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Block Designs Balanced for Second Order Interference Effects from Neighbouring Experimental Units

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Abstract

This paper deals with block model with interference effect arising from the neighbouring units up to distance 2 (or second order). The information matrices for estimating direct as well as interference effects have been derived. Further, some classes of balanced and strongly balanced block designs with second order interference effect have been obtained with reference to both complete and incomplete blocks and their characterization properties have been studied. The designs so obtained are totally balanced for estimating direct and interference effects of treatments.

Key words: Block design; Interference; Neighbouring units; Second order; Strongly balanced; Totally balanced

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1 Introduction

In classical block model, it is assumed that the response from a unit/ plot to a particular treatment is not affected by the treatment applied on the neighbouring plots. However, in agricultural field experiments conducted in smaller units with no gaps, the estimates of treatment differences may deviate because of interference by the treatments applied in neighbouring units. For example, in varietal trials, the yields of shorter varieties may be depressed due to shading from taller neighbouring varieties (Kempton and Lockwood, 1984). Here, the neighbouring plots interfere (compete) with one another and induce serious source of bias in the evaluation of treatments. The interference from neighbouring units may contribute to variability in experimental results and lead to loss in efficiency. In order to avoid bias when comparing the effects of treatments in this situation, it is important to ensure that no treatment is unduly disadvantaged by its neighbours. This is done by using the designs that are combinatorially balanced for neighbours wherein the allocation of treatments is such that each treatment occurs equally often with every other treatment as neighbour. Hence, it is important to include the interference effects in the model for proper specification. These interference effects may not only arise from the treatments applied to the immediate neighbouring units but also from the treatments applied to the units at higher distance. Hence, interference may not be restricted to immediately adjacent units but may extend further, as with the spread of inoculum in disease screening trials (Kempton, 1992). Further, it may also operate on a block basis, as described by Pearce (1957) for experiments with fruit trees where a treatment applied to a branch (unit) affected the response of all other branches on the same tree (block). Block designs balanced for interference effects at higher distance are thus needed. Iqbal et al. (2006) constructed designs balanced for neighbour effects upto distance 2 (second order neighbour effects) for $3 \leq k \leq 7$ in circular blocks of size k using method of cyclic shifts. Mingyao et al. (2007) constructed all order neighbour balanced designs for odd prime $v = 2m + 1$ in 2m circular blocks through the initial block for total effects. Akhtar and Ahmed (2009) generated second-order neighbor designs for different configurations in circular binary blocks. Third-order and fourth-order neighbor designs for some cases are also constructed.

In this article, we have considered block model with interference effects arising from neighbouring units on both sides (left and right) at distance 2. The experimental setup has been defined and the information matrices for estimating direct as well as interference effects have been derived. Methods of constructing series of complete/ incomplete block designs balanced for second order interference effects have been discussed and their characterization properties have been investigated.

2 Experimental Setup and Model

We consider a class of block designs with v treatments, b blocks and n experimental units. Let Y_{ij} be the response from the i^{th} plot in the j^{th} block ($i = 1, 2,\ldots, k_j$; $j = 1, 2,\ldots, b$). It is assumed that the experiment is conducted in small plots in well separated blocks with no guard areas between the plots in a block. The blocks are circular i.e., the treatment on the immediate left border plot is same as the treatment on the right end inner plot of the block and treatment on the left border plot at distance 2 (leaving one plot from the first plot of the block) is same as the treatment on the second last inner plot from the right side. Similarly, treatments on the immediate right border plot is same as the treatment on the left end inner plot of the block and treatment on the right border plot at distance 2 (leaving one plot from the last plot of the block) is same as the treatment on the second last inner plot from the left side.

Following fixed effects additive model is considered for analyzing a block design with second order interference effects:

$$\mathbf{Y} = \mu\mathbf{1} + \mathbf{\Delta}'\boldsymbol{\tau} + \mathbf{\Delta}_1'\boldsymbol{\delta} + \mathbf{\Delta}_2'\boldsymbol{\gamma} + \mathbf{\Delta}_3'\boldsymbol{\alpha} + \mathbf{\Delta}_4'\boldsymbol{\eta} + \mathbf{D}'\boldsymbol{\beta} + \mathbf{e}, \qquad \ldots (2.1)$$

