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Research Note

Development of a Novel Web-based WebECGR Package for Estimation of Compound Growth Rates for Monotonically Non-decreasing Situations

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Abstract

The compound growth rate (CGR) is generally estimated by assuming that the path of response variable can be described by either Malthus model or monotonically non-decreasing nonlinear growth models, like monomolecular, logistic and Gompertz models. In this article, the methodology for its estimation is discussed by employing more advanced parametric nonlinear growth models exhibiting sigmoid behaviour, viz. Richards and mixed-influence models. As the methodology is quite complicated, an online user-friendly web-based application, viz. WebECGR package has been developed, and the same has been uploaded under the link 'Online Analysis of Data' at I.A.S.R.I. website *http://iasri.res.in/cgr*. A brief description of this package has been provided in this paper. For illustrating its use, the CGR for India's wheat yield has been estimated during the period 1950-51 to 2011-12 through Richards model using WebECGR package.

Key words: CGR, WebECGR package, nonlinear growth models, Richards models, wheat yield

JEL Classification: C88

Introduction

Compound growth rate (CGR) is a key indicator to measure agricultural growth and can be used for forecasting *y(t)*, the response variable at time *t*, say production, yield, and area. This, in turn, plays a vital role in framing optimal policies, like import and export policies for various agricultural commodities. To estimate CGR, the traditional methodology based on Malthusian law, was proposed by Panse (1964). The disturbing feature is that this procedure is beset with many pitfalls. Firstly, the underlying model is Malthusian model, whose drawback is that the response variable $y(t) \rightarrow \infty$ as $t \rightarrow \infty$, which cannot happen in reality. Secondly, the assumption of multiplicative errors is usually made only for mathematical convenience. In fact, it is valid only when variability of response variable *y(t)* increases with its increasing values, which is rarely satisfied. The third drawback is that goodness of fit of the original nonlinear model is assessed on the basis of the coefficient of determination, R^2, for the corresponding linearized model. Prajneshu and Chandran (2005) have discussed all these aspects in great details. The authors have described the correct procedure that should be followed based on more realistic parametric nonlinear growth models, viz. monomolecular, logistic and Gompertz models. Apart from these, there also exist some more advanced parametric non-decreasing nonlinear growth models, like Richards model and mixed-influence model (Seber and Wild, 2007).

In this article, it is advocated that these models should also be employed for estimation of CGR. However, the methodology using nonlinear growth models is quite complex. Further, the existing statistical

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packages cannot meet this challenge for all types of users as it requires indepth statistical knowledge. Now-a-days, online web-based applications are emerging as the best solution to overcome this scenario for global users. In this direction, a novel web solution for estimation of CGR has been developed, which is known as 'WebECGR package' and is reported in this paper. As an illustration, CGR has been estimated for India's wheat yield during the period 1950-51 to 2011-12 through Richards model using this novel WebECGR Package.

Materials and Methods

Let $y(t)$ denote the response variable at time t, like agricultural production, yield, or area; let r be the intrinsic growth rate, and K be the carrying capacity. When the path of response variable is monotonically non-decreasing, Prajneshu and Chandran (2005) have discussed three nonlinear growth models, viz. monomolecular, logistic and Gompertz models, all having three parameters each. However, in order to describe the above type of behaviour, two important four-parameter advanced parametric nonlinear growth models are the Richards and mixed-influence models, which are briefly discussed below (Seber and Wild, 2007):

(i) Richards Model

This model, which has sigmoid type of behaviour, is given by the differential equation (1):

$$dy/dt = ry\,(K^m - y^m) / (mK^m) \quad \ldots(1)$$

Integrating Equation (1), we get

$$y(t) = Ky_0/[y_0{}^m + (K^m - y_0^m)\exp(-rt)]^{1/m} \quad \ldots(2)$$

Point of inflexion, i.e. the point at which the tangent to the curve changes direction, is at the point $y(t) = K / (m + 1)^{1/m}$. Further, monomolecular, logistic and Gompertz models are the particular cases of this model when $m = -1$, 1, and 0, respectively.

