Journal of Applied Statistics
Vol. 32, No. 1, 87–97, January 2005

Routledge
Taylor & Francis Group

On Totally Balanced Block Designs for Competition Effects

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ABSTRACT *Competition between neighbouring units in field experiments is a serious source of bias. The study of a competing situation needs construction of an environment in which it can happen and the competing units have to appear in a predetermined pattern. This paper describes methods of constructing incomplete block designs balanced for neighbouring competition effects. The designs obtained are totally balanced in the sense that all the effects, direct and neighbours, are estimated with the same variance. The efficiency of these designs has been computed as compared to a complete block design balanced for neighbours and a catalogue has also been prepared.*

KEY WORDS: Competition effects, circular design, totally balanced design, MOLS

Introduction

Field experiments are usually performed to assess the effect of several management factors or genetic factors, or both, on crop performance. The experimental plots allocated to different treatments and subjected to different production techniques are commonly placed side by side. As a consequence, the response from a given plot may be affected by the treatments applied to its neighbouring plots besides the treatment applied to the plot itself. Interdependence of adjacent plots because of their common needs is referred to as the competition effect. Competition or interference between neighbouring units is a serious source of bias. Understanding the structure of these effects helps in minimizing such bias to a great extent. The study of a competing situation needs construction of an environment in which it can happen and the competing units have to appear in a predetermined pattern. This involves construction of a design in which two competing treatments occur together in some order.

Studies of interference between neighbouring units under laboratory conditions began with the work on neighbour designs by Rees (1967) on designing of plots to diffusion tests in virus research. Martin (1973) developed beehive designs in which plants of two species are arranged on a hexagonal grid such that for one species the number of neighbouring plants of the second species varies between zero and six. These designs allow the experimenter to carry out the investigations in a much smaller area and each plant is either a recorded plant or a competing plant. Martin (1986) has investigated the design of field experiments in which the correlation between adjoining plots is taken

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0266-4763 Print/1360-0532 Online/05/010087–11
DOI: 10.1080/0266476042000305177

into account and hence the errors are considered to be spatially correlated under different correlation structures.

Dyke & Shelley (1976) introduced serial designs that allow the independent estimation of the effects of treatments to neighbouring plots and have constructed serial designs based on a computer program. Assuming that competition occurs only between the test plot and its immediate neighbouring plots on either side and that the effects are the same for the left- and right-hand-side arrangements, Lin *et al.* (1985) introduced a similar treatment sequence and computer-aided non-random designs. Azais *et al.* (1993) obtained a series of designs that are balanced in $t-1$ blocks of size t and t blocks of size $t-1$, where t is the number of treatments.

In this study, it is assumed that the effect of a treatment applied to a given plot is the sum of the direct effect due to the treatment applied to the plot, a left-neighbour effect due to the competition with the treatment applied to the immediate left-neighbour plot and a right-neighbour effect due to the competition with the treatment applied to the immediate right-neighbour plot. The purpose of this paper is to give some methods of constructing incomplete block designs that are totally balanced for estimating the direct effects as well as the left- and right-neighbour effects. These methods are based on the development of initial blocks and mutually orthogonal Latin squares. The incomplete block designs so obtained are totally balanced in the sense that all the effects, direct and neighbours, are estimated with the same variance. The efficiency of these designs has been computed as compared to a complete block design balanced for neighbours and a catalogue has also been prepared.

Model and Definition

Let v be the number of treatments whose effects are to be studied. Considering the three effects obtained from a plot, the competition can be studied from an ordered triplet. The direct effect can be obtained from the treatment appearing in the middle, while the two treatments appearing as the immediate neighbour plots to it provide the left and right competition effects. Under the block design set-up with b blocks of sizes $k_1, k_2, \ldots, k_b$, respectively, the following model has been considered for analysing a design with competition effects:

$$Y_{ij} = \mu + \tau_{(i,j)} + \beta_j + \delta_{(i-1,j)} + \rho_{(i+1,j)} + e_{ij} \tag{1.1}$$

where Y_{ij} is the response from the i^{th} plot in the j^{th} block ($i = 1, 2, \ldots, k_j; j = 1, 2, \ldots, b$), μ is the general mean, $\tau_{(i,j)}$ is the direct effect of the treatment in the i^{th} plot of the j^{th} block, β_j is the effect of the j^{th} block, $\delta_{(i-1,j)}$ is the left-neighbour effect due to the treatment in the $(i-1)^{\text{th}}$ plot of the j^{th} block and $\rho_{(i+1,j)}$ is the right-neighbour effect due to the treatment in the $(i+1)^{\text{th}}$ plot of the j^{th} block. e_{ij} are error terms independently and normally distributed with mean zero and variance σ^2.

