# Universally Optimal Block Designs for Diallel Crosses 

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#### Abstract

Summary Optimal block designs for diallel cross experiments are proposed for the situation when the interest of the experimenter is in estimating the general combining ability effects assuming that the specific combining ability effects are excluded from the model. Some general methods of construction of universally optimal block designs are given for both proper as well as non-proper settings. The designs hitherto known in the literature fall out as particular cases of the general methods. Catalogues of universally optimal designs are reported.

Some key words and phrases: Mating design, balanced incomplete block design, partially balanced incomplete block design, nested balanced incomplete block design, nested balanced block design, diallel crosses, general combining ability, universal optimality.


## 1. Introduction

Diallel crossing is a very useful method for conducting plant-breeding experiments. The diallel cross is a type of mating design used to study the genetic properties of a set of inbred lines. Suppose there are p-inbred lines and it is desired to perform a diallel crossing experiment involving $p(p-1) / 2$ crosses of the type (ixj) for $i<j, i, j=1,2, \cdots, p$. This is the type IV mating design of Griffing(1956), who studied the detailed analysis of such mating designs laid out in a randomized complete block design.

The number of crosses in such mating designs increases rapidly with increase in the number of lines; for $p=4$, there are only 6 crosses while for $p=8$, the number of crosses is 28 . In literature one way advocated for designing such 'experiments is to treat crosses as treatments and use the usual block designs for estimating line effects. Laying out the design, as a randomized complete block design, even a moderately large number of lines, will, result in large blocks and consequently large intra-block variances.

In order to overcome this problem, one may use incomplete block designs like balanced incomplete block (BIB) designs, partially balanced incomplete block (PBIB) designs with two associate classes, cyclic designs, etc. by treating the crosses as treatments. For instance; a BIB design has been used by identifying crosses as treatments [see e.g., Das and Giri(1986, pp441-442); Ceranka and Mejza, (1988)]. These designs have interesting optimality properties when making inferences on a complete set of orthonormalised treatment contrasts. However, in diallel cross experiments the interest

[^0]of the experimenter is in making comparisons among general combining ability (gca) effects of lines and not of crosses and, therefore, using these designs as mating designs may result into poor precision of the comparisons among lines. Further, the analysis of a diallel cross experiment in incomplete blocks depends on the incidence of lines in blocks, rather than the incidence of the crosses as treatments, in blocks. Another approach advocated in literature is to start with an incomplete block design, write all the pairs of treatments within a block, identify these pairs of treatments as crosses by treating treatments of the original incomplete block design as lines and use the resulting design as a design for diallel crosses. Sharma (1996) used this approach for complete diallel crosses experiments by using balanced lattice designs. This was, however, also advocated by Das and Giri(1986), in the context of BIB designs, and a balanced lattice is also a BIB design. Ghosh and Divecha(1997) used this for PBIB designs to obtain designs for partial diallel crosses and Sharma(1998) obtained designs for partial diallel crosses through circular designs. However, this approach also does not seem to do well as will become clear through the following examples:

An experimenter is interested in generating a mating design for comparing 7 -inbred lines on the basis of their gca effects. A mating design for diallel crosses experiment, $D$, with $2 /$ crosses can be obtained by writing all possible pairs of treatments within a block of the BIB design, $D_{0}$, with parameters $v=b=7, r=k=3, \lambda=l$ and treating the treatments as lines and paired treatments as crosses. Here the number of crosses is $v=$ $21, b=7, r=1, k=3$. The designs, with rows as blocks, are

|  | $D_{0}$ |  | $D$ |  |  | $D^{*}$ |  |  |
| :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- |
| 1 | 2 | -4 | $1 \times 2$ | $1 \times 4$ | $2 \times 4$ | $1 \times 7$ | $2 \times 6$ | $3 \times 5$ |
| 2 | 3 | 5 | $2 \times 3$ | $2 \times 5$ | $3 \times 5$ | $1 \times 2$ | $3 \times 7$ | $4 \times 6$ |
| 3 | 4 | 6 | $3 \times 4$ | $3 \times 6$ | $4 \times 6$ | $2 \times 3$ | $1 \times 4$ | $5 \times 7$ |
| 4 | 5 | 7 | $4 \times 5$ | $4 \times 7$ | $5 \times 7$ | $3 \times 4$ | $2 \times 5$ | $1 \times 6$ |
| 5 | 6 | 1 | $5 \times 6$ | $1 \times 5$ | $1 \times 6$ | $4 \times 5$ | $3 \times 6$ | $2 \times 7$ |
| 6 | 7 | 2 | $6 \times 7$ | $2 \times 6$ | $2 \times 7$ | $5 \times 6$ | $4 \times 7$ | $1 \times 3$ |
| 7 | 1 | 3 | $1 \times 7$ | $3 \times 7$ | $1 \times 3$ | $6 \times 7$ | $1 \times 5$ | $2 \times 4$ |

The $\mathrm{C}=\mathrm{G}-\mathrm{NK}^{-1} \mathrm{~N}^{\prime}$ matrix of the design $D$ is $\mathrm{C}=\frac{7}{3}\left(\mathrm{I}_{7}-\frac{1}{7} \mathrm{~J}_{7}\right)$, and the variance of the Best Linear Unbiased Estimator (B.L.U.E.) of any elementary contrast among lines (gca) is $\frac{6}{7} \sigma^{2}$. Here $\mathbf{G}$ is a matrix with diagnoal elements as replication number of lines and offdiagonal elements as replication number of crosses, $\mathbf{N}$ is the incidence matrix of lines $V_{S}$ blocks, $K$ is the diagonal matrix with elements as block sizes, $I_{V}$ is an
identity matrix of order $v, J_{v}$, a $v x v$ matrix of all elements ones, and $\sigma^{2}$ is the per plot variance.

Another mating design generated through a different method is $D^{*}$. The $\mathrm{C}=\mathrm{G}-\mathrm{NK}^{-1} \mathrm{~N}^{\prime}$ matrix of the design $D^{*}$ is $\mathrm{C}=\frac{14}{3}\left(\mathrm{I}_{7}-\frac{1}{7} \mathrm{~J}_{7}\right)$, and the variance of the B.L.U.E. of any elementary contrast among lines (gca) is $\frac{3}{7} \sigma^{2}$. Thus, one can see that the design $D^{*}$ estimates the gea with twice the precision as obtained through the design $D$ although both the designs are variance balanced for estimating any normalized contrast of gca effects.

Consider another situation when the experimenter is interested in designing an experiment with $p=9$ lines. Sharma (1998) generated a mating design $D$, from a cyclic design with parameters $v=b=9, r=k=3$, by developing the initial block $(1,2,3) \bmod$ 9 and then taking all the possible $\binom{k}{2}$ crosses from each block. The variances of the B.L.U.E. of any elementary contrast among lines (gca) is $0.81045 \sigma^{2}, 1.04574 \sigma^{2}$, $1.29411 \sigma^{2}$ and $1.39866 \sigma^{2}$. Each of these variances is for 9 different B.L.U.E. of the gca effects and the average variance is given by $1.13724 \sigma^{2}$. A similar type of mating design $D_{i}^{*}$ can be obtained from a cyclic design with parameters $v=b=9, r=k=3$ by developing the initial block ( $1,2,4$ ) mod 9. The variances of the B.L.U.E. of any elementary contrast among lines (gca) is $0.87543 \sigma^{2}, 0.89409 \sigma^{2}, 0.89718 \sigma^{2}$ and $1.02492 \sigma^{2}$, and the average variance is given by $0.92291 \sigma^{2}$. The design $D_{1}^{*}$ has smaller average variance of B.L.U.E. of elementary contrasts of gca effects as compared to $D_{1}$. The design $D_{\text {I }}^{*}$ seems to have an intuitive appeal also as it contains more number of distinct crosses as compared to the design $D$, although the size of both the designs is the same in terms of the total number of observations. Hence this design is more useful for same number of experimental units.

Consider another situation when the experimenter is interested in designing an experiment with $p=12$ lines. Ghosh and Divecha(1997) generated a mating design $D_{2}$ from a group divisible design with parameters $v=12, b=9, r=3, k=4, \lambda_{1}=0, \lambda_{2}=1$, $m=4, n=3$ (Clatworthy, 1973; SR41), by taking all possible $\binom{k}{2}$ crosses of treatments within each block, by treating treatments in the original design as lines. The variances of the B.L.U.E. of gca effects are $0.44444 \sigma^{2}$, for the first associates ( $/ 2$ in number) and $0.40741 \sigma^{2}$, for the second associates (54 in number). The average variance is $0.41414 \sigma^{2}$. A similar type of mating design $D_{2}^{*}$ is obtained from a different method. The variances of the B.L.U.E. of any elementary contrast among gca effects is
$0.22222 \sigma^{2}$, for the first associates ( 12 in number) and $0.25926 \sigma^{2}$ for the second associates ( 54 in number). The average variance is $0.25253 \sigma^{2}$. One can easily see that in the two mating designs the precision of the B.L.U.E. of gcas is different and, therefore, the choice of an appropriate mating design is important.

| $D_{2}^{*}$ |  |  |  |  |  |  |  |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| Blocks |  |  |  |  |  | Blocks |  |  |  |  |  |
| $1 \times 2$ | $5 \times 6$ | $9 \times 10$ | $3 \times 4$ | $7 \times 8$ | $11 \times 12$ | $1 \times 8$ | $5 \times 12$ | $9 \times 4$ | $2 \times 7$ | $6 \times 11$ | $10 \times 3$ |
| $1 \times 3$ | $5 \times 7$ | $9 \times 11$ | $2 \times 4$ | $6 \times 8$ | 10x12 | $1 \times 10$ | $5 \times 2$ | $9 \times 6$ | $3 \times 12$ | $7 \times 4$ | $11 \times 8$ |
| $1 \times 4$ | $5 \times 8$ | $9 \times 12$ | $2 \times 3$ | $6 \times 7$ | 10x11 | $1 \times 11$ | $5 \times 3$ | $9 \times 7$ | $2 \times 12$ | $6 \times 4$ | 10x8 |
| $1 \times 6$ | $5 \times 10$ | $9 \times 2$ | $3 \times 8$ | $7 \times 12$ | $11 \times 4$ | $1 \times 12$ | $5 \times 4$ | $9 \times 8$ | $2 \times 11$ | $6 \times 3$ | 10x7 |
| 187 | $5 \times 11$ | $9 \times 3$ | $2 \times 8$ | $6 \times 12$ | 10x4 |  |  |  |  |  |  |

It is clear from the above discussions that for making comparisons of gca effects of p-inbred lines, the choice of an appropriate design is important. This paper addresses this and similar problems.

The problem of generating optimal mating designs for experiments with diallel crosses has been recently investigated by several authors [see e.g., Gupta and Kageyama (1994), Dey and Midha (1996), Mukerjee (1997), Das, Dey and Dean (1998)]. These authors used nested balanced incomplete block (NBIB) designs of Preece (1967) for this purpose. This paper derives general methods of construction of mating designs, essentially generated from nested variance balanced block (NBB) designs. The optimality aspects have also been investigated under a non-proper setting as well. The model considered here involves only the gca effects, the specific combining ability effects being excluded from the model. The designs obtained are variance balanced in the sense that the variances of the B.L.U.E. of elementary contrasts among general combining ability effects are all same,

## 2. Some Preliminaries

Let $d$ be a block design for a diallel cross experiment of the type mentioned in Section 1 involving p-inbred lines, $b$ blocks such that the $j^{\text {sh }}$ block is of size $k_{j}$. This means that there are $k_{j}$ crosses or $2 k_{j}$ lines, in each block of $d$. It may be mentioned here that the designs for diallel crosses have two types of block sizes, $k_{1}^{(0)}$, the block sizes with respect to crosses and $k_{2}^{(\varrho)}$, the block sizes with respect to the lines and $k_{2}^{(\omega)}=2 k_{1}^{(\text {e }}$. It therefore, follows that the block designs for diallel crosses may also be viewed as nested block designs with sub blocks of size 2 each and the pair of treatments in each sub block form the crosses, the treatments being the lines. Further, let $r_{d i}$ denote the number of times the $i^{\text {th }}$ cross appears in $d, i=1,2, \ldots p(p-1) / 2$ and similarly $s_{d / f}$ denotes the number
of times the $t^{\text {th }}$ line occurs in the crosses in the whole design $d, l=l, 2, \ldots, p$. Then it is easy to see that

$$
\begin{aligned}
& \sum_{i=1}^{p(p-1) / 2} r_{d i}=\sum_{j=1}^{b} k_{j}=n, \text { the total number of observations, and } \\
& \sum_{l=1}^{p} s_{d l}=2 \sum_{j=1}^{b} k_{j}, \text { (because in every cross there are two lines). }
\end{aligned}
$$

For the data obtained from the design $d$, we postulate the model

$$
\begin{equation*}
\mathbf{Y}=\mu 1_{n}+\Delta_{1}^{\prime} \mathbf{g}+\Delta_{2}^{\prime} \beta+\mathrm{e} \tag{2.1}
\end{equation*}
$$

where Y is the $n x I$ vector of observed responses, $\mu$ is a general mean effect, $1_{n}$ denotes an $n$-component column vector of all ones, $\mathbf{g}$ and $\beta$ are vectors of $p$ gca effects and $b$ block effects, respectively. $\Delta_{1}^{\prime}$ and $\Delta_{2}^{\prime}$ are the corresponding $n x p$ and $n x b$ design matrices respectively, i.e., the $(s, t)^{\text {th }}$ element of $\Delta_{1}^{\prime}$ is $I$ if the $s^{\text {th }}$ observation pertains to the $t^{t h}$ line and is zero otherwise. Similarly $(s, t)^{\text {th }}$ element of $\Delta_{2}^{\prime}$ is $l$ if the $s^{\text {th }}$ observation comes from the $t^{\text {th }}$ block and is zero otherwise : e is the random error which follows a $N_{n}\left(0, \sigma^{2} I_{n}\right)$.

In the model (2.1) we have not included the specific combining ability effects. Under model (2.1), it can be shown that the coefficient matrix for reduced normal equations for estimating linear functions of gca effects using a design $d$ is

$$
\mathrm{C}_{d}=\mathrm{G}_{d}-\mathrm{N}_{d} \mathrm{~K}_{d}^{-1} \mathrm{~N}_{d}^{\prime}
$$

where $\mathrm{G}_{d}=\left(\left(g_{d i i^{\prime}}\right)\right), \mathrm{N}_{d}=\left(\left(\mathrm{n}_{d i j}\right)\right), \mathrm{g}_{d i i}=s_{\| l}$ and for $i \neq i^{\prime}, \mathrm{g}_{d i i^{\prime}}$ is the number of times the cross (ixi') appears in $d ; n_{d j}$ is the number of times line $i$ occurs in the block $j$.

