comparisons. *Indian J. Genet*., **73**(2): 186-193.

Taksande, N., Sharma, A., Varghese, C., Jaggi, S., Lal, S.B. (2012). Web-enabled software for generation and analysis of partial diallel crosses. *Journal of Indian Society Agricultural Statistics*, **66 (2)**, 343–350.

Uto, N. P. and Ekpenyong, M. E. (2014). On contrasts for discriminating the optimal semi-Latin squares with side six and block size two. *Science World Journal*, **9(3)**, 1597-6343.

Varghese, C. and Jaggi, S. (2011). Generalized row-column designs for agricultural experiments**.** *IASRI/PR-02/2011*, IASRI, New Delhi.

Varghese, C., Rao, A. R. and Sharma, V. K. (2002). Robustness of Williams Square change-over designs. *Metrika*, **55**, 199-208.

Varghese, E., Jaggi, S. and Varghese, C. (2014). Neighbor-Balanced Row-column Designs *Communications in Statistics - Theory and Methods*, 43(6), 1261–1276.

Web generation of polycross designs (webPD). http://iasri.res.in/webPD. 2015.

Williams, E. R. (1986). Row and column designs with contiguous replicates. *Australian Journal of Statistics*, **28**, 154-163.

Yates, F. (1936). Incomplete Latin squares. *Journal of Agricultural Science*, **26**, 301-315.

Youden, W.J. (1937). Use of incomplete block replications in estimating tobacco mosaic virus. *Contribution from Boyce Thompson Institute*, **9,** 317-326.

ANNEXURE I

SAS CODE FOR OBTAINING THE C-MATRIX AND THE HARMONIC MEAN OF NON-ZERO EIGEN-VALUES OF C-MATRIX OF ORIGINAL DESIGN AND THE RESIDUAL DESIGN FOR GRC DESIGN

```
proc iml;

/*design [put non-zero values]*/

a={

1    6    2    7    3    8    4    9    5    0    ,
2    8    3    9    4    10   5    6    1    0    ,
3    10   4    6    5    7    1    8    2    0    ,
4    7    5    8    1    9    2    10   3    0    ,
5    9    1    10   2    6    3    7    0    0

};

/*define cell sizes*/

b={

2    2    2    2    1    ,
2    2    2    2    1    ,
2    2    2    2    1    ,
2    2    2    2    1    ,
2    2    2    2    0

};

cc=b[+, ];

dd=b[ ,+];

bb=j(nrow(b)*ncol(b),1,0);

k=1;

do i=1 to nrow(b);

do j=1 to ncol(b);

bb[k, ]= b[i,j];

k=k+1;
```

```
end;
end;
b1=bb[loc(bb>0),];
*print b1;
aa=j(nrow(a)*ncol(a),1,0);
k=1;
do i=1 to nrow(a);
do j=1 to ncol(a);
aa[k, ]= a[i,j];
k=k+1;
end;
end;
m1=j(nrow(a)*ncol(a),1,1);/*mean vector*/
dir=j(nrow(a)*ncol(a),max(a),0);/*design matrix
                    obseravation VS treatment*/
k=1;
do i=1 to nrow(a);
do j=1 to ncol(a);
if a[i,j]>0 then
          do;
          dir[k,a[i,j]]=1;
          k=k+1;
          end;
end;
end;
r=j(nrow(a)*ncol(a),nrow(dd),0);/*design matrix observation
VS row*/
k=1;
do i=1 to nrow(a);
do j=1 to ncol(a);
r[k,i]=1;
k=k+1;
```

```
end;
end;
c=j(nrow(a)*ncol(a),ncol(b),0);/*design  matrix  observation
VS column*/
k=1;
do i=1 to nrow(b);
do j=1 to ncol(b);
do l=1 to b[i,j];
c[k,j]=1;
k=k+1;
end;
end;
end;
cell=j((nrow(a)*ncol(a)),nrow(b1),0);/*design matrix
                              observation VS cell*/
kk=1;
z=0;
do k=1 to nrow(b1);
do j=1 to b1[k];
if aa[z+j, ]>0 then
     do;
     cell[kk,k]=1;
     kk=kk+1;
     end;
end;
z=z+b1[k];
end;
x=m1||dir||r||c;/*design matrix*/
*print x[format=3.0];
x1=dir;
x2=m1||r||c;
c_mat=(x1`*x1)-(x1`*x2*(ginv(x2`*x2))*x2`*x1)/*C matrix*/;
```

```
print c_mat;
eig=eigval(c_mat);
eig1=eig[loc(eig>0.005),];/*positive eigen values*/
eig2=1/eig1;
HM1=nrow(eig2)/sum(eig2);
print HM1;
quit;
```

