Model Assisted Statistics and Applications 6 (2011) 317–324
DOI 10.3233/MAS-2011-0190
IOS Press

Variance balanced circular designs involving sequences of treatments with first and second residuals

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Abstract. Designs involving sequences of treatments (also called changeover, crossover, or repeated measurements designs) balanced for first and second residuals available in literature are usually large even for a moderate number of treatments. Besides, most of these designs require number of periods at least equal to the number of treatments. In this paper a class of variance balanced circular designs for prime or prime power number of treatments, v ($= mp + 1$; m, a positive integer) using p ($\geqslant 4$, $< v$) periods and only mv experimental units has been proposed. A simple method of analysis of these designs is given and the efficiency factors relative to the orthogonal designs have been tabulated for $v \leqslant 31$ and $p \geqslant 4, \leqslant 12$.

Keywords: Change-over designs, first residual effects, repeated measurements designs, second residual effects

1. Introduction

Designs in which every experimental unit receives a sequence of treatments over periods of time, one treatment per period and observations are recorded in each period, have been called by various names in the literature like changeover designs, crossover designs, repeated measurements designs or designs involving sequences of treatments. Cochran et al. [6] were the first to use such a design in a cow feeding experiment to estimate the direct effect of feeds in the presence of first order residual effects, the carryover effects that persist up to one period after the period of direct treatments application. Subsequently, these designs have been advantageously used in several fields of research like sensory trials [10], clinical trials [7], biological assays [11]. Assuming the presence of first order residual effects, a vast literature on these designs exists, a detailed account of which can be found in Stufken [28] and Jones and Kenward [15].

Designs with second and higher order residual effects have been studied by Williams [31,32], Patterson [20], Quenouille [22]; (see also [16]), Patterson and Lucas [21], Sharma [25], Bose and Mukherjee [5], Sharma et al. [26], Varghese et al. [30], Aggarwal et al. [3], etc. Recently, Iqbal et al. [14] constructed circular changeover designs balanced for first and second-order residual effects. However, most of the existing designs are large even for moderate values of v. Besides, many of these require at least v periods which may not always be feasible. Here, we present a family of balanced circular designs for first and second residuals for prime or prime power values of v of the form $(mp + 1)$ in incomplete periods, p ($\geqslant 4$, $< v$) and using *only* mv experimental units; m being a positive integer.

In the following section we present a method of constructing the designs, which will be followed by the sections on a simple method of analysis and conclusion.

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ISSN 1574-1699/11/$27.50

2. Method of constructing balanced circular designs with first and second residuals

The concept of balance in the context of changeover designs has been defined differently by various researchers (see e.g. [4,12,20,24]), we use the following definitions:

Definition 1. A design involving sequences of treatments considering first and second-order residual effects is called balanced, if the variance of all estimated elementary contrasts among direct effects is the same, say α, the variance of all estimated elementary contrasts among first residual effects is the same, say β and the variance of all estimated elementary contrasts among second residual effects is also same, say γ. The constants α, β and γ need not necessarily be equal. However, if $\alpha = \beta = \gamma$, then the design is called *totally balanced.*

Circular designs in the presence of first residuals have been studied by Lawless [18], Magda [19], Afsarinejad [1, 2], Kunert [17], Varghese and Sharma [29], Druilhet and Tinsson [9], Iqbal and Tahir [13], etc. We define the balanced circular designs with first and second-residuals as follows:

Definition 2. A design involving sequences of treatments with first and second-order residual effects having two pre-periods each of the same duration as the other periods (observations from the pre-periods are not used in the analysis) is called circular, if each subject receives the treatment of the last but one period in the first pre-period and that of the last period in the second pre-period; the last but one period is the period just preceding the last period.

Evidently, in circular designs each subject receives direct, first as well as second order residual effect of all the treatments included in the sequence.

2.1. The model

We consider a circular balanced design with p periods and n experimental units for v treatments. Let Y_{hijkl} be the effect observed from the l^{th} experimental unit in the h^{th} period, when the treatment i is applied to it and is immediately preceded by the j^{th} treatment and by two positions by k^{th} treatment. We postulate the following linear additive fixed effects model for the observations:

$$Y_{hijkl} = \mu + \pi_h + \tau_i + \rho_j + \chi_k + \Psi_l + \varepsilon_{hijkl}, 1 \leqslant h \leqslant p; 1 \leqslant i, j, k \leqslant v; 1 \leqslant l \leqslant mv \tag{1}$$

where μ, π_h, ψ_l, τ_i, ρ_j and χ_k denote respectively, the general mean, effect of h^{th} period, effect of l^{th} unit, direct effect of i^{th} treatment, first residual effect of j^{th} treatment and second residual effect of k^{th} treatment. And ε_{hijkl} are random errors assumed to be identically and independently distributed normally with mean zero and constant variance, σ^2.