A design $d$ is said to be connected if and only if Rank $\left(\mathrm{C}_{d}\right)=p-1$, or equivalently, if and only if all elementary comparisons among the gca effects are estimable using $d$. A connected design $d$ is variance balanced if and only if all the diagonal elements of the matrix $\mathrm{C}_{d}$ are equal and all the off diagonal elements of the matrix $\mathrm{C}_{d}$ are equal. In other words, the matrix $\mathrm{C}_{d}$ is completely symmetric. For given positive integers $p, b, n, \mathrm{D}_{0}$ ( $p, b, n$ ) will denote the class of all connected block designs $d$ with $p$ lines, $b$ blocks and $n$ experimental units. Here the block sizes are arbitrary but for a given design $d \in \mathbf{D}_{0}(p, b, n)$, the block sizes are $k_{d 1}, k_{d 2}, \cdots, k_{d b}$. Similarly, $\mathrm{D}\left(p, b, k_{1}, \ldots, k_{b}\right)$ will denote the class of all connected block designs $d$ with $p$ lines, $b$ blocks such that $j^{\text {th }}$ block is of size $k_{j}$. We may allow $k_{i}>2 p$ for some or all $j=1,2, \cdots, b$.

We shall now state the following Lemmas that shall be useful in establishing the universal optimality of designs for diallel crosses.

Lemma 2.1: For given positive integers $s$ and $t$, the minimum of $\sum_{i=1}^{3} n_{i}^{2}$ subject to $\sum_{i=1}^{s} n_{i}=t$, where the $n_{i}$ 's are non-negative integers, is obtained when $t-s\{\operatorname{int}(t / s)\}$ of the $n_{i}$ are equal to $\operatorname{int}(t / s)+1$ and $s-t+\mathrm{s}\{\operatorname{int}(t / s)\}$ are equal to $\operatorname{int}(t / s)$. The corresponding minimum is $t(2 \operatorname{int}(t / s)+1)-s \operatorname{int}(t / s)(\operatorname{int}(t / s)+1)$. Here $\operatorname{int}(x)$ denotes the largest integer part of $x$.

Lemma 2.2: (Kiefer, 1975). A design $d^{*} \in \mathrm{D}$ is universally optimal in D if
(i) $\mathrm{C}_{i /}^{*}$ is completely symmetric, and
(ii) $\operatorname{trace}\left(\mathbf{C}_{d}^{*}\right)=\max _{d \in D} \operatorname{trace}\left(\mathbf{C}_{d}\right)$.

It is well known that a universally optimal design is A-optimal, D-optimal, E-optimal and optimal according to several other families of optimality criteria.

## 3. Main Results

In this Section, we present the main results in the form of Theorems and Corollaries. These results enable us to generate universally optimal block designs for diallel crosses.
Theorem 3.1: (i) For any design $d \in \mathrm{D}\left(\mathrm{p}, \mathrm{b}, \mathrm{k}_{1}, \ldots, \mathrm{k}_{\mathrm{b}}\right)$

$$
\operatorname{trace}\left(\mathbf{C}_{d}\right) \leq 2 \sum_{j=1}^{b} k_{j}-\sum_{j=1}^{b} \frac{1}{k_{i}}\left[2 k_{j}\left(2 x_{j}+1\right)-p x_{j}\left(x_{j}+1\right)\right],
$$

the equality holding if and only if $n_{d j j}=x_{j}$ or $x_{j}+1$ for all $i=1, \ldots, p, j=1, \ldots, b$, where $x_{j}=\operatorname{int}\left(2 k_{j} / p\right)$.
(ii) For any design $d \in \mathbf{D}_{0}(p, b, n)$, trace $\left(\mathbf{C}_{d}\right) \leq 2(n-b)$, with equality for $n_{d j l}=0$ or 1 .

Proof: (i) For any $d \in \mathbf{D}\left(p, b, k_{1}, \ldots, k_{h}\right)$

$$
\operatorname{trace}\left(\mathbf{C}_{d}\right)=\sum_{i=1}^{p} s_{d i}-\sum_{i=1}^{p} \sum_{j=1}^{\mathrm{b}} \mathrm{n}_{d j}^{2} / k_{d j}=2 \sum_{i=1}^{h} k_{j}-\sum_{i=1}^{n} \frac{1}{k_{j}} \sum_{i=1}^{\mathrm{p}} n_{d j j}^{2} .
$$

Now using the fact that $\sum_{i=1}^{p} n_{d j j}=2 k_{j}$, and using Lemma 2.1, we have
$\sum_{j=1}^{b} \frac{1}{k_{j}} \sum_{i=1}^{p} \mathrm{n}_{\mathrm{dij}}^{2} \geq \sum_{\mathrm{j}=1}^{\mathrm{b}} \frac{1}{\mathrm{k}_{j}}\left[2 \mathrm{k}_{j}\left(2 x_{i}+1\right)-p x_{j}\left(x_{j}+1\right)\right]$.
Therefore, $\operatorname{trace}\left(\mathrm{C}_{\mathrm{d}}\right) \leq 2 \sum_{j=1}^{b} \mathrm{k}_{j}-\sum_{j=1}^{b} \frac{1}{k_{j}}\left[2 k_{j}\left(2 x_{j}+1\right)-p x_{j}\left(x_{j}+1\right)\right]$,
and equality is achieved if and only if

$$
n_{d j j}=\operatorname{int}\left(2 k_{j} / \mathrm{p}\right) \operatorname{or} \operatorname{int}\left(2 k_{\mathrm{j}} / p\right)+1 \quad \forall \mathrm{i}=1, \ldots, \mathrm{p}, \mathrm{j}=1, \ldots, \mathrm{~b} .
$$

(ii) Now if $2 k_{d j}<p$, then $x_{j}=0 \forall \mathrm{j}=1, \cdots, \mathrm{~b}$, and then $\operatorname{trace}\left(\mathrm{C}_{\mathrm{d}}\right) \leq 2(n-b)$.

Hence the proof.
Further, using Lemma 2.2, we have the following theorem:
Theorem 3.2: Let $d^{*} \in \mathbb{D}\left(p, b, k_{1}, \ldots, k_{b}\right)\left[\mathbb{D}_{0}(p, b, n)\right]$ be a block design for diallel crosses and suppose that $d^{*}$ satisfies
i) $\operatorname{trace}\left(\mathbf{C}_{d}\right)=2 \sum_{j=1}^{b} k_{j}-\sum_{j=1}^{b} \frac{1}{k_{j}}\left[2 k_{j}\left(2 x_{j}+1\right)-p x_{j}\left(x_{j}+1\right)\right]$
or

$$
\left[\operatorname{trace}\left(\mathbf{C}_{v^{*}}\right)=2(n-b)\right]
$$

ii) $\mathrm{C}_{d^{*}}$ is completely symmetric.

Then $d^{*}$ is universally optimal over $\mathbf{D}\left(p, b, k_{1}, \ldots, k_{b}\right)\left[\mathbf{D}_{0}(p, b, n)\right]$
A block design $d$ for diallel crosses is said to be variance balanced if and only if its information matrix is completely symmetric. In particular, a variance balanced block design for diallel crosses is said to be a generalized binary variance balanced block (GBBB) design if in addition to completely symmetric information matrix, $n_{d j}=x$, or $x_{j}+1$ and is said to be binary variance balanced block (BBB) design, if $n_{d i j}=0$ or 1 .

