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Modelling transmission of potato price volatility in West Bengal markets : MGARCH approach

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ABSTRACT

Volatility is a common phenomenon which can be observed in the financial market. Volatility is often considered to be same as risk, but the truth is, risk deals with only negative shocks whereas volatility takes care of both negative and positive shocks. It is important to model and forecast volatility efficiently as it involves a large domain of stakeholders. In present global scenario where markets are no more operating in isolation and trade taking place across markets at domestic and international levels, there exist an influence of one market upon the other. Under these circumstances the very popular GARCH model [1] which is univariate in nature seems restrictive in modelling and forecasting volatility. Hence, the multivariate GARCH (MGARCH) models were introduced to capture the movement of volatility among different markets. Various MGARCH models have been proposed in the literature. The most commonly used ones are BEKK (Baba, Engle, Kraft and Kroner), CCC (Constant Conditional Correlation) and DCC (Dynamic Conditional Correlation) modifications. In this paper, various MGARCH models have been explained in details highlighting their usefulness in capturing different volatility transmission process. Further, real data sets have been analysed using these models and the volatility transmission processes explained as a part of an illustration.

Keywords : GARCH, MGARCH, BEKK, CCC, DCC, price transmission, volatility,

1. INTRODUCTION

Modelling and prediction of volatile time series data has been extensively studied since the seminal paper (Engle, 1982). There are two very popular ways of modelling volatility, one being the ARCH (Autoregressive Conditional Heteroscedascity) type models and the other being SV (Stochastic Volatility) models. In this chapter we have focused only into the ARCH type models and its variations in multivariate framework. ARCH and its generalised version GARCH has been extensively used for modelling and forecasting of volatile time series data for last few decades. These models proved to be very efficient hence a large number of variations have been proposed in literature such as Exponential GARCH (EGARCH), Glosten, Jagannathan, and Runkle GARCH (GJR-GARCH), Threshold GARCH (TGARCH), Integrated GARCH (IGARCH), *etc*. Each of these models has been proposed to capture some specific phenomenon of financial series. For example EGARCH is very useful to model the asymmetric volatility pattern in the series. In present scenario due to globalisation and liberalisation of trade, these models are unable to meet the requirements. The basic reason behind it being that GARCH models are univariate in nature, thus they can only model single series at a time. And currently a financial series is being governed by another series directly or indirectly. For example the price movement of various stocks at Indian market is affected by the behaviour in the stock market at USA or China. The reason being that the markets are now not working independently on its own, its price dynamics are being dependent on many other factors as well. Thus, the necessity to have models with multivariate framework that can capture the movement of volatility among different related financial series was felt. This lead to the development of MGARCH (Multivariate GARCH) models which has the ability to model more than one volatile series at a time and can capture the volatility process present between them. A large number of MGARCH models have been discussed widely in literature (Bollerslev, 1986). These models have fond considerable amount of application in agricultural domain too (Lama *et al.*, 2016 ; Sinha *et al.*, 2017). The remaining part of paper has been structured as methodology, results and discussion and conclusions.

2. MATERIALS AND METHODS

2.1. MGARCH models

The rationale behind inception of MGARCH class of models has been explained in the earlier section and it's beyond doubt that these models are need of the hour. But, there are few problems associated with the formulation of MGARCH models. To begin with, the number of parameters in the model increases rapidly with the dimension of the series used. This results into model estimation problems and becomes challenging to maintain parsimony and

