çfro'kh ÁHkkoksa dh mifLFkfr esa fo"ke fjLi,UI ljQs l vfHkdYiuk,¡

vfdrk oekZ] lhek tXxh] fluh oxhZl] vi.k Hkkfed*] vufnrk nRrk ,oa ,Yn® oxhZt[1]

Hkk-—vu-i- dæh; leæh eRL; vul/kku lLFkku] dkfPp-682 018] djy

ikIrh viy 2020 Lohdrh tykbZ 2020

lkjka'k

iLrr y[ks e] fudVorh bdkbZ;ksa l çfro'kh çHkkoksa dk lfEefyr dju oky fo"ke fjLi,UI ljQs e,My ij v/;;u fd;k x;k gA çfro'kh çHkkoksa d lkFk $2^n \times 3$ Q,e d f}rh; v,Mj fo"ke fjLi,UI ljQs d fy, i)fr fodflr dh xbZ gA ekinMk ¼ijkfeVjl½ d leku Álj.k ,oa fudV&v,Fkkxkuy vkdyu d fy, vko';d Áfrc/kksa çkIr fd; x; gA $2^n \times 3$ d fy, ucj bQDVl d lkFk f}rh; v,Mj ,flefVdy jkVVcy fMtkbu (SOARDNE) cuku dh fof/k fodflr dh xbZ gA

'kCn dqath%

Asymmetrical response surface designs in the presence of neighbour effects

Ankita Verma, Seema Jaggi, Cini Varghese Arpan Bhowmik*,
Anindita Datta and Eldho Varghese[1]

ICAR-Central Marine Fisheries Research Institute, Kochi - 682 018, Kerala

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ABSTRACT

In this article, we have discussed a general methodology for constructing Asymmetrical Response Surface Designs in the presence of Neighbour Effects. Methodology of form $2^n \times 3$ has been derived. Conditions have been derived for the near orthogonal estimation of coefficients of response model. Further, conditions for rotatability under these models have also been obtained. A method of constructing asymmetrical response surface designs i.e.$2^n \times 3$ for fitting second order response surface in the presence of neighbour effects has been developed.

Key words:

çLrkouk

—f"k v/;;u e db ,lh ifjfLFkfr;ksa g¨rh g tgk mí'; dÃ0;k[;kRed pj vkj ,d ;k ,d l vfèkd vkIr nr dk çfrfØ;k pj/pj® d chp lcèk dk fuèkkfjr djuk g¨rk gA dk;ç.kkyh tk lokZ"Ke çfrfØ;k çkIr dju e enn djrh g ml çfrfØ;k lrg dk;ç.kkyh ¼RSM½ dgk tkrk gA bu rjhdk dk mi;kx çfrfØ;k vkj çfrfØ;k dk çHkkfor dju oky dkjdk d chp lcèkk dh tkp dju d fy, fd;k tkrk g, tl vfèkdre mit vkj U;ure ykxr d® çkIr dju d fy, dkjd Lrjk dk lcl vPNk lV [kktukA vkj-,l-,e-dk mi;kx fofHkUu xkfQdy] lkf[;dh; vkj xf.krh; rduhdk dk mi;kx djd fdlh çfØ;k dk fodflr dju] lèkkju ;k vudfyr dju d fy, fd;k tkrk gA

vDlj] vkj-,l-,e- e çfrfØ;k pj vkj Lor= pj d chp d lcèk dk :i Kkr ugÈ g¨rk g vkj blfy, bl çk;kfxd {k= d Hkhrj] pj e mi;ä fMxh d cgin }kjk çkIr fd;k tkrk gA cgin e,My tk i;kIr :i l lgh ek=k&çfrfØ;k lcèk dk çfrfufèkRo djr g] mUg çfrfØ;k lrg ¼fjLi,UI ljQl½ e,My dgk tkrk gA fMtkbu tk çfrfØ;k lrgk dh fQ;Vx dh vuefr nr g vkj mudh i;kIrrk d ijh{k.k d fy, ,d mik; çnku djr g] mUg çfrfØ;k lrg fMtkbu dgk tkrk gA vkj-,l-,e- ij fooj.k d fy,] c,Dl vkj Mij ¼1987½ [kjh vkj d,uy ¼1996½ vkj [kjh ¼2017½ dk mYy[k dj ldr gA

