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Determination of Thermal Diffusivity of Freshwater Fish during Ice Storage by using a One-dimensional Fourier Cylindrical Equation

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Thermal diffusivities of Indian major carp Catla (Catla catla) and Rohu (Labeo rohita) fish were determined by using a one-dimensional (1D) Fourier equation applied to a cylinder. Experiments were conducted by cooling the different quantities of individual fish with ice in an insulated box. Time–temperature records were used to determine the thermal diffusivity. The values of thermal diffusivities of fish cooling with ice were ranged from $6{\cdot}6012$ to $3{\cdot}2475 \times 10^{-8}\,m^{-2}\,s^{-1}$ and $6{\cdot}6481$ to $5{\cdot}4267 \times 10^{-8}\,m^{-2}\,s^{-1}$ for Catla and Rohu fish, respectively. It was observed that the thermal diffusivity decreases with an increase in the weight of the fish. A logarithmic model adequately described the relationship between thermal diffusivity α and mass of fish m, with values for the coefficient of determination of 0·9949 and 0·9996, and standard errors of 0·088 and 0·0006 for Catla and Rohu fish, respectively.

1. Introduction

Fish is highly perishable and has a short storage life. The one factor, which has the most important role to play in helping to retain the quality of the fish, is temperature. In order to land the best quality fresh fish, it essential to cool fish as quickly as possible to a low temperature just above freezing point. Only a few hours delay at 15–20 °C will reduce the storage life of fish by several days (Hansen & Jensen, 1982). Temperature is a very important factor in accelerating the process of spoilage. The spoilage reactions connecting on the death of the fish proceed at very rapid rate. Cooling is a common and important preservation technique to maintain the quality and prevent the spoilage of the products (Dincer, 1995). The simplest method of cooling of fish is icing (Govindan, 1985) and this is widely accepted as the most economical and readily available method (Jain et al., 2005). Ice keeps the chilled fish moist and glossy and prevents the dehydration (Graham et al., 1992).

Knowledge of food thermo-physical properties and that of their variations is very important for making heat transfer analysis and for designing heat transfer equipment. Convective heat transfer coefficient and diffusivity are the most important thermal properties in any heat and mass transfer process. Various studies have been conducted to determine the convective heat transfer coefficient (Jain et al., 2005; Jain, 2006) and diffusivities (Jain & Pathare, 2007) of fish. Wang et al. (1998, 2000) investigated the salt diffusivity in farmed Atlanta Salmon muscles with different stages of rigor mortis.

Thermal diffusivity includes the effects of properties like mass density, thermal conductivity and specific heat capacity. Thermal diffusivity, which is involved in all unsteady heat-conduction problems, is a property of the solid object. The time rate of change of temperature depends on its numerical value. The physical significance of thermal diffusivity is associated with the diffusion of heat into the medium during changes of temperature with time. The higher thermal diffusivity coefficient signifies the faster penetration of the heat into the medium and the less time required to remove the heat from the solid (Dincer & Dost, 1995). There are numerous methods to measure the thermal diffusivity in

Notation

A, B	regression coefficients	T	temperature, °C
e_s	standard error	T_f	temperature of the centre of fish, °C
F_o	Fourier number ($=\alpha t/r^2$)	$T_{f,0}$	initial temperature of the centre of fish, °C
J_0, J_1	Bessel's Function, first kind order 0 and 1	m	mass of fish, kg
K	constant	x	axial coordinate, m
R^2	coefficient of determination	α	thermal diffusivity, $m^2\,s^{-1}$
r	radial cylindrical coordinate, m	β	root of Bessel's function
r_f	mean radius of fish, m	θ	dimensionless temperature
t	time, s		

the literature. Yang et al. (2002) presents the thermal diffusivity of borage seeds. Weidenfeller et al. (2004) propose a method of thermal diffusivity measurement for filler materials like magnetite, barite, talc, copper and glass fibres. Nevertheless, most of the methods require relatively complex instrumentation or experimental assemblies and demand an expertise of the thermal phenomena that is not within the reach of many users. The one-dimensional (1D) solution of Fourier's equation is the reliable method established by means of consistent repetitive measurements with an acceptable margin of error (Baïri et al., 2007).