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flexibility of the model at same time. Secondly, it's required to satisfy the condition of positive definiteness of the conditional covariance matrices which becomes numerically difficult for large systems. Finally, the issue of numerical optimisation of the likelihood function which contains the conditional covariance matrix becomes a hindrance in the estimation of the model. As the conditional covariance matrix is time dependent and it has to be inverted every time in the optimisation procedure, thus making it a time consuming and numerically unstable procedure (Silvennoinen and Terasvirta, 2009). Keeping these challenges in mind for the estimation of MGARCH models researchers has used different ways to tackle them. In literature we find four different approaches for it each having some advantages over the other. In first approach the conditional covariance matrix H_t is modelled directly as is done in BEKK model. In the second approach it is assumed that is an outcome of smaller unobservable heterocedastic factors. These are called factor models and the main emphasis is on the parsimony of the models. In this chapter we will not discuss much on this class of models. The third approach is based on the idea of modelling the conditional variance and correlation instead of modelling the conditional covariance matrix directly. CCC and DCC models fall in this class. The final approach is the semi and non-parametric procedures for estimation of the model parameters. Before moving into the details of the MGARCH models we define a multivariate series as follows :

$$y_t = H_t^{1/2}\varepsilon_t \tag{1}$$

where, $H_t^{1/2}$ is k x k positive-definite matrix and of the conditional variance of y_t. k is the number of series and t = 1, 2,…,n (number of observations).

2.2. BEKK specification

Engle and Kroner [7] introduced the BEKK model which is the direct generalization of the univariate GARCH model. The resulting variance is dependent on the amount of currently available information. A general GARCH (p, q) model [1] can be defined as :

$$h_t = \alpha_0 + \alpha_1\varepsilon^2_{t-1} + \cdots + \alpha_p\varepsilon^2_{t-p} + \beta_1 h_{t-1} + \cdots + \beta_q h_{t-q}$$
$$\alpha_i > 0, \beta_i > 0,\ \alpha_i + \beta_i < 1 \tag{2}$$

where, $\square_t$ is the conditional variances which depends on the previous error terms as well as previous conditional variances of the process.

Equation (2) can be transferred into multivariate GARCH model with a generalization of the resulting variance matrix H_t

$$H_t = \begin{pmatrix} h_{11} & h_{12} & h_{13} \\ h_{21} & h_{22} & h_{23} \\ h_{31} & h_{32} & h_{33} \end{pmatrix}$$

Each element of H_t depends on the p delayed values of the squared ε_t, the cross product of ε_t and on the q delayed values of elements from H_t. In general, multivariate GARCH (1, 1) model can be written as :

$$H_t = C_0' C_0 + \cdots + \begin{pmatrix} a_{11} & 0 & 0 \\ 0 & a_{22} & 0 \\ 0 & 0 & a_{33} \end{pmatrix} \begin{pmatrix} \varepsilon^2{}_1 & \varepsilon_1\varepsilon_2 & \varepsilon_1\varepsilon_3 \\ \varepsilon_2\varepsilon_1 & \varepsilon^2{}_2 & \varepsilon_2\varepsilon_3 \\ \varepsilon_3\varepsilon_1 & \varepsilon_3\varepsilon_2 & \varepsilon^2{}_3 \end{pmatrix} \begin{pmatrix} a_{11} & 0 & 0 \\ 0 & a_{22} & 0 \\ 0 & 0 & a_{33} \end{pmatrix} + \ldots$$

$$\cdots + \begin{pmatrix} b_{11} & 0 & 0 \\ 0 & b_{22} & 0 \\ 0 & 0 & b_{33} \end{pmatrix} \begin{pmatrix} h^2{}_1 & h_1h_2 & h_1h_3 \\ h_2h_1 & h^2{}_2 & h_2h_3 \\ h_3h_1 & h_3h_2 & h^2{}_3 \end{pmatrix} \begin{pmatrix} b_{11} & 0 & 0 \\ 0 & b_{22} & 0 \\ 0 & 0 & b_{33} \end{pmatrix}$$

In compact form, the above equation can also be written as

$$H_t = C_0' C_0 + A'\varepsilon_{t-1}\varepsilon'_{t-1}A + B' H_{t-1} B \tag{3}$$

