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Robustness of Standard Re-inforced Balanced Incomplete Block Designs against Exchange of a Test Treatment

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SUMMARY

In agricultural field experiments, during the laying out of the experiment, mistakes like attaching an incorrect label to the seed packet or plot may occur and that may remain undetected till the end of experiment. These kinds of mistakes may result in poor efficiency of the resulting design. Thus, there is a need to look for designs that are insensitive or robust to such kind of disturbances. Present investigation deals with robustness of Standard Reinforced Balanced Incomplete Block (SR-BIB) design against exchange of test treatment. Two situations have been considered viz., (i) when exchange of test treatment occurs with a test treatment belonging to same block resulting in non-binary design and (ii) when exchange of test treatment takes place with a test treatment not belonging to the same block. As a particular case efficiencies of SR-BIB designs obtainable by adding a control treatment to each block of BIB designs with $r \leq 20$ for asymmetric BIB designs and $r \leq 30$ for symmetric BIB designs have been worked out when such an accident has taken place. The designs have been found to be highly efficient against such kind of disturbances especially in first situation.

Key Words : Balanced Incomplete Block design, Balanced Treatment Incomplete Block design, Efficiency, Harmonic mean, Robustness, Standard Reinforced Balanced Incomplete Block design.

1. INTRODUCTION

Mechanical errors like wrong labeling of treatments are very common in field trials. Two very common situations are (i) interchange of treatments and (ii) exchