Linear Programming techniques for fisheries management

Chandrasekar.V*, Geethlakshmi.V & Nikita Gopal

ICAR-Central Institute of Fisheries Technology, Kochi-682029, Kerala
*vcsecon@gmail.com

Introduction:

Linear programming an optimsation technique is one of the most used modern tools for the mathematical society where all involved functions are linear. It is the subject of studying and solving the linear programs, we all are make decision for the sole purpose of maximize our quality of life productivity, time and welfare or some way or another to optimizing our activities. It was first conceived by Dantzig, around 1947 at the end of the Second World War used for military logistic problems. Very historically, the work of a Russian mathematician first had taken place in 1939 but since it was published in 1959, Dantzig was still credited with starting linear programming. In fact Dantzig did not use the term linear programming. His first paper was titled 'Programming in Linear Structure'. Much later, the term 'Linear Programming' was coined by Koopmans. The Simplex method which is the most popular and powerful tool to solve linear programming problems, was published by Dantzig in 1949. So this is the brief history of this field called Linear Programming.

- A Linear Programming model seeks to maximize or minimize a linear function, subject to a set of linear constraints.
- The linear model consists of the following components:
  - A set of decision variables.
  - An objective function.
  - A set of constraints.

The Importance of Linear Programming used in many real world problems lend themselves to linear programming modeling and it can be approximated by linear models. There are well-known successful applications in Manufacturing, Marketing, Finance (investment), Advertising, Agriculture

- Assumptions of the linear programming model are
  - The parameter values are known with ***certainty.***
  - The objective function and constraints exhibit ***constant returns to scale.***
  - There are ***no interactions*** between the decision variables (the additivity assumption).
  - The ***Continuity*** assumption: Variables can take on any value within a given feasible range.

Essential Steps in Linear programming:

To define an optimization model in Excel the following steps are to be followed:

1. Organize the data for your problem in the spreadsheet in a logical manner.
2. Choose a spreadsheet cell to hold the value of each decision variable in your model.
3. Create a spreadsheet formula in a cell that calculates the objective function for your model.
4. Create a formulas in cells to calculate the left hand sides of each constraint.
5. Use the dialogs in Excel to tell the Solver about your decision variables, the objective, constraints, and desired bounds on constraints and variables.
6. Run the Solver to find the optimal solution.

Within this overall structure, There is a great deal of flexibility in choosing cells to hold the model's decision variables and constraints, and which formulas and built-in functions are to be use. In general, the goal should be to create a spreadsheet that communicates its purpose in a clear and understandable manner.

Linear programming problem

Let x be the number of Cutla fish & y be the number of Rohu fish reared in the pond, The fish farmer has constant investment of maximum amount of Rs 10000 and he can store maximum 1000 fishes (storage constant) and it is know that x & y are non-negative i.e $x,y \geq 0$. Mathematically it can be stated as

$100x + 80y \leq 10000$ ----- (i)

$x + y \leq 1000$ ---------- (ii)

Here the goal is to maximize the profit $Z = 100x + 80y$ i.e $Z = f(x,y)$ is the objective function and subject to constrain of investment and storage

The essential characterstics of a linear programming model are explained below

For a given problem situation, there are certain essential conditions that need to be solved by using linear programming.

1. *Limited resources:* limited number of labor, material equipment and finance
2. *Objective:* refers to the aim to optimize (maximize the profits or minimize the costs).
3. *Linearity:* increase in labor input will have a proportionate increase in output.
4. *Homogeneity:* the products, workers' efficiency, and machines are assumed to be identical.
5. *Divisibility:* it is assumed that resources and products can be divided into fractions. (in case the fractions are not possible, like production of one-third of a computer, a modification of linear programming called integer programming can be used).

The following properties form the linear programming model:

1. Seek to minimize or maximize
2. Include "constraints" or limitations
3. There must be alternatives available
4. All equations are linear

FORMULATION OF LINEAR PROGRAMMING PROBLEM EXAMPLES

Formulation of linear programming is the representation of problem situation in a mathematical form. It involves well defined decision variables, with an objective function and set of constraints.

Objective function

The objective of the problem is identified and converted into a suitable objective function. The objective function represents the aim or goal of the system (i.e., decision variables) which has to be determined from the problem. Generally, the objective in most cases will be either to maximize resources or profits or, to minimize the cost or time.

For example, assume that a furniture manufacturer produces tables and chairs. If the manufacturer wants to maximize his profits, he has to determine the optimal quantity of tables and chairs to be produced.

Let

x_1= Optimal production of tables

P_1= Profit from each table sold

X_2= Optimal production of chairs

P_2= Profit from each chair sold.

Hence,

Total profit from tables =$P_1 X_1$

Total profit from chairs = $P_2 X_2$

The objective function is formulated as below,

Maximize Z or Z max = $P_1 X_1 + P_2 X_2$

Constraints

When the availability of resources is in surplus, there will be no problem in making decisions. But in real life, organizations normally have scarce resources within which the job has to be performed in the most effective way. Therefore, problem situations are within confined limits in which the optimal solution to the problem must be found.

Considering the previous example of furniture manufacturer, let w be the amount of wood available to produce tables and chairs. Each unit of table consumes w_1 unit of wood and each unit of chair consumes w_2 units of wood.

For the constraint of raw material availability, the mathematical expression is,

$w_1 x_1 + w_2 x_2 \leq w$

In addition to raw material, if other resources such as labor, machinery and time are also considered as constraint equations.

Non-negativity constraint

Negative values of physical quantities are impossible, like producing negative number of chairs, tables, etc., so it is necessary to include the element of non-negativity as a constraint

i.e., $x_1, x_2 \geq 0$

GENERAL LINEAR PROGRAMMING PROBLEM

A general representation of LP model is given as follows:

Maximize or Minimize, $Z = p_1 x_1 + p_2 x_2 \ldots\ldots\ldots\ldots\ldots p_n x_n$

Subject to constraints,

$w_{11} x_1 + w_{12} x_2 + \ldots\ldots\ldots\ldots\ldots w_{1n} x_n \leq$ or = or $\geq w_1 \ldots\ldots\ldots\ldots\ldots$(i)

$w_{21} x_1 + w_{22} x_2 \ldots\ldots\ldots\ldots\ldots w_{2n} x_n \leq$ or = or $\geq w_2 \ldots\ldots\ldots\ldots\ldots$(ii)

. .

. .

. .

$w_{m1} x_1 + w_{m2} x_2 + \ldots\ldots\ldots\ldots w_{mn} x_n \leq$ or $= \geq w_m \ldots\ldots\ldots\ldots$(iii)

Non-negativity constraint,

$x_i \geq o$ (where i = 1,2,3 .....n)

Example :1

A seafood processing unit plans to produce two types of products, one with a round shape and another with a square shape. The following resources are used in processing of shrimp product,

(i) Raw material, of which daily availability is 150 kg.

(ii) Machinery, of which daily availability is 25 machine hours.

(iii) Labor, of which daily availability is 40 man-hours.

The resources used are shown in Table. If the unit profit of round and square biscuits is Rs 3.00 and Rs 2.00 respectively, how many round and square biscuits should be produced to maximize total profit?

Resources Used

Resources	Requirement/Unit		Daily availability
	Round	Square	
Raw Material	100	115	1500 grams
Machine	10	12	720 minutes
Manpower	3	2	240 minutes

Solution:

Key Decision:

To determine the number of round and square product to be produced.

Decision Variables:

Let x_1 be the number of round product to be produced daily, and

x_2 be the number of square product to be produced daily

Objective function:

It is given that the profit on each unit of round product is Rs 3.00 and of square product is Rs. 2.00. The objective is to maximize profits, therefore, the total profit will be given by the equation,

$Zmax = 3x_1+2x_2$

Constraints:

Now, the manufacturing process is imposed by a constraint with the limited availability of raw material. For the production of round product, $100x_1$of raw material is used daily and for the production of square product, $115x_2$ of raw material is used daily.

It is given that the total availability of raw material per day is 1500 grams. Therefore, the constraint for raw material is,

$100x_1 + 115x_2 \leq 1500$

Similarly, the constraint for machine hours is,

$10x_1+12x \leq 720$

and for the manpower is, $3x_1+2x_2 \leq 240$

Since the resources are to be used within or below the daily available level, inequality sign of less than or equal sign (≤) is used. Further, we cannot produce negative number of units of product which is a non-negative constraint expressed as,

Thus, the linear programming model for the given problem is,

Maximize $Z = 3x_1 + 2x_2$

Subject to constraints,

$100x_1+115x_2 \leq 1500$ ....(i)

$10x_1+12x_2 \leq 720$ ....(ii)

$3x_1+2x_2 \leq 240$ ....(iii)

where $x_1 \geq 0$, $x_2 \geq 0$

Example : 2 An agricultural urea company must daily produce 500 kg of a mixture consisting of ingredients x_1, x_2 and x_3. Ingredient x_1 costs Rs. 30 per kg, x_2Rs. 50 per kg and x_3 Rs. 20 per kg. Due to raw material constraint, not more than 100 kg of x_1, 70 kg of x_2 and 45 kg of x_3 must be used. Determine how much of each ingredient should be used if the company wants to minimize the cost.