çfrfØ;k lrg ç;kxk e vkerkj ij ;g ekuk tkrk g fd voykdu Lor= g vkj vkIUu iMkh bdkbZ;k e ykx mipkjk l çHkkfor ugÈ gA yfdu ,lh fLFkfr e

Corresponding author's E-mail: arpan.stat@gmail.com
[1]ICAR-Indian Agricultural Statistics Research Institute, New Delhi-110 012;
1Hkk-—vu-i- - Hkkjrh; —f"k lkf[;dh vul/kku lLFkku] ub fnYyh - 110 012

tc bdkb;k dk ,d jf[kd rjhd l cgr djhc 0;ofLFkr fd;k tkrk g] çk;kfxd bdkb;k vkl Vu bdkb;k@ Hk[kMk ij ykx mipkjk l iMkl h çHkkok dk vuHko dj ldrh g ¼Mij ,o xVeu] 1980; tXxh ,V-,y-] 2010; oxht ,V-,y-] 2016; lkfjdk ,V-,y-] 2009½A mnkgj.k d fy,] ,d {k= ç;kx e ;fn ,d Hk[kM ij ,d jkl k;fud mipkj ykx fd;k tkrk g] rk iou d cgko d dkj.k jkl k;fud mipkj dk çHkko iMkl h Hk[kMk ij Qy ldrk g ;k feêh d r;kj gku l ,d Hk[kM l fu"Qy feêh fefJr gk ldrh g vxy Hk[kM l xj&ck> feêh d lkFkA blfy,] ;g è;ku j[kuk vko';d g fd fdl h fo'k"k Hk[kM l çkIr çfrfØ;k u doy ml Hk[kM ij ykx mipkj l;ktu l çHkkfor gkrh g] cfYd iMkl h Hk[kMk ij ykx mipkj l;ktuk }kjk Hkh çHkkfor gkrh gA vDlj] ,l çHkkok dk de dju d fy, lkoèkkfu;k cjrh tkrh gA ;g v/;;u l Hko iMkl h fgrk dk çfrfufèkRo dju d rjhdk dh ryk'k dju vkj ;g tkpu d fy, g fd D;k okLro e] o l Hkh ektn gA blfy,] e,My e iMkl h bdkb;k d çHkko dk 'kkfey djuk egRoi.k g rkfd çfrfØ;k dk vfèkd lVhd vueku yxk;k tk ldA

lkexh ,oa ijh{k.k fof/k

fuEufyf[kr ,d lkekU; çfrfØ;k e,My g ft le rRdky ckb vkj nkb iMkl h bdkb;k l çHkko 'kkfey g%

$$y_{u'}=\sum_{u=1}^{N} g_{uu'} f(x_u)+e_{u'},\ u'=1,2,...,N \qquad (1)$$

;gk $g_{uu'}=1$, ;fn $u=u'$

$=\alpha$, $|\alpha|<1$, ;fn $|u-u'|=1$

= 0, vU;Fkk

$X_1, X_2, ..., X_v$ Lor= pj] y çfrfØ;k pj vkj N dy voykdu gA ;gk α] ck, vkj nk, iMkl h bdkb;k l iMkl h çHkko dk çfrfuf/kRo djrk g vkj 0 l 1 rd dh bldh lhek, gA $f(x_u)$ og QD'ku g ft ld }kjk çfrfØ;k vkj buiV pj lcf/kr gA

;gk ij n dkjdk d¨ Lrj nk ij vkj ,d dkjd d¨ Lrj rhu ij] ;kuh ,d $2^n \times 3$ QDV®fj;y] ,d fo"ke çfrfØ;k lrg ij fopkj fd;k x;k g A ekuk tku okyk e,My bVjD'ku dh 'krk d fcuk ,d nljk v,Mj dk e,My gA bldk :i bl çdkj g%

$$f(x_u)=\beta_0+\sum_{i=1}^{n+1}\beta_i x_i+\beta_{n+1,n+1}x_{n+1}^2$$

β_0, β_i ¼n+1 dkjdk d jf[kd çHkko½ vkj $\beta_{n+1,n+1}$ [¼n+1½o dkjd dk f}?kkr çHkko] vuekfur fd, tku oky ijkeHkVj gA

