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ORIGINAL ARTICLE

TESTING OF VARIANCE COMPONENTS FOR CONTINUOUS DATA FROM NESTED UNBALANCED DESIGNS

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Abstract: Under unbalanced design, testing of variance ratios are generally neither independent nor distributed as chi-square variates and does not follow standard F-distribution. In this case, exact testing of variance ratios is not available in the literature. Procedure for unbalanced data (generally not independent and are not distributed as chi-square variates) has been developed for testing the variance components in one way and two way unbalanced nested designs.

Key words: Unbalanced data, Variance components, Nested design, Effect size, Approximate tests.

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1. Introduction

In case of random effects models for balanced designs, the analysis is simple and no problem is encountered in testing the variance components since the sums of squares are independent, sums of squares are chi-square variates, ratio of variance components follow standard F-distribution and hence exact testing is possible. When a random effects model is considered in unbalanced designs, analysis of variance technique rarely produce exact tests for testing the hypothesis. Under the conventional normality assumptions, except for the error component, the analysis of variance fails to decompose the total sums of squares into independently distributed sums of squares. Also, sums of squares are neither chi-square variates nor multiple of chi-square variate. The sums of squares are not independent either. Another standardized measure that quantifies the difference between means and relationship between independent and the dependent variable is effect-size measure. Two generally used statistics for computing effect-size are eta and omega squared statistics. But, these statistics do not yield correct estimate of effect-size that are comparable across different designs [Bakeman (2005)]. In that scenario, generalized eta and omega statistics given by Olejnik and Algina (2003) can be used. There was a conversation on two-way factorial ANOVA with mixed effects and interactions [Nelder (1977, 1982, 1994, 2008)]. The major assessments about the two-way factorial ANOVA model is no substantial rationale for the imposed constraints on random interactions and a lack of clear interpretation of its variance components, especially for the main random effects in respect of the response [Nelder (1977), Wolfinger and Stroup (2000), Lencina *et al*. (2007)]. As a result, the usual model is more widely used nowadays. The unbalanced mixed ANOVA models are often analyzed under the linear mixed models (LMM) framework using the restricted maximum likelihood (REML) or generalized least squares approaches [Littell (2002), Stroup (2013), Jiang (2017)]. Kaur and Garg (2020) attempted for Computer aided construction of rectangular PBIB

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designs. Gupta and Sharma (2020) constructed a set of balanced incomplete block designs (BIBD) against the loss of two blocks where loss of some observations lie in between at most two common treatments. Gupta (2021) worked on nested partially balanced incomplete block designs and its analysis. Singh *et al.* (2021 presented mixture designs generated using orthogonal arrays.

In this study, the one way random effects model for unbalanced nested design in which we have given the model, hypothesis to be tested, sums of squares and testing procedure for the hypothesis along with analysis of variance table. In the next section, we have explained model, hypothesis testing, sums of squares, hypothesis testing procedure and analysis of variance table for two way unbalanced nested design. Since in two way unbalanced case the means squares are generally not independent and are not distributed as chi-square variates, exact testing is not available for the main class variance component. We have obtained the expected size of approximate tests and the actual size for both conventional and approximate tests. Then with the help of a simulated data we found out the numerical for actual size of the conventional test and the actual and expected size of the approximate tests for some assumed values of the variance components.

2. One-way Nested Design

The linear model for one way unbalanced nested design can be expressed as

$$y_{ij} = m + a_i + e_{ij}, \ (j = 1, 2, ..., n_i;\ i = 1, 2, ..., g;\ \Sigma_i n_i = N) \quad (1)$$

where, y_{ij} represents the jth observation in the ith group, m is the overall mean, a_i represents the random effect owing to ith group, e_{ij} is the error component. It is assumed that the variables $\{a_i\}$ and $\{i_{ij}\}$ are independently distributed as normal variates with zero mean and variances as σ_α^2 and $\sigma_\in^2$, respectively. The variances σ_α^2 and $\sigma_\in^2$ are known as group and error variance components, respectively, in the model