Further if B=AD, D being a diagonal matrix then equation (3) becomes 'diagonal BEKK' model. For the sake of better understanding we have defined the volatility equations for bivariate case. In that condition the model is represented as follows :

$$H_t = \begin{bmatrix} c_{11} & c_{12} \\ c_{21} & c_{22} \end{bmatrix} + \begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{bmatrix}' \begin{bmatrix} \varepsilon^2_{1,t-1} & \varepsilon_{1,t-1}\varepsilon_{2,t-1} \\ \varepsilon_{2,t-1}\varepsilon_{1,t-1} & \varepsilon^2_{2,t-1} \end{bmatrix} \begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{bmatrix} + \begin{bmatrix} g_{11} & g_{12} \\ g_{21} & g_{22} \end{bmatrix}' H_{t-1} \begin{bmatrix} g_{11} & g_{12} \\ g_{21} & g_{22} \end{bmatrix}$$

$$h_{11,t} = c_{11} + a_{11}^2\varepsilon_{1,t-1}^2 + 2a_{11}a_{21}\varepsilon_{1,t-1}\varepsilon_{2,t-1} + a_{21}^2\varepsilon_{2,t-1}^2 + g_{11}^2 h_{11,t-1} + 2g_{11}g_{21}h_{12,t-1} + g_{21}^2 h_{22,t-1}$$

$$h_{12,t} = c_{12} + a_{11}a_{21}\varepsilon_{1,t-1}^2 + (a_{21}a_{12} + a_{11}a_{22})\varepsilon_{1,t-1}\varepsilon_{2,t-1} + a_{21}a_{22}\varepsilon_{2,t-1}^2 + g_{11}g_{12}h_{11,t-1} + (g_{21}g_{12} + g_{11}g_{22})h_{12,t-1} + g_{21}g_{22}h_{22,t-1}$$

$$h_{22,t} = c_{22} + a_{12}^2\varepsilon_{1,t-1}^2 + 2a_{12}a_{22}\varepsilon_{1,t-1}\varepsilon_{2,t-1} + a_{22}^2\varepsilon_{2,t-1}^2 + g_{12}^2 h_{11,t-1} + 2g_{12}g_{22}h_{12,t-1} + g_{22}^2 h_{22,t-1}$$

Here $h_{11,t}$ and $h_{22,t}$ are the volatility equations for first and second series respectively and $h_{12,t}$ is the volatility spillover equation between the two series under consideration.

Though BEKK is considered as a direct generalization of the univariate GRACH model, yet there are problems in its estimation. Large number of parameters along with the non-linear nature of the model causes concern for obtaining convergence. For full BEKK model the number of parameters are $(p+q)kN^2+N(N+1)$ or $(p+q)kN + N(N+1)/2$ for diagonal BEKK model. The parameters of the models are estimated using the maximum likelihood estimation (MLE) procedure.

2.3. CCC and DCC specification

A relatively flexible approach is the CCC model introduced by Bollerslev,1990. In this type of specification the conditional covariance matrix is decomposed into conditional standard deviations and correlations. This model assumes the conditional correlations to be constant. This restriction strongly reduces the number of unknown parameter and thus simplified the estimation. In case of CCC model H_t is represented as follows:

$$h_{ij,t} = \rho_{ij}\sqrt{h_{ii,t}h_{jj,t}} \quad (5)$$

where, R is a symmetric positive-definite matrix whose elements are (constant) conditional correlations $\rho_{ij}, i, j = 1,2,...k(\rho_{ij} = 1, i = j)$ and $D_t = diag(h_{11,t}^{1/2},...,h_{kk,t}^{1/2})$. Thus each conditional covariance is given by :

$$H_t = D_t R D_t \quad (6)$$

Details regarding the CCC models and its extensions can be found in Anderson *et al.*, 2009. The simplicity in estimation procedure for CCC model has attracted users. But, in practical applications we rarely come across situations where the correlations among the series are constant over time. To do away with this DCC model was proposed by Engle, 2002. In case of DCC the R matrix is time varying thus making it dynamic. Lagrange Multiplier (LM) test has been proposed by Tse, 2000 for testing the presence of dynamic correlation.The representation of the DCC model is as follows :

$$H_t = D_t R_t D_t$$
$$R_t = diag(Q_t)^{-1/2}\, Q_t diag(Q_t)^{-1/2} \quad (7)$$

where, $Q_t = (1-\alpha-\beta)R + \alpha u_{t-1}u'_{t-1} + \beta Q_{t-1}$ and $u_t = D_t^{-1}y_t$.