2.2. Construction

Let $v(= mp + 1; m \geqslant 1, p \geqslant 4)$ be the number of treatments, which is a prime or prime power and let α be a primitive root of GF (v), the Galois field with v elements [for definition of Galois field, see e.g., Raghavarao [23, pp. 338–341] or Das and Giri [8, pp. 104–105]. Form m Latin rectangles of $p + 2$ rows and v columns each by developing the following m initial sequences (mod v):

$$\{\alpha^{i+(p-2)m}, \alpha^{i+(p-1)m}, \alpha^{i}, \alpha^{i+m}, \alpha^{i+2m}, \ldots, \alpha^{i+(p-1)m}\}, i = 0, 1, \ldots, m-1. \tag{2}$$

Number the rows of all the rectangles from top to bottom as 0_1, 0_2, 1, ..., p and columns from left to right of the first rectangle from 1 to v, of the second rectangle from $v + 1$ to $2v$, and so on, the columns of the m^{th} rectangle from $(m - 1)v + 1$ to $n(= mv)$. If rows represent the periods and columns, the experimental units, the rectangles form a balanced circular design for v treatments in p periods and mv experimental units; 0_1, and 0_2 being the first and second pre-period, respectively. It may be noted that in the initial sequences given at (2), the treatment in the first position is the same as that in the last but one position and the treatment in second position is the same as that in the last position.

Example: Let $v = 13$ and $p = 6$. This gives $m = 2$. A primitive root of GF (13) is 2. Therefore, the two initial sequences of 8 ($= p + 2$) elements each for $i = 0$ and $i = 1$, respectively are:

$$\{2^8 = 9, 2^{10} = 10, 2^0 = 1, 2^2 = 4, 2^4 = 3, 2^6 = 12, 2^8 = 9, 2^{10} = 10\}$$

and

$$\{2^{1+8} = 5, 2^{1+10} = 7, 2^{1+0} = 2, 2^{1+2} = 8, 2^{1+4} = 6, 2^{1+6} = 11, 2^{1+8} = 5, 2^{1+10} = 7\};$$

elements being mod 13. The design with initial sequences in bold figures is given in Table 1.

Table 1
Circular design balanced for first and second order residual effects with the parameters $v = 13$, $p = 6$, $n = 26$

		Experimental Units												
		1	2	3	4	5	6	7	8	9	10	11	12	13
Periods	0_1	**9**	10	11	12	0	1	2	3	4	5	6	7	8
	0_2	**10**	11	12	0	1	2	3	4	5	6	7	8	9
	1	**1**	2	3	4	5	6	7	8	9	10	11	12	0
	2	**4**	5	6	7	8	9	10	11	12	0	1	2	3
	3	**3**	4	5	6	7	8	9	10	11	12	0	1	2
	4	**12**	0	1	2	3	4	5	6	7	8	9	10	11
	5	**9**	10	11	12	0	1	2	3	4	5	6	7	8
	6	**10**	11	12	0	1	2	3	4	5	6	7	8	9

		Experimental Units												
		14	15	16	17	18	19	20	21	22	23	24	25	26
Periods	0_1	**5**	6	7	8	9	10	11	12	0	1	2	3	4
	0_2	**7**	8	9	10	11	12	0	1	2	3	4	5	6
	1	**2**	3	4	5	6	7	8	9	10	11	12	0	1
	2	**8**	9	10	11	12	0	1	2	3	4	5	6	7
	3	**6**	7	8	9	10	11	12	0	1	2	3	4	5
	4	**11**	12	0	1	2	3	4	5	6	7	8	9	10
	5	**5**	6	7	8	9	10	11	12	0	1	2	3	4
	6	**7**	8	9	10	11	12	0	1	2	3	4	5	6

Remark 1. If the period classification and the residual effects are ignored and the experimental units are treated as blocks, then the design (without pre-periods) reduces to a BIB design with the parameters $v = mp + 1$, $b = mv$, $r = mp$, $k = p$, $\lambda = (p - 1)$. The solution of this BIB design is given by Sprott [27] which ensures the existence of the proposed design.

Remark 2. It can be easily seen that without pre-periods the design does not retain the property of balance for the estimation of direct, first as well as second-order residual effects.

