



Numerical calculation of psychrometric properties on a calculator

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Abstract

Simple and precise equations for calculation of saturation vapour pressure in three different temperature ranges are presented in the paper. Input parameters required for calculation of psychrometric properties are wet- and dry-bulb temperatures, dew point and dry-bulb temperatures and relative humidity and dry-bulb temperature. In this paper, the equations used for computing saturation vapour pressure may also be used for determining dew point temperature for the entire pressure range. © 2002 Published by Elsevier Science Ltd.

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1. Introduction

Knowledge of psychrometric properties is the basic requirement for environmental measurements and drying processes. An understanding of physical and thermodynamic properties of an air–water vapour mixture is called psychrometrics. It is fundamental to the design of environmental control systems for plants, crops, animals and human beings. The equations presented in the paper enable an engineer to calculate all or some of the psychrometric properties, if any two independent psychrometric properties of an air–water vapour mixture are known in addition to the atmospheric pressure. Generally, one may have the values of dry-bulb temperature and another psychrometric property such as, wet-bulb temperature, relative humidity or dew point temperature. From these parameters, one can easily determine other psychrometric properties such as, relative humidity, enthalpy, humidity ratio, specific volume or vapour pressure. The determination of wet-bulb temperature is somewhat difficult. Either it is solved by trial or the secant method (Conte, Elementary Numerical Analysis, McGraw-Hill, New York, 1965) can be used, which is cumbersome. Computational problems may occur with this method at temperatures of 100°C and above (Wilhelm, Numerical calculation of

psychrometric properties in SI units, ASAE, Paper No. 75-4019, 1976). Hence, there is a need to develop a simple method for the determination of the wet-bulb temperature.

2. Equations for the determination of psychrometric properties

2.1. Saturation vapour pressure

The water vapour saturation pressure is an essential part of psychrometric calculations. Although a number of formulas for computing vapour pressure are available, only accurate relationships should be used. The calculation of vapour pressure should be accurate as it is used in determining other psychrometric properties. Several formulas for different temperature ranges were evaluated to find the most suitable and accurate formulas for this analysis. They are very complex and lengthy. However, vapour pressure was calculated by using the equation given by Dille [1] which is simple and precise. His equation for computing saturation vapour pressure in Pascal (Pa) was found to be accurate for the temperature range 0–63°C. However, it required slight modification for other temperature ranges as given below:

$$P_{wsT} = 610.78 \exp \left[\frac{17.269 T}{237.3 + T} \right] \quad (0^\circ\text{C} < T < 63.0^\circ\text{C})$$

$$\text{and } (610.78 \text{ Pa} < P_{wsT} \text{ or } P_w < 22870.52 \text{ Pa}). \quad (1a)$$

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Nomenclature

T	temperature, °C	h	enthalpy, J g ⁻¹
v	specific volume, m ³ kg ⁻¹	C_p	specific heat at constant pressure, J g ⁻¹ °C ⁻¹
γ	relative humidity, fraction		
R_a	gas constant for air	<i>Subscript</i>	
P	atmospheric pressure, Pa	a	dry air
P	partial pressure, Pa	wb	wet-bulb
h_{fg}	latent heat of vapourization, J kg ⁻¹	dp	dew point
B	psychrometric constant	w	condition for water vapour
Δ	slope of saturation pressure line,	s	condition at saturation
W	humidity ratio		

For temperatures above 63°C, the following equation is used:

$$P_{wsT} = 610.78 \exp \left[\frac{17.269T}{236.3 + 1.01585T} \right]$$

$$(63^\circ\text{C} < T < 110^\circ\text{C})$$

$$\text{and } (22870.52 \text{ Pa} < P_{wsT} \text{ or } P_w < 143292.97 \text{ Pa}).$$

(1b)