The joint information matrix for estimating the direct effects, left-neighbour effects and right-neighbour effects of treatments is:

$$\mathbf{C} = \begin{bmatrix} \mathbf{R}_\tau - \mathbf{N}_3\mathbf{K}^{-1}\mathbf{N}_3' & \mathbf{N}_1 - \mathbf{N}_3\mathbf{K}^{-1}\mathbf{N}_5' & \mathbf{N}_2 - \mathbf{N}_3\mathbf{K}^{-1}\mathbf{N}_6' \\ \mathbf{N}_1' - \mathbf{N}_5\mathbf{K}^{-1}\mathbf{N}_3' & \mathbf{R}_\delta - \mathbf{N}_5\mathbf{K}^{-1}\mathbf{N}_5' & \mathbf{N}_4 - \mathbf{N}_5\mathbf{K}^{-1}\mathbf{N}_6' \\ \mathbf{N}_2' - \mathbf{N}_6\mathbf{K}^{-1}\mathbf{N}_3' & \mathbf{N}_4' - \mathbf{N}_6\mathbf{K}^{-1}\mathbf{N}_5' & \mathbf{R}_\rho - \mathbf{N}_6\mathbf{K}^{-1}\mathbf{N}_6' \end{bmatrix} \tag{1.2}$$

where $\mathbf{N}_1$, $\mathbf{N}_2$ and $\mathbf{N}_4$ are the $v \times v$ incidence matrices of direct versus left-neighbour treatments, direct versus right-neighbour treatments and left- versus right-neighbour treatments, respectively. $\mathbf{N}_3$, $\mathbf{N}_5$ and $\mathbf{N}_6$ are the $v \times b$ incidence matrices of direct treatments versus blocks, left-neighbour treatments versus blocks and right-neighbour treatments versus blocks, respectively. Further, $\mathbf{R}_\tau = \mathrm{diag}(r_1, r_2, \ldots, r_v)$; $\mathbf{R}_\delta = \mathrm{diag}(r_{11}, r_{12}, \ldots, r_{1v})$; $\mathbf{R}_\rho = \mathrm{diag}(r_{21}, r_{22}, \ldots, r_{2v})$, $r_{1i}(r_{2i})$ being the number of times the treatments in the design have i^{th} treatment as left (right) neighbour and $\mathbf{K} = \mathrm{diag}(k_1, k_2, \ldots, k_b)$. The $3v \times 3v$ matrix $\mathbf{C}$ is symmetric, non-negative definite with zero row and column sums. The information matrix for estimating the direct effects of treatments and neighbour competing effects can be obtained easily from equation (1.2). We now give some definitions useful for obtaining the designs.

Definition 1.1.1. A block design for competition effects is combinatorially balanced if every treatment has every other treatment appearing a constant number of times as a right neighbour and as a left neighbour.

Definition 1.1.2. A block of treatments with border plots is left circular if the treatment in the left border is the same as the treatment in the right-end inner plot and right circular if the treatment in the right border is the same as the treatment in the left-end inner plot. A circular block is a left- and right-circular block and a circular design is a design with all its blocks circular.

Definition 1.1.3. A block design with circular blocks, permitting the estimation of direct and neighbour effects, is called variance balanced if the variance of any estimated elementary contrast among the direct effects is constant, say V_1, the variance of any estimated elementary contrast among the left-neighbour effects is constant, say V_2, and the variance of any estimated elementary contrast among the right-neighbour effects is constant, say V_3. The constants V_1, V_2 and V_3 may not be equal. A block design is totally balanced if $V_1 = V_2 = V_3$.