where $\mathbf{Y}$ is a $n \times 1$ vector of observations, μ is the general mean, $\mathbf{1}$ is a $n \times 1$ vector of ones, $\mathbf{\Delta}'$ is a $n \times v$ matrix of observations versus direct treatments, $\boldsymbol{\tau}$ is a $v \times 1$ vector of direct treatment effects, $\mathbf{\Delta}_1'$ is a $n \times v$ matrix of observations versus interference effect from treatment on the immediate left neighbour units i.e. treatment at distance 1, $\boldsymbol{\delta}$ is $v \times 1$ vector of left neighbour interference effects at distance 1, $\mathbf{\Delta}_2'$ is a $n \times v$ incidence matrix of observations versus interference effect from treatment on the immediate right neighbour units i.e. treatment at distance 1, $\boldsymbol{\gamma}$ is $v \times 1$ right neighbor interference effects at distance 1, $\mathbf{\Delta}_3'$ is a $n \times v$ incidence matrix of observations versus interference effect from left neighbour treatments at distance 2 (leaving one plot), $\boldsymbol{\alpha}$ is $v \times 1$ vector of left interference effects at distance 2, $\mathbf{\Delta}_4'$ is a $n \times v$ incidence matrix of observations versus interference effect from right neighbour treatments at distance 2, $\boldsymbol{\eta}$ is $v \times 1$ vector of right neighbour interference effects at distance 2, $\mathbf{D}'$ is a $n \times b$ incidence matrix of observations versus blocks, $\boldsymbol{\beta}$ is a $b \times 1$ vector of block effects and $\mathbf{e}$ is a $n \times 1$ vector of errors with $E(\mathbf{e}) = 0$ and $D(\mathbf{e}) = \sigma^2 \mathbf{I}_n$.

Let, $\mathbf{r} = (r_1, r_2, \ldots, r_v)'$ be the $v \times 1$ replication vector of direct treatments with r_s ($s = 1, 2, \ldots, v$) being the number of times the s^{th} treatment appears in the design. $\mathbf{r}_1 = (r_{11}, r_{12}, \ldots, r_{1v})'$ be the $v \times 1$ replication vector of the immediate left neighbour treatments with r_{1s} being the number of times the treatments in the design has s^{th} treatment as immediate left neighbour. $\mathbf{r}_2 = (r_{21}, r_{22}, \ldots, r_{2v})'$ be the $v \times 1$ replication vector of the immediate right neighbour treatments with r_{2s} being the number of times the treatments in the design has s^{th} treatment as immediate right neighbour. $\mathbf{r}_3 = (r_{31}, r_{32}, \ldots, r_{3v})'$ be the $v \times 1$ replication vector of the left neighbour treatments at distance 2 with r_{3s} being the number of times the treatments in the design has s^{th} treatment as left neighbour at distance 2. $\mathbf{r}_4 = (r_{41}, r_{42}, \ldots, r_{4v})'$ be the $v \times 1$ replication vector of the right neighbour treatments at distance 2 with r_{4s} being the number of times the treatments in the design has s^{th} treatment as right neighbour at distance 2.

Further,

$$\boldsymbol{\Delta}\boldsymbol{\Delta}' = \mathbf{R}_\tau = \text{diag}(r_1, r_2, ..., r_v),\ \boldsymbol{\Delta}_1\boldsymbol{\Delta}_1' = \mathbf{R}_\delta = \text{diag}(r_{11}, r_{12}, ..., r_{1v}),$$
$$\boldsymbol{\Delta}_2\boldsymbol{\Delta}_2' = \mathbf{R}_\gamma = \text{diag}(r_{21}, r_{22}, ..., r_{2v}),\ \boldsymbol{\Delta}_3\boldsymbol{\Delta}_3' = \mathbf{R}_\alpha = \text{diag}(r_{31}, r_{32}, ..., r_{3v}),$$
$$\boldsymbol{\Delta}_4\boldsymbol{\Delta}_4' = \mathbf{R}_\eta = \text{diag}(r_{41}, r_{42}, ..., r_{4v}),\ \mathbf{DD}' = \mathbf{K} = \text{diag}(k_1, k_2, ..., k_b),$$