For temperatures below 0°C, the following relationship is used:

$$P_{wsT} = 610.78 \exp \left[\frac{21.874T}{265 + 0.9615T} \right]$$

$$(0^\circ\text{C} > T > -40^\circ\text{C})$$

$$\text{and } (610.78 \text{ Pa} > P_{wsT} \text{ or } P_w > 12.838 \text{ Pa}). \quad (1c)$$

The above equations calculate not only saturation vapour pressure as a function of temperature but also temperature as a function of saturation vapour pressure. These equations [1(a)–(c)] should not be used outside the stated temperature or pressure ranges. The relationship is within 0.067 per cent of the correct values for the temperature range 0–110°C and 0.175 per cent of the values given by Wilhelm for the temperature range 0°C to –40°C.

By using the perfect gas relationship and definitions, the following relationships can be developed.

2.2. Humidity ratio

Weight of the water vapour in kg/kg of dry air is given as

$$W = \frac{\text{Mass of water vapour } (m_w)}{\text{Mass of dry air } (m_a)}$$

$$= \frac{(PV/RT)_w}{(PV/RT)_a}$$

$$= \frac{P_w R_a}{P_a R_w} = \frac{(R_a/R_w)P_w}{(P - P_w)}$$

(the values of V and T are same for water vapour and dry air)

$$= \frac{0.62198P_w}{P - P_w}, \quad (2)$$

where, (R_a/R_w) is the ratio of mole masses of air and water (0.62198).

Eq. (2) can be rewritten for actual vapour pressure

$$P_w = \frac{PW}{0.62198 + W} \quad (3)$$

2.3. Specific volume

Space occupied by a given mass of an air–water vapour mixture expressed as m³ of mixture per kg of dry air.

$$P_a V = m_a R_a (T_a + 273.16)$$

$$v = \frac{V}{m_a} = \frac{R_a (T_a + 273.16)}{P_a} = \frac{287.05(T_a + 273.16)}{P - P_w}. \quad (4a)$$

On substituting the value of P_w from Eq. (3) into Eq. 4(a) we get,

$$\text{Specific volume} = \frac{287.05(T_a + 273.16)}{P_a} \left(1 + \frac{W}{0.62198} \right). \quad (4b)$$

2.4. Relative humidity

Ratio of actual vapour pressure to saturated vapour pressure at the same temperature is given as

$$\gamma = \frac{P_w}{P_{wsT_a}}. \quad (5)$$

2.5. Dew point temperature

The dew point temperature can be calculated by using saturation vapour pressure equations 1(a)–(c) and putting the value of actual vapour pressure (P_w) or saturation vapour pressure at dew point temperature. In this instance, the above three equations are required for the entire pressure ranges. The salient feature of the saturation vapour pressure equations is that the same equations can be used for computing dew point temperature.

2.6. Wet-bulb temperature

The wet-bulb temperature can be estimated from the following well-known psychrometric relation:

$$P_w = P_{sT_{wb}} - B(T_a - T_w) \quad (6)$$

and the approximation

$$P_{sT_{wb}} - P_w = \Delta(T_{wb} - T_{dp}). \quad (7)$$

From Eqs. (6) and (7) we have

$$B(T_a - T_{wb}) = \Delta(T_{wb} - T_{dp}).$$

Or

$$T_w = \frac{BT_a + \Delta T_{dp}}{B + \Delta}, \quad (8)$$

where B is the thermodynamic value of the psychrometric constant given as

$$B = \frac{1006.925[P - P_s(T_{wb})](1 + (0.15557/P)P_w)}{0.62194 h_{fg}} \quad (9)$$

and

$$h_{fg} = [3161.36 - 2.406(T_{wb} + 273.16)]1000. \quad (10)$$

2.7. Wet-bulb depression

This can be approximated as,

$$W_{bd} = \frac{0.1533T_a + 5.1}{0.9}(1 - \gamma) \quad (\text{for } T_a, 0\text{--}30^\circ\text{C}) \quad (11a)$$

and

$$W_{bd} = \frac{0.4355T_a + 5.1}{0.9}(1 - \gamma) \quad (\text{for } T_a, 0\text{--}110^\circ\text{C}) \quad (11b)$$

$$T_{wb1} = T_a - W_{bd}$$

If $T_{wb1} < T_{dp}$, T_{wb1} is taken as $(T_{dp} + 0.5)$.