For a detailed review of different types of balance in different contexts, reference may be made to Preece (1982).

Methods of Constructing Totally Balanced Block Designs for Competition Effects

We give here methods of constructing incomplete block designs that are totally balanced for neighbouring competition effects through initial block solution and making use of mutually orthogonal Latin squares (MOLS).

Method 2.1 Let the number of treatments $v = sm + 1$ (prime or prime power) where $m > 3$. The sv blocks of size $k = m$ with replication of each treatment being sm are obtained by developing the following initial blocks modulo v:

$$x^{i+(m-1)s} | x^i, x^{i+s}, x^{i+2s}, \ldots, x^{i+(m-1)s} | x^i \tag{2.1}$$

$i = 0, 1, 2, \ldots, s-1$ and x is the primitive element of GF$(sm+1)$.

The series of incomplete block designs obtained is totally balanced for the estimation of direct, left- and right-neighbour effects. Border plots have been added to make the blocks circular. The structure of the incidence matrices is as follows:

$$\begin{aligned} \mathbf{N}_1 &= \mathbf{N}_2 = \mathbf{N}_4 = \mathbf{J} - \mathbf{I}, \\ \mathbf{N}_3\mathbf{N}_3' &= \mathbf{N}_5\mathbf{N}_5' = \mathbf{N}_6\mathbf{N}_6' = \mathbf{N}_3\mathbf{N}_5' = \mathbf{N}_3\mathbf{N}_6' = \mathbf{N}_5\mathbf{N}_6' \\ &= (v-m)\mathbf{I} + (m-1)\mathbf{J} \end{aligned}$$

The **C** matrix as given in equation (1.2) reduces here to:

$$\mathbf{C} = \begin{bmatrix} \frac{v(k-1)}{k}\left[\mathbf{I}-\frac{\mathbf{J}}{v}\right] & \frac{-v}{k}\left[\mathbf{I}-\frac{\mathbf{J}}{v}\right] & \frac{-v}{k}\left[\mathbf{I}-\frac{\mathbf{J}}{v}\right] \\ \frac{-v}{k}\left[\mathbf{I}-\frac{\mathbf{J}}{v}\right] & \frac{v(k-1)}{k}\left[\mathbf{I}-\frac{\mathbf{J}}{v}\right] & \frac{-v}{k}\left[\mathbf{I}-\frac{\mathbf{J}}{v}\right] \\ \frac{-v}{k}\left[\mathbf{I}-\frac{\mathbf{J}}{v}\right] & \frac{-v}{k}\left[\mathbf{I}-\frac{\mathbf{J}}{v}\right] & \frac{v(k-1)}{k}\left[\mathbf{I}-\frac{\mathbf{J}}{v}\right] \end{bmatrix} \tag{2.2}$$

Therefore, the individual information matrices for estimating the direct effects ($\mathbf{C}_\tau$), left-neighbour effects ($\mathbf{C}_\delta$) and right-neighbour effects ($\mathbf{C}_\rho$) of treatments are obtained as given below:

$$\mathbf{C}_\tau = \mathbf{C}_\delta = \mathbf{C}_\rho = \frac{v(k-3)}{k-2}\left[\mathbf{I}-\frac{\mathbf{J}}{v}\right] \tag{2.3}$$

Example 2.1. Let $s = 2$ and $m = 5$; therefore $v = sm + 1 = 11$. Developing the two initial blocks mod 11 along with border plots by taking $i = 0$ and 1, the following block design in 22 blocks of size 5 each is obtained:

3	**1**	**4**	**5**	**9**	**3**	1		6	**2**	**8**	**10**	**7**	**6**	2
4	2	5	6	10	4	2		7	3	9	11	8	7	3
5	3	6	7	11	5	3		8	4	10	1	9	8	4
6	4	7	8	1	6	4		9	5	11	2	10	9	5
7	5	8	9	2	7	5		10	6	1	3	11	10	6
8	6	9	10	3	8	6		11	7	2	4	1	11	7
9	7	10	11	4	9	7		1	8	3	5	2	1	8
10	8	11	1	5	10	8		2	9	4	6	3	2	9
11	9	1	2	6	11	9		3	10	5	7	4	3	10
1	10	2	3	7	1	10		4	11	6	8	5	4	11
2	11	3	4	8	2	11		5	1	7	9	6	5	1

Remark 2.1.1. In the above method if $s = 1$ and $m = v - 1$, then the method reduces to obtaining balanced designs of Azais *et al.* (1993) for v treatments in v blocks of size $v - 1$ each.