$$\boldsymbol{\Delta}\boldsymbol{\Delta}_1' = \mathbf{M}_1,\ \boldsymbol{\Delta}\boldsymbol{\Delta}_2' = \mathbf{M}_2,\ \boldsymbol{\Delta}_1\boldsymbol{\Delta}_2' = \mathbf{M}_3,\ \boldsymbol{\Delta}\boldsymbol{\Delta}_3' = \mathbf{M}_4,\ \boldsymbol{\Delta}\boldsymbol{\Delta}_4' = \mathbf{M}_5,$$
$$\boldsymbol{\Delta}_1\boldsymbol{\Delta}_3' = \mathbf{M}_6,\ \boldsymbol{\Delta}_1\boldsymbol{\Delta}_4' = \mathbf{M}_7,\ \boldsymbol{\Delta}_2\boldsymbol{\Delta}_3' = \mathbf{M}_8,\ \boldsymbol{\Delta}_2\boldsymbol{\Delta}_4' = \mathbf{M}_9,\ \boldsymbol{\Delta}_3\boldsymbol{\Delta}_4' = \mathbf{M}_{10},$$
$$\boldsymbol{\Delta}\mathbf{D}' = \mathbf{N}_1,\ \boldsymbol{\Delta}_1\mathbf{D}' = \mathbf{N}_2,\ \boldsymbol{\Delta}_2\mathbf{D}' = \mathbf{N}_3,\ \boldsymbol{\Delta}_3\mathbf{D}' = \mathbf{N}_4,\ \boldsymbol{\Delta}_4\mathbf{D}' = \mathbf{N}_5,$$

where, $\mathbf{M}_1$ is v × v incidence matrix of direct treatments versus immediate left neighbour treatments, $\mathbf{M}_2$ is a v × v incidence matrix of direct treatments versus immediate right neighbour treatments, $\mathbf{M}_3$ is a v × v incidence matrix of immediate left neighbour treatments versus immediate right neighbour treatments, $\mathbf{M}_4$ is a v × v incidence matrix of direct treatments versus left neighbour treatments at distance 2, $\mathbf{M}_5$ is a v × v incidence matrix of direct treatments versus right neighbour treatments at distance 2, $\mathbf{M}_6$ is a v × v incidence matrix of immediate left neighbour treatments versus left neighbour treatments at distance 2, $\mathbf{M}_7$ is a v × v incidence matrix of immediate left neighbour treatments versus right neighbour treatments at distance 2, $\mathbf{M}_8$ is a v × v incidence matrix of immediate right neighbour treatments versus left neighbour treatments at distance 2, $\mathbf{M}_9$ is a v × v incidence matrix of immediate right neighbour treatments versus right neighbour treatments at distance 2, $\mathbf{M}_{10}$ is a v × v incidence matrix of left neighbour treatments at distance 2 versus right neighbour treatments at distance 2, $\mathbf{N}_1$ is a v × b incidence matrix of direct treatments versus blocks. $\mathbf{N}_2$ is a v × b incidence matrix of immediate left neighbour treatments versus blocks. $\mathbf{N}_3$ is a v × b incidence matrix of immediate right neighbour treatments versus blocks. $\mathbf{N}_4$ is a v × b incidence matrix of left neighbour treatments at distance 2 versus blocks and $\mathbf{N}_5$ is a v × b incidence matrix of right neighbour treatments at distance 2 versus blocks.

The 5v × 5v symmetric, nonnegative definite joint information matrix for estimating the direct effects of treatment and interference effects from the neighbouring units up to distance 2 is obtained as:

$$\mathbf{C} = \begin{bmatrix} \mathbf{R}_\tau - \mathbf{N}_1\mathbf{K}^{-1}\mathbf{N}_1' & \mathbf{M}_1 - \mathbf{N}_1\mathbf{K}^{-1}\mathbf{N}_2' & \mathbf{M}_2 - \mathbf{N}_1\mathbf{K}^{-1}\mathbf{N}_3' & \mathbf{M}_4 - \mathbf{N}_1\mathbf{K}^{-1}\mathbf{N}_4' & \mathbf{M}_5 - \mathbf{N}_1\mathbf{K}^{-1}\mathbf{N}_5' \\ \mathbf{M}_1' - \mathbf{N}_2\mathbf{K}^{-1}\mathbf{N}_1' & \mathbf{R}_\delta - \mathbf{N}_2\mathbf{K}^{-1}\mathbf{N}_2' & \mathbf{M}_3 - \mathbf{N}_2\mathbf{K}^{-1}\mathbf{N}_3' & \mathbf{M}_6 - \mathbf{N}_2\mathbf{K}^{-1}\mathbf{N}_4' & \mathbf{M}_7 - \mathbf{N}_2\mathbf{K}^{-1}\mathbf{N}_5' \\ \mathbf{M}_2' - \mathbf{N}_3\mathbf{K}^{-1}\mathbf{N}_1' & \mathbf{M}_3' - \mathbf{N}_3\mathbf{K}^{-1}\mathbf{N}_2' & \mathbf{R}_\gamma - \mathbf{N}_3\mathbf{K}^{-1}\mathbf{N}_3' & \mathbf{M}_8 - \mathbf{N}_3\mathbf{K}^{-1}\mathbf{N}_4' & \mathbf{M}_9 - \mathbf{N}_3\mathbf{K}^{-1}\mathbf{N}_5' \\ \mathbf{M}_4' - \mathbf{N}_4\mathbf{K}^{-1}\mathbf{N}_1' & \mathbf{M}_6' - \mathbf{N}_4\mathbf{K}^{-1}\mathbf{N}_2' & \mathbf{M}_8' - \mathbf{N}_4\mathbf{K}^{-1}\mathbf{N}_3' & \mathbf{R}_\alpha - \mathbf{N}_4\mathbf{K}^{-1}\mathbf{N}_4' & \mathbf{M}_{10} - \mathbf{N}_4\mathbf{K}^{-1}\mathbf{N}_5' \\ \mathbf{M}_5' - \mathbf{N}_5\mathbf{K}^{-1}\mathbf{N}_1' & \mathbf{M}_7' - \mathbf{N}_5\mathbf{K}^{-1}\mathbf{N}_2' & \mathbf{M}_9' - \mathbf{N}_5\mathbf{K}^{-1}\mathbf{N}_3' & \mathbf{M}_{10}' - \mathbf{N}_5\mathbf{K}^{-1}\mathbf{N}_4' & \mathbf{R}_\eta - \mathbf{N}_5\mathbf{K}^{-1}\mathbf{N}_5' \end{bmatrix}$$

$$=\begin{bmatrix} \mathbf{C}_{11} & \mathbf{C}_{12} \\ \mathbf{C}_{21} & \mathbf{C}_{22} \end{bmatrix}, \qquad \ldots (2.2)$$

where

$$\mathbf{C}_{11} = \mathbf{R}_{\tau} - \mathbf{N}_1\mathbf{K}^{-1}\mathbf{N}_1',$$

$$\mathbf{C}_{12} = \begin{bmatrix} \mathbf{M}_1 - \mathbf{N}_1\mathbf{K}^{-1}\mathbf{N}_2' & \mathbf{M}_2 - \mathbf{N}_1\mathbf{K}^{-1}\mathbf{N}_3' & \mathbf{M}_4 - \mathbf{N}_1\mathbf{K}^{-1}\mathbf{N}_4' & \mathbf{M}_5 - \mathbf{N}_1\mathbf{K}^{-1}\mathbf{N}_5' \end{bmatrix}$$

and

$$\mathbf{C}_{22} = \begin{bmatrix} \mathbf{R}_{\delta} - \mathbf{N}_2\mathbf{K}^{-1}\mathbf{N}_2' & \mathbf{M}_3 - \mathbf{N}_2\mathbf{K}^{-1}\mathbf{N}_3' & \mathbf{M}_6 - \mathbf{N}_2\mathbf{K}^{-1}\mathbf{N}_4' & \mathbf{M}_7 - \mathbf{N}_2\mathbf{K}^{-1}\mathbf{N}_5' \\ \mathbf{M}_3 - \mathbf{N}_3\mathbf{K}^{-1}\mathbf{N}_2' & \mathbf{R}_{\gamma} - \mathbf{N}_3\mathbf{K}^{-1}\mathbf{N}_3' & \mathbf{M}_8 - \mathbf{N}_3\mathbf{K}^{-1}\mathbf{N}_4' & \mathbf{M}_9 - \mathbf{N}_3\mathbf{K}^{-1}\mathbf{N}_5' \\ \mathbf{M}_6' - \mathbf{N}_4\mathbf{K}^{-1}\mathbf{N}_2' & \mathbf{M}_8' - \mathbf{N}_4\mathbf{K}^{-1}\mathbf{N}_3' & \mathbf{R}_{\alpha} - \mathbf{N}_4\mathbf{K}^{-1}\mathbf{N}_4' & \mathbf{M}_{10} - \mathbf{N}_4\mathbf{K}^{-1}\mathbf{N}_5' \\ \mathbf{M}_7' - \mathbf{N}_5\mathbf{K}^{-1}\mathbf{N}_2' & \mathbf{M}_9' - \mathbf{N}_5\mathbf{K}^{-1}\mathbf{N}_3' & \mathbf{M}_{10}' - \mathbf{N}_5\mathbf{K}^{-1}\mathbf{N}_4' & \mathbf{R}_{\eta} - \mathbf{N}_5\mathbf{K}^{-1}\mathbf{N}_5' \end{bmatrix}.$$