Solution:

Let

x_1 be the kg of ingredient x_1 to be used

x_2 be the kg of ingredient x_2 to be used

x_3 be the kg of ingredient x_3 to be used

The objective is to minimize the cost,

Minimize $Z = 30x_1 + 50x_2 + 20x_3$

Subject to constraints,

$x_1+ x_2+ x_3 = 500$ (total production) ....(i)

$x_1 \leq 100$ (max. use of x_1) ....(ii)

$x_2 \leq 70$ (max. use of x_2) ....(iii)

$x_3 \leq 45$ (max. use of x_3) ....(iv)

where

$x_1, x_2, x_3 \geq 0$ (non-negativity)

Example :3

Rose Perfume Company produces both perfumes and body spray from two flower extracts F1 and F2. The following data is provided:

Data Collected

	Litres of Extract		
	Perfume	Body Spray	Daily Availability (litres)
Flower Extract, F_1	8	4	20
Flower Extract, F_2	2	3	8
Profit Per litre (Rs.)	7	5	

The maximum daily demand of body spray is 20 bottles of 100 ml each. A market survey indicates that the daily demand of body spray cannot exceed that of perfume by more than 2 litres. The company wants to find out the optimal mix of perfume and body spray that maximizes the total daily profit. Formulate the problem as a linear programming model.

Solution:

Let

x_1 be the litres of perfume produced daily

x_2 be the litres of body spray produced daily

Objective function:

The company wants to increase the profit by optimal product mix

$$Zmax = 7x_1+5x_2$$

Constraints:

The total availability of flower extract F1 and flower extract F2 are 20 and 8 litres respectively. The sum of flower extract F1 used for perfume and body spray must not exceed 20 litres. Similarly, flower extract F2 must not exceed 8 litres daily.

The constraints are,

$8x_1+4x_2 \leq 20$ (Flower extract F1)

$2x_1+3x_2 \leq 8$ (Flower extract F2)

The daily demand of body spray x_2 is limited to 20 bottles of 100ml each (i.e, 20 * 100 = 2000 ml = 2 litres)

Therefore, $x_2 \leq 2$

Again, there is an additional restriction, that the difference between the daily production of perfume and body spray , $x_2 - x_1$ does not exceed 2 litres, which is expressed as

$x_2 - x_1 \leq 2$

(or)

$-x_1 + x_2 \leq 2$.

The model for Rose perfumes company is,

Maximize , $Z = 7x_1 + 5x_2$

Subject to constraints,

$8x_1 + 4x_2 \leq 20$ ....(i)

$2x_1 + 3x_2 \leq 8$ ....(ii)

$-x_1 + x_2 \leq 2$ ....(iii)

$x_2 \leq 2$ ....(iv)

where $x_1, x_2 \geq 0$

Feasible Solution:

Any values of x_1 and x_2 that satisfy all the constraints of the model constitute a feasible solution. For example, in the above problem if the values of $x_1 = 2$ and $x_2 = 1$ are substituted in the constraint equation, we get

(i) $8(2) + 4(1) \leq 20$

$20 \leq 20$

(ii) $2(2) + 3(1) \leq 8$

$7 \leq 8$

(iii) $-2 + 1 \leq 2$

$-1 \leq 2$

(iv) $1 \leq 2$

All the above constraints (including non-negativity constraint) are satisfied. The objective function for these values of x_1= 2 and x_2= 1, are

Zmax = 7(2) + 5(1)

= 14 + 5 = Rs. 19.00

As said earlier, all the values that do not violate the constraint equations are feasible solutions. But, the problem is to find out the values of x_1 and x_2 to obtain the optimum feasible solution that maximizes the profit. These optimum values of x_1 and x_2 can be found by using the Graphical Method or by Simplex Method.

