

SAS Macro for Generation of Symmetric Factorial with Minimally Changed Run Sequences

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Factorial experiments are experiments wherein two or more factors each at two or more levels are used simultaneously. Such an experiment allows studying the effect of each factor as well as the effects of interactions between factors on the response variable. Factorial experiments find profound applications in many field of agricultural and industrial research. It is always advisable that the order of execution of factorial designs should be random as randomization of run sequences can avoid bias in the estimates of the effects of interest which might result from a time trend. However under factorial experiment setup, randomization can induce a large number of changes in factor levels which will ultimately make the experimentation expensive, time-consuming and difficult. The number of level changes is of serious concern to experimenters in many agricultural, post-harvest and processing, engineering and industrial experiments as in such experiments one may come across some situations where it is physically very difficult to change levels of some factors. So, in such situations, instead of randomizing the run sequences, one alternative is to go for factorial experiments with minimum number of level changes i.e. one should construct run sequences where total number of change in the level of factor is minimum.

The search for experimentation run orders that offer the minimum number of factor level changes and at the same time minimize the possible influence of undesirable factors is nothing new. Cox (1951) began the study of systematic designs for the replicated varietal trials. Draper and Stoneman (1968) have tabulated run orders for the 2^3 factorial and for various fractional factorial designs also requiring eight runs in which the estimates of the main effects are free or nearly free of a linear time trend and the number of factor level changes is also small. Dickinson (1974) extended the work of Draper and Stoneman (1968) to 2^4 and 2^5 complete factorial plans with the search restricted to minimum cost run orders. De León *et al.* (2005) proposed experimentation orders for designs with 8 or 16 runs that combine an excellent level of protection against the influence of unknown factors, with the minimum number of changes in factor levels. Correa *et al.* (2009) propose a method to obtain the matrix of a 2^{k+1} design, using a 2^k design with the minimum number of level changes as a starting point. The matrix obtained has the minimum number of changes and a maximum bias

which doubles that of the initial design. Correa *et al.* (2012) discussed the best possible sequences for designs with 32 experiments, as well as sequences that offer excellent properties for designs with 64 and 128 experiments. Bhowmik *et al.* (2015) obtained symmetric factorial designs with minimum level changes along with half replicate of 2-level factorial designs and also 2 level confounded factorial in two blocks with minimum level changes in run sequences. A detailed investigation on factorial experiments with minimum changes in run sequences was carried out by Bhowmik *et al.* (2016). Further, Bhowmik *et al.* (2017) gave a detailed methodology for obtaining a mixed level factorial designs with minimum level changes in run orders. An exhaustive search algorithm has also been developed for obtaining all possible minimally changed run sequences.

Here, for easy accessibility of end users, a SAS Macro has been developed to generate a symmetric factorial with minimally changed run sequences. The macro which is based on a particular method, would also enhance the application potential of these run sequences. This macro not only generates the minimally changed run sequences but also generates the factor-wise number of level changes and total number of changes in the run sequences which is equal to $s^k - 1$ for s^k factorial experiment [where k is the number of factors and s is the levels for each factor] with minimally changed run sequences. For a particular factor, the levels for different values of s are denoted by

- $s/2$, $-(s-2)/2$, $-(s-4)/2$, ..., $-[s-(s-2)]/2$, $[s-(s-2)]/2$, ..., $(s-4)/2$, $(s-2)/2$, $s/2$ for even s and
 - $(s-1)/2$, $-(s-3)/2$, $-(s-5)/2$, ..., $-[s-(s-2)]/2$, 0 , $[s-(s-2)]/2$, ..., $(s-5)/2$, $(s-3)/2$, $(s-1)/2$ for odd s .

SAS macro for the generation of symmetric factorial with minimally changed run sequences

Here, user need to enter the number of factors as k (≥ 2) and the number of levels per factor as s (≥ 2). If user run the macro after entering any number (≥ 2) as the value of k and also as the value of s which may or may not be equal to the value of k , then the SAS Macro will generate a particular symmetric factorial design with minimally changed run sequences corresponding to the value of k and s under the heading "Minimally changed run sequences for symmetric factorial experiment". Along with the design, the macro will also generate two different arrays for factor wise number of changes in the run sequences and total number of changes in the run sequences under the heading "Factor-wise number of changes in the run sequence" and "Total number of changes in the run sequence" respectively corresponding to the generated symmetric factorial with minimally changed run sequences

Once user run the macro, everytime the SAS macro would also generate a word file containing the output. User can then save the word file.