The information matrix for estimating the direct effects can be obtained as follows:

$$\mathbf{C}_{\tau} = \mathbf{C}_{11} - \mathbf{C}_{12}\mathbf{C}_{22}^{-}\mathbf{C}_{21}, \qquad \ldots (2.3)$$

where $\mathbf{C}^{-}_{22}$ is a g-inverse of $\mathbf{C}_{22}$. Similarly the information matrices for estimating the second order interference effects from the neighbouring units can be obtained.
Following are some general definitions associated with the block design with interference effects:

Definition 2.1: A block design is *balanced for second order interference effects* from the neighbouring units if every treatment has every other treatment (except itself) as both left and right neighbour up to distance 2 constant number of times (say μ_1). Further, a block design with both sided interference effects is *strongly balanced* if each treatment has itself as both left and right neighbours up to distance 2 a constant number of times (say μ_2).

Definition 2.2: A block design with interference effects from neighbouring units up to distance 2 is called *variance balanced* if the variance of any estimated elementary contrast among the direct effects is constant (say V_1), the variance of any estimated elementary contrast among the interference effects arising from the immediate left neighbouring units is constant (say V_2), the variance of any estimated elementary contrast among the interference effects arising from the immediate right neighbouring units is constant (say V_3), the variance of any estimated elementary contrast among the interference effects arising from the left neighbouring units at distance 2 is constant (say V_4) and the variance of any estimated elementary contrast among the interference effects arising from the right neighbouring units at distance 2 is constant (say V_5).
A block design is *totally balanced* if $V_1 = V_2 = V_3 = V_4 = V_5$.

3 Block Designs Balanced for Interference Effects

In this section, some methods of constructing complete and incomplete circular balanced and strongly balanced block designs with second order interference effects from the neighbouring units have been described.

Method 3.1

Series 1: Let there be v (prime) treatments labeled as 0, 1, 2, …, v-1. A series of complete block design strongly balanced for second order interference effects can be obtained by developing the blocks of the design as follows for all q = 0, 1, …, (v-1) and p = 1, 2, …, (v-1)/2:

q, q + p, q + 2p,…, q + (v-2)p, q + (v-1)p, q + (v-2)p,…, q + 2p, q + p, q (modulo v)

The parameters of the design so obtained are v, b = v(v-1)/2, r = (v-1)(2v-1)/2, k = 2v-1, μ_1 = v-1 and μ_2 = (v-1)/2. Here, we have to consider border plots at both ends up to distance 2 in order to make the design *circular*.
For this class of designs,

$$\mathbf{R}_\tau = \mathbf{R}_\delta = \mathbf{R}_\gamma = \mathbf{R}_\alpha = \mathbf{R}_\eta = \frac{(\text{v-1})(2\text{v}-1)}{2}\mathbf{I}_\text{v}, \mathbf{K} = \text{k}\,\mathbf{I}_\text{b} = (2\text{v}-1)\mathbf{I}_\text{b},$$

$$\mathbf{M}_1 = \mathbf{M}_2 = \mathbf{M}_3 = \mathbf{M}_4 = \mathbf{M}_5 = \mathbf{M}_6 = \mathbf{M}_7 = \mathbf{M}_8 = \mathbf{M}_9 = \mathbf{M}_{10} = \frac{(\text{v-1})}{2}(2\mathbf{11'} - \mathbf{I}_\text{v}),$$

$$\mathbf{N}_\text{u}\mathbf{N}'_{\text{u}'} = \frac{(\text{v-1})}{2}\left[\mathbf{I}_\text{v} + 4(\text{v-1})\mathbf{11'}\right], \text{ u, u}'=1, 2,\dots, 5 \qquad \dots (3.1)$$