Remark 2.1.2. The designs obtained by this method are minimally balanced in the sense that every treatment has every other treatment as a neighbour on both sides once.

Remark 2.1.3. If border plots are not considered, the design is a balanced incomplete block design.

Method 2.2. Let $v = s$ ($s > 4$) be a prime or prime power and the s treatments be denoted by 1, 2, ..., s. Develop $s - 1$ mutually orthogonal Latin squares (MOLS) of order s by multiplying the first principal row by the elements of GF(s), except 1 and s, and adding corresponding entries in each cell. Juxtapose these MOLS so that we obtain an arrangement of s symbols in $s(s - 1)$ rows and s columns. Deleting the last i columns ($i = 1$, 2, ..., s − 4) and taking rows as blocks along with border plots, to make the blocks circular, would result in an incomplete block design totally balanced for competition effects. This series of designs can be obtained for any k, $4 \le k \le s - 1$. The parameters of the design are $v = s$, $b = s(s-1)$, $r = (s-1)(s-i)$ and $k = s - i$. Here $\mathbf{N}_1 = \mathbf{N}_2 = \mathbf{N}_4 = (s-i)[\mathbf{J} - \mathbf{I}]$ and $\mathbf{N}_3\mathbf{N}_3' = \mathbf{N}_5\mathbf{N}_5' = \mathbf{N}_6\mathbf{N}_6' = \mathbf{N}_3\mathbf{N}_5' = \mathbf{N}_3\mathbf{N}_6' = \mathbf{N}_5\mathbf{N}_6' = (s-i)[i\mathbf{I} + (s-i-1)\mathbf{J}]$.

The $\mathbf{C}$ matrix as given in equation (1.2) reduces to:

$$\mathbf{C} = \begin{bmatrix} v(k-1)\left[\mathbf{I} - \dfrac{\mathbf{J}}{v}\right] & -v\left[\mathbf{I} - \dfrac{\mathbf{J}}{v}\right] & -v\left[\mathbf{I} - \dfrac{\mathbf{J}}{v}\right] \\ -v\left[\mathbf{I} - \dfrac{\mathbf{J}}{v}\right] & v(k-1)\left[\mathbf{I} - \dfrac{\mathbf{J}}{v}\right] & -v\left[\mathbf{I} - \dfrac{\mathbf{J}}{v}\right] \\ -v\left[\mathbf{I} - \dfrac{\mathbf{J}}{v}\right] & -v\left[\mathbf{I} - \dfrac{\mathbf{J}}{v}\right] & v(k-1)\left[\mathbf{I} - \dfrac{\mathbf{J}}{v}\right] \end{bmatrix} \tag{2.4}$$

Therefore, the information matrix for estimating individually the direct, left- and right-neighbour effects of treatments is:

$$\mathbf{C}_\tau = \mathbf{C}_\delta = \mathbf{C}_\rho = \frac{vk(k-3)}{k-2}\left[\mathbf{I} - \frac{\mathbf{J}}{v}\right] \tag{2.5}$$

Example 2.2. Let $v = s = 5$. Juxtaposing four MOLS of order 5, deleting the last column ($i = 1$) and taking rows as blocks along with border plots would result in the following block design for competition effects with parameters $v = 5$, $b = 20$, $r = 16$ and $k = 4$:

3	5	1	2	3	5
4	1	2	3	4	1
5	2	3	4	5	2
1	3	4	5	1	3
2	4	5	1	2	4
1	5	2	4	1	5
2	1	3	5	2	1
3	2	4	1	3	2
4	3	5	2	4	3
5	4	1	3	5	4
4	5	3	1	4	5
5	1	4	2	5	1
1	2	5	3	1	2
2	3	1	4	2	3
3	4	2	5	3	4
2	5	4	3	2	5
3	1	5	4	3	1
4	2	1	5	4	2
5	3	2	1	5	3
1	4	3	2	1	4