Graphical method to solve Linear Programming problem (LPP) helps to visualize the procedure explicitly. It also helps to understand the different terminologies associated with the solution of LPP. Linear programming problems with two variables can be represented and solved graphically with ease. Though in real-life, the two variable problems are practiced very little, the interpretation of this method will help to understand the simplex method. The solution method of solving the problem through graphical method is discussed with an example given below.

Example 4:

A company manufactures two types of boxes, corrugated and ordinary cartons. The boxes undergo two major processes: cutting and pinning operations. The profits per unit are Rs. 6 and Rs. 4 respectively. Each corrugated box requires 2 minutes for cutting and 3 minutes for pinning operation, whereas each carton box requires 2 minutes for cutting and 1 minute for pinning. The available operating time is 120 minutes and 60 minutes for cutting and pinning machines. Determine the optimum quantities of the two boxes to maximize the profits.

Solution:

Key Decision:

To determine how many (number of) corrugated and carton boxes are to be manufactured.

Decision variables:

Let x_1 be the number of corrugated boxes to be manufactured.

x_2 be the number of carton boxes to be manufactured

Objective Function:

The objective is to maximize the profits. Given profits on corrugated box and carton box are Rs. 6 and Rs. 4 respectively.

The objective function is,

Zmax = $6x_1 + 4x_2$

Constraints:

The available machine-hours for each machine and the time consumed by each product are given.

Therefore, the constraints are,

$2x_1 + 3x_2 \leq 120$ ....(i)

$2x_1 + x_2 \leq 60$ ....(ii)

where $x_1, x_2 \geq 0$

Graphical Solution:

As a first step, the inequality constraints are removed by replacing 'equal to' sign to give the following equations:

$2x_1 + 3x_2 = 120$ ....(1)

$2x_1 + x_2 = 60$ ....(2)

Find the co-ordinates of the lines by substituting $x_1 = 0$ and $x_2 = 0$ in each equation. In equation (1), put $x_1 = 0$ to get x_2 and vice versa

$2x_1 + 3x_2 = 120$

$2(0) + 3x_2 = 120, x_2 = 40$

Similarly, put $x_2 = 0$,

$2x_1 + 3x_2 = 120$

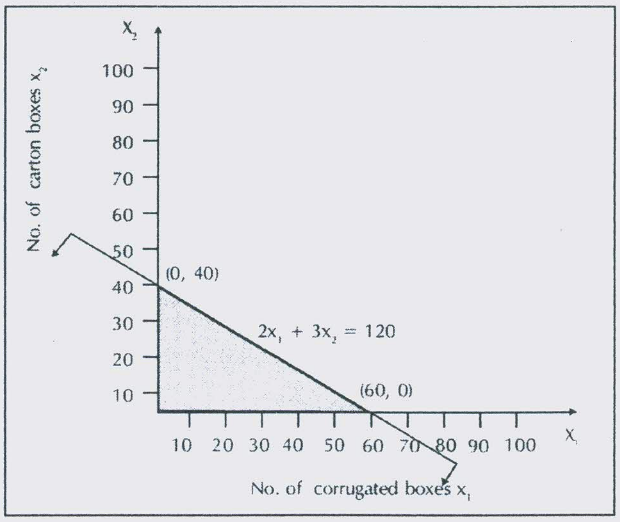
$2x_1 + 3(0) = 120, x_1 = 60$

The line $2x_1 + 3x_2 = 120$ passes through co-ordinates (0, 40) (60, 0).

The line $2x_1 + x_2 = 60$ passes through co-ordinates (0, 60) (30, 0).

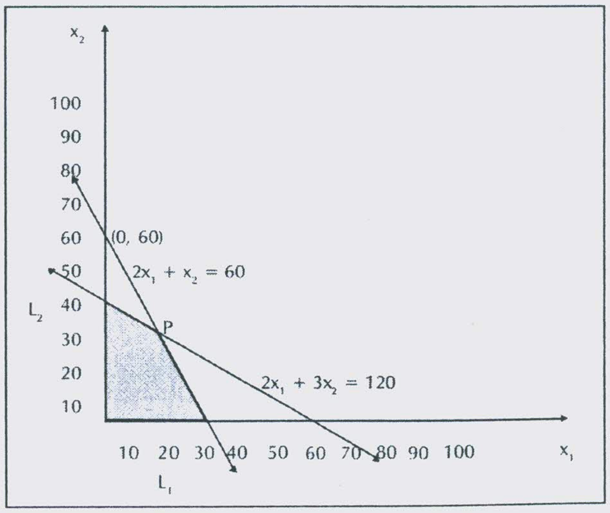
The lines are drawn on a graph with horizontal and vertical axis representing boxes x_1 and x_2 respectively. Figure shows the first line plotted.