The joint information matrix for estimating the direct as well as interference effects from the neighbouring units is obtained and the information matrix for estimating the direct effects is

$$\mathbf{C}_\tau = \frac{2\text{v}(\text{v-1})(\text{v-3})}{(2\text{v}-5)}\left(\mathbf{I}_\text{v} - \frac{\mathbf{11'}}{\text{v}}\right), \text{ v} > 3 \qquad \dots (3.2)$$

Similarly, the information matrices for estimating the immediate left interference effects, immediate right interference effects , left interference effects at distance 2 and right interference at distance 2 effects from the neighbouring units are obtained as

$$\mathbf{C}_\delta = \mathbf{C}_\gamma = \mathbf{C}_\alpha = \mathbf{C}_\eta = \frac{2\text{v}(\text{v-1})(\text{v-3})}{(2\text{v}-5)}\left(\mathbf{I}_\text{v} - \frac{\mathbf{11'}}{\text{v}}\right), \text{ v} > 3 \qquad \dots (3.3)$$

The design is thus variance balanced for estimating the contrast pertaining to direct effects of treatments and interference effects from the neighbouring units up to distance 2. Also, since $V_1 = V_2 = V_3 = V_4 = V_5$ the series of design obtained is totally balanced for estimating the contrasts pertaining to direct effects of treatments and second order interference effects.

Example 3.1: For v = 5, following is a strongly balanced complete block design with second order interference effects from the neighbouring units with v = 5, b = 10, r = 18, k = 9, $\mu_1 = 4$, $\mu_2 = 2$.

0	1	2	3	4	3	2	1	0
1	2	3	4	0	4	3	2	1
2	3	4	0	1	0	4	3	2
3	4	0	1	2	1	0	4	3
4	0	1	2	3	2	1	0	4
0	2	4	1	3	1	4	2	0
1	3	0	2	4	2	0	3	1
2	4	1	3	0	3	1	4	2
3	0	2	4	1	4	2	0	3
4	1	3	0	2	0	3	1	4

Series 2: A series of incomplete block design strongly balanced for interference effects can also be obtained by developing the blocks of the design as follows for all q = 0, 1, …, (v-1) and p = 1, 2, …, (v-1)/2:

q, q + p, q + 2p,…, q + (v-3)p, q + (v-2)p, q + (v-3)p,…, q + 2p, q + p, q (modulo v)

The parameters of this class of designs are v, b = v(v-1)/2, r = (v-1)(2v-3)/2, k = 2v-3, μ_1 = v-2 and μ_2 = (v-1)/2. For this class,

$$\mathbf{R}_\tau = \mathbf{R}_\delta = \mathbf{R}_\gamma = \mathbf{R}_\alpha = \mathbf{R}_\eta = \frac{(\text{v-1})(2\text{v}-3)}{2}\mathbf{I}_\text{v}, \mathbf{K} = \text{k}\mathbf{I}_\text{b} = (2\text{v}-3)\mathbf{I}_\text{b},$$

$$\mathbf{M}_1 = \mathbf{M}_2 = \mathbf{M}_3 = \mathbf{M}_4 = \mathbf{M}_5 = \mathbf{M}_6 = \mathbf{M}_7 = \mathbf{M}_8 = \mathbf{M}_9 = \mathbf{M}_{10} = (\text{v-2})\mathbf{11}' - \frac{(\text{v-3})}{2}\mathbf{I}_\text{v},$$

$$\mathbf{N}_\text{u}\mathbf{N}'_{\text{u}'} = 2(\text{v-2})^2\mathbf{11}' + \frac{(5\text{v}-9)}{2}\mathbf{I}_\text{v}, \text{ u, u}'=1, 2,\ldots,5 \qquad \ldots (3.4)$$

The joint information matrix for estimating the direct as well as interference effects from the neighbouring units is obtained and the information matrix for estimating the direct effects is

$$\mathbf{C}_\tau = \frac{2v(v-2)(v-4)}{(2v-7)}\left(\mathbf{I}_v - \frac{\mathbf{11}'}{v}\right), v>4 \qquad \ldots (3.5)$$

