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Estimation of Compound Growth Rates for Non-Monotonic Situations through Nonlinear Growth Models using WebECGR Package

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SUMMARY

Compound growth rate is widely employed in the field of Agriculture as it has important policy implications. Generally, this is computed by assuming that the path of response variable can be described by monotonically non-decreasing nonlinear growth models, like Malthus model and logistic model. However, in reality, data sets in agriculture need not always depict steady upward or downward movement over time, *i.e.* sometimes these are non-monotonic in nature. In such cases, it is not appropriate to employ above growth models. In this article, three nonlinear growth models, viz. over-damped, under-damped and critically-damped are considered, which have the capability to describe increasing and then decreasing or vice-versa type of behaviour of the response variable. The methodology to estimate compound growth rate by using these growth models is discussed. As it is very difficult to apply, an online user-friendly web-based application, viz. WebECGR package is developed. Finally, as an illustration, this package is employed for estimation of compound growth rate for India's total lentil production data during the period 1980-81 to 2010-11.

Keywords: Compound growth rate, Critically damped model, India's total lentil production, Non-monotonic situations, WebECGR package.

1. INTRODUCTION

Compound growth rate is a key indicator to measure agricultural growth and can be used for forecasting production, productivity, area, etc. of various commodities. This, in turn, plays a vital role in framing of optimal agricultural policies, like import and export policies for various agricultural commodities. The current methodology to compute compound growth rate, based on Malthusian law, was proposed by Panse (1964). However, it has several limitations, as discussed by Prajneshu and Chandran (2005). These authors also described the correct procedure based on more realistic parametric nonlinear growth models, viz. monomolecular, logistic and Gompertz models. However, unlike economic data, a peculiarity of data from agriculture is that response variable sometimes moves in both upward and downward directions, *i.e.* these are non-monotonic in nature. Accordingly, in such situations, conventional nonlinear growth models indicated above are not appropriate. However, an alternative class of nonlinear growth models based on a second-order homogeneous differential equation (Gilligan 1990) is able to capture satisfactorily the non-monotonic behavior. In the present investigation, procedure for estimation of compound growth rates in respect of such situations is thoroughly discussed for three possibilities, viz. over-damped, under-damped, and critically damped. As the underlying methodology is quite complicated to apply, WebECGR package which is a web solution for estimating compound growth rate is developed. Features and functionalities of the online application are described. As an illustration, using this package, compound growth rate is estimated for India's total lentil production during the period 1980-81 to 2010-11.

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2. NONLINEAR GROWTH MODELS FOR NON-MONOTONIC SITUATIONS

Let $y(t)$ denote the response variable at time t, *e.g.* agricultural production, productivity, or area. A class of nonlinear growth models (Gilligan 1990) expressible in terms of second-order differential equation (with constant coefficients) capable of describing increasing as well as decreasing behaviour is:

$$d^2y/dt^2 + 2\lambda dy/dt + \mu y = 0. \quad (1)$$

There are following three possibilities:

Case (i): $\lambda^2 > \mu$

The roots of the auxiliary equation $\alpha^2 + 2\lambda\alpha + \mu = 0$ are real and the solution of eq.(1) is the sum of two exponentials

$$y(t) = Be^{\alpha_1 t} + Ce^{\alpha_2 t}, \quad (2)$$

where B and C are arbitrary constants and $\alpha_{1,2}$ are rate parameters given by $\alpha_{1,2} = -\lambda \pm (\lambda^2 - \mu)^{1/2}$. If $\lambda > 0$, α_1 and α_2 are both negative. Differentiating eq. (2) with respect to t, we get

$$dy/dt = B\alpha_1 e^{\alpha_1 t} + C\alpha_2 e^{\alpha_2 t}. \quad (3)$$

Eq. (2) is called 'Over-damped model'.

Case (ii): $\lambda^2 < \mu$

The roots of the auxiliary equation are complex and

$$y(t) = (B \cos \beta t + C \sin \beta t)e^{-\lambda t} \quad (4)$$

where B and C are arbitrary constants and $|\beta| = |(\lambda^2 - \mu)^{1/2}|$. If $\lambda > 0$, there are decaying oscillations as $t \to \infty$. From eq. (4), differentiating y with respect to t, we get

$$dy/dt = e^{-\lambda t}[(C\beta - \lambda B) \cos(\beta t) - (\lambda C + B\beta) \sin(\beta t)]. \quad (5)$$

Eq. (4) is called 'Under-damped model'.