The present study propose a simple method to determine thermal diffusivity of freshwater fish cooled in ice based on 1D solution of Fourier's equation of heat. Which has very important application in the design of fish cold storehouse.

2. Theoretical consideration

Fish samples were considered as an infinite cylinder. The initial temperature of the object is considered to be uniform. When heat is transferred by conduction through solid material it generates a temperature field can be described by Fourier equation in cylindrical coordinates:

$$\frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial T}{\partial r}\right)+\frac{\partial^2 T}{\partial x^2}=\frac{1}{\alpha}\frac{\partial T}{\partial t} \tag{1}$$

where: T is temperature in °C; r is radius in m; x is axial coordinate in m; α thermal diffusivity in $m^2\,s^{-1}$; and t is time in s.

Considering the very small internal temperature gradient, the derivative $\partial^2 T/\partial x^2$ is ignored. As the cylinder is long enough to consider that the heat exchanges are made quasi-exclusively through the lateral surface, the problem is simplified and becomes 1D. The result aims to retain the procedure presented in this work, which is based on the analytical solution of 1D Fourier's equation in cylindrical coordinates:

$$\frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial T}{\partial r}\right)=\frac{1}{\alpha}\frac{\partial T}{\partial t} \tag{2}$$

The above equation can be solved by the method developed by Baïri and Laraqi (2003) as

$$\theta=\frac{T_f-T_{ice}}{T_{f,0}-T_{ice}}=2\sum_{n=1}^{\infty}\exp(-\lambda_n^2F_o)\frac{1}{\lambda_n}\frac{J_1(\lambda_1)}{J_0^2(\lambda_1)+J_1^2(\lambda_1)}J_0(\lambda_n r) \tag{3}$$

where: θ is dimensionless temperature ratio; $T_{f,0}$ is initial temperature of fish in °C; T_f is temperature of centre of fish in °C; T_{ice} is temperature of ice in °C; F_o is Fourier number in dimensionless; J_0 and J_1 are the Bessel function of first kind of order 0 and 1; and λ_1 and λ_n are the positive roots of the Bessel's equation. Equation (1) can be approximated for situations where the cylindrical material is exposed for long time by retaining only first term of the series solution. The value for the first root λ_1 is 2·405. Thus, the Eqn (1) can be rewritten as

$$\theta=\frac{T_f-T_{ice}}{T_{f,0}-T_{ice}}=K\exp(-\lambda_1^2F_o) \tag{4}$$

where the constant K is given by

$$K=2\frac{1}{\lambda_n}\frac{J_1(\lambda_1)}{J_0^2(\lambda_1)+J_1^2(\lambda_1)}J_0(\lambda_n r) \tag{5}$$

Therefore,

$$\theta=K\exp\left(-\lambda_1^2\frac{\alpha t}{r_f^2}\right) \tag{6}$$

where r_f is mean radius of fish in m.

Plotting the ln θ versus time will give a straight line. Measuring carefully the slope of the curve, thermal diffusivity can be calculated by the following expression:

$$\alpha = \frac{F_o}{\left(t/r_f^2\right)} \tag{7}$$

where t is time in s.

3. Material and methods

Freshwater Indian major carp Catla fish (Catla catla) and Rohu (Labeo rohita) was used for the present study. The live fish were procured from the local fishpond, where the cultural practices were same. Fish were kept free from stress in a water tank for 1–2 days until used. The fish were killed by stunning the head for sudden death (Abbas et al., 2005). The moisture content was determined by the method of drying at temperature of 130 °C described by Gerasimov and Antonova (1979) and observed as 3·219 kg [H_2O] kg^{-1} [dry matter] for Catla fish and 3·329 kg [H_2O] kg^{-1} [dry matter] for Rohu fish.