Corollary 3.1: In view of the above definitions, we have the following results:
(i) A GBBB design for diallel crosses, whenever existent, is universally optimal over $\mathrm{D}\left(p, b, k_{1}, \ldots, k_{b}\right)$
(ii) A BBB design for diallel crosses, whenever existent, is universally optimal over $\mathrm{D}_{0}(p, b, n)$.

It may be noted that a design $d^{*}$ that is universally optimal over $\mathbf{D}\left(p, b, k_{1}, \ldots, k_{b}\right)$ is also universally optimal over $\mathbf{D}_{0}(p, b, n)$ provided all $2 k_{j} \leq p$ for all $j=1,2, \cdots, b$. Similarly, a design $d^{*}$ that is universally optimal over $\mathrm{D}_{0}(p, b, n)$ is also universally optimal over $\mathrm{D}\left(p, b, k_{1}, \ldots, k_{h}\right)$ provided $k_{\|^{*} j}=k_{j}$ for all $j=1,2, \cdots, b$.

As a consequence of Corollary 3.1, all the designs known hitherto in the literature as universally optimal over $\mathbf{D}(p, b, k)$ are also universally optimal over $\mathbf{D}_{0}(p, b, n)$.

Therefore, now our problem of obtaining universally optimal block designs for diallel crosses, over $\mathrm{D}\left(p, b, k_{1}, \ldots, k_{b}\right)$ or $\mathrm{D}_{0}(p, b, n)$ reduces to obtaining GBBB designs/BBB designs for diallel crosses.

## 4. Methods of Construction

The purpose of this Section is to give some methods of construction of GBBB/BBB designs for diallel crosses that are universally optimal over $\mathrm{D}\left(p, b, k_{1}, \ldots, k_{h}\right)$ or $\mathbf{D}_{0}(p, b, n)$.

### 4.1. Method Of Construction Of Universally Optimal, Non-Proper Block Designs For Diallel Crosses

We give below a method of construction of designs for diallel crosses that are both BBB and GBBB designs and are, therefore, universally optimal over both $\mathrm{D}_{0}(p, b, n)$ and $\mathbf{D}\left(p, b, k_{1}, \ldots, k_{b}\right)$.

Method 4.1: Let $\mathrm{N}_{i}, i=1,2$ be the incidence matrix of a PBIB design with two associate classes and with parameters $v=p, b_{i}, r_{i}, k_{i}, n_{1}, n_{2}, \lambda_{i 1}, \lambda_{i 2}$, where the symbols have their usual meaning. Suppose that there exist two series of NBIB designs, Series / and Serjes 2, with parameters, $k_{1}, b_{1}^{*}, b_{2}^{*}, r^{*}, k_{1}^{*}, k_{2}^{*}=2, \lambda_{1}^{*}, \lambda_{2}^{*}=1$ and $k_{2 p} b_{1}^{*}, b_{2}^{*}, r^{* *}, k_{1}^{*}, k_{2}^{*}=2$, $\lambda_{1}^{* *}, \lambda_{2}^{*}=l$, respectively. Rewrite the block contents of each block of the $i^{t h}$ PBIB design as NBIB design using Series 1 and Series 2 and take copies of the blocks so obtained in the ratio $\theta_{1}: \theta_{2}:: \alpha_{1}: \alpha_{2}$, where $\alpha_{1}=\theta_{1} / c, \alpha_{2}=\theta_{2} / c$ and $c$ is the highest common factor of $\theta_{1}$ and $\theta_{2}$, such that the following conditions are satisfied:

$$
\frac{\alpha_{1}}{\alpha_{2}}=\frac{k_{1}^{*}}{k_{1}^{*}-2 \lambda_{1}^{*}} \frac{k_{1}^{* *}-2 \lambda_{1}^{* *}}{k_{1}^{* *}} \frac{\lambda_{22}-\lambda_{21}}{\lambda_{11}-\lambda_{12}} .
$$

The resulting design is a NBB design with parameters

$$
\begin{aligned}
& v=p, \quad b_{1}^{+}=\alpha_{1} b_{1} b_{1}^{*}+\alpha_{2} b_{2} b_{1}^{* *}, \quad b_{2}^{+}=\alpha_{1} b_{1} b_{2}^{*}+\alpha_{2} b_{2} b_{2}^{\prime *}, \quad r^{*}=\alpha_{1} r_{1} r^{*}+\alpha_{2} r_{2} r^{*}, \\
& \mathrm{k}_{1}^{+}=\left(k_{1}^{\prime} 1_{\alpha_{1}, b_{1}^{*}}^{\prime}, k_{1}^{*} 1_{a_{2} b_{b} u_{1}^{\prime}}^{\prime}\right), \mathrm{k}_{2}^{*}=\left(21_{b_{3}^{*}}^{\prime}\right),
\end{aligned}
$$

and hence the mating design for diallel crosses with parameters $p=v_{,}, b=b_{1}^{*}, \quad \mathrm{k}=\frac{1}{2} \mathrm{k}_{1}^{*}$.
As a particular case consider the following two series of NBIB designs that always exist:

Series 1: $v^{\delta}=2 t, b_{1}^{\delta}=2 t-1, b_{2}^{\delta}=t(2 t-1), r^{\delta}=2 t-1, k_{1}^{\delta}=2 t, k_{2}^{\delta}=2, \lambda_{1}^{\delta}=2 t-1, \lambda_{2}^{\delta}=1$.
Series 2: $v^{\delta}=2 t+l=b_{1}^{\delta}, b_{2}^{\delta}=t(2 t+l), r^{\delta}=2 t, k_{1}^{\delta}=2 t, k_{2}^{\delta}=2, \lambda_{1}^{\delta}=2 t-l, \lambda_{2}^{\delta}=1$.
Here $t$ is any positive integer larger than $l$.

For each block of the $i^{\text {th }}$ PBIB design with two associate classes for $k_{i}$ even (odd) rewrite the block contents as NBIB design using Series 1 (Series 2). Taking copies of the blocks so obtained in the ratio $\theta_{1}: \theta_{2}:: \alpha_{1}: \alpha_{2}$, such that the following conditions are satisfied, we get a universally optimal design for diallel crosses:

Case I: Both $k_{1}$ and $k_{2}$ are even.

$$
\frac{\alpha_{1}}{\alpha_{2}}=\frac{k_{1}}{2-k_{1}} \frac{2-k_{2}}{k_{2}} \frac{\lambda_{22}-\lambda_{21}}{\lambda_{11}-\lambda_{12}}
$$

Case II: $k_{1}$ is odd and $k_{2}$ is even.

$$
\frac{\alpha_{1}}{\alpha_{2}}=\frac{k_{1}-1}{3-k_{1}} \frac{2-k_{2}}{k_{2}} \frac{\lambda_{22}-\lambda_{21}}{\lambda_{11}-\lambda_{12}}
$$

Case III: $k_{1}$ is even and $k_{2}$ is odd.

$$
\frac{\alpha_{1}}{\alpha_{2}}=\frac{k_{1}}{2-k_{1}} \frac{3-k_{2}}{k_{2}-1} \frac{\lambda_{22}-\lambda_{21}}{\lambda_{11}-\lambda_{12}}
$$

Case IV: Both $k_{l}$ and $k_{2}$ are odd.

$$
\frac{\alpha_{1}}{\alpha_{2}}=\frac{k_{1}-1}{3-k_{1}} \frac{3-k_{2}}{k_{2}-1} \frac{\lambda_{22}-\lambda_{21}}{\lambda_{11}-\lambda_{12}}
$$