The null hypothesis to be tested is

$$H_0 : \sigma_a^2 = 0 \quad (2)$$

Under model (1), sums of squares are written as

$$SSB = \sum_i n_i \left(\bar{y}_i - \bar{y}\right)^2 \text{ and } WSS = \Sigma_i \Sigma_j \left(\bar{y}_i - \bar{y}\right)^2 \quad (3)$$

where, $\bar{y}_i = \Sigma_j \dfrac{y_{ij}}{n_i}$ and $\bar{y} = \Sigma_i \Sigma_j \dfrac{y_{ij}}{N}$

The expected values of the sums of squares are calculated as follows

$$E(SSB) = (g-1)\left[\sigma_e^2 + n_0\sigma_a^2\right] \text{ and}$$

$$E(WSS) = (N-g)\sigma_e^2 \quad (4)$$

The between sums of squares is distributed as chi-square with p (= g – 1) degrees of freedom and error sums of squares are distributed as chi-square with q(= N – g) degrees of freedom, under the null hypothesis H_0.

The test statistic for testing whether or not the null hypothesis H_0 is true is defined as

$$F = MSB/WMS \quad (5)$$

where, MSB(= SSB/p) and WMS (= WSS/q) are mean squares of the main class and error component, respectively.

The F-ratio in (5) is distributed as standard F with p and q degrees of freedom, under the null hypothesis H_0.

As a result, the testing of the variance component in a one-way random effect models may be done precisely in both balanced and unbalanced circumstances.

The null hypothesis is rejected if calculated $F \geq F_{(\alpha,\, p,\, q)}$, where $F_{(\alpha,\, p,\, q)}$ is α upper tail point of F distribution with degrees of freedom (p, q).

Table 1 shows the analysis of variance of one way unbalanced nested design.

3. Two Way Unbalanced Nested Model

The model for two-way nested unbalanced design is

$$y_{ijk} = m + a_i + b_{ij} + e_{ijk}$$

Table 1: ANOVA for One Way Nested Unbalanced Design.

Source	df	SS	E_0(MS)	Test -Statistic	Reference distribution
Between Groups	g – 1	SSB	σ_a^2	$F_{(g-1,\, N-g)}$	
Error	N – g	WSS	σ_e^2		

$(i = 1, 2, ..., a; j = 1, 2, ..., c_i; k = 1, 2, ..., n_{ij})$ (6)

where, y_{ijk} is the kth observation in the jth sub class within the ith main class, m is the overall mean, a_i is the random effect due to ith main class, b_{ij} is the effect due to jth subclass within the ith main class and e_{ijk} is the error variable associated with y_{ijk}. It is assumed that the variables $\{a_i\}$, $\{b_{ij}\}$ and $\{e_{ijk}\}$ are independently distributed as normal variates with zero mean and variances σ_a^2, σ_b^2 and σ_e^2, respectively. The variances, σ_a^2, σ_b^2 and σ_e^2, are known as variance components in the model.

The null hypotheses to be tested for the main class and subclass are

$$H_{01}:\sigma_a^2 = 0 \text{ and } H_{02}:\sigma_b^2 = 0 \tag{7}$$

The sums of squares under model (6) are defined as

$$\mathrm{SSA} = \Sigma_i\, n_i\left(\bar{y}_i - \bar{y}\right)^2,\ \mathrm{SSAB} = \Sigma_i\, \Sigma_j n_{ij}\left(\bar{y}_{ij} - \bar{y}_i\right)^2,$$

$$\mathrm{SSE} = \Sigma_i \Sigma_j \Sigma_k\left(y_{ijk} - \bar{y}_{ij}\right)^2 \tag{8}$$

where, $n_i = \sum_i n_{ij}$, $N = \sum_i n_i$, $b = \sum_i c_i$ and $\bar{y}_{ij}$, $\bar{y}_i$ and $\bar{y}$ are the usual mean values.