The work was started by weighing the fish on an electronic balance an accuracy of ±0·1 g. Volume of the fish was measured by the water displacement method (Rahman, 1995). The fish was divided longitudinally into equal segment of 20 mm width. The perimeter of each side of segment of fish was measured to the nearest millimetre using a flexible measuring tape. The mean perimeter was calculated of each segment. Then mean radius was calculated.

An experimental set-up used for fish cooling is shown in Fig. 1. An aluminium box of 400 mm length, 150 mm breadth and 170 mm height with insulation all around by 50 mm thick thermo-coal sheet is used for cooling the fish with ice. The insulated aluminium box was initially filled with small ice flakes up to 50 mm thickness. In order to insert the temperature probe at the desired depth (centre) of the fish thickness, a needle was pierced into the fish from its vertebral side. The temperature probe thus inserted through this whole into the fish and penetrates slightly into the dorsal muscles. It was considered as thermal centre for longest time of cooling. The fish was immediately kept in ice after taking the physical observation. The fish was placed in the box with pierced temperature probe as down the dorsal side and the box was stuffed with ice flakes to ensure the perfect contact between ice and fish (Fig. 1). Experiments were conducted with different weight ranging from 0·200 to 1·200 kg of fish, since this is the most common range of weight of fish for harvesting from pond and marketing. Replications with the same weight were not possible as difficult to obtain the fish with the same weight. Twenty experiments were conducted and the nine experiments were used for present analysis for each variety of fish. The temperature inside the fish flesh were measured and recorded in a data logger (ADAM 4520, Century Instruments, Chandigarh, India) at an equal time interval of 300 s. The initial temperature of the fish was observed as 29·5 °C and the fish was cooled to 0·0 °C.

4. Result and discussion

4.1. Thermal diffusivity

The logarithmic dimensionless temperature (ln θ) was plotted against cooling time for Catla and Rohu fish with three variable weights (Figs 2, 3). The regression analyses were done for relating the cooling time and logarithmic dimensionless temperature. Tables 1 and 2 illustrate the linear relationship parameters and correlation coefficients between cooling time and logarithmic dimensionless temperature for Catla and Rohu fish, respectively. The value for the coefficient of determination R^2 ranged as 0·9890–0·9988 and 0·9908–0·9972, standard error of 0·05624–0·09219 and 0·04867–0·13395 for Catla and Rohu fish cooling, respectively. The better correlation parameters represent the accuracy of computation of thermal diffusivity.

Thermal diffusivity α in $m^2 s^{-1}$ was calculated by using Eqn (7) with the knowledge of numerical value of slope and mean radius of individual fish. The values of thermal diffusivities α ranged from $6{\cdot}6012 \times 10^{-8}$ to $3{\cdot}2475 \times 10^{-8} m^2 s^{-1}$ and $6{\cdot}6481 \times 10^{-8}$ to $5{\cdot}4267 \times 10^{-8} m^2 s^{-1}$ for Catla and Rohu fish, respectively. These values are within the general range of 10^{-8}–$10^{-7} m^2 s^{-1}$ for cooling the fish reported by Rahman (1995).

4.2. Effect of weight of fish on thermal diffusivity

Figure 4 presents the relation of thermal diffusivity with the mass of fish in the range of 0·1952–1·2106 kg and 0·2150–1·1650 kg for Catla and Rohu fish, respectively. The values of thermal diffusivity decreased with increase in weight of fish during ice cooling. The value of α for Rohu fish is higher than Catla fish for the same weight. It indicates that Rohu fish needs less cooling time compared to Catla fish. It also revealed that thermal diffusivity is the function of mass of the fish within a particular variety and on the other way depends upon the composition of fish variety.