Case (iii): $\lambda^2 = \mu$

The two roots of the auxiliary equation are identical and

$$y(t) = (B + Ct)e^{-\lambda t}. \quad (6)$$

From eq. (6), differentiating y with respect to t, we get

$$dy/dt = e^{-\lambda t}[C - \lambda B - \lambda Ct]. \quad (7)$$

Eq. (6) is known as 'Critically damped model'.

The shapes of over-damped, under-damped, and critically damped models are exhibited in Fig. 1. It may be noted that these models have been proposed deterministically. In order to apply these to data, an

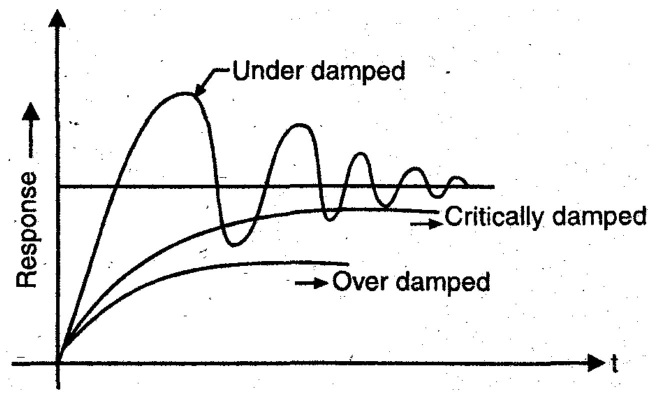

Fig. 1. Shapes of various models

error term is added on the right hand side of each one of these, thereby making these as 'Nonlinear statistical models' (Draper and Smith 1998). Thereafter, error term is assumed to be independently and identically distributed with equal variances. Nonlinear estimation procedures, like Gauss-Newton algorithm is required to be employed for fitting the models. Goodness of fit of fitted models is examined by computing Mean square error (MSE). Finally, for over-damped, under-damped and critically damped models, annual growth rates ($y^{-1}dy/dt$) pertaining to the period (t_i, t_{i+1}), $i = 0, 1, ..., n - 1$ where n denotes the number of data points are respectively computed using the following formulae:

$$R_t^{od} = \frac{B\alpha_1 e^{\alpha_1 t} + C\alpha_2 e^{\alpha_2 t}}{A + Be^{\alpha_1 t} + Ce^{\alpha_2 t}} \quad (8)$$

$$R_t^{ud} = \frac{(C\beta - \lambda B)\cos(\beta t) - (\lambda C + B\beta)\sin(\beta t)}{B\cos(\beta t) + C\sin(\beta t)} \quad (9)$$

$$R_t^{cd} = -\lambda + C/(B + Ct). \quad (10)$$

Taking arithmetic mean of the annual growth rates, requisite compound growth rate over a given time-period is estimated.

3. WebECGR PACKAGE

WebECGR package is a user-friendly web-based application for estimation of compound growth rate.

3.1 WebECGR Architecture and Requirements

This web-based solution is developed employing ASP.NET with .NET Framework (Walther *et al.* 2011). ASP.NET is a powerful and flexible technology for creating dynamic web pages. The statistical software *R for Windows 2.15.2* is integrated into .NET environment by using noncommercial version of *statconnDCOM* as shown in Fig. 2. For embedding *R* into .NET, the DCOM technology (Baier and Neuwirth 2007) is used.

WebECGR is hosted from the server having Internet Information Services (IIS) installed in it. The other prerequisites for running this web application include *R* software, *statconnDCOM* Server and .NET Framework, Version 4.0. The only requirement at the client side is a webbrowser. WebECGR package can be accessed over the internet through the web address *http://iasri.res.in/cgr*.