In the following section, we present a simple method of analysis of the design comprising of m Latin rectangles balanced for first and second residuals for $v(= mp + 1)$ treatments in p periods using mv experimental units and show that the design is variance balanced.

3. Analysis

3.1. Notations and ANOVA

G = Grand total
P_h = Total of values of the observations in the h^{th} period
T_i = Sum of values of the observations that contain the direct effect of i^{th} treatment
R_i = Sum of values of the observations that contain the first residual effect of i^{th} treatment

Q_i = Sum of values of observations that contain the second residual effect of i^{th} treatment
U_k = Sum of values of the observations from the k^{th} unit
$\sum_k U_k^{(i)}$ = Sum of those units that contain the direct (first residual or second residual) effect of treatment i

$$T_i' = T_i - p^{-1}\sum_k U_k^{(i)}$$

$$R_i' = R_i - p^{-1}\sum_k U_k^{(i)}$$

$$Q_i' = Q_i - p^{-1}\sum_k U_k^{(i)}$$

Minimization of the residual sum of squares with respect to the parameters in model (1) leads to a set of normal equations which under the restrictions

$$\sum \pi_h = \sum \tau_i = \sum \rho_j = \sum \chi_k = \sum \psi_l = 0, \tag{3}$$

can be solved to give

$$\hat{\mu} = \frac{G}{mp(mp+1)},$$

$$\hat{\pi}_h = \frac{P_h}{m(mp+1)} - \frac{G}{mp(mp+1)},$$

$$\hat{\tau}_i = \frac{(p-2)T_i' + R_i' + Q_i'}{(mp+1)(p-3)},$$

$$\hat{\rho}_i = \frac{T_i' + (p-2)R_i' + Q_i'}{(mp+1)(p-3)},$$

$$\hat{\chi}_i = \frac{T_i' + R_i' + (p-2)Q_i'}{(mp+1)(p-3)},$$

and

$$\hat{\psi}_k = p^{-1}U_k - \hat{\mu} - p^{-1}\sum\nolimits_1 \hat{t}_\theta,$$

where $\hat{t}_i = \hat{\tau}_i + \hat{\rho}_i + \hat{\chi}_i$ is the total effect of i^{th} treatment and $\sum_1 \hat{t}_\theta$ is the sum of total effects ($\hat{t}_\theta = \hat{\tau}_\theta + \hat{\rho}_\theta + \hat{\chi}_\theta$) of those treatments that are applied to the k^{th} unit; θ being one of the treatments received by the k^{th} unit. The total effect of a treatment has also been called permanent effect by Peterson and Lucas [21]. Furthermore, the variances and co-variances between estimated elementary contrasts are seen to be

$$\mathrm{Var}\,(\hat{\tau}_i - \hat{\tau}_{i'}) = \mathrm{Var}\,(\hat{\rho}_i - \hat{\rho}_{i'}) = \mathrm{Var}\,(\hat{\chi}_i - \hat{\chi}_{i'}) = \frac{2(p-2)\sigma^2}{(mp+1)(p-3)}, \tag{4}$$

$$\mathrm{cov}\,(\hat{\tau}_i - \hat{\tau}_{i'}, \hat{\rho}_i - \hat{\rho}_{i'}) = \mathrm{cov}\,(\hat{\rho}_i - \hat{\rho}_{i'}, \hat{\chi}_i - \hat{\chi}_{i'}) = \mathrm{cov}\,(\hat{\tau}_i - \hat{\tau}_{i'}, \hat{\chi}_i - \hat{\chi}_{i'}) = \frac{2\sigma^2}{(mp+1)(p-3)},$$

and

$$\mathrm{Var}\,(\hat{t}_i - \hat{t}_{i'}) = \frac{6p\sigma^2}{(mp+1)(p-3)},$$

$$1 \leqslant h \leqslant p; 1 \leqslant i, i' \leqslant (mp+1); 1 \leqslant k \leqslant m(mp+1).$$

Variance expression in Eq. (4) shows that the design is totally balanced. If second order residual effects are not found to be significant, treatment effects ignoring second residuals become important. The least-squares solution for direct and first order residuals are then seen to be

$$\tilde{\tau}_i = \frac{(p-1)T_i' + R_i'}{(p-2)(mp+1)}, \text{ and} \tag{5}$$

$$\tilde{\rho}_i = \frac{T_i' + (p-1)R_i'}{(p-2)(mp+1)}$$

with variances and co-variances of estimated elementary contrasts in direct and first residual effects as

$$\mathrm{Var}(\tilde{\tau}_i - \tilde{\tau}_{i'}) = \mathrm{Var}(\tilde{\rho}_i - \tilde{\rho}_{i'}) = \frac{2(p-1)\sigma^2}{(p-2)(mp+1)}, \text{ and} \tag{6}$$

$$\mathrm{cov}(\tilde{\tau}_i - \tilde{\tau}_{i'}, \tilde{\rho}_i - \tilde{\rho}_{i'}) = \frac{2\sigma^2}{(p-2)(mp+1)}.$$