Similarly, the information matrices for estimating the immediate left interference effects, immediate right interference effects, left interference effects at distance 2 and right interference effects at distance 2 respectively from the neighbouring units are:

$$\mathbf{C}_\delta = \mathbf{C}_\gamma = \mathbf{C}_\alpha = \mathbf{C}_\eta = \frac{2v(v-2)(v-4)}{(2v-7)}\left(\mathbf{I}_v - \frac{\mathbf{11}'}{v}\right), v>4 \qquad \ldots(3.6)$$

Hence, the design is variance balanced and also totally balanced for estimating the contrast pertaining to direct effects of treatments and interference effects from the neighbouring units up to distance two.

Example 3.2: For v = 5, following is a strongly balanced incomplete block design with second order interference effects from the neighbouring units with v = 5, b = 10, r = 14, k = 7, μ_1= 3, μ_2 = 2.

0	1	2	3	2	1	0
1	2	3	4	3	2	1
2	3	4	0	4	3	2
3	4	0	1	0	4	3
4	0	1	2	1	0	4
0	2	4	1	4	2	0
1	3	0	2	0	3	1
2	4	1	3	1	4	2
3	0	2	4	2	0	3
4	1	3	0	3	1	4

Method 3.2

The designs obtained by Azais et al. (1993) in v-1 blocks of size v (> 5) each are shown to be balanced for second order interference effects. For v (prime) treatments, the contents of the v-1 complete blocks of the design can be obtained by writing the treatments in systematic order within a block with a difference of 1,2,…,v-1 between the treatments (modulo v) in the consecutive blocks. The first block is formed by taking the difference of one between treatments, the second block by taking the difference of two and so on, the $(v-1)^{th}$ block by taking the difference of (v-1). Once the design is written the blocks are made circular up to distance 2 which will result in a series of complete block design balanced for second order interference effect from the neighbouring units with parameters v = k, b = (v-1) = r, μ_1= 1. For this class of designs,

$$\mathbf{R}_\tau = \mathbf{R}_\delta = \mathbf{R}_\gamma = \mathbf{R}_\alpha = \mathbf{R}_\eta = (\text{v-1})\mathbf{I}_v, \mathbf{K} = k\mathbf{I}_b = v\,\mathbf{I}_b,$$
$$\mathbf{M}_1 = \mathbf{M}_2 = \mathbf{M}_3 = \mathbf{M}_4 = \mathbf{M}_5 = \mathbf{M}_6 = \mathbf{M}_7 = \mathbf{M}_8 = \mathbf{M}_9 = \mathbf{M}_{10} = (\mathbf{11}' - \mathbf{I}_v),$$
$$\mathbf{N}_u \mathbf{N}'_{u'} = (\text{v-1})\mathbf{11}', \text{ u, u}' = 1, 2, \ldots 5 \qquad \ldots (3.7)$$

The information matrix for estimating the direct effects is

$$\mathbf{C}_\tau = \frac{v(v\text{-}5)}{(v\text{-}4)}\left(\mathbf{I}_v - \frac{\mathbf{11}'}{v}\right), \; v > 5 \qquad \ldots (3.8)$$

Similarly, the information matrices for estimating the immediate left interference effects, immediate right interference effects, left interference effects at distance 2, and right interference effects at distance 2 respectively from the neighbouring units is

$$\mathbf{C}_\delta = \mathbf{C}_\gamma = \mathbf{C}_\alpha = \mathbf{C}_\eta == \frac{v(v\text{-}5)}{(v\text{-}4)}\left(\mathbf{I}_v - \frac{\mathbf{11}'}{v}\right), v>5 \qquad \ldots (3.9)$$

Hence, the design is totally balanced for estimating the contrasts pertaining to direct effects of treatments and second order interference effects arising from the neighbouring units.