ANNEXURE II

SAS Code PROC IML to calculate the information matrix (C-matrix) of direct treatment effects and neighbour treatment effects

```
proc iml;
/*design [put non-zero values]*/
a={
1 2 3 1 3 5 1 4 7 1 5 2 1 6 4 1 7 6,
2 3 4 2 4 6 2 5 1 2 6 3 2 7 5 2 1 7,
3 4 5 3 5 7 3 6 2 3 7 4 3 1 6 3 2 1 ,
4 5 6 4 6 1 4 7 3 4 1 5 4 2 7 4 3 2,
5 6 7 5 7 2 5 1 4 5 2 6 5 3 1 5 4 3,
6 7 1 6 1 3 6 2 5 6 3 7 6 4 2 6 5 4 ,
7 1 2 7 2 4 7 3 6 7 4 1 7 5 3 7 6 5

};
/*define cell sizes*/
b={3 3 3 3 3 3 ,
3 3 3 3 3 3 3
3 3 3 3 3 3 ,
3 3 3 3 3 3,
3 3 3 3 3 3 ,
3 3 3 3 3 3 ,
3 3 3 3 3 3
};
cc=b[+, ];
dd=b[ ,+];
bb=j(nrow(b)*ncol(b),1,0);
k=1;
do i=1 to nrow(b);
do j=1 to ncol(b);
bb[k, ]= b[i,j];
k=k+1;
end;
end;
b1=bb[loc(bb>0),];
*print b1;
aa=j(nrow(a)*ncol(a),1,0);
k=1;
do i=1 to nrow(a);
do j=1 to ncol(a);
aa[k, ]= a[i,j];
k=k+1;
end;
end;
*print aa;
m1=j(nrow(a)*ncol(a),1,1);/*mean vector*/
/*print m1;*/
dir=j(nrow(a)*ncol(a),max(a),0);/*design matrix -obs VS direct treatment*/
k=1;
do i=1 to nrow(a);
do j=1 to ncol(a);
if a[i,j]>0 then
            do;
```

```
            dir[k,a[i,j]]=1;
            k=k+1;
            end;
end;
end;
print dir;
r=j(nrow(a)*ncol(a),nrow(dd),0);/*design matrix -obs VS row*/
k=1;
do i=1 to nrow(a);
do j=1 to ncol(a);
r[k,i]=1;
k=k+1;
end;
end;
*print r;
c=j(nrow(a)*ncol(a),ncol(b),0);/*design matrix -  obs VS column*/
k=1;
do i=1 to nrow(b);
do j=1 to ncol(b);
do l=1 to b[i,j];
c[k,j]=1;
k=k+1;
end;
end;
end;
*print c;
cell=j((nrow(a)*ncol(a)),nrow(b1),0);/*design matrix -  obs VS cell*/
kk=1;
z=0;
do k=1 to nrow(b1);
do j=1 to b1[k];
if aa[z+j, ]>0 then
      do;
      cell[kk,k]=1;
      kk=kk+1;
      end;
end;
z=z+b1[k];
end;
*print cell;

l_neig = j(nrow(a)*ncol(a),max(a),0);
k=2;
z=0;
do i = 1 to nrow(b1);
do j = 1 to b1[i]-1;

      if aa[z+j, ]>0 then l_neig[k,aa[z+j, ]]=l_neig[k,aa[z+j, ]]+1;
      k=k+1;
end;
z=z+b1[i];
k=k+1;
end;

*print l_neig;

r_neig = j(nrow(a)*ncol(a),max(a),0);
```

```
k=1;
z=0;
do i = 1 to nrow(b1);
do j = 2 to b1[i];

      if aa[z+j, ]>0 then r_neig[k,aa[z+j, ]]=r_neig[k,aa[z+j, ]]+1;
      k=k+1;
end;
z=z+b1[i];
k=k+1;
end;

*print r_neig;

neigh=l_neig+r_neig;
x1=dir||neigh;
x2=m1||r||c;
c_mat=(x1`*x1)-(x1`*x2*(ginv(x2`*x2))*x2`*x1)/*C matrix*/;
print c_mat;


c11=j(max(a),max(a),0);
do i=1 to max(a);
do j=1 to max(a);
c11[i,j]=c_mat[i,j];
end;
end;
*print c11;

c12=j(max(a),ncol(c_mat)-max(a),0);
do i=1 to max(a);
k=1;
do j=max(a)+1 to ncol(c_mat);
c12[i,k]=c_mat[i,j];
k=k+1;
end;
end;
*print c12;

c22=j(nrow(c_mat)-max(a),nrow(c_mat)-max(a),0);
k=1;
do i=max(a)+1 to nrow(c_mat);
kk=1;
do j=max(a)+1 to nrow(c_mat);
c22[k,kk]=c_mat[i,j];
kk=kk+1;
end;
k=k+1;
end;
*print c22;
c_dir=c11- c12*ginv(c22)*c12`;
print c_dir;
eig=eigval(c_mat);
*print eig;
eig1=eig[loc(eig>0.0000001),];/*positive eigen values*/
rep=dir`*dir;
eig2=eig1/(rep[1,1]);
```

```
eig3=1/eig2;
CanEffFactor=nrow(eig3)/sum(eig3);
*print CanEffFactor;
quit;
```