The expected values of sums of squares are obtained as

$$E(SSA) = (a-1)\left\{\sigma_e^2 + k_2\sigma_b^2 + k_1\sigma_a^2\right\},$$

$$E(SSAB) = (b-a)\left\{\sigma_e^2 + k_3\sigma_b^2\right\},$$

$$E(SSE) = (N-b)\sigma_e^2 \tag{9}$$

where, $k_1 = \left[N - \Sigma_i n_i^2/N\right]/(a-1)$,

$$k_2 = \left[\sum_{ij} n_{ij}^2\{(1/n_i) - (1/N)\}\right]\Big/(a-1),$$

$$k_3 = \left[N - \sum_{ij} n_{ij}/n_i\right]\Big/(b-a).$$

The sums of squares, SSA, SSAB and SSE are independently distributed as some multiple of chi-square in the complete balanced case, that is, $n_{ij} = n$ and $c_i = c$. The error sum of squares SSE is distributed as a multiple of chi-square in the unbalanced case, whereas the sums of squares SSA and SSAB are neither independent nor have a distribution that is a multiple of chi-square. Even the null distribution of SSA is not a multiple of chi-square except, when $\sigma_b^2 = 0$. The null distribution of SSAB is, however, some multiple of chi-square. These derived expressions cannot be used for practical applications due to involvement of parameters. We, here use approximate distributions of SSA and SSB.

3.1 Testing Procedure

The main class, subclass within main class and error mean squares are defined as

$$MSA = SSA/(a-1),\ MSAB = SSAB/(b-a) \text{ and}$$

$$MSE = SSE/(N-b) \tag{10}$$

Exact testing is available for both variance components σ_a^2 and σ_b^2 under the balanced case. The null hypothesis, $H_{01}:\sigma_a^2 = 0$ is tested by using the variance ratio

$$F_{10} = MSA/MSAB \tag{11}$$

with reference distribution as F on degrees of freedom (a-1, b-a). Similarly, the null hypothesis $H_{02}:\sigma_b^2 = 0$ is tested by using the variance ratio

$$F_B = MSAB/MSE \tag{12}$$

with reference distribution as F on degrees of freedom (b-a, N-b).

Under unbalanced case the exact testing is available for $H_{02}:\sigma_b^2 = 0$ by using (12), but it is not available for $H_{01}:\sigma_a^2 = 0$ particularly when $\sigma_b^2 \neq 0$. The exact testing for null hypothesis $H_{01}:\sigma_a^2 = 0$ is, however, available under the last stage uniformity, that is when, $n_{ij} = n$, $\forall i$ and j. The mean squares MSA and MSAB under H_{01} are independently distributed as some multiple of chi-square when the last stage uniformity is assumed, for example, when the number of progeny per dam is considered to be equal. For testing $H_{01}:\sigma_a^2 = 0$, the test statistic and reference distribution are the same as in (11).

when $\sigma_b^2 = 0$, the testing for $\sigma_a^2 = 0$ can be carried out by using the following variance ratio in the general unbalanced situation

$$F_A = \frac{MSA}{MSE_0},$$

where, $MSE_0 = (SSA + SSE)/(N - a)$ (13)

With reference distribution as F on degrees of freedom $(a - 1, N - a)$.

When, σ_b^2 is non-zero, the mean squares are no longer independent and do not exhibit a constant times chi-square distribution. Tietjen and Moore (1968) established a technique say F_1 for testing the main class variance component by producing a denominator with the same expected value as the numerator under the null hypothesis. In a simulation study by Tietjen (1974) discovered that when mean squares are negatively engaged in the generated denominator, the F_1 test does not perform well. For such scenarios, Cummings and Gaylor (1974) devised another approximate test say F_2 and discovered that the perturbations in the expected size of the test are minor. Tan and Cheng (1984) suggested a new approximate test say F_3 by building both the numerator and denominator as a linear combination of mean squares with the same expected values under the null hypothesis and demonstrating that their statistic produces better results. They also discovered that power of testing of these approximate tests are similar.

The reported size (α) in these research was approximated by the size of the test using the expected mean square, which is not the actual size but the expected size. From the null distributions of test statistics generated from simulated normal samples under the model for some apriori parametric values, we have obtained the expected size of approximate tests (F_1, F_2 and F_3), as well as the actual size for both conventional (F_{10}) and approximate tests (F_1, F_2 and F_3). For these approximate tests, the effects of unbalancedness on the actual and expected size have been investigated.