(ii) Mixed-Influence Model

It is a combination of monomolecular and logistic models and is given by the differential equation (3):

$$dy/dt = (a + by)\,(K - y) \quad \ldots(3)$$

Equation (3) reduces to logistic model, if $a = 0$ and to monomolecular model, if $b = 0$. Integrating Equation (3), we get

$$y(t) = \frac{K(a+by_0) - a(K-y_0)e^{-(a+bK)t}}{(a+by_0) + b(K-y_0)e^{-(a+bK)t}} \quad \ldots(4)$$

The point of inflexion is at the point $y(t) = [K/2] - [a / (2b)]$ and the graph of $y(t)$ versus t is again a sigmoid.

It may be pointed out that the monomolecular model is able to describe those data sets in which there is no point of inflexion. Logistic model is appropriate when the point of inflexion is symmetric, i.e. at half the carrying capacity. For asymmetric point of inflexion, either Gompertz model or more flexible models, viz. Richards and mixed-influence models should be tried.

In respect of Richards and mixed-influence models, proceeding along similar lines as Prajneshu and Chandran (2005), annual growth rates ($y^{-1}\,dy/dt$) pertaining to the period $(t_i - t_{i-1})$, $i = 0,1,\ldots,n-1$, where, n denotes the number of data points, can respectively be computed using the formulae (5) and (6):

$$R_t^R = r\left[K^m - y^m(t)\right]\big/\left(mK^m\right) \quad \ldots(5)$$

$$R_t^{MI} = a\left[\left\{K/y(t) - 1\right\} + b\left\{1 - y(t)/K\right\}\right] \quad \ldots(6)$$

Taking arithmetic mean of the annual growth rates, requisite CGR over a given time-period can be estimated.

It may be noted that the above models have been proposed deterministically. However, in order to apply these to data, an error term is added on the right hand side of these models, thereby making these as 'Nonlinear statistical models'. The errors are assumed to be independently and identically distributed with equal variances. The nonlinear estimation procedure is to be employed for fitting the models and their goodness of fit is examined by computing the mean square error (MSE) (Draper and Smith, 1998).

WebECGR Package

In order to carry out the above-mentioned tasks, we have developed WebECGR package, which is a user-friendly web-based application for the estimation of CGR. It is accessible over the internet through web address *http://iasri.res.in/cgr* for global users. This web-based solution has been developed employing ASP.NET with .NET Framework, Version 4.0 (Walther *et al.*, 2010). The statistical software *R* has been integrated into .NET environment by using the non-commercial version of *statconnDCOM*. For embedding *R* as background statistical engine into .NET, the DCOM technology (Baier and Neuwirth, 2007) has been used. In WebECGR package, *R* function nls() (Ritz and Streibig, 2008) has been used to obtain nonlinear least-squares estimates of the parameters of nonlinear growth models. In this function, Gauss-Newton algorithm has been used to solve the nonlinear least squares estimation problem.

The WebECGR package contains the following components: About WebECGR, Analyze, Help, Sample Data and Contact Us. The 'About WebECGR' page contains general description about WebECGR package. The 'Help' section provides guidance to the users for preparing input data file as well as estimating CGR through step-by-step procedures. Users can download sample data for estimating CGR based on datasets in the 'Sample Data' page. In 'Contact Us' page, users can get email ids of concerned persons related to this web application. Users may send emails to the development team in case of any query related to WebECGR package. The CGR can be estimated in the 'Analyze' tab of WebECGR package.

An Illustration

As an illustration, India's wheat yield data (in kg/ha) for the period 1950-51 to 2011-12 were taken from the website *www.agricoop.nic.in* and the same were uploaded in WebECGR package under 'Analyze' tab and are exhibited in Figure 1. The uploaded data were represented through a graph (Figure 2) to have an idea about the pattern of data. To estimate CGR, users need to select the methodology as either 'Parametric' or 'Nonparametric'. In the present case, 'Parametric' approach was employed. Since graph of the data (Figure 2) was found to be monotonically non-decreasing and sigmoid, therefore monomolecular model was not tried.