The reason for variation in thermal diffusivity with mass of fish, since the radius (r_f) of the fish varies with the mass of fish. The centre of bigger fish takes more time to be cooled. The heat removal from the bigger fish is slow. Therefore, the thermal diffusivity of the bigger

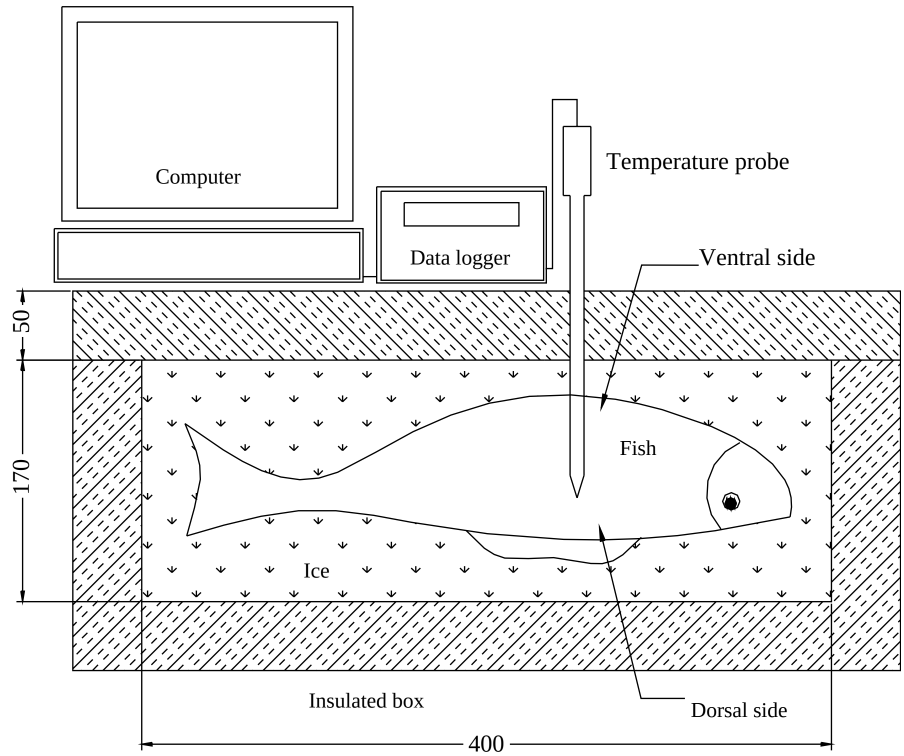

Fig. 1. Cut-away diagram of fish thermal diffusivity determination with ice cooling; all dimensions in mm (drawing not to scale)

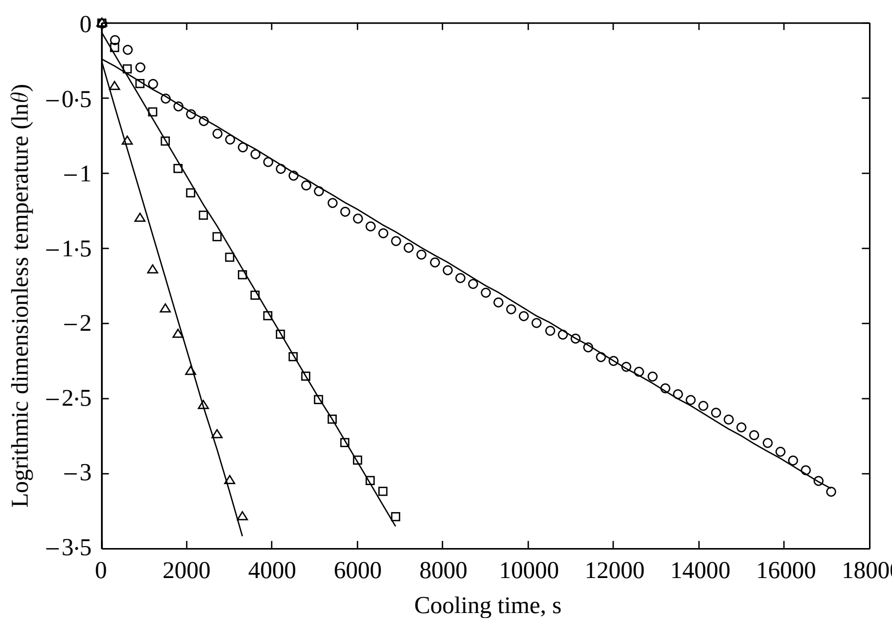

Fig. 2. Evolution of dimensionless temperature (θ) for Catla fish for different weight △, 0·1952 kg; □, 0·4018 kg; ○, 1·2106 kg

fish is lower than the smaller fish. As it is clear that the lower thermal diffusivity coefficient signifies the lower penetration of the heat into the medium and the more time required to remove the heat.