The conventional test statistic for testing $H_0 : \sigma_a^2 = 0$ is F_{10} is defined in (11). The test statistic proposed by Tietjen and Moore (1968) is defined as

$$F_1 = MSA/[(1 - r_1)MSE + r_1 MSAB] \quad (14)$$

with reference distribution as F on degrees of freedom (a-1) and f_1, where

$$f_1 = [E\{(1-r_1)MSE + r_1 MSAB\}]^2 / \left[\{(1-r_1)E(MSE)\}^2/(N-b) + \{r_1 E(MSAB)\}^2/(b-a)\right]$$ and $r_1 = k_2/k_3$.

The test statistic proposed by Cummings and Gaylor (1974) is defined as

$$F_2 = [r_2 MSA + (1 - r_2)MSE]/MSAB \quad (15)$$

with assumed distribution as F on degrees of freedom f_2 and (b-a), where

$$f_2 = [E_0\{(1-r_2)MSE + r_2 MSA\}]^2 / \left[\{(1-r_2)E(MSE)\}^2/(N-b) + \{r_2 E_0(MSA)\}^2/(a-1)\right]$$ and $r_2 = k_3/k_2$.

The test statistic proposed by Tan and Cheng (1984) is given by

$$F_3 = (MSA + r_1 MSE)/(r_1 MSAB + MSE) \quad (16)$$

with reference distribution as F on degrees of freedom as f_3 and f_4, where

Here, E_0 indicates the expectation under H_{01}.

4. Numerical Results

The size of a test with test statistic F_1 is defined as

$$\alpha(F_i) = P_r(F_i \geq c_{0i}),\ i = 1, 2, 3 \quad (17)$$

Under the assumed distribution of test statistic F_i, c_{0i} is the critical point with upper tail probability α. For some apriori values of variance components and design parameters, the sampling distribution of test statistic F_i is constructed from 1000 normal samples simulated under the model (1). The estimated degrees of freedom were calculated using the mean squares (MSA and MSAB) corresponding to the 50th value of F_i sorted in descending order. The predicted degrees of freedom were calculated using the expected mean squares for the same apriori variance component and design parameter values as in the simulation. The upper critical points (c_{0i}) for estimated and expected degrees of freedom were separately noted from the standard F distribution table. The number of F_i values greater or equal to c_{0i} corresponding to expected and estimated degrees of freedom in the simulated distributions were taken as actual and expected size of approximate F_i tests, respectively, for assigned values of variance components and design parameters. The numerical values for actual size of conventional test and the actual and expected size of approximate tests are presented in Tables 3, 4 and 5, respectively, for the following apriori values of variance components and design parameters

$$\alpha = 0.05,\ \sigma_a^2 = 0,\ r = (\sigma_b^2/\sigma_e^2) = 0, 0.5, 1.0, 5.0, 15.0$$

I. $a = 3$, $c_1 = c_2 = 2$, $c_3 = 5$

Table 2: ANOVA for Two Way Nested Unbalanced Design.

Source	df	SS	E_0(MS)	Test -Statistic	Reference distribution
Main class	a-1	SSA	$\sigma_e^2 + k_2\sigma_b^2 + k_1\sigma_a^2$	F_1 or F_2	
Sub class	b-a	SSAB	$\sigma_e^2 + k_3\sigma_b^2$	F_B	$F_{[b-a,(N-b)]}$
Error	N-b	SSE	σ_e^2		

Table 3: Actual size of the conventional test for stated size 0.05.