Therefore, attempts were made to fit the remaining four models, viz. logistic, Gompertz, Richards and mixed-influence models. A snapshot of WebECGR package exhibiting selection of model and parameter value initialization is given in Figure 3. Several sets of initial values for the parameters (y_0, r, K, m) were tried and it was found that the final estimates remained the same, thereby ensuring global convergence. Estimates of parameters for fitted logistic, Gompertz, and Richards models along with their standard errors were computed and the same are reported in Table 1. A perusal of this table indicated that the standard errors of all parameter estimates of fitted models were less than the corresponding estimates, which is highly desirable. However, for the mixed-influence model,

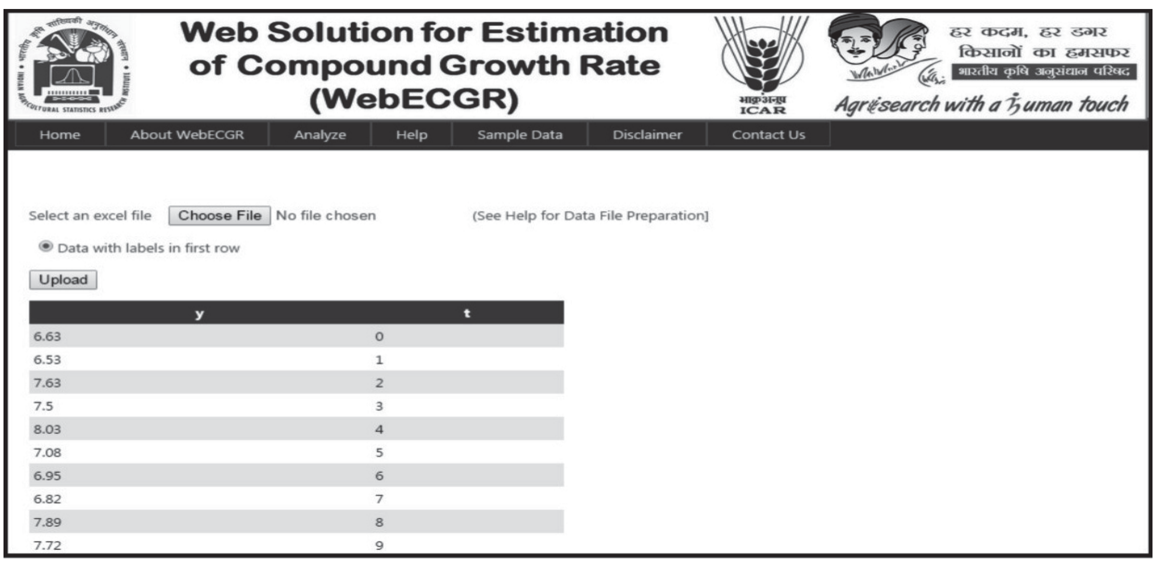

Figure 1. Uploading of data into WebECGR package

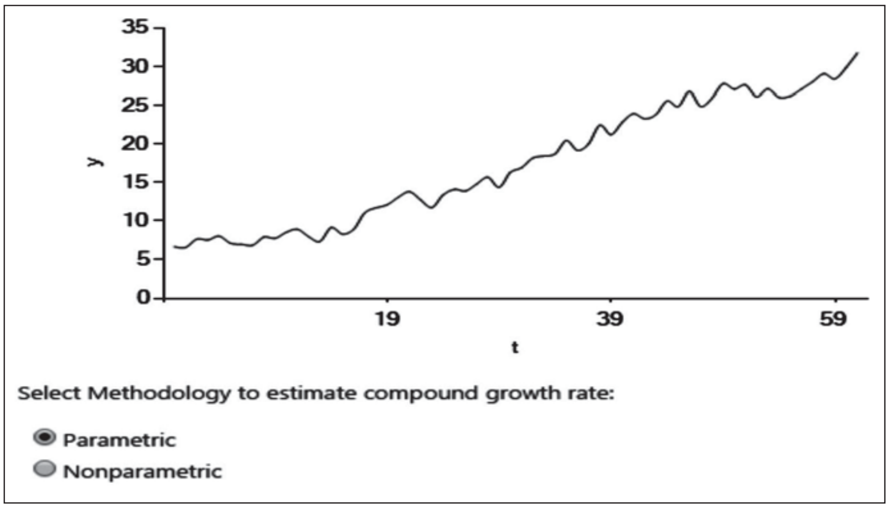

Figure 2. Representation of data given in Figure 1 in a chart

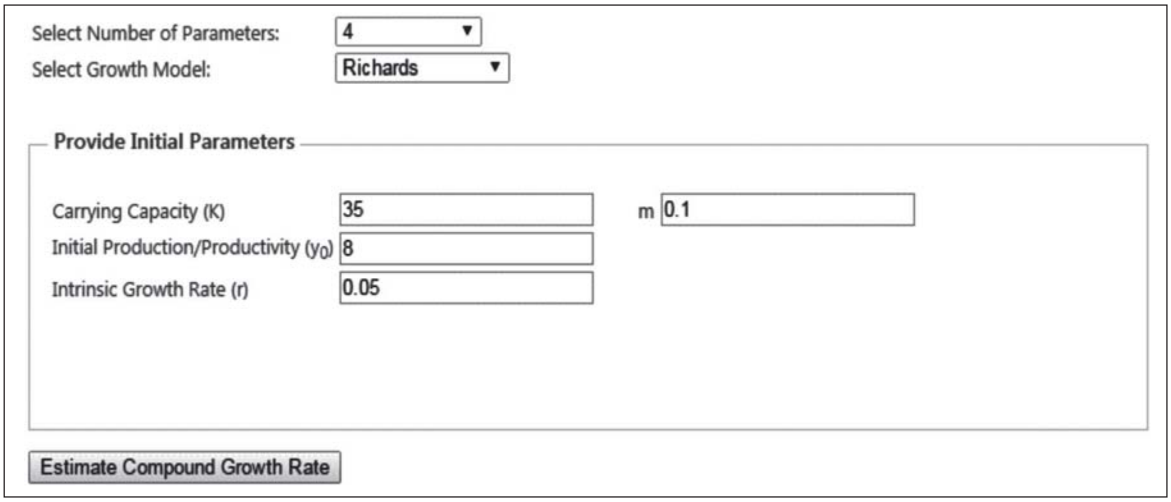

Figure 3. Selection of model and parameter value initialization

Table 1. Parameter estimates for logistic, Gompertz and Richards models

Parameter	Logistic model		Gompertz model		Richards model	
	Estimate	Standard error	Estimate	Standard error	Estimate	Standard error