e,My (1) d¨ fuEufyf[kr efVDl Qke e fy[k tk ldrk g%

$$\mathbf{Y}=\mathbf{GX}\boldsymbol{\beta}+\mathbf{e},$$

G ,d $N\times(N+2)$ dk ucj efVDl g] **X** ,d $(N+2)\times(n+3)$ fMtkbu efVDl g] $\boldsymbol{\beta}$,d $(n+3)\times 1$ ijkeHkVj oDVj ,o **e** ,d $N\times 1$ $N(0,\sigma^2\mathbf{I})$,jj oDVj g A

bl ifjfLFkfr d fy, fMtkbu efVDl bl Ádkj g

$$\mathbf{X}=\begin{bmatrix}
1 & x_{1N} & x_{2N} & \cdots & x_{iN} & \cdots & x_{nN} & x_{n+1N} & x_{n+1N}^2\\
1 & x_{11} & x_{21} & \cdots & x_{i1} & \cdots & x_{n1} & x_{n+1,1} & x_{n+1,1}^2\\
1 & x_{12} & x_{22} & \cdots & x_{i2} & \cdots & x_{n2} & x_{n+1,2} & x_{n+1,2}^2\\
\vdots & \vdots & \vdots & & \vdots & & \vdots & \vdots & \vdots\\
\vdots & \vdots & \vdots & & \vdots & & \vdots & \vdots & \vdots\\
\vdots & \vdots & \vdots & & \vdots & & \vdots & \vdots & \vdots\\
1 & x_{1i} & x_{2i} & \cdots & x_{ii} & \cdots & x_{ni} & x_{n+1,i} & x_{n+1,i}^2\\
\vdots & \vdots & \vdots & & \vdots & & & \vdots & \vdots\\
\vdots & \vdots & \vdots & & \vdots & & & \vdots & \vdots\\
\vdots & \vdots & \vdots & & \vdots & & & \vdots & \vdots\\
1 & x_{1N} & x_{2N} & \cdots & x_{iN} & \cdots & x_{nN} & x_{n+1N} & x_{n+1N}^2\\
1 & x_{11} & x_{21} & \cdots & x_{i1} & \cdots & x_{n1} & x_{n+1,1} & x_{n+1,1}^2
\end{bmatrix}$$

$$\hat{\boldsymbol{\beta}}=(\mathbf{Z}'\mathbf{Z})^{-1}\mathbf{Z}'\mathbf{Y}$$

;gk

$$\mathbf{Z}=\mathbf{GX},$$

$$D(\hat{\boldsymbol{\beta}})=\sigma^2(\mathbf{Z}'\mathbf{Z})^{-1}$$

çfrfØ;k e,My d x.kkdk d fudV v,Fkkxkuy vkdyu d fy, vkj Álj.k dh fLFkjrk d fy, vko';d 'kr çkIr dh xb gA bld vykok] bu e,Myk d j®VVcy gku dh fLFkfr Hkh çkIr dh xb gA

vuekfur fjLi,Ul dk Álj.k $\sum_{i=1}^{n} x_{io}^2$ ij fuHkj djrk gA

ijh.kke ,oa foospuk

fn, x, α d fy, og fcn ftl d fy, leku g] vuekfur fjLi,Ul dk Álj.k leku gkxkA ;fn fdlh fcn x ij vuekfur fjLi,Ul dk Álj.k doy fMtkbu dæ fcn l x dh njh ij fuHkj djrk g] rk fMtkbu j@VVcy gkrk gA bl x.k dk lr"V dju oky fMtkbu dk ;gk f}rh; v,Mj ,flefVdy fjLi,Ul ljQl fMtkbu (SOARDNE) c@yk x;k g A