Efficiency of Incomplete Block Designs for Competition Effects

The incomplete block designs for competition effects obtained above are compared with a complete block design balanced for competition effects. The variance (V_c) of the best linear unbiased estimates of treatment differences for a complete block design balanced

Table 1. Block designs for competition effects obtained through initial blocks

v (s, m)	b	r	k	Initial blocks (to be developed mod v)	Efficiency
5 (1, 4)	5	4	4	3 |1, 2, 4, 3| 1	0.750
7 (1, 6)	7	6	6	5 |1, 3, 2, 6, 4, 5| 1	0.937
11 (1, 10)	11	10	10	6 |1, 2, 4, 8, 5, 10, 9, 7, 3, 6| 1	0.984
11 (2, 5)	22	10	5	3 |1, 4, 5, 9, 3| 1; 6 |2, 8, 10, 7, 6| 2	0.750
13 (1, 12)	13	12	12	7 |1,2, 4, 8, 3, 6, 12, 11, 9, 5, 10, 7| 1	0.990
13 (2, 6)	26	12	6	10 |1, 4, 3, 12, 9, 10 |1; 7 |2, 8, 6, 11, 5, 7| 2	0.825
13 (3, 4)	39	12	4	5 |1, 8, 12, 5| 1; 10 |2, 3, 11, 10| 2; 7 |4, 6, 9, 7| 4	0.550
17 (1, 16)	17	16	16	6 |1, 3, 9,10, 13, 5, 15, 11, 16, 14, 8, 7, 4, 12, 2, 6| 1	0.995
17 (2, 8)	34	16	8	2 |1, 9, 13, 15, 16, 8, 4, 2| 1; 6 |3, 10, 5, 11, 14, 7, 12, 6| 3	0.893
17 (4, 4)	68	16	4	4 |1, 13, 16, 4| 1; 12 |3, 5, 14, 12| 3; 2 |9, 15, 8, 2| 9; 6 |10, 11, 7, 6| 10	0.536
19 (1, 18)	19	18	18	10 |1, 2, 4, 8, 16, 13, 7, 14, 9, 18, 17, 15, 11, 3, 6, 12, 5, 10| 1	0.996
19 (2, 9)	38	18	9	5 |1, 4, 16, 7, 9, 17, 11, 6, 5| 1; 10 |2, 8, 13, 14, 18, 15, 3, 12, 10| 2	0.911
19 (3, 6)	57	18	6	11 |1, 8, 7, 18, 11, 12| 1; 5 |2, 16, 14, 17, 3, 5| 2; 10 |4, 13, 9, 15, 6, 10| 4	0.797
23 (1, 22)	23	22	22	14 |1, 5, 2, 10, 4, 20, 8, 17, 16, 11, 9, 22, 18, 21, 13, 19, 3, 15, 6, 7, 12, 14| 1	0.997