ANNEXURE III

SAS CODE FOR ANALYSIS OF NBGRC DESIGN

```
Data NBGRC;
Input row column treatment neighbour Yield;
Cards;
1	1	1	2	27.84
1	1	2	1	23.20
1	1	3	2	34.03
2	1	2	3	46.41
2	1	3	2	37.13
2	1	4	3	23.51
3	1	3	4	43.57
3	1	4	3	25.47
3	1	5	4	26.93
4	1	4	5	42.39
4	1	5	4	29.01
4	1	1	5	50.20
5	1	5	1	14.34
5	1	1	5	53.79
5	1	2	1	31.64
1	2	1	3	21.27
1	2	3	1	14.18
1	2	5	3	16.07
2	2	2	4	45.70
2	2	4	2	21.38
2	2	1	4	27.42
3	2	3	5	47.48
3	2	5	3	18.99
3	2	2	5	64.75
4	2	4	1	12.47
4	2	1	4	31.99
4	2	3	1	18.05
5	2	5	2	23.91
5	2	2	5	74.71
5	2	4	2	26.72
1	3	1	4	40.22
1	3	4	1	15.68
1	3	2	4	67.03
2	3	2	5	95.63
2	3	5	2	30.60
2	3	3	5	70.13
3	3	3	1	26.81
3	3	1	3	40.22
3	3	4	1	18.53
4	3	4	2	39.90
4	3	2	4	85.31
4	3	5	2	35.70
```

5	3	5	3	35.06
5	3	3	5	87.66
5	3	1	3	46.41
1	4	1	5	65.74
1	4	5	1	17.53
1	4	4	5	55.52
2	4	2	1	42.19
2	4	1	2	50.63
2	4	5	1	19.13
3	4	3	2	67.03
3	4	2	3	83.79
3	4	1	2	54.84
4	4	4	3	45.72
4	4	3	4	78.20
4	4	2	3	90.23
5	4	5	4	51.80
5	4	4	5	75.70
5	4	3	4	83.79

;

PROC glm data=NBGRC;
Class row column treatment neighbour;
Model YIELD = row column treatment neighbour/ss2;
Run;

ANNEXURE IV

SAS MACRO FOR GENERATION OF NEIGHBOUR BALANCED GRC DESIGNS AND ITS OUTPUT

```
%let v=7;/* Enter the number of teatments (Treament number should be odd
number)*/
%let s=3;/*Enter the cell sizes(it varies from 2 to (v-1)*/
ods rtf  file= 'output.rtf' startpage=no;
proc iml;
TRT1=j(&v,&s*(&v-1),0);
k=1;
do i=1 to &s;
do j=1 to &v;
TRT1[j,i]=(j+(i-1));
if TRT1[j,i]>&v then TRT1[j,i]=TRT1[j,i]-&v;
end;
end;
kk=&s+1;
do k=1 to &v-1;
do i=1 to &s;
do j=1 to &v;
TRT1[j,kk]=TRT1[j,kk-(&s)]+(i-1);
if TRT1[j,kk]>&v then do;
TRT1[j,kk]=TRT1[j,kk]-&v;
end;
end;
kk=kk+1;
end;
end;
varNames2= "Column1":"Column"+strip(char(&v-1));
varNames3= "Row1":"Row"+strip(char(&v));
do i=1 to (&v-1);
do j=1 to &s;
columns=varNames2[ ,i];
columns1=columns1||columns;
end;
end;
GRC_Design=char(TRT1,5,0);
print 'Neighbour Balanced Generalized Row Column (GRC) Design';
print GRC_Design[rowname=varNames3 colname=columns1];
print 'Number of Rows =' &v;
print 'Number of Columns ='(&v-1);
print 'Number of treatments in each Row-Column Intersection is =' &s;
ods rtf close;
quit;
```

SAS output for generation of a Neighbour Balance GRC designs for *v* = 5 and *k* = 3

The SAS System

Neighbour Balanced Generalized Row Column (GRC) Design

GRC_Design												
	Column1	**Column1**	**Column1**	**Column2**	**Column2**	**Column2**	**Column3**	**Column3**	**Column3**	**Column4**	**Column4**	**Column4**
Row1	1	2	3	1	3	5	1	4	2	1	5	4
Row2	2	3	4	2	4	1	2	5	3	2	1	5
Row3	3	4	5	3	5	2	3	1	4	3	2	1
Row4	4	5	1	4	1	3	4	2	5	4	3	2
Row5	5	1	2	5	2	4	5	3	1	5	4	3

Number of Rows =	5

Number of Columns =	4

Number of treatments in each Row-Column Intersection is =	3