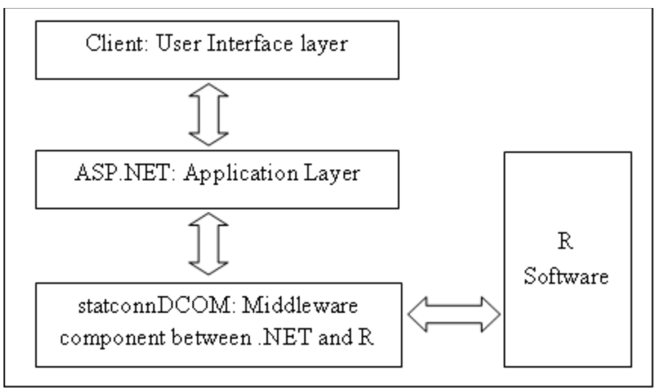

Fig. 2. WebECGR architecture

3.2 WebECGR Functionalities

In WebECGR, *R* functions (Ritz and Streibig 2008) are employed and entangled into .NET environment using *C#* programming language for statistical computations. The online application has the following components: Home, About WebECGR, Analyze, Help, Sample Data and Contact Us. 'About WebECGR' page contains the description about this web solution. 'Help' section guides users for preparing input data file as well as estimating compound growth rates by providing step-by-step procedures. Users can download sample data for estimating compound growth rates based on datasets in the 'Sample Data' page. Users may send email to the development team in case of any query related to this package. The compound growth rate can be estimated in the 'Analyze' page of WebECGR. The exclusive feature which makes this web solution unique is that it can compute 'compound growth rates' instantly using different methodologies once data is uploaded online. No other existing software, either commercial or noncommercial, is provided with such functionality. Moreover, it is freely accessible to online users throughout the globe. Therefore, this package is extremely useful to its stakeholders, viz. Agricultural scientists and policy makers.

4. AN ILLUSTRATION USING WebECGR PACKAGE

As an illustration, India's total lentil production data (in Million tonnes) during the period 1980-81 to 2010-11, collected from Directorate of Economics and Statistics, Department of Agriculture and Cooperation, Government of India, New Delhi are considered. At first, the data in MS-Excel format are uploaded into WebECGR package under 'Analyze' page, as shown in Fig. 3. The uploaded data are represented through a

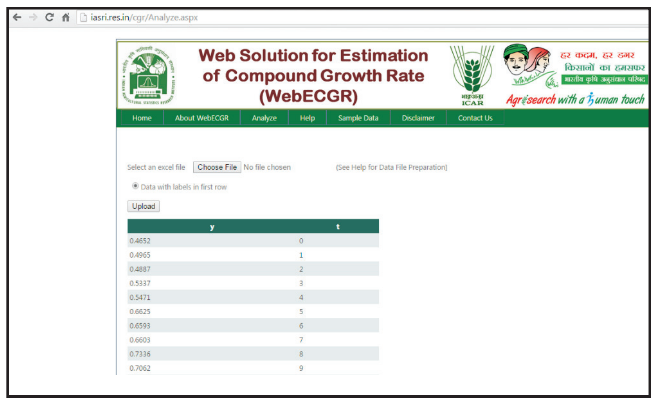

Fig. 3. Uploading of data into WebECGR package

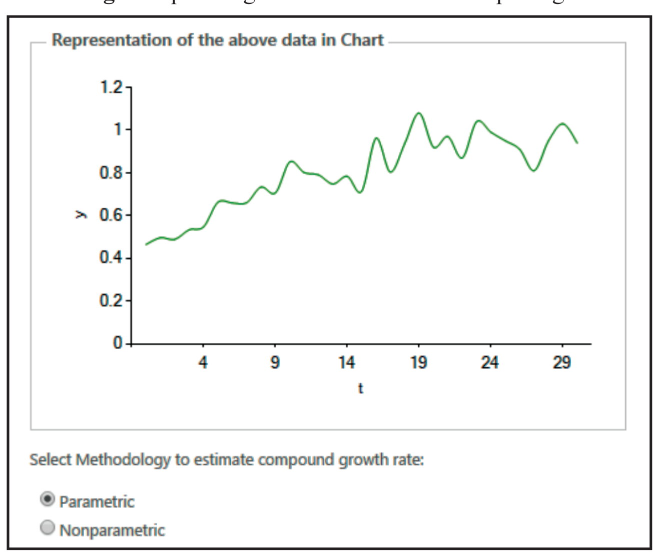

Fig. 4. Representation of data into chart and selection of methodology

graph (Fig. 4) so that users can have an idea about pattern of the data. To estimate compound growth rate, users need to select the methodology either as 'Parametric' or 'Nonparametric'. In the present illustration, 'Parametric' approach is employed (Fig. 4). Since the graph of the data (Fig. 4) is somewhat similar to that of Critically damped model (Fig. 1), therefore attempts are made to fit this model (Fig. 5). In order to specify appropriate initial value for parameter B, it is noted from eq. (6), that $y(0) = B$. Further, if the data are mainly increasing, initial value of parameter λ should be negative, otherwise it should be positive. Subsequently, initial value for the remaining parameter C can be obtained by noting, from eq. (7), that value of t for which $y(t)$ is maximum satisfies $t_{max} = (C - \lambda B)/(\lambda C)$. Several sets of initial values for the parameters B, C and λ in the neighbourhood of these initial values may be tried (Fig. 5) and if it is found that the final estimates obtained are the same, global convergence is ensured. Finally, compound growth rate is estimated by pressing the 'Estimate Compound Growth Rate' button (Fig. 6) and the output is displayed on the same web page. Several other computations related to estimation of compound growth rate are displayed through tabular and graphical representations in the package; these are combined, as shown in Fig. 6. The output, thus obtained, can be saved as a webpage for future reference. The results can also be copied manually into Excel or Word document easily.