y_0	5.2333	0.2705	4.9843	0.3471	5.8401	0.2624
r	0.0548	0.0032	0.0249	0.0029	0.1606	0.0431
K	36.4653	1.6547	50.9392	5.7897	29.3955	0.8494
m	-	-	-	-	4.5437	1.4550

Hessian was found to be singular, implying thereby that this model was not suitable for describing the given data set. However, before taking the final decision, assumption of independence of errors was examined by employing 'Run test' on residuals. The results obtained in respect of fitted logistic, Gompertz, and Richards models, using WebECGR package, are reported in Table 2. A perusal of Columns 1 and 2 in Table 2 indicated that for the fitted logistic and Gompertz models, null hypothesis of independence of errors was rejected at 5 per cent level of significance as calculated values of absolute values of Z-statistic

Table 2. Run tests for logistic, Gompertz and Richards models

Particulars	Logistic model	Gompertz model	Richards model
Total cases	62	62	62
Number of runs	19	15	29
Z	– 3.329	– 4.350	– 0.768
Asymptotic significance (2-tailed)	0.001	0.000	0.442

Table 3. ANOVA for Richards model

Source	Sum of squares	Degrees of freedom	Mean squares
Regression	22,586.3787	4	5,646.5947
Residual	59.1783	58	1.0203
Uncorrected total	22,645.5570	62	
Corrected total	3,843.8005	61	