We get NBB designs with the following parameters:
Case I: $v=p, \quad b_{1}^{*}=\alpha_{1} b_{1}\left(2 t_{1}-1\right)+\alpha_{2} b_{2}\left(2 t_{2}-1\right), \quad b_{2}^{*}=\alpha_{1} b_{1} t_{1}\left(2 t_{1}-1\right)+\alpha_{2} b_{2} t_{2}\left(2 t_{2}-1\right)$, $r^{*}=\alpha_{1} r_{1}\left(2 t_{1}-1\right)+\alpha_{2} r_{2}\left(2 t_{2}-1\right), \mathrm{k}_{j}^{\prime}=\left(2 t_{1} 1_{\alpha, h_{1}\left(2 t_{1}-1\right)}^{\prime}, 2 t_{2} 1_{\alpha_{2} h_{2}\left(2 t_{2}-1\right)}^{\prime}\right), \mathrm{k}_{2}^{*^{\prime}}=\left(21_{h_{1}}^{\prime}\right)$,
and hence the mating design for diallel crosses with parameters $p=\nu_{\text {. }}$ $b^{*}=\alpha_{1} b_{1}\left(2 t_{1}-1\right)+\alpha_{2} b_{2}\left(2 t_{2}-1\right), \mathrm{k}^{.^{\prime}}=\left(t_{1} 1_{a_{1} b_{1}\left(z_{1}-1\right)}^{\prime}, t_{2} 1_{a_{2} b_{2}\left(2 z_{2}-1\right)}^{\prime}\right)$.

Case II: $v=p, \quad b_{1}^{*}=\alpha_{1} b_{1}\left(2 t_{1}+1\right)+\alpha_{2} b_{2}\left(2 t_{2}-1\right), \quad b_{2}^{*}=\alpha_{1} b_{1} t_{1}\left(2 t_{1}+1\right)+\alpha_{2} b_{2} t_{2}\left(2 t_{2}-1\right)$.
$r^{*}=\alpha_{1} r_{1}\left(2 t_{1}\right)+\alpha_{2} r_{2}^{\prime}\left(2 t_{2}-1\right), \mathrm{k}_{i}^{\prime}=\left(2 t_{1} 1_{\alpha, h_{1}\left(2 t_{1}+1\right)}^{\prime}, 2 t_{2} 1_{\alpha_{2} b_{3}\left(2 t_{2}-1\right)}^{\prime}\right), \mathrm{k}_{3}^{\prime}=\left(21_{b b_{1}^{\prime}}^{\prime}\right)$,
and hence the mating design for diallel crosses with parameters $p=\nu_{\text {. }}$ $b^{*}=\alpha_{1} b_{1}\left(2 t_{1}+1\right)+\alpha_{2} b_{2}\left(2 t_{2}-1\right), \mathrm{k}^{{ }^{\prime}}=\left(t_{1} 1_{a_{1} h_{1}\left(2 t_{1}+1\right)}^{\prime}, t_{2} 1_{a_{2} b_{2}\left(2 t_{2}-1\right)}^{\prime}\right)$.

Case III: $v=p, b_{1}^{*}=\alpha_{1} b_{1}\left(2 t_{1}-1\right)+\alpha_{2} b_{2}\left(2 t_{2}+1\right), \quad b_{2}^{*}=\alpha_{1} b_{1} t_{1}\left(2 t_{1}-1\right)+\alpha_{2} b_{2} t_{2}\left(2 t_{2}+1\right)$, $r^{*}=\alpha_{1} r_{1}\left(2 t_{1}-1\right)+\alpha_{2} r_{2}\left(2 t_{2}\right), \mathrm{k}_{t}^{\prime}=\left(2 t_{2} 1_{\alpha_{1} h_{1}\left(2 t_{1}-11\right.}^{\prime}, 2 t_{2} 1_{\alpha, h_{1}\left(2 t_{2}+1\right)}^{\prime}\right), \mathrm{k}_{2}^{\prime}=\left(21_{p_{j}^{\prime}}^{\prime}\right)$,
and hence the mating design for diallel crosses with parameters $p=v_{1}$ $b^{*}=\alpha_{1} b_{1}\left(2 t_{1}-1\right)+\alpha_{2} b_{2}\left(2 t_{2}+1\right), \mathrm{k}^{*}=\left(t_{1} 1_{\alpha, b_{1}\left(2 t_{1}-1\right)}^{\prime}, t_{2} 1_{a b_{2}\left(2 t_{2}+1\right)}^{\prime}\right)$.

Case IV: $v=p, b_{1}^{*}=\alpha_{1} b_{1}\left(2 t_{1}+1\right)+\alpha_{2} b_{2}\left(2 t_{2}+1\right), \quad b_{2}^{*}=\alpha_{1} b_{1} t_{1}\left(2 t_{1}+1\right)+\alpha_{2} b_{2} t_{2}\left(2 t_{2}+1\right)$, $r^{*}=\alpha_{1} r_{1}\left(2 t_{1}\right)+\alpha_{2} r_{2}\left(2 t_{2}\right), \mathrm{k}_{i}^{\prime}=\left(2 t_{2} 1_{a_{1} b_{1}\left(2 t_{1}+1\right)}^{\prime}, 2 t_{2} 1_{\alpha_{2} b_{2}\left(2 t_{2}+1\right)}^{\prime}\right), \mathrm{k}_{2}^{\prime}=\left(21_{/ i_{1}^{\prime}}^{\prime}\right)$.
and hence the mating design for diallel crosses with parameters $p=v$. $b^{*}=\alpha_{1} b_{1}\left(2 t_{1}+1\right)+\alpha_{2} b_{2}\left(2 t_{2}+1\right), \mathrm{k}^{\prime}=\left(t_{1} 1_{\alpha, \beta_{1}\left(2 t_{1}+1\right)}^{\prime}, t_{2} 1_{\alpha_{2} b_{2}\left(2 t_{2}+1\right)}^{\prime}\right)$.

Using the method just described, a large number of non-proper block designs for diallel crosses can be obtained using PBIB designs from Clatworthy (1973) and Sinha (1991).

Remark 4.1. It is interesting to note that in the above method, we get unequal replications of different crosses.
Remark 4.2. In the above method, if in place of two PBIB designs we take a pairwise balanced design with parameters $v=p, b, k_{p}, k_{2}, \lambda$ and repeat the above process for each of the four cases as above, then selecting copies in the ratio $\frac{\alpha_{1}}{\alpha_{2}}=\frac{k_{1}}{2-k_{1}} \frac{2-k_{2}}{k_{2}}$ in case I, $\frac{\alpha_{1}}{\alpha_{2}}=\frac{k_{1}-1}{3-k_{1}} \frac{2-k_{2}}{k_{2}}$ in case II, $\frac{\alpha_{1}}{\alpha_{2}}=\frac{k_{1}}{2-k_{1}} \frac{3-k_{2}}{k_{2}-1}$ in case III and $\frac{\alpha_{1}}{\alpha_{2}}=\frac{k_{1}-1}{3-k_{1}} \frac{3-k_{2}}{k_{2}}$ in case IV will suffice.

When $\left|k_{i}-k_{i}\right|=1$ (larger being the odd), $i_{i} i^{\prime}=1,2$, we get an universally optimal proper block design for diallel crosses. However, when $\left|k_{i}-k_{f}\right| \geq 2$ or $\left|k_{i}-k_{i}\right|=1$. (larger being the even), $i, i^{\prime}=1,2$, we get a universally optimal non-proper block design for diallel crosses. However, through this method one can never get a design with minimum number of observations.