$$\mathbf{X}_{26\times 5} = \begin{bmatrix} 1 & -1 & -1 & -1 & 1 \\ 1 & 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 0 & 0 \\ 1 & 1 & 1 & -1 & 1 \\ 1 & 1 & -1 & 1 & 1 \\ 1 & 1 & -1 & 0 & 0 \\ 1 & 1 & -1 & -1 & 1 \\ 1 & -1 & 1 & 1 & 1 \\ 1 & -1 & 1 & 0 & 0 \\ 1 & -1 & 1 & -1 & 1 \\ 1 & -1 & -1 & 1 & 1 \\ 1 & -1 & -1 & 0 & 0 \\ 1 & -1 & -1 & -1 & 1 \\ 1 & 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 0 & 0 \\ 1 & 1 & 1 & -1 & 1 \\ 1 & -1 & 1 & 1 & 1 \\ 1 & -1 & 1 & 0 & 0 \\ 1 & -1 & 1 & -1 & 1 \\ 1 & 1 & -1 & 1 & 1 \\ 1 & 1 & -1 & 0 & 0 \\ 1 & 1 & -1 & -1 & 1 \\ 1 & -1 & -1 & 1 & 1 \\ 1 & -1 & -1 & 0 & 0 \\ 1 & -1 & -1 & -1 & 1 \\ 1 & 1 & 1 & 1 & 1 \end{bmatrix}$$

$2^n\times 3$ SOARDNE d fuek.k dh fof/k% $2^n\times 3$ l;ktu dk yDlkxkfQd Øe ;k fjol yDlkxkfQd Øe e0;ofLFkr djA ¼n-1½½$2^n\times 3$½ vfrfjDr l;ktu ¼ju½ 2^n xV d LrHkk dk ?keku l çkIr djA çfro'kh çHkkok d fy, lhek fcn d :i e nk vfrfjä fcn tkM tkr gA ifj.kkeLo:i n¼$2^n\times 3$½ ju e ,d SOARDNE fMtkbu ÁkIr g@xkA vuekfur fd, tku oky ekinMk dh dy l[;k n+3 gA

$2^2\times 3$ d fy, SOARDNE: n ¾ 2 d ifj.kkeLo:i 3 dkjd ¼X_1, X_2 vkj X_3½ g] dkjd X_1 vkj X_2 n@Lrjk ij çR;d vkj X_3 rhu Lrjk ¼$2^2\times 3$½ ij A igy fjol yfDldkxkfQd Øe e0;ofLFkr i.k QDVkfj;y l 12 ju çkIr djr gA bld ckn] X_2 l lcf/kr nlj d,ye dh lkexh dk X_1 l lcf/kr igy d,ye d uhp vkj igy d,ye l lcf/kr lkexh dk nlj d,ye d uhp fy[kk x;k gA vr e] lhek ju tkM tkr gA çkIr SOARDNE 24 ju e j@VVcy gkxkA ikp d,ye $(1 x_1 x_2 x_3 x^2_3)$ vkj nk lhek fcnvk d lkFk 24 fcnvk d lkFk fMtkbu efVDl X bl çdkj g%

$V(\hat{\beta}_0) = 0.34375,\ \sigma^2 V(\hat{\beta}_1) = V(\hat{\beta}_2) = 0.0166,$
$V(\hat{\beta}_3) = 0.25,\ \sigma^2 V(\hat{\beta}_{33}) = 0.75$
$\mathrm{Cov}(\hat{\beta}_0, \hat{\beta}_{33}) = -0.5$

$V(\hat{y}_0) = 0.3771$ t¨ x e lHkh fcnvk d fy, leku gA

fu"d"kZ

fodflr dh xb lHkh vfHkdYiuk, vuekfur fjLi,Ul d Álj.k dh fLFkjrk d@ lfuf'pr djrh gA blfy, fMtkbu j@VVcy gA

$$\mathbf{Z'Z} = \begin{bmatrix} 96 & 0 & 0 & 0 & 64 \\ 0 & 60 & 0 & 0 & 0 \\ 0 & 0 & 60 & 0 & 0 \\ 0 & 0 & 0 & 4 & 0 \\ 64 & 0 & 0 & 0 & 44 \end{bmatrix} \qquad (\mathbf{Z'Z})^{-1} = \begin{bmatrix} 0.3437 & 0 & 0 & 0 & -0.5 \\ 0 & 0.0166 & 0 & 0 & 0 \\ 0 & 0 & 0.0166 & 0 & 0 \\ 0 & 0 & 0 & 0.25 & 0 \\ -0.5 & 0 & 0 & 0 & 0.75 \end{bmatrix}$$

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