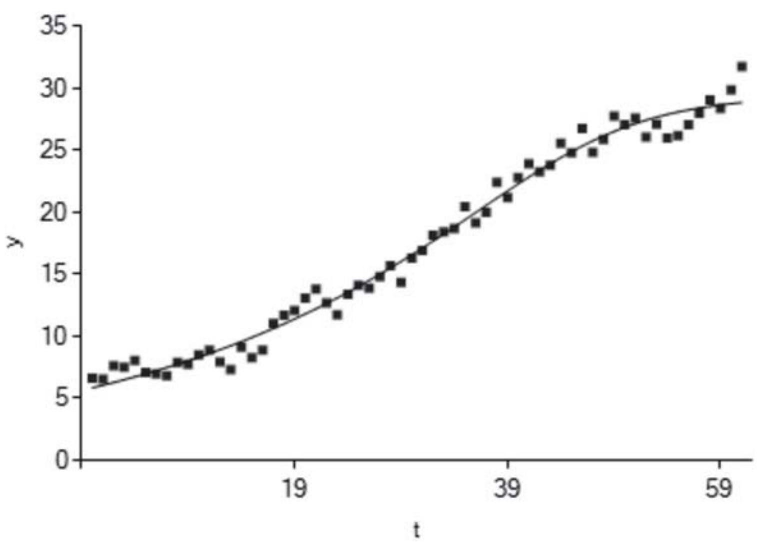

Figure 4. Graph of fitted Richards model along with data

were greater than the tabulated value, viz. 1.960. Thus, it was not possible to fit the logistic and Gompertz models for describing the given data set. Further, from Column 3 in Table 2, null hypothesis of independence of errors was not rejected at 5 percent level, indicating that Richards model was properly fitted to the given data. The Analysis of Variance (ANOVA) results for this model are given in Table 3, which shows that the goodness of fit for this model, computed through MSE, is 1.020, which is quite low. In order to get a visual idea, a graph of fitted Richards model along with data, produced by WebECGR package, has been exhibited in Figure 4. Evidently, the fitted values were very close to the actual values. Thus, it was concluded that Richards model was appropriate for describing the given data set.

Using Equation (5) for Richards model, the annual CGRs for the data were computed by pressing the ‘Estimate Compound Growth Rate’ button in WebECGR package (Figure 3) and the same are reported in Table 4. Their arithmetic mean, again using the package, gave the CGR for India’s wheat yield during the period 1950-51 to 2011-12 as 2.59 per cent.

Finally, attempt was made to compute CGR for the given data through the traditional approach based

Table 4. Computation of annual compound growth rates

t	Observed *y(t)* (kg/ha)	Predicted *y(t)* (kg/ha)	Residuals (kg/ha)	Annual compound growth rate
0	6.630	5.840	0.790	0.0353
1	6.530	6.050	0.480	0.0353
2	7.630	6.268	1.362	0.0353
3	7.500	6.493	1.007	0.0353
4	8.030	6.726	1.304	0.0353
5	7.080	6.968	0.112	0.0353
6	6.950	7.218	0.268	0.0353
7	6.820	7.478	0.657	0.0353
8	7.890	7.746	0.144	0.0353
9	7.720	8.024	0.304	0.0353
10	8.510	8.312	0.198	0.0352
11	8.900	8.610	0.290	0.0352
12	7.930	8.918	0.988	0.0353
13	7.300	9.238	1.937	0.0353
14	9.130	9.568	0.438	0.0352
15	8.270	9.910	1.640	0.0352
16	8.870	10.264	1.394	0.0352
17	11.030	10.630	0.400	0.0349
18	11.690	11.008	0.682	0.0348
19	12.080	11.399	0.681	0.0347
20	13.070	11.803	1.267	0.0345
21	13.800	12.220	1.580	0.0342
22	12.710	12.651	0.059	0.0346
23	11.720	13.095	1.375	0.0348
24	13.380	13.553	0.173	0.0344
25	14.100	14.025	0.075	0.0341
26	13.870	14.510	0.640	0.0342
27	14.800	15.009	0.209	0.0338
28	15.680	15.521	0.158	0.0333
29	14.360	16.046	1.686	0.0340
30	16.300	16.583	0.283	0.0329
31	16.910	17.131	0.221	0.0325
32	18.160	17.689	0.470	0.0314
33	18.430	18.256	0.173	0.0311
34	18.700	18.831	0.131	0.0308
35	20.460	19.411	1.048	0.0285
36	19.160	19.994	0.834	0.0303
37	20.020	20.578	0.558	0.0292
38	22.440	21.160	1.279	0.0250
39	21.210	21.737	0.527	0.0273

contd.