Remark 4.3. One trivial method of generating universally optimal non-proper block designs for diallel crosses is that of taking the union of two universally optimal proper block designs for diallel crosses.

Remark 4.4. Das, Dey and Dean (1998) reported a method (family 5) of construction of block designs for diallel crosses. These designs are essentially NBB designs. Therefore, in the method 4.1 given above, if we start with two PBIB designs with block sizes as $k_{\%}=$ $2 t_{1}+l$ and $k_{2}=2 t_{2}+l$ and replace the contents of each of the blocks of the two PBIB designs with the NBB designs of Das, Dey and Dean(1998) and take copies of the block: so obtained in the ratio $O_{1}: O_{2}:: \alpha_{1}: \alpha_{2}$, we get a NBB design for diallel crosses if
$\frac{\alpha_{1}}{\alpha_{2}}=\frac{k_{1}}{k_{1}-4 i_{1}} \frac{k_{2}-4 t_{2}}{k_{2}} \frac{\lambda_{22}-\lambda_{21}}{\lambda_{11}-\lambda_{12}}$.

These designs are universally optimal over $\mathbf{D}\left(p, b, \mathbf{k}^{*}\right)$, where $b=\alpha_{1} t_{1} b_{1}+\alpha_{2} t_{2} b_{2}$, $\mathrm{k}^{*^{\prime}}=\left(\mathrm{k}_{1} 1_{\alpha, b_{1}, t}^{\prime}, k_{2} 1_{\alpha}^{\prime}{ }_{c, b_{2},}^{\prime}\right)$.

### 4.2. Method Of Construction Of Universally Optimal, Proper Block Designs For Diallel Crosses

We give below a method of construction of designs for diallel crosses that are BBB designs and are therefore universally optimal over $\mathbf{D}_{0}(p, b, n)$.

Family 1: Let $v=p=m t+/$ be a prime or prime power and $x$ be a primitive element of the Galois field of order $p$, GF $(p)$, where $m=2 u$ for $u \geq 2$ and $t \geq 1$. Consider $t$ initial blocks

$$
\left\{\left(\mathrm{x}^{i}, x^{i+u t}\right)\left(x^{i+1}, x^{i+(u+1) i}\right), \ldots ;\left(x^{i+(u-1) t}, x^{i+(2 u-1) x}\right)\right\} \forall \mathrm{i}=0,1, \ldots, t-1 .
$$

These initial blocks when developed $\bmod p$, give rise to a NBIB design with parameters $v=p=m t+1, b_{1}=t(m t+1), b_{2}=u t(m t+1), k_{1}=m=2 u, k_{2}=2, r=m t, \lambda_{1}$ $=m-l, \lambda_{2}=1, n=2 u t(m t+l)$.

If we identify the treatments of $d$ as lines of a diallel crosses experiment and perform crosses among the lines appearing in the same sub-block of size 2 in $d$, we get a universally optimal block design for diallel crosses over $\mathbf{D}_{0}(p, b, n)$ with minimal number of experimental units and with parameters as $p=(m t+l), b=t(m t+l), k=u, n=$ $u t(m t+I)$ such that each of the crosses is replicated once in the design.

For $m=4$ and $m=6$, we get respectively Family 1 and Family 2 designs of Das, Dey and Dean (1998). For $t=1$, we get the same designs as reported by Gupta and Kageyama (1994).

Example 4.1: For $m=8$ and $t=2$, i.e. $v=p=17$, the primitive root of $\mathrm{G} \mathrm{F}(17)$ is 3 . Therefore developing the initial blocks

$$
\begin{aligned}
& {[(1,16) ;(9,8) ;(13,4) ;(15,2)]} \\
& {[(3,14) ;(10,7) ;(5,12) ;(11,6)]}
\end{aligned}
$$

$\bmod I 7$, we get a universally optimal diallel cross design over $\mathbf{D}_{0}(p, b, n)$ with $p=17, b$ $=34, k=4, n=136$.

Catalogues of designs for $p \leq 30$, obtained through this method are reported in Table 1 in appendix. This table also includes universally optimal proper block designs for complete diallel crosses known hitherto in the literature.

Family 2: Suppose there exists a BIB design with parameters $v=p, b, r, k, \lambda$ and there also exists an NBIB design with parameters $k_{,} b_{p}, b_{2}, k_{p}, k_{2}=2, r^{*}, \lambda_{p}, \lambda_{z}$ Then writing each of the block contents of BIB design as NBIB design, we get an NBIB design with parameters $p, b_{1}{ }^{*}=b b_{1}, b_{2}^{*}=b b_{2}, k_{1}^{*}=k_{1}, k_{2}{ }^{*}=2, r^{* *}=r r^{*}, \lambda_{1}{ }^{*}=\lambda_{1}, \lambda_{2}{ }^{*}=\lambda \lambda_{2}$ and hence a universally optimal design for diallel crosses over $\mathbf{D}\left(p, b^{*}, n\right)$, and with
parameters $p, b^{*}=b b_{1}, k^{*}=k_{1} / 2, n=b b_{1} k^{*}$. Now if $\lambda_{2}=\lambda=1$, then we get a design in minimal number of observations.

This is a fairly general method of construction and the existence of any NBIB design and a BIB design satisfying the conditions mentioned above implies the existence of a NBIB design for diallel crosses. Some particular cases of interest are:

## Particular Cases

Case I: Suppose there exists a BIB design $v=p, b, r, k=2 t, \lambda$ and a NBIB design with parameters $v_{l}=2 t, b_{1}=2 t-1, b_{2}=t(2 t-l), r=2 t-1, k_{1}=2 t, k_{2}=2, \lambda_{1}=2 t-l, \lambda_{2}=1$ always exists. Therefore, we can always get a universally optimal design for diallel crosses over D with parameters $p, b^{*}=b(2 t-1), k^{*}=t, n=b^{*} k^{*}$.
Example 4.2: Consider a BIB design with parameters $p=16, b=20, r=5, k=4, \lambda=1$ and a NIB design with parameter, $v^{*}=k=4, b_{1}{ }^{*}=3, b_{2}{ }^{*}=6, k_{1}{ }^{*}=4, k_{2}{ }^{*}=2, r=3, \lambda_{1}$ $=3, \lambda_{2}=1$. Then we get a universally optimal design for diallel crosses $\mathrm{D}(16,60,120)$ with parameters $p=16, b=60, k=2, n=120$.

This design is not obtainable by the methods given by Gupta and Kageyama (1994), Dey and Midha (1996), Das, Dey and Dean (1998) for these values of $p=16$ and $k=2$.

Case II: If there exists a BIB design with parameters $p, b, r, k=2 t+1, \lambda$, where $t$ is a positive integer, and an NBIB design with parameters $v^{*}=2 t+1, b_{i}^{*}=2 t+1, b_{2}^{*}=t(2 t+1), k_{1}^{*}=2 t, \mathrm{k}_{2}^{*}=2, r^{*}=2 t, \lambda_{1}=2 t-1, \lambda_{2}=1$, we can always get a universally optimal design over $\mathbf{D}\left(p, b^{*}, n\right)$ for diallel crosses with parameters $p, b^{*}=b(2 t+1), k^{*}=t, n=\mathrm{bt}(2 \mathrm{t}+1)$.