Design	a	r_1	τ = 0.0	0.5	1.0	5.0	15.0
D1	3	0.70	0.052	0.036	0.029	0.032	0.030
D2	3	0.72	0.049	0.037	0.032	0.025	0.023
D3	3	0.83	0.047	0.048	0.042	0.032	0.024
D4	3	1.57	0.047	0.078	0.082	0.090	0.088
D5	3	1.69	0.046	0.082	0.091	0.092	0.093
D6	3	2.69	0.051	0.137	0.152	0.172	0.180
D7	5	0.88	0.053	0.045	0.040	0.040	0.040
D8	5	0.92	0.052	0.044	0.046	0.044	0.040
D9	5	0.93	0.040	0.039	0.040	0.047	0.042
D10	5	1.57	0.051	0.092	0.105	0.120	0.121
D11	5	1.56	0.043	0.096	0.107	0.122	0.122
D12	5	2.69	0.040	0.201	0.237	0.288	0.316

Table 4: Expected size of approximate tests for stated size 0.05.

Design	a	r_1	Test	τ = 0.0	0.5	1.0	5.0	15.0
D1	3	0.7	F1	0.041	0.045	0.051	0.054	0.057
			F3	0.032	0.041	0.048	0.051	0.052
D2	3	0.72	F1	0.047	0.044	0.043	0.044	0.053
			F3	0.047	0.039	0.038	0.038	0.049
D3	3	0.83	F1	0.038	0.055	0.053	0.041	0.044
			F3	0.041	0.050	0.049	0.038	0.041
D4	3	1.57	F2	0.052	0.034	0.044	0.038	0.042
			F3	0.030	0.036	0.039	0.038	0.041
D5	3	1.69	F2	0.046	0.043	0.040	0.043	0.041
			F3	0.033	0.029	0.035	0.047	0.041
D6	3	2.69	F2	0.047	0.030	0.027	0.025	0.024
			F3	0.021	0.026	0.025	0.025	0.021
D7	5	0.88	F1	0.050	0.055	0.051	0.058	0.055
			F3	0.045	0.054	0.056	0.057	0.053
D8	5	0.92	F1	0.053	0.049	0.053	0.051	0.053
			F3	0.039	0.050	0.050	0.047	0.050
D9	5	0.93	F1	0.039	0.041	0.049	0.052	0.050
			F3	0.034	0.038	0.050	0.053	0.048
D10	5	1.57	F2	0.048	0.041	0.032	0.032	0.033

Table 4 continued...

Table 4 continued...

			F3	0.032	0.036	0.028	0.027	0.030
D11	5	1.56	F2	0.043	0.037	0.031	0.030	0.032
			F3	0.033	0.030	0.028	0.029	0.030
D12	5	1.56	F2	0.039	0.030	0.025	0.023	0.018
			F3					

Table 5: Actual size of approximate tests for stated size 0.05.

Design	a	r_1	Test	$\tau = 0.0$	0.5	1.0	5.0	15.0
D1	3	0.7	F1	0.059	0.073	0.07	0.082	0.072
			F3	0.05	0.06	0.099	0.09	0.087
D2	3	0.72	F1	0.058	0.055	0.061	0.062	0.073
			F3	0.058	0.066	0.09	0.099	0.089
D3	3	0.83	F1	0.052	0.079	0.071	0.075	0.066
			F3	0.027	0.064	0.103	0.1	0.084
D4	3	1.57	F2	0.054	0.053	0.056	0.051	0.053
			F3	0.104	0.076	0.07	0.083	0.07
D5	3	1.69	F2	0.049	0.054	0.051	0.058	0.054
			F3	0.045	0.085	0.069	0.066	0.072
D6	3	2.69	F2	0.044	0.034	0.033	0.038	0.041
			F3	0.13	0.057	0.057	0.064	0.069
D7	5	0.88	F1	0.057	0.064	0.064	0.065	0.065
			F3	0.046	0.058	0.08	0.086	0.091
D8	5	0.92	F1	0.056	0.058	0.062	0.06	0.062
			F3	0.046	0.077	0.069	0.07	0.073
D9	5	0.93	F1	0.056	0.043	0.057	0.061	0.059
			F3	0.039	0.046	0.084	0.069	0.082
D10	5	1.57	F2	0.049	0.042	0.033	0.033	0.034
			F3	0.033	0.037	0.029	0.028	0.031
D11	5	1.56	F2	0.054	0.041	0.041	0.045	0.042
			F3	0.071	0.054	0.065	0.075	0.049
D12	5	2.69	F2	0.047	0.034	0.033	0.028	0.028
			F3	0.086	0.087	0.062	0.039	0.035