A logarithmic relationship between thermal diffusivity α m^2 s^{-1} and mass m kg of fish presented in Eqns (8) and (9) for Catla and Rohu fish, respectively. The acceptability of the model is based on a value for the

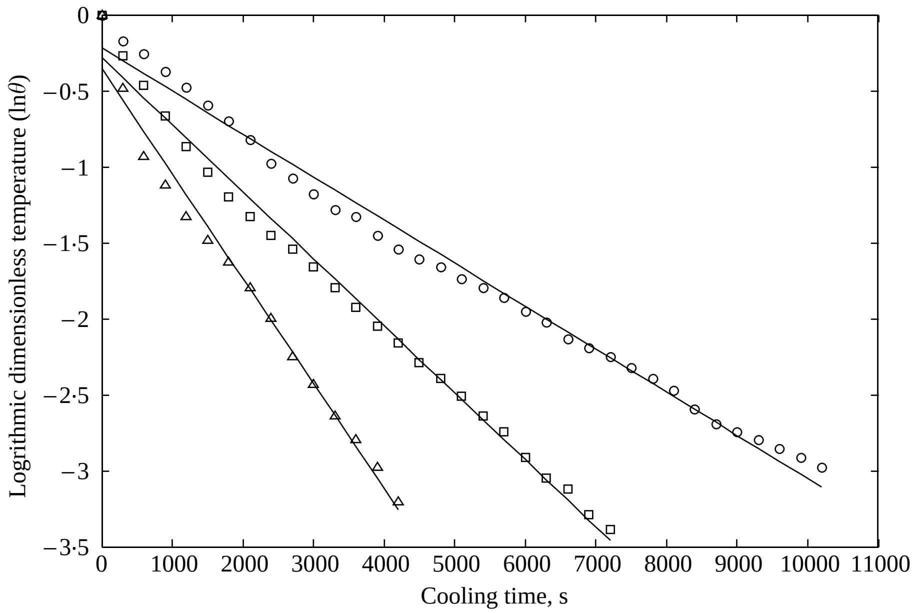

Fig. 3. Evolution of dimensionless temperature (θ) for Rohu fish for different weight △, 0·215 kg; □, 0·6364 kg; ○, 0·860 kg

Table 1
Relationship between dimensionless temperature ln θ and cooling time t for Catla fish [ln θ = A+B]

Mass of fish, kg	Coefficient		R^2	e_s
	A	B		
0·1952	0·0009545	−0·2659	0·9890	0·13395
0·218	0·0009471	−0·1422	0·9953	0·07798
0·2763	0·0006824	−0·2858	0·9938	0·09966
0·4018	0·0004762	−0·06713	0·9988	0·04867
0·6720	0·0002385	−0·3064	0·9895	0·1155
0·7469	0·0002300	−0·3137	0·9892	0·1168
0·8879	0·0002131	−0·2880	0·9930	0·09587
0·9864	0·0001991	−0·2841	0·9953	0·08101
1·2106	0·0001673	−0·2410	0·9973	0·12811

R^2, coefficient of determination; e_s, standard error.