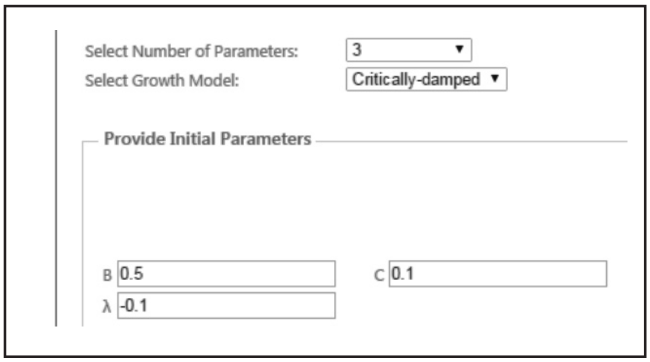

Fig. 5. Selection of model and initialization of parameter values

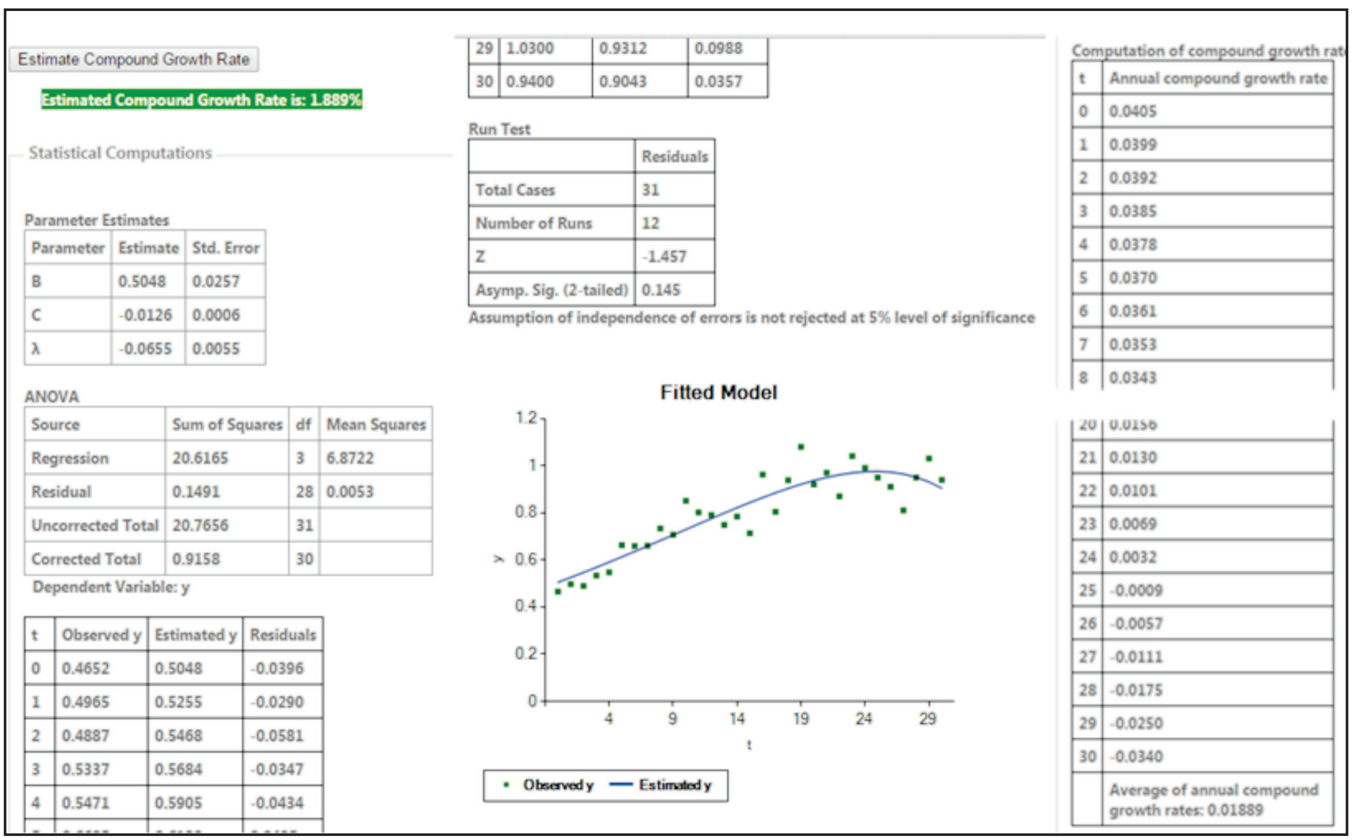

Fig. 6. Output for estimation of compound growth rate

5. RESULTS AND DISCUSSION

From second column of Table 3, $y(0) = 0.47$, therefore initial value of B should be near this value, say 0.50. A perusal of the data shows that it is mainly increasing, therefore initial value of parameter λ should be negative. Further, in a span of 30 years, the value of $y(t)$ has increased from 0.47 to 1.08, therefore a reasonable initial value for λ should be around $-(1.08 - 0.47)/30$, say -0.02. From second column of Table 3, it is noticed that $t_{max} = 19 = (C + 0.02 \times 0.50)/(-0.02C)$ *i.e.* $C = -0.007$. Final estimates of parameters of Critically damped model along with their standard errors, using WebECGR package, are reported in Table 1. Analysis of variance (ANOVA), reported in Table 2, lists down sources of variations, sums of squares and degrees of freedom of those sources. It gives the Mean Square Error (MSE) value as 0.005, which being quite low, reflects that Critically damped model provides a good fit to the data. This can also be seen visually from the graph of fitted model along with data (Fig. 6). Using eq. (10), Annual compound growth rates for the data are computed by WebECGR package and the same are reported in the last column of Table 3. Their arithmetic mean gives the Compound growth rate for total lentil production in India during the period 1980-81 to 2010-11 as 1.89%.

Table 1. Estimates of Parameters

Parameter	Estimate	Standard error
B	0.505	0.026
C	–0.013	0.001
λ	–0.066	0.006

Table 2. ANOVA

Source	Sum of squares	Degrees of freedom	Mean squares
Regression	20.616	3	6.872