In this case, the estimate of total treatment effect (permanent effect) of i^{th} treatment is given by

$$\tilde{t}_i = \tilde{\tau}_i + \tilde{\rho}_i, \tag{7}$$

with variance of estimated elementary contrast

$$\mathrm{Var}(\tilde{t}_i - \tilde{t}_{i'}) = \frac{4p\sigma^2}{(p-2)(mp+1)}. \tag{8}$$

If both the residual effects are not found to be significant, then the least-squares solution for treatment effects is seen to be

$$\tilde{\tilde{\tau}}_i = \frac{pT_i'}{(p-1)(mp+1)}, \tag{9}$$

with the variance of estimated elementary contrast in treatment effects,

$$\mathrm{Var}(\tilde{\tilde{\tau}}_i - \tilde{\tilde{\tau}}_{i'}) = \frac{2p\sigma^2}{(p-1)(mp+1)}. \tag{10}$$

The partitioning of the total sum of squares is given in Tables 2A and Table 2B shows further partitioning of the treatment sum of squares into meaningful components.

Table 2A
Partitioning of the total sum of squares for a balanced circular design with the parameters $v = mp+1, p, n = m(mp+1)$

Source	df	SS
Units	$m(mp+1)-1$	$\sum U_k^2/p - G^2/mp(mp+1)$
Periods	$p-1$	$\sum P_h^2/m(mp+1) - G^2/mp(mp+1)$
Treatments	$3mp$	$\sum \hat{\tau}_i T_i' + \sum \hat{\rho}_i R_i' + \sum \hat{\chi}_i Q_i'$
Error	$(mp-1)^2 - (m^2p+m+p)$	By subtraction
Total	$mp(mp+1)-1$	$\sum y_{hijkl}^2 - G^2/mp(mp+1)$

The first three partitioning are sufficient to test the significance of second residuals, first residuals and direct effects. However, first and fourth partitioning will suffice, if second residuals are not found to be significant. In case both the residual effects are non-significant, first partitioning will be enough. Other ways of partitioning the total treatments sum of squares are not considered because of practical importance. For example, it is unlikely to have second order residual effects, if first residuals are absent.

Table 2B
Partitioning of treatments sum of squares

	Source	df	SS
1.	Direct effects ignoring first and second residuals	mp	$\frac{p\sum T_i'^2}{(mp+1)(p-1)}$
	First residuals elim. direct effects ignoring second residuals	mp	$\frac{\sum\left[T_i'+(p-1)R_i'\right]^2}{(mp+1)(p-1)(p-2)}$
	Second residuals elim. direct and first residuals	mp	$\frac{\sum\left[T_i'+R_i'+(p-2)Q_i'\right]^2}{(mp+1)(p-2)(p-3)}$
2.	Direct effects ignoring first and second residuals	mp	$\frac{p\sum T_i'^2}{(mp+1)(p-1)}$
	Second residuals elim. direct effects ignoring first residuals	mp	$\frac{\sum\left[T_i'+(p-1)Q_i'\right]^2}{(mp+1)(p-1)(p-2)}$
	First residuals elim. direct and second residuals	mp	$\frac{\sum\left[T_i'+(p-2)R_i'+Q_i'\right]^2}{(mp+1)(p-2)(p-3)}$
3.	Second residuals ignoring direct and first residuals	mp	$\frac{p\sum Q_i'^2}{(mp+1)(p-1)}$
	First residuals elim. second residuals ignoring direct effects	mp	$\frac{\sum\left[(p-1)R_i'+Q_i'\right]^2}{(mp+1)(p-1)(p-2)}$
	Direct effects elim. first and second residuals	mp	$\frac{\sum\left[(p-2)T_i'+R_i'+Q_i'\right]^2}{(mp+1)(p-2)(p-3)}$
4.	First residuals ignoring direct and second residuals	mp	$\frac{p\sum R_i'^2}{(mp+1)(p-1)}$
	Direct effects elim. first residuals ignoring second residuals	mp	$\frac{\sum\left[(p-1)T_i'+Q_i'\right]^2}{(mp+1)(p-1)(p-2)}$
	Second residuals elim. direct and first residuals	mp	$\frac{\sum\left[T_i'+R_i'+(p-2)Q_i'\right]^2}{(mp+1)(p-2)(p-3)}$