Example 3.3: For v = 7, following is a complete block design balanced for second order interference effects from the neighbouring units with parameters v = 7 = k, b = 6 = r, μ_1= 1:

1	2	3	4	5	6	0
1	3	5	0	2	4	6
1	4	0	3	6	2	5
1	5	2	6	3	0	4
1	6	4	2	0	5	3
1	0	6	5	4	3	2

Remark 3.1: For the afore mentioned class of designs one can generalize the result by taking interference effects up to distance i ($1 \leq i \leq k\text{-}1$). The information matrix for estimating the direct effects and interference effects is thus obtained as follows:

$$\mathbf{C} = \frac{v[v\text{-}(2i+1)]}{(v\text{-}2i)}\left(\mathbf{I}_v - \frac{\mathbf{11}'}{v}\right), \; v > (2i+1)$$

Method 3.3

Tomar et al. (2005) obtained a series of block design balanced for adjacent left and right neighboring units for v = mt + 1 (prime or prime power (m > 3)), b = tv, r = mt, k = m and μ_1= 1 by developing following initial blocks modulo v and augmenting the whole set of blocks generated from each initial block one after another:

$$x^{w}, x^{w+t}, x^{w+2t}, \ldots x^{w+(m-1)t} \;;\quad \text{for } w = 0,1, \ldots, t-1,$$

where x is the primitive element of GF(v). This class of design is also found to be balanced for second order neighbour effects after making the blocks circular up to distance two.

For this class of designs:

$$\mathbf{R}_\tau = \mathbf{R}_\delta = \mathbf{R}_\gamma = \mathbf{R}_\alpha = \mathbf{R}_\eta = (v\text{-}1)\mathbf{I}_v, \mathbf{K} = k\,\mathbf{I}_b,$$

$$\mathbf{M}_1 = \mathbf{M}_2 = \mathbf{M}_3 = \mathbf{M}_4 = \mathbf{M}_5 = \mathbf{M}_6 = \mathbf{M}_7 = \mathbf{M}_8 = \mathbf{M}_9 = \mathbf{M}_{10} = (\mathbf{11}' - \mathbf{I}_v)$$

$$\mathbf{N}_u \mathbf{N}'_{u'} = (v\text{-}k)\mathbf{I}_v + (k\text{ - }1)\mathbf{11}', \ u, u' = 1, 2, \ldots 5 \qquad \ldots (3.10)$$

The information matrix for estimating the direct effects is:

$$\mathbf{C}_\tau = \frac{v(k\text{-}5)}{(k\text{-}4)}\left(\mathbf{I}_v - \frac{\mathbf{11}'}{v}\right), k > 5 \qquad \ldots (3.11)$$

The information matrices for estimating the immediate left interference effects, immediate right interference effects, left interference effects at distance 2 and right interference effects at distance 2 respectively from the neighbouring units are

$$\mathbf{C}_\delta = \mathbf{C}_\gamma = \mathbf{C}_\alpha = \mathbf{C}_\eta = \frac{v(k\text{-}5)}{(k\text{-}4)}\left(\mathbf{I}_v - \frac{\mathbf{11}'}{v}\right), k > 5 \qquad \ldots (3.12)$$

It is seen that the design is totally balanced for estimating the contrast pertaining to direct effects of treatments and second order interference effects from the neighbouring units.

Example 3.4: Let m = 6 and t = 2, then we get following two initial blocks modulo 11 for w = 0 and w = 1:

1 4 3 12 9 10 and 2 8 6 11 5 7

Developing these blocks we obtain the following incomplete block design balanced for second order interference effects with v = 13, b = 26, r = 11, k = 6 and μ_1= 1:

1	4	3	12	9	10
2	5	4	0	10	11
3	6	5	1	11	12
4	7	6	2	12	0
5	8	7	3	0	1
6	9	8	4	1	2
7	10	9	5	2	3
8	11	10	6	3	4
9	12	11	7	4	5
10	0	12	8	5	6
11	1	0	9	6	7
12	2	1	10	7	8
0	3	2	11	8	9
2	8	6	11	5	7
3	9	7	12	6	8
4	10	8	0	7	9
5	11	9	1	8	10
6	12	10	2	9	11
7	0	11	3	10	12
8	1	12	4	11	0
9	2	0	5	12	1
10	3	1	6	0	2
11	4	2	7	1	3
12	5	3	8	2	4
0	6	4	9	3	5
1	7	5	10	4	6

Remark 3.2: The above class can be generalized by taking interference effects up to distance i ($1 \leq i \leq k-1$). The information matrix for estimating the direct effects and interference effects is thus obtained as follows:

$$\mathbf{C} = \frac{v\left[k-(2i+1)\right]}{(k-2i)}\left(\mathbf{I}_v - \frac{\mathbf{11}'}{v}\right),\ k > (2i+1).$$

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