SAS Code

```

options nodate nonumber ;
%let k=3; /* Enter the number of factors (it should be >=2)*/
%let s=4; /* Enter the number of levels (it should be >=2)*/
ods rtf file= 'output.rtf' startpage=no;
proc iml;
m=mod(&s,2);
a=j(&s,1,0);
do i=1 to &s;
do j=i to &s;
if m=1 then
do;
a[j,1]=-((&s-1)/2)+(i-1);
end;
else
do;
if -(&s/2)+(i-1)<0 then do;
a[j,1]=-(&s/2)+(i-1);
end;
else do;
a[j,1]=-(&s/2)+i;
end;
end;
end;
end;
*print a;
aa=j(&s*&s,2,0);
k=1;
do i=1 to &s;
do j=1 to &s;
aa[k,1]=a[i,1];
aa[k,2]=a[j,1];
k=k+1;
end;
end;
*print aa;
do i=1 to &s;
if mod(i,2)=0 then
do;
do j=&s*(i-1)+1 to i*&s;
aa[j,2]=-aa[j,2];
k=k+1;
end;
end;
end;
end;
*print aa;
if &k=2 then do;
print 'Minimally changed run sequences for symmetric factorial experiment';
Run_Sequence=aa;
print Run_sequence;
Total_Change=nrow(aa)-1;
Factor_Change=j(1,&k,0);
do i=1 to &k;
Factor_Change[1,i]=(&s-1)*(&s**(i-1));

```

```

end;
print 'Factor-wise number of changes in the run sequence';
print Factor_Change;
print 'Total number of changes in the run sequence';
print Total_Change;
end; else do;
do j=1 to &k-2;
do i=1 to nrow(aa);
kk=repeat(aa[i, ],&s,1);
if i=1 then do;
aaa=kk;
end;else do;
aaa=aaa//kk;
end;
end;
if mod(&s,2)=0 then do;
hh=repeat(aa[ ,ncol(aa)],&s,1);
aa=aaa||hh;
end;
else do;
gg=aa[ ,ncol(aa)]//aa[ ,ncol(aa)]*-1;
hh1=repeat(gg,(&s-1)/2,1);
hh=hh1//aa[ ,ncol(aa)];
aa=aaa||hh;
end;
end;
print 'Minimally changed run sequences for symmetric factorial experiment';
Run_Sequence=aa;
print Run_sequence;
Total_Change=nrow(aa)-1;
Factor_Change=j(1,&k,0);
do i=1 to &k;
Factor_Change[1,i]=(&s-1)*(&s**(i-1));
end;
print 'Factor-wise number of changes in the run sequence';
print Factor_Change;
print 'Total number of changes in the run sequence';
print Total_Change;
end;
run;
ods rtf close;
quit;

```

SAS Output

Following is the output corresponding to 4^3 factorial with minimally changed run sequences obtained through the macro for $k = 3$ and $s = 4$

Minimally changed run sequences for symmetric factorial experiment

Run_Sequence		
-2	-2	-2
-2	-2	-1
-2	-2	1
-2	-2	2
-2	-1	2
-2	-1	1
-2	-1	-1
-2	-1	-2
-2	1	-2
-2	1	-1
-2	1	1
-2	1	2
-2	2	2
-2	2	1
-2	2	-1
-2	2	-2
-1	2	-2
-1	2	-1
-1	2	1
-1	2	2
-1	1	2
-1	1	1
-1	1	-1
-1	1	-2

Run_Sequence		
-1	-1	-2
-1	-1	-1
-1	-1	1
-1	-1	2
-1	-2	2
-1	-2	1
-1	-2	-1
-1	-2	-2
1	-2	-2
1	-2	-1
1	-2	1
1	-2	2
1	-1	2
1	-1	1
1	-1	-1
1	-1	-2
1	1	-2
1	1	-1
1	1	1
1	1	2
1	2	2
1	2	1
1	2	-1
1	2	-2
2	2	-2
2	2	-1
2	2	1
2	2	2
2	1	2
2	1	1

Run_Sequence		
2	1	-1
2	1	-2
2	-1	-2
2	-1	-1
2	-1	1
2	-1	2
2	-2	2
2	-2	1
2	-2	-1
2	-2	-2

Factor-wise number of changes in the run sequence

Factor_Change		
3	12	48

Total number of changes in the run sequence

Total_Change
63

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