Graph Considering First Constraint



The inequality constraint of the first line is (less than or equal to) ≤ type which means the feasible solution zone lies towards the origin. The no shaded portion can be seen is the feasible area shown in Figure (Note: If the constraint type is ≥ then the solution zone area lies away from the origin in the opposite direction). Now the second constraints line is drawn.

Graph Showing Feasible Area



When the second constraint is drawn, you may notice that a portion of feasible area is cut. This indicates that while considering both the constraints, the feasible region gets reduced further. Now any point in the shaded portion will satisfy the constraint equations. For example, let the solution point be (15,20) which lies in the feasible region. If the points are substituted in all the equations, it should satisfy the conditions.

$2x_1 + 3x_2 \leq 120 = 30 + 60 \leq 120 = 90 \leq 120$

$2x_1 + x_2 \leq 60 = 30 + 20 \leq 60 = 50 \leq 60$

Now, the objective is to maximize the profit. The point that lies at the furthermost point of the feasible area will give the maximum profit. To locate the point, we need to plot the objective function (profit) line.

Equate the objective function for any specific profit value Z,

Consider a Z-value of 60, i.e.,

$6x_1 + 4x_2 = 60$

Substituting $x_1 = 0$, we get $x_2 = 15$ and

if $x_2 = 0$, then $x_1 = 10$

Therefore, the co-ordinates for the objective function line are (0, 15), (10, 0) as indicated by dotted line L1 in Figure. The objective function line contains all possible combinations of values of xl and x_2.

The line L1 does not give the maximum profit because the furthermost point of the feasible area lies above the line L1. Move the line (parallel to line L1) away from the origin to locate the furthermost point. The point P, is the furthermost point, since no area is seen further. Take the corresponding values of $x_1$and x_2 from point P, which is 15 and 30 respectively, and are the optimum feasible values of x_1 and x_2.

Therefore, we conclude that to maximize profit, 15 numbers of corrugated boxes and 30 numbers of carton boxes should be produced to get a maximum profit. Substituting $x_1 = 15$ and $x_2 = 30$ in objective function, we get

$Zmax = 6x_1 + 4x_2$

$= 6(15) + 4(30)$

Maximum profit: Rs. 210.00

SUMMARY OF GRAPHICAL METHOD IN QUANTITATIVE TECHNIQUES

Step 1: Convert the inequality constraint as equations and find co-ordinates of the line.

Step 2: Plot the lines on the graph.

(***Note:*** If the constraint is ≥ type, then the solution zone lies away from the centre. If the constraint is ≤ type, then solution zone is towards the centre.)

Step 3:

Obtain the feasible zone.

Step 4:

Find the co-ordinates of the objectives function (profit line) and plot it on the graph representing it with a dotted line.

Step 5:

Locate the solution point.

(***Note:*** If the given problem is maximization, Zmax then locate the solution point at the far most point of the feasible zone from the origin and if minimization, Zmin then locate the solution at the shortest point of the solution zone from the origin).

***Step 6:* Solution type**

i. If the solution point is a single point on the line, take the corresponding values of x_1 and x_2.

ii. If the solution point lies at the intersection of two equations, then solve for x_1 and x_2 using the two equations.

iii. If the solution appears as a small line, then a multiple solution exists.

iv. If the solution has no confined boundary, the solution is said to be an unbound solution.

LIMITATIONS OF GRAPHICAL METHOD IN LINEAR PROGRAMMING

- Linear programming is applicable only to problems where the constraints and objective function are linear i.e., where they can be expressed as equations which represent. In real life situations, when constraints or objective functions are not linear, this technique cannot be used.
- Factors such as uncertainty, weather conditions etc. are not taken into consideration.
- There may not be an integer as the solution, e.g., the number of men required may be a fraction and the nearest integer may not be the optimal solution.i.e., Linear programming technique may give practical valued answer which is not desirable.
- Only one single objective is dealt with while in real life situations, problems come with multi-objectives.
- Parameters are assumed to be constants but in reality they may not be so

Further readings:

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