Table 4. Computation of annual compound growth rates — Contd.

t	Observed $y(t)$ (kg/ha)	Predicted $y(t)$ (kg/ha)	Residuals (kg/ha)	Annual compound growth rate
40	22.810	22.306	0.503	0.0242
41	23.940	22.863	1.076	0.0214
42	23.270	23.405	0.135	0.0231
43	23.800	23.930	0.129	0.0218
44	25.590	24.432	1.158	0.0165
45	24.830	24.910	0.080	0.0189
46	26.790	25.362	1.427	0.0122
47	24.850	25.786	0.936	0.0189
48	25.900	26.180	0.280	0.0155
49	27.780	26.543	1.236	0.0080
50	27.080	26.876	0.204	0.0110
51	27.620	27.179	0.441	0.0087
52	26.100	27.452	1.352	0.0148
53	27.130	27.698	0.567	0.0108
54	26.020	27.917	1.897	0.0150
55	26.190	28.111	1.921	0.0144
56	27.080	28.282	1.202	0.0110
57	28.020	28.433	0.413	0.0069
58	29.070	28.565	0.504	0.0017
59	28.390	28.680	0.290	0.0052
60	29.880	28.780	1.099	0.0027
61	31.770	28.867	2.903	0.0150

on fitting the linearized form of Malthus model. Subsequently, the method of least squares was employed for estimation of parameters and the underlying assumption was that the errors were independent. It was found that for given data, it was not satisfied as calculated value of Z was –5.838. Thus, it was not possible to compute CGR for the given data based on this approach.

Concluding Remarks

This paper has reported about the existence and utility of a novel package, which is capable of estimating CGRs for monotonically non-decreasing situations through nonlinear growth models, viz. monomolecular, logistic, Gompertz, Richards and mixed-influence models. This package known as 'WebECGR Package', can also compute CGR when the underlying path is non-monotonic. Specifically, in this regard, there are three models, viz. over-damped, under-damped and critically-damped in the package. Thus, under a parametric set-up, the WebECGR package has eight different nonlinear models. If the underlying path cannot be satisfactorily described by any of these models, then WebECGR package has also got the provision of employing non-parametric approach. Under this set-up, one methodology for the State domain analysis and four methodologies for Time domain analysis, viz. moving averages, kernel smoothing, iterative plug-in algorithm, and local linear smoothing have been included in the package. All these would be discussed separately in due course of time. The main advantage of WebECGR package is that it is user-friendly and so the entire data analysis can be carried out without putting much effort in understanding intrinsic complicacies of the underlying procedures.

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References

Baier, T. and Neuwirth, E. (2007) Excel :: COM :: R. *Computational Statistics*, **22**: 91-108.

Draper, N. R. and Smith, H. (1998) *Applied Regression Analysis*, 3rd edn. John Wiley & Sons, New York, USA.

Panse, V.G. (1964) Yield trends of rice and wheat in first two Five-Year Plans in India. *Journal of Indian Society of Agricultural Statistics*, **16**: 1-50.

Prajneshu and Chandran, K.P. (2005) Computation of CGRs in agriculture: Revisited. *Agricultural Economics Research Review*, **18**: 317-24.

Ritz, C. and Streibig, J.C. (2008) *Nonlinear Regression with R*. Springer, New York, USA.

Seber, G.A.F. and Wild, C.J. (2007) *Nonlinear Regression*, 2nd edn. John Wiley & Sons, New York, USA.

Walther, S., Hoffman, K. and Dudek, N. (2010) *ASP.NET 4 Unleashed*, 1st edn., Sams Publishing, Indianapolis, USA.

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