23 (2, 11)	46	22	11	12 |1, 2, 4, 8, 16, 9, 18, 13, 3, 6, 12| 1; 14 |5,10, 20,17,11, 22, 21,19,15, 7,14| 5	0.933
31 (1, 30)	31	30	30	21 |1, 3, 9, 27, 19, 26, 16, 17, 20, 29, 25, 13, 8, 24, 10, 30, 28, 22, 4, 12, 5, 15, 14, 11, 2, 6, 18, 23, 7, 21| 1	0.999
31 (2, 15)	62	30	15	7 |1, 9, 19, 16, 20, 25, 8, 10, 28, 4, 5, 14, 2, 18, 7| 1; 21 |3, 27, 26, 17, 29, 13, 24, 30, 22, 12, 15, 11, 6, 23, 21| 3	0.956
31 (3, 10)	63	30	10	23 |1, 27, 16, 29, 8, 30, 4,0 15, 2, 23| 1; 7 |3, 19, 17, 25, 24, 28, 12, 14, 6, 7| 3; 21 |9, 26, 20, 13, 10, 22, 5, 11, 18, 21| 9	0.906
31 (5, 6)	155	30	6	6 |1, 26, 25, 30, 5, 6| 1; 18 |3, 16, 13, 28, 15, 18| 3; 23 |9, 17, 8, 22, 14, 23| 9; 7 |27, 20, 24, 4, 11, 7| 27; 21 |19, 29, 10, 12, 2, 21| 19	0.777
31 (6, 5)	186	30	5	2 |1, 16, 8, 4, 2| 1; 6 |3, 17, 24, 12, 6| 3; 18 |9, 20, 10, 5, 18| 9; 23 |27, 29, 30, 15, 23| 27; 7 |19, 25, 28, 14, 7| 19; 21 |26, 13, 22, 11, 21| 26	0.690
37 (1, 36)	37	36	36	19 |1, 2, 4, 8, 16, 32, 27, 17, 34, 31, 25, 13, 26, 15, 30, 23, 9, 18, 36, 35, 33, 29, 21, 5, 10, 20, 3, 6, 12, 24, 11, 22, 7, 14, 28, 19| 1	0.999
37 (2, 18)	74	36	18	28 |1, 4, 16, 27, 34, 25, 26, 30, 9, 36, 33, 21, 10, 3, 12, 11, 7, 28| 1; 19 |2, 8, 32, 17, 31,13, 15, 23,18, 35, 29, 5, 20, 6, 24, 22, 14, 19| 2	0.965

(*continued*)

Table 1. *Continued*

v (s, m)	b	r	k	Initial blocks (to be developed mod v)	Efficiency
37 (3, 12)	111	36	12	14 |1, 8, 27, 31, 26, 23, 36, 29, 10, 6, 11, 14| 1; 28 |2, 16, 17, 25, 15, 9, 35, 21, 20, 12, 22, 28| 2; 19 |4, 32, 34, 13, 30, 18, 33, 5, 3, 24, 7, 19| 4	0.926
37 (4, 9)	148	36	9	7 |1, 16, 34, 26, 9, 33, 10, 12, 7| 1; 14 |2, 32, 31, 15, 18, 29, 20, 24, 14| 2; 19 |4, 27, 25, 30, 36, 21, 3, 11, 28| 4; 19 |8, 17, 13, 23, 35,5, 6, 22, 19| 8	0.882
37 (6, 6)	222	36	6	11 |1, 27, 26, 36, 10, 11| 1; 22 |2, 17, 15, 35, 20, 22| 2; 7 |4, 34, 30, 33, 3, 7| 4; 14 |8, 31, 23, 29, 6, 14| 8; 28 |16, 25, 9, 21, 12, 28| 16; 19 |32, 13, 18, 5, 24, 19| 32	0.772
37 (9, 4)	333	36	4	6 |1, 31, 36, 6| 1; 12 |2, 25, 35, 12| 2; 24 |4, 13, 33, 24| 4; 11 |8, 26, 29, 11| 8; 22 |16, 15, 21, 22| 16; 7 |32, 30, 5, 7| 32; 14 |27, 23, 10, 14| 27; 38 |17, 9, 20, 38| 17; 19 |34, 18, 3, 19| 34	0.515
41 (1, 40)	41	40	40	6 |1, 7, 8, 15, 23, 38, 20, 17, 37, 13, 9, 22, 31, 12, 2, 14, 16, 30, 5, 35, 40, 34, 33, 26, 18, 3, 21, 24, 4, 28, 32, 19, 10, 29, 39, 27, 25, 11, 36, 6| 1	0.999