R is the unconditional covariance matrix of u_t. And the conditional covariances are given as follows :

$$h_{ij,t} = q_{ij,t}\sqrt{h_{ii,t}h_{jj,t}} \,/\, \sqrt{q_{ii,t}q_{jj,t}} \quad (8)$$

Q_t is written as GARCH(*1,1*) type equation and then transformed to get R_t. The parameters of the models are estimated using the maximum likelihood estimation (MLE) procedure.

3. RESULTS AND DISCUSSION

In this study we have used daily price of Potato from two markets in West Bengal namely Mednipur and Hooghly collected from http://agmarknet.gov.in. Prices are considered from 01st, January, 2016 to 31st, August, 2018 (Fig .1). We have analysed the data in univariate as well as in multivariate framework. Starting with univariate analysis we first identified the mean model of the individual series using ARIMA model. The parameter estimates obtained are reported in table 1. After which we diagnosed the residuals and found presence of ARCH effect in it. Hence, we proceed with fitting the series using GARCH model. Estimates so obtained from GARCH models are reported in table 2. Further, we analysed the series under multivariate framework, to do so we started with the mean model. For modelling the mean of multivariate series we can either opt for VAR or VEC model. To decide which model needs to be applied we employed Johansen test for Co-integration and we found the series are co-integrated. Thus we proceed with VEC model; the estimates obtained are reported in table 3. The results show that the coefficients of error correction term in Mednipur market fulfils the condition of negativity and significant for Hooghly market which indicates that when Mednipur market deviates from equilibrium level, Hooghly market tends to correct back towards long run equilibrium level in the next period. After, fitting the mean model we proceed to estimate the transmission of volatility among the series using MGARCH models. The BEKK model helps us to understand the volatility transmission phenomenon, whereas DCC model allows us to capture the dynamic conditional correlation in the volatility structure. The parameter estimates of BEKK and DCC models are reported in table 4 and 5 respectively. It was observed that transmission of volatility in price is taking place from Hooghly to Mednipur market (-0.23), which is indicative that if prices in Hooghly market rises than the prices in Mednipur market is likely to fall.