Δ can be estimated from Eq. (7) by substituting T_{wb1} for T_{wb} , which is given as

$$\Delta = \frac{P_s T_{wb1} - P_w}{T_{wb1} - T_{dp}}. \quad (12)$$

By substituting the values of B and Δ into Eq. (8), T_w can be estimated. T_w becomes T_{wb1} for the next step for which B and Δ are estimated again and substituted into Eq. (8). The process continues till we get a constant value of T_w .

2.8. Enthalpy

It is the heat energy content of an air–water vapour mixture

The enthalpy of moist air can be given as [2]

$$h = 1.006T_a + W(2501 + 1.77T_a) \quad (-50^\circ\text{C} \leq T_a \leq 110^\circ\text{C}). \quad (13)$$

3. Application of equations

The above equations from (1) to (13) are used for the determination of psychrometric properties, if any two psychrometric properties of an air–water mixture are known. (see Table 1).

- (1) Dry- and wet-bulb temperature, or
- (2) dry-bulb and dew point temperature, or
- (3) dry-bulb temperature and relative humidity.

Thus,

Case(1): When T_a and T_w are known:

- (1) P_{wsT_a} and $P_{sT_{wb}}$ can be calculated by Eq. 1(a), (b) or (c).
- (2) P_w using Eqs. (6), (9) and (10).
- (3) W using Eq. (2) and P_w .
- (4) T_{dp} using Eq. 1(a), (b) or (c) and P_w .
- (5) v using Eq. 4(a) or (b).
- (6) h using Eq. (13).
- (7) γ using Eq. (5).

Case(2): When T_a and γ are known:

- (1) P_{wsT_a} can be calculated by Eq. 1(a), (b) or (c).
- (2) P_w using Eq. (5).
- (3) W using Eq. (2) and P_w .
- (4) T_{dp} using Eq. 1(a), (b) or (c) and P_w .
- (5) v using Eq. 4(a) or (b).
- (6) h using Eq. (13).
- (7) γ using Eq. (5).

Case(3): When T_a and T_{dp} are known:

- (1) P_{wsT_a} and P_w can be calculated by Eq. 1(a), (b) or (c).
- (2) W using Eq. (2) and P_w .
- (3) v using Eq. 4(a) or (b).
- (4) h using Eq. (13).
- (5) γ using Eq. (5).
- (6) T_{wb} as per given procedure.

3.1. Accuracy and limitations

The accuracy of the calculated values of dew point temperature depends on the limits of temperature and vapour pressure. Within the limits specified Eqs. 1(a), (b) or (c) compute the accurate values of dew point temperature, the maximum deviation being only 0.08°C . But if the calculations are to be made outside the stated limits, the above