41 (2, 20)	82	40	20	36 |1, 8, 23, 20, 37, 9, 31, 2, 16, 5, 40, 33, 18, 21, 4, 32, 10, 39, 25, 36| 1; 6 |7, 15, 38, 17, 13, 22, 12, 14, 30, 35, 34, 26, 3, 24, 38, 19, 29, 27, 11, 6| 7	0.969
41 (4, 10)	164	40	10	25 |1, 23, 37, 31, 16, 40, 18, 4, 10, 25| 1; 11 |7, 38, 13, 12, 30, 34, 3, 28, 29, 11| 7; 36 |8, 20, 9, 2, 5, 33, 21, 32, 39, 36| 8; 6 |15, 17, 22, 14, 35, 26, 19, 27, 6| 15	0.898
41 (5, 8)	205	40	8	27 |1, 38, 9, 14, 40, 3, 32, 27| 1; 25 |7, 20, 22, 16, 34, 21, 19, 25| 7; 11 |8, 17, 31, 30, 33, 24, 10, 11| 8; 36 |15, 37, 12, 5, 26, 4, 29, 36| 15; 6 |23, 13, 2, 35, 18, 28, 39, 6| 23	0.855
41 (8, 5)	328	40	5	10 |1, 37, 16, 18, 10| 1; 29 |7, 13, 30, 3, 29| 7; 39 |8, 9, 5, 21, 39| 8; 27 |15, 22, 35, 24, 27| 15; 25 |23, 31, 40, 4, 25| 23; 11 |38, 12, 34, 28, 11| 38; 36 |20, 2, 33, 32, 36| 20; 6 |7, 14, 26, 19, 6| 7	0.684
41 (10, 4)	410	40	4	32 |1, 9, 40, 32| 1; 19 |7, 22, 34, 19| 7; 10 |8, 31, 33, 10| 8; 29 |15, 12, 26, 29| 15; 27 |38, 14, 3, 27| 38; 25 |20, 16, 21, 25| 20; 11 |17, 30, 24, 11| 17; 36 |37, 5, 4, 36| 37; 6 |13, 35, 28, 6| 13	0.513

Table 2. Block designs for competition effects obtained through MOLS

v	b	i	r	k	Efficiency
5	20	1	16	4	0.750
7	42	1	36	6	0.938
7	42	2	30	5	0.833
7	42	3	24	4	0.625
9	72	1	64	8	0.972
9	72	2	56	7	0.933
9	72	3	48	6	0.875
9	72	4	40	5	0.778
9	72	5	32	4	0.583
11	110	1	100	10	0.984
11	110	2	90	9	0.964
11	110	3	80	8	0.938
11	110	4	70	7	0.900
11	110	5	60	6	0.844
11	110	6	50	5	0.750
11	110	7	40	4	0.563
13	156	1	144	12	0.990
13	156	2	132	11	0.978
13	156	3	120	10	0.963
13	156	4	108	9	0.943
13	156	5	96	8	0.917
13	156	6	84	7	0.880
13	156	7	72	6	0.825
13	156	8	60	5	0.733
13	156	9	48	4	0.550
17	272	1	256	16	0.995
17	272	2	240	15	0.989
17	272	3	224	14	0.982
17	272	4	208	13	0.974
17	272	5	192	12	0.964
17	272	6	176	11	0.952
17	272	6	176	11	0.952
17	272	7	160	10	0.938
17	272	8	144	9	0.918
17	272	9	128	8	0.893
17	272	10	112	7	0.857
17	272	11	96	6	0.804
17	272	12	80	5	0.714
17	272	13	64	4	0.536
19	342	1	324	18	0.996
19	342	2	306	17	0.992
19	342	3	288	16	0.987
19	342	4	270	15	0.981
19	342	5	252	14	0.974
19	342	6	234	13	0.966
19	342	7	216	12	0.956
19	342	8	198	11	0.944
19	342	9	180	10	0.930

(*continued*)

Table 2. *Continued*

v	b	i	r	k	Efficiency
19	342	10	162	9	0.911
19	342	11	144	8	0.885
19	342	12	126	7	0.850
19	342	13	108	6	0.797
19	342	14	90	5	0.708
19	342	15	72	4	0.531

for neighbours is:

$$V_c = \frac{2(v-2)}{v(v-3)}\sigma^2$$

Therefore, the efficiency (E) of the block design obtained is as given below:

$$E = \frac{(v-2)(k-3)}{(k-2)(v-3)}$$

Table 1 gives a list of block designs for $v < 42$ obtained through Method 2.1. The table also contains the efficiency and the initial blocks that will generate the final design. Table 2 lists the block designs for $v < 20$ obtained using Method 2.2 by deleting different numbers of rows. The series of designs obtained here are equally efficient for estimating all the three effects.

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