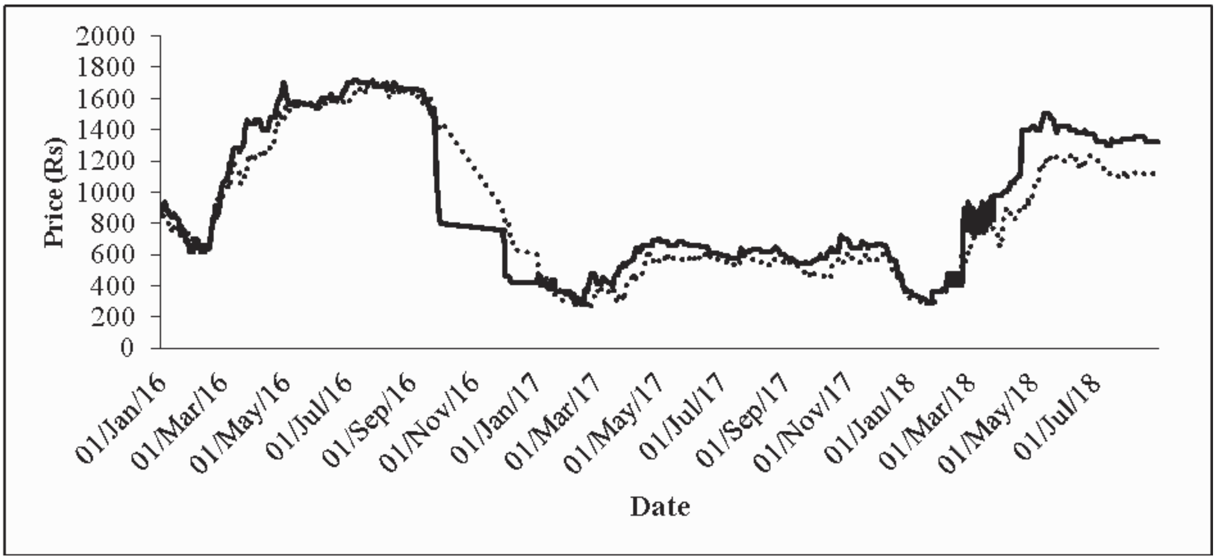

Fig.1. Time plot of potato price in Mednipur (dashed) and Hooghly (Bold) markets.

Table 1: ARIMA model estimates

Potato price series	AR(1) (S.E.)	AR(2) (S.E.)	MA(1) (S.E.)	MA(2) (S.E.)	MA(3) (S.E.)
Hooghly market	-0.9277	———	0.7897	0.1454	———
	0.0095	**———**	**0.0193**	**0.0173**	**——-**
Mednipur market	1.4374	-0.5426	-1.4108	0.3503	0.2019
	0.1747	**0.2185**	**0.1768**	**0.2424**	**0.0479**

Table 2: GARCH model estimates

Potato price series	Omega(S.E.)	Alpha (S.E.)	Beta(S.E.)
Hooghly market	430.90	0.67	0.18
	0.001	**0.01**	**0.02**
Mednipur market	212.30	0.11	0.80
	0.001	**0.01**	**0.01**

Table 3: VEC model estimates

Potato price series	Co-integration Rank	ECT
Mednipur market	1	-0.135(0.013)
Hooghly market	1	-0.053(0.017)

Table 4: MGARCH-BEKK model estimates for Mednipur and Hooghly market potato prices

Coefficients	Estimate	Std. Error	t value	P value
C11	89.78	11.14	8.06	<0.01
C21	89.64	8.47	10.54	<0.01
C22	27.45	2.23	11.85	<0.01
A11	0.96	0.06	24.28	<0.01
A21	0.01	0.06	0.09	0.93
A12	-0.10	0.05	-0.22	0.83
A22	0.98	0.06	26.95	<0.01
B11	0.50	0.08	0.64	0.52
B21	-0.23	0.07	-3.21	<0.01
B12	0.04	0.07	0.08	0.93
B22	0.29	0.08	3.53	<0.01

Table 5. MGARCH-DCC model estimates for Mednipur and Hooghly market Potato prices

Coefficients	Estimate	Std. Error	t value	P value
C1	910.00	3.53	258.03	<0.01
ω1	46.37	56.51	0.82	0.41
A11	0.16	0.07	2.32	0.02
B11	0.83	0.05	15.81	<0.01
C2	870.13	<0.01	354.11	<0.01
ω2	29.72	28.46	1.04	0.30
A22	0.17	0.09	1.92	0.05
B22	0.83	0.07	11.18	<0.01
δ12	0.04	<0.01	<0.01	0.97
δ12	0.92	0.15	6.03	<0.01

4. CONCLUSION

In this study we have attempted to model the presence and transmission of volatility in the potato prices of two markets in West Bengal. We also explored the presence of integration among the marketed and we found that the Hooghly and Mednipur markets are co-integrated among themselves. A significant amount of volatility transmission from Hooghly to Mednipur market has also been identified. Thus from this study we infer that the prices of Potato among these two markets do not operate in isolation.

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