$D_1 = \{n_{11} = n_{12} = 3;\ n_{21} = n_{22} = 2;\ n_{31} = n_{32} = 4;\ n_{33} = 7,\ n_{34} = n_{35} = 5\}$

$D_2 = \{n_{il} = n_{i2} = 3;\ i = 1, 2, 3;\ n_{33} = 11,\ n_{34} = n_{35} = 5\}$

$D_3 = \{n_{i1} = n_{i2} = 3;\ i = 1, 2, 3;\ n_{33} = 9,\ n_{34} = n_{35} = 5\}$

II. a = 3, $c_1 = c_2 = c_3 = 3$

$D_4 = \{n_{i1} = n_{i2} = 3;\ n_{i3} = 9;\ i = 1, 2, 3\}$

$D_5 = \{n_{11} = n_{12} = 3;\ n_{13} = 9;\ n_{21} = n_{22} = 2;\ n_{23} = 11;\ n_{31} = n_{32} = 4,\ n_{33} = 7\}$

$D_6 = \{n_{il} = n_{i2} = 2;\ n_{i3} = 11;\ i = 1, 2, 3\}$

III. $a = 5$, $c_1 = c_2 = c_3 = c_4 = c_5 = 5$

$D_7 = \{n_{11} = n_{12} = 3;\ n_{21} = n_{22} = 2;\ n_{31} = n_{32} = 4;\ n_{41} = 2,\ n_{42} = 4,\ n_{43} = 9,\ n_{44} = n_{45} = 5,\ n_{51} = 3,\ n_{52} = 5,\ n_{53} = 7,\ n_{54} = n_{55} = 5$

$D_8 = \{n_{i1} = n_{i2} = 3;\ i = 1, 2, 3, 4, 5;\ n_{43} = n_{53} = 9;$

$n_{44} = n_{45} = n_{54} = n_{55} = 5\}$

$D_9 = \{n_{i1} = n_{i2} = 2; i = 1, 2, 3, 4, 5; n_{43} = n_{53} = 11; n_{14} = n_{45} = n_{54} = n_{55} = 5\}$

IV. $a = 5, c_1 = c_2 = c_3 = c_4 = c_5 = 3$

$D_{10} = \{n_{i1} = n_{i2} = 3; n_{i3} = 9; i = 1, 2, 3, 4, 5\}$

$D_{11} = \{n_{11} = n_{12} = 3; n_{13} = 9; n_{21} = n_{22} = 2; n_{23} = 11; n_{31} = n_{32} = 4, n_{33} = 7, n_{41} = 2, n_{42} = 4, n_{43} = 9; n_{51} = 3, n_{52} = 5, n_{53} = 7\}$

4. Conclusion

The numerical results reveal that when $\tau > 0$ the actual size of conventional test (F_{10}) over estimated the stated size for unbalanced situations having $r_1 > 1$ and under estimated for unbalanced situations having $r_1 < 1$ and this over and under estimation increase with increase in the value of τ, the ratio of subclass to error variance components. The disturbance in expected size with respect to the stated size (0.05) is small for all approximate tests preferably for $a = 5$. The disturbance in the actual size is not small with respect to the stated size particularly for approximate test F_3. These results imply that the approximate test F_2 for situations with $r_1 > 1$ and the approximate test F_1 for other situations may be used for analysis of data from nested designs under model (1) for practical situations. The analysis of variance is presented in Table 2.

The null hypothesis H_{02} is tested by comparing F_B with upper α% tabulated value of F distribution on $[b - a, N - b]$ degrees of freedom. If H_{02} is not rejected then H_{01} is tested by comparing F_A with upper α% tabulated value of distribution on [a–1, N – a] degrees of freedom. If H_{02} is rejected then H_{01} is tested by comparing F_2 or F_1 with upper α% tabulated value of F distribution on $[a-1, f_2]$ or $[a-1, f_1]$ degrees of freedom.

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