Example 4.3: Consider a BIB design with parameters $p=b=6, r=k=5, \lambda=4$ and a corresponding NBIB design with parameters $v^{*}=2 t+1=5, b_{j}^{*}=5, b_{2}^{*}=10, k_{i}^{*}=4$, $k_{2}^{*}=2, r=4, \lambda_{1}=3, \lambda_{2}=1$. Following the above procedure, we get a universally optimal design for diallel crosses over $\mathbf{D}(p=6, b=30, n=60)$ with parameters as $p=$ $6, b=30, k=2, n=60$.

Catalogue of designs obtained through this method using Case- I and Case -II for $p \leq 30$ and $n \leq 1000$, is reported in Table 2 given in appendix. The designs obtainable by taking copies of the designs reported are not included in the catalogue.
Remark 4.5: Agarwal and Das (1987) gave an application of balanced $n$-ary designs in the construction of incomplete block designs for evaluating the gca effects from complete diallel system IV of Griffing (1956) using BIB designs with $v=p, b=p(p-I) / 2, r=p$ $l, k=2, \lambda=l$ and triangular designs with parameters $v=p(p-l) / 2, b, r, k, \lambda_{i}, n_{i}, p_{j k}^{\prime}$, ( $i, j, k=1,2$ ). Although the authors do not discuss the optimality aspects of these designs, indeed some of their designs are universally optimal. In fact the design obtained in the

Example given by the authors is universally optimal using the conditions of Das, Dey and Dean (1998).

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## APPENDIX

Table 1 : Proper block designs for diallel crosses : I

| Sl.No. | p | b | k | n | Method of Construction |
| :--- | :--- | :--- | :--- | :--- | :--- |
| $1^{\mathrm{a}, \mathrm{b}}$ | 5 | 5 | 2 | 10 | Family $1: \mathrm{m}=4$ |
| $2^{\mathrm{b}}$ | 9 | 18 | 2 | 36 | Family $1: \mathrm{m}=4$ |
| $3^{\mathrm{b}}$ | 13 | 39 | 2 | 78 | Family $1: \mathrm{m}=4$ |
| $4^{\mathrm{b}}$ | 17 | 68 | 2 | 136 | Family $1: \mathrm{m}=4$ |
| $5^{\mathrm{b}}$ | 25 | 150 | 2 | 300 | Family $1: \mathrm{m}=4$ |
| $6^{\mathrm{b}}$ | 29 | 203 | 2 | 406 | Family $1: \mathrm{m}=4$ |
| $7^{\mathrm{a}, \mathrm{c}}$ | 7 | 7 | 3 | 21 | Family $1: \mathrm{m}=6$ |
| $8^{\mathrm{c}}$ | 13 | 26 | 3 | 78 | Family $1: \mathrm{m}=6$ |
| $9^{\mathrm{c}}$ | 19 | 57 | 3 | 171 | Family $1: \mathrm{m}=6$ |
| $10^{\mathrm{c}}$ | 25 | 100 | 3 | 300 | Family $1: \mathrm{m}=6$ |
| $11^{\mathrm{a}}$ | 9 | 9 | 4 | 36 | Family $1: \mathrm{m}=8$ |
| 12 | 17 | 34 | 4 | 136 | Family $1: \mathrm{m}=8$ |
| 13 | 25 | 75 | 4 | 300 | Family $1: \mathrm{m}=8$ |
| $14^{\mathrm{a}}$ | 11 | 11 | 5 | 55 | Family $1: \mathrm{m}=10$ |
| $15^{\mathrm{a}}$ | 13 | 13 | 6 | 78 | Family $1: \mathrm{m}=12$ |
| 16 | 25 | 50 | 6 | 300 | Family $1: \mathrm{m}=12$ |
| 17 | 29 | 58 | 7 | 406 | Family $1: \mathrm{m}=14$ |
| $18^{\text {a }}$ | 17 | 17 | 8 | 136 | Family $1: \mathrm{m}=16$ |
| $19^{\text {a }}$ | 19 | 19 | 9 | 171 | Family $1: \mathrm{m}=18$ |
| $20^{\text {a }}$ | 23 | 23 | 11 | 253 | Family $1: \mathrm{m}=22$ |
| $21^{\mathrm{a}}$ | 25 | 25 | 12 | 300 | Family $1: \mathrm{m}=24$ |
| $22^{\mathrm{a}}$ | 27 | 27 | 13 | 351 | Family $1: \mathrm{m}=26$ |
| $23^{\mathrm{a}}$ | 29 | 29 | 14 | 406 | Family $1: \mathrm{m}=28$ |
| 24 | 15 | 15 | 7 | 105 | Series $1: \mathrm{Gupta}$ and Kageyama (1994) |
| 25 | 21 | 21 | 10 | 210 | Series $1: \mathrm{Gupta}$ and Kageyama $(1994)$ |
| 26 | 4 | 3 | 2 | 6 | Series $2: \mathrm{Gupta}$ and Kageyama (1994) |
| 27 | 6 | 5 | 3 | 15 | Series $2: \mathrm{Gupta}$ and Kageyama (1994) |
| 28 | 8 | 7 | 4 | 28 | Series $2: \mathrm{Gupta}$ and Kageyama (1994) |


| SI. No. | p | b | k | n | Method of Construction |
| :---: | :---: | :---: | :---: | :---: | :---: |
| 29 | 10 | 9 | 5 | 45 | Series 2 : Gupta and Kageyama (1994) |
| 30 | 12 | 11 | 6 | 66 | Series 2 : Gupta and Kageyama (1994) |
| 31 | 14 | 13 | 7 | 91 | Series 2: Gupta and Kageyama (1994) |
| 32 | 16 | 15 | 8 | 120 | Series 2 : Gupta and Kageyama (1994) |
| 33 | 18 | 17 | 9 | 153 | Series 2 : Gupta and Kageyama (1994) |
| 34 | 20 | 19 | 10 | 190 | Series 2 : Gupta and Kageyama (1994) |
| 35 | 22 | 21 | 11 | 231 | Series 2 : Gupta and Kageyama (1994) |
| 36 | 24 | 23 | 12 | 276 | Series 2 : Gupta and Kageyama (1994) |
| 37 | 26 | 25 | 13 | 325 | Series 2 : Gupta and Kageyama (1994) |
| 38 | 28 | 27 | 14 | 378 | Series 2 : Gupta and Kageyama (1994) |
| 39 | 30 | 29 | 15 | 435 | Series 2 : Gupta and Kageyama (1994) |
| 40 | 19 | 95 | 2 | 171 | Family 3 : Das, Dey and Dean (1998) |
| 41 | 5 | 10 | 2 | 20 | Family 4 : Das, Dey and Dean (1998) |
| 42 | 7 | 21 | 2 | 42 | Family 4 : Das, Dey and Dean (1998) |
| 43 | 9 | 36 | 2 | 72 | Family 4 : Das, Dey and Dean (1998) |
| 44 | 11 | 55 | 2 | 110 | Family 4 : Das, Dey and Dean (1998) |
| 45 | 13 | 78 | 2 | 156 | Family 4 : Das, Dey and Dean (1998) |
| 46 | 17 | 136 | 2 | 272 | Family 4 : Das, Dey and Dean (1998) |
| 47 | 19 | 171 | 2 | 342 | Family 4 : Das, Dey and Dean (1998) |
| 48 | 23 | 253 | 2 | 506 | Family 4 : Das, Dey and Dean (1998) |
| 49 | 25 | 300 | 2 | 600 | Family 4 : Das, Dey and Dean (1998) |
| 50 | 27 | 351 | 2 | 702 | Family 4 : Das, Dey and Dean (1998) |
| 51 | 29 | 406 | 2 | 812 | Family 4 : Das, Dey and Dean (1998) |
| 52 | 5 | 2 | 5 | 10 | Family 5 : Das, Dey and Dean (1998) |
| 53 | 7 | 3 | 7 | 21 | Family 5: Das, Dey and Dean (1998) |
| 54 | 9 | 4 | 9 | 36 | Family 5 : Das, Dey and Dean (1998) |
| 55 | 11 | 5 | 11 | 55 | Family 5 : Das, Dey and Dean (1998) |
| 56 | 13 | 6 | 13 | 78 | Family 5 : Das, Dey and Dean (1998) |
| 57 | 15 | 7 | 15 | 105 | Family 5 : Das, Dey and Dean (1998) |
| 58 | 17 | 8 | 17 | 136 | Family 5 : Das, Dey and Dean (1998) |
| 59 | 19 | 9 | 19 | 171 | Family 5 : Das, Dey and Dean (1998) |
| 60 | 21 | 10 | 21 | 210 | Family 5 : Das, Dey and Dean (1998) |
| 61 | 23 | 11 | 23 | 253 | Family 5 : Das, Dey and Dean (1998) |
| 62 | 25 | 12 | 25 | 300 | Family 5 : Das, Dey and Dean (1998) |
| 63 | 27 | 13 | 27 | 351 | Family 5 : Das, Dey and Dean (1998) |
| 64 | 29 | 14 | 29 | 406 | Family 5 : Das, Dey and Dean (1998) |
| 65 | 5 | 30 | 3 | 90 | Theorem 4.1 : Das, Dey and Dean (1998) |