Table 2
Relationship between dimensionless temperature ln θ and cooling time t for Rohu fish [ln θ = At + B]

Mass of fish, kg	Coefficient		R^2	e_s
	A	B		
0·2150	0·0006904	−0·3550	0·9908	0·06839
0·2455	0·0006507	−0·3304	0·9951	0·06946
0·4184	0·0005940	−0·2746	0·9945	0·06979
0·4845	0·0005655	−0·2223	0·9957	0·07714
0·6364	0·0004410	−0·2818	0·9957	0·06808
0·6850	0·0004220	−0·2171	0·9972	0·05624
0·7657	0·0002913	−0·3119	0·9949	0·06824
0·8600	0·0002831	−0·2181	0·9949	0·07450
1·1650	0·0002483	−0·2454	0·9945	0·09219

R^2, coefficient of determination; e_s, standard error.

coefficient of determination R^2 which should be close to one, and low values for the standard error e_s. The coefficients of determination R^2 were 0·9949 and 0·9996 and standard error e_s 0·088 and 0·0006 for Catla and Rohu fish, respectively:

$$\alpha = [8{\cdot}143 \exp(-4{\cdot}073m) + 3{\cdot}039] \times 10^{-8} \quad (8)$$

$$\alpha = [3{\cdot}623 \exp(-5{\cdot}04m) + 5{\cdot}412] \times 10^{-8} \quad (9)$$

Thus, the thermal diffusivity can be predicted on the basis of above logarithmic model with the greater accuracy on the basis of mass of the Catla and Rohu fish (whole fish) for the given range of determinations. This has very important application in the design of fish storehouse.

5. Conclusions

Thermal diffusivities of Catla and Rohu fish were determined using a one-dimensional (1D) solution of Fourier equation of heat. Thermal diffusivity of fish depends on the mass of the fish and decreases with the increase in mass. Thermal diffusivity for Rohu fish is higher than Catla fish. A logarithmic model could adequately describe the thermal diffusivity of fishes on the basis of statistical parameters such as coefficient of determination and standard error. Thermal diffusivities ranged from $6{\cdot}6012$–$3{\cdot}2475 \times 10^{-8}\,m^{-2}\,s^{-1}$ and $6{\cdot}6481$–$5{\cdot}4267 \times 10^{-8}\,m^{-2}\,s^{-1}$ for Catla and Rohu fish, respectively.

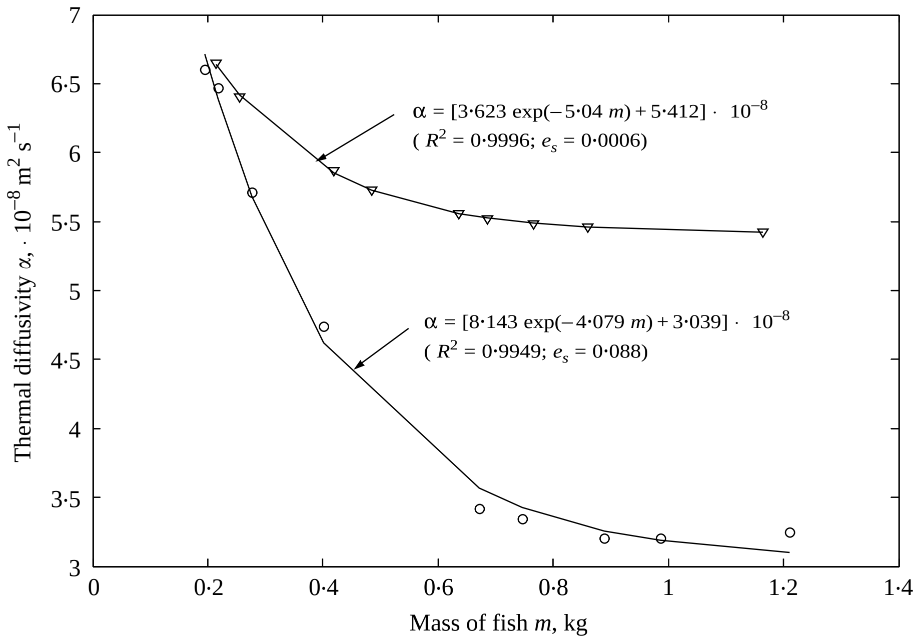

Fig. 4. Thermal diffusivity of freshwater fish ○, Catla fish; ▽, Rohu fish; R^2, coefficient of determination; e_s, standard error

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