Table 1
Comparison of computed psychrometric properties with tabulated values

Source	T_a	T_w	T_{dp}	γ	W	v
Table ^a	-30	b	-40	0.338	0.0000793	0.689
Wet-bulb	-30	-30.37	-39.92	0.342	0.0000796	0.689
Humidity	-30	-30.37	-40.01	0.338	0.0000787	0.689
Dew point	-30	-30.37	-40.01	0.338	0.0000786	0.689
Table ^a	-10	b	-17.75	0.492	0.000783	0.746
Wet-bulb	-10	-11.53	-17.75	0.491	0.000783	0.746
Humidity	-10	-11.53	-17.75	0.491	0.000783	0.746
Dew point	-10	-11.53	-17.75	0.491	0.000783	0.746
Table ^a	0	b	-1.47	0.886	0.00335	0.777
Wet-bulb	0	-0.61	-1.47	0.886	0.00334	0.778
Humidity	0	-0.61	-1.47	0.886	0.00334	0.778
Dew point	0	-0.61	-1.47	0.886	0.00334	0.778
Table ^a	20	b	-9.50	0.116	0.00168	0.832
Wet-bulb	20	7.87	-9.50	0.116	0.00168	0.832
Humidity	20	7.89	-9.50	0.116	0.00168	0.832
Dew point	20	7.89	-9.50	0.116	0.00168	0.832
Table ^a	20	b	18.03	0.884	0.130	0.848
Wet-bulb	20	18.68	18.03	0.884	0.130	0.848
Humidity	20	18.68	18.03	0.884	0.130	0.848
Dew point	20	18.68	18.03	0.884	0.130	0.848
Table ^a	35	b	-0.35	0.106	0.00368	0.878
Wet-bulb	35	16.10	-0.32	0.106	0.00367	0.878
Humidity	35	16.10	-0.32	0.106	0.00367	0.878
Dew point	35	16.10	-0.32	0.106	0.00367	0.878
Table ^a	35	b	32.97	0.893	0.0326	0.918
Wet-bulb	35	33.35	32.97	0.893	0.0325	0.919
Humidity	35	33.35	32.97	0.893	0.0325	0.919
Dew point	35	33.35	32.97	0.893	0.0325	0.919
Table ^a	70	b	29.59	0.133	0.0266	1.01
Wet-bulb	70	36.96	29.63	0.133	0.0266	1.014
Humidity	70	36.94	29.63	0.133	0.0266	1.014
Dew point	70	36.94	29.63	0.133	0.0266	1.014
Table ^a	70	b	68.85	0.951	0.260	1.38
Wet-bulb	70	68.89	68.84	0.951	0.260	1.38
Humidity	70	68.88	68.84	0.951	0.260	1.38
Dew point	70	68.88	68.84	0.951	0.260	1.38
Table ^a	100	b	46.00	0.100	0.0692	1.17 ^c
Wet-bulb	100	51.19	46.08	0.100	0.0691	1.175
Humidity	100	51.19	46.08	0.100	0.0691	1.175
Dew point	100	51.19	46.08	0.100	0.0691	1.175
Table ^a	110	b	90.00	0.489	1.4 ^c	3.6 ^c
Wet-bulb	110	90.15	90.01	0.489	1.395	3.52
Humidity	110	90.17	90.01	0.489	1.395	3.52
Dew point	110	90.17	90.01	0.489	1.395	3.52

^aTabulated values are same as those used by Wilhelm (1976).

^bWet-bulb values are not determined.

^cTaken from psychrometric chart.

equations need to be reformulated. Eq. 1(b) has the accuracy of 0.067 per cent of the correct values of saturation vapour pressure for the temperature range (63–110°C). The errors go on accumulating, if we go for higher temperatures, i.e., 0.075 per cent for 115°C and 0.125 per cent for 120°C. Similarly, for the temperature below freezing (0°C to -20°C), Eq. 1(c) gives the values of saturation vapour pressure within 0.072 per cent of the values given by Wilhelm and 0.175 per cent for the temperature range 0°C to -40°C. There are other formulas available in the literature for calculating saturation vapour pressure [3,4,2]. But the

formula given by Fernandez and Chargoy is not accurate for large range of temperatures and those given by Keenan and Keyes and Wilhelm are lengthy.

4. Summary

The paper presents three equations for the calculation of saturation vapour pressure in three different temperature ranges 0°C to -40°C, 0°C to 63°C and 63°C to 110°C. The same equations also calculate dew point temperature. The

relationships proposed here are valid from -40°C to 110°C . Outside the stated limits, the equations need to be reformulated. The paper also defines various physical and thermodynamic properties of air–water mixture and determines their values by direct calculations.

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