Residual	0.149	28	0.005
Uncorrected Total	20.766	31	
Corrected Total	0.916	30	

Table 3. Computation of Annual Compound Growth Rates

t	Observed $y(t)$	Predicted $y(t)$	Residuals	Annual compound growth rate
0	0.470	0.505	–0.035	0.041
1	0.500	0.526	–0.026	0.040
2	0.490	0.547	–0.057	0.039
3	0.530	0.569	–0.039	0.039
4	0.550	0.591	–0.041	0.038
5	0.660	0.613	0.047	0.037
6	0.660	0.636	0.024	0.036
7	0.660	0.659	0.001	0.035
8	0.730	0.682	0.048	0.034
9	0.710	0.706	0.004	0.033
10	0.850	0.729	0.121	0.032
11	0.800	0.752	0.048	0.031
12	0.790	0.776	0.014	0.030
13	0.750	0.799	–0.049	0.029
14	0.780	0.821	–0.041	0.027
15	0.710	0.843	–0.133	0.026
16	0.960	0.864	0.096	0.024
17	0.800	0.884	–0.084	0.022
18	0.940	0.903	0.037	0.020
19	1.080	0.920	0.160	0.018
20	0.920	0.936	–0.016	0.016
21	0.970	0.950	0.020	0.013
22	0.870	0.961	–0.091	0.010
23	1.040	0.969	0.071	0.007
24	0.990	0.974	0.016	0.003
25	0.950	0.975	–0.025	–0.001
26	0.910	0.972	–0.062	–0.006
27	0.810	0.964	–0.154	–0.011
28	0.950	0.951	–0.001	–0.017
29	1.030	0.932	0.098	–0.025
30	0.940	0.905	0.035	–0.034

6. CONCLUDING REMARKS

In this article, the methodology is described for estimation of Compound growth rates for non-monotonic situations through over-damped, under-damped, and critically damped nonlinear growth models. Further, features and functionalities for online application of the user-friendly WebECGR package are also discussed. As an illustration, compound growth rate for total lentil production in India during the period 1980-81 to 2010-11 is estimated. It is hoped that Agricultural Scientists would start using web-based WebECGR package for estimation of compound growth rates for data sets exhibiting non-monotonic behaviour.

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