3.2. *Efficiency factors*

In the proposed design, number of replications for direct, first or second residual effect of each treatment is mp. Therefore, the variance of the estimated elementary contrast in treatment effects for orthogonal design is $2\sigma^2 / mp$. Following Patterson and Lucas [21], the efficiency factors for direct (E_d), first residual (E_{r1}), second residual (E_{r2}) and permanent effects (E_p) are seen to be

$$E_d = E_{r1} = E_{r2} = \frac{(mp+1)(p-3)}{mp(p-2)}, \text{ and } E_p = \frac{(mp+1)(p-3)}{mp^2}.$$

If second residuals are not found to be significant, then the efficiency factors for direct effects, E_d^*, first order residual effects, E_{r1}^*, and permanent effects, E_p^* are

$$E_d^* = E_{r1}^* = \frac{(mp+1)(p-2)}{mp(p-1)}, \text{ and } E_p^* = \frac{(mp+1)(p-2)}{mp^2}.$$

If both the residuals are ignored, then the efficiency factor for treatment effects, E_t^{**} is

$$E_t^{**} = \frac{(mp+1)(p-1)}{mp^2}.$$

Table 3
Efficiency factors for the balanced circular designs with first and second residuals

S. No.	v	p	n	$E_d = E_{r1} = E_{r2}$	E_p	$E_d^* = E_{r1}^*$	E_p^*	E_t^{**}
1#	5	4	5	0.6250	0.3125	0.8333	0.6250	0.9375
2	7	6	7	0.8750	0.5833	0.9333	0.7778	0.9722
3	8	7	8	0.9143	0.6531	0.9524	0.8163	0.9796
4	9	4	18	0.5625	0.2813	0.7500	0.5625	0.8438
5	9	8	9	0.9375	0.7031	0.9643	0.8438	0.9844
6	11	5	22	0.7333	0.4400	0.8250	0.6600	0.8800
7	11	10	11	0.9625	0.7700	0.9778	0.8800	0.9900
8	13	4	39	0.5417	0.2708	0.7222	0.5417	0.8125
9	13	6	26	0.8125	0.5417	0.8667	0.7222	0.9028
10	13	12	13	0.9750	0.8125	0.9848	0.9028	0.9931
11	16	5	48	0.7111	0.4267	0.8000	0.6400	0.8533
12	17	4	68	0.5313	0.2656	0.7083	0.5313	0.7969
13	17	8	34	0.8854	0.6641	0.9107	0.7969	0.9297
14	19	6	57	0.7917	0.5278	0.8444	0.7037	0.8796
15	19	9	38	0.9048	0.7037	0.9236	0.8210	0.9383
16	23	11	46	0.9293	0.7603	0.9409	0.8554	0.9504
17	25	4	150	0.5208	0.2604	0.6944	0.5208	0.7813
18	25	6	100	0.7813	0.5208	0.8333	0.6944	0.8681
19	25	8	75	0.8681	0.6510	0.8929	0.7813	0.9115
20	25	12	50	0.9375	0.7813	0.9470	0.8681	0.9549
21	29	7	116	0.8286	0.5918	0.8631	0.7398	0.8878
22	31	10	93	0.9042	0.7233	0.9185	0.8267	0.9300

Note: # The design is to be replicated to have enough degrees of freedom for error. v, number of treatments; p, number of periods; n, number of units; E_d, E_{r1}, E_{r2}, E_p denote efficiency factors for direct, first residual, second residual and permanent effects respectively; E_d^*, E_{r1}^*, E_p^* denote efficiency factors for direct, first residual and permanent effects respectively ignoring the second residual effects; E_t^{**} stands for efficiency factors for the treatment effects ignoring both the residual effects.

The efficiency factors of various treatment effects for the designs with $v \leqslant 31$ and $p \geqslant 4, \leqslant 12$ have been presented in Table 3. It can be seen from the table that the efficiency of the designs increases with the increase in the number of periods indicating thereby that degree of non-orthogonality of treatment effects with experimental unit effects declines as the number of periods increase.

Acknowledgements

The financial support provided by the Indian Council of Agricultural Research to the first author is duly acknowledged. The authors are grateful to the referees for their valuable comments that improved the paper immensely.

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