Table 2: Proper Block Designs for diallel crosses : II

| SI.No. | p | b | k | n | Reference Design | Method of Construction |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 1 | 5 | 15 | 2 | 30 | 55443 | Family 2 : Case 1 |
| 2 | 6 | 30 | 2 | 60 | 66554 | Family 2: Case II |
| 3 | 7 | 21 | 2 | 42 | 77442 | Family 2 : Case I |
| 4 | 7 | 35 | 3 | 105 | 77665 | Family 2 : Case I |
| 5 | 8 | 42 | 2 | 84 | 814743 | Family 2 : Case I |
| 6 | 8 | 56 | 3 | 168 | 88776 | Family 2: Case II |
| 7 | 9 | 54 | 2 | 108 | 918843 | Family 2 : Case I |
| 8 | 9 | 60 | 3 | 180 | 912865 | Family 2 : Case 1 |
| 9 | 9 | 63 | 4 | 252 | 99887 | Family 2 : Case I |
| 10 | 10 | 45 | 2 | 90 | 1015642 | Family 2 : Case I |
| 11 | 10 | 75 | 3 | 225 | 1015965 | Family 2 : Case I |
| 12 | 10 | 90 | 4 | 360 | 1010998 | Family 2: Case II |
| 13 | 11 | 55 | 2 | 110 | 1111552 | Family 2: Case II |
| 14 | 11 | 55 | 3 | 165 | 1111663 | Family 2 : Case I |
| 15 | 11 | 99 | 5 | 495 | 111110109 | Family 2 : Case I |
| 16 | 12 | 99 | 2 | 198 | 12331143 | Family 2 : Case 1 |
| 17 | 12 | 110 | 3 | 330 | 12221165 | Family 2 : Case I |
| 18 | 12 | 132 | 5 | 660 | 1212111110 | Family 2: Case II |


| Sl. No. | p | b | k | n | Reference Design | Method of Construction |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 19 | 13 | 39 | 2 | 78 | 1313441 | Family 2 : Case I |
| 20 | 13 | 130 | 3 | 390 | $13 \quad 261265$ | Family 2 : Case I |
| 21 | 13 | 117 | 4 | 468 | 1313996 | Family 2: Case II |
| 22 | 13 | 143 | 6 | 858 |  | Family 2 : Case I |
| 23 | 14 | 182 | 3 | 546 | 14261376 | Family 2: Case II |
| 24 | 15 | 105 | 2 | 210 | 1521752 | Family 2; Case II |
| 25 | 15 | 175 | 3 | 525 | 15351465 | Family 2 : Case I |
| 26 | 15 | 105 | 3 | 315 | 1515773 | Family 2: Case II |
| 27 | 15 | 105 | 4 | 420 | 1515884 | Family 2 : Case I |
| 28 | 15 | 189 | 5 | 945 | 152114109 | Family 2 : Case I |
| 29 | 16 | 60 | 2 | 120 | 1620541 | Family 2 : Case I |
| 30 | 16 | 80 | 3 | 240 | 1616662 | Family 2 : Case I |
| 31 | 16 | 120 | 3 | 360 | 1624963 | Family 2 : Case I |
| 32 | 16 | 200 | 3 | 600 | 16401565 | Family 2 : Case I |
| 33 | 16 | 210 | 4 | 840 | 16301587 | Family 2 : Case I |
| 34 | 16 | 144 | 5 | 720 |  | Family 2 : Case I |
| 35 | 17 | 204 | 2 | 408 | $\begin{array}{llllllllllllll}17 & 68 & 163\end{array}$ | Family 2 : Case I |
| 36 | 17 | 238 | 4 | 952 | 17341687 | Family 2 : Case I |
| 37 | 18 | 255 | 3 | 765 | 18511765 | Family 2 : Case I |
| 38 | 19 | 171 | 2 | 342 | 19571242 | Family 2 : Case I |
| 39 | 19 | 285 | 3 | 855 | 19571865 | Family 2 : Case I |
| 40 | 19 | 171 | 4 | 684 | 1919994 | Family 2: Case II |
| 41 | 19 | 171 | 5 | 855 | $\begin{array}{lllllllllll}19 & 19 & 10 \quad 10 & 5\end{array}$ | Family 2 : Case 1 |
| 42 | 20 | 285 | 2 | 570 | 20951943 | Family 2 : Case I |
| 43 | 20 | 380 | 2 | 760 | 20761954 | Family 2: Case II |
| 44 | 21 | 105 | 2 | 210 | 2121551 | Family 2: Case II |
| 45 | 21 | 140 | 3 | 420 | 2128862 | Family 2 : Case I |
| 46 | 21 | 210 | 3 | 630 | 21421263 | Family 2 : Case I |
| 47 | 22 | 231 | 2 | 462 | 22771442 | Family 2 : Case I |
| 48 | 22 | 154 | 3 | 462 | 2222772 | Family 2: Case II |
| 49 | 22 | 231 | 4 | 924 |  | Family 2 : Case I |
| 50 | 24 | 414 | 2 | 828 | 241382343 | Family 2 : Case I |
| 51 | 25 | 150 | 2 | 300 | 2550841 | Family 2 : Case I |
| 52 | 25 | 225 | 4 | 900 | 2525993 | Family 2: Case II |
| 53 | 26 | 325 | 3 | 975 | 26651563 | Family 2: Case I |


| Sl. No. | p | b | k | n | Reference Design | Method of Construction |  |  |
| :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- |
| 54 | 28 | 189 | 2 | 378 | 28 | 63 | 9 | 4 |
| 1 | Family 2: Case I |  |  |  |  |  |  |  |
| 55 | 28 | 252 | 3 | 756 | 28 | 36 | 9 | 7 |
| 2 | Family 2: Case II |  |  |  |  |  |  |  |
| 56 | 29 | 203 | 4 | 812 | 29 | 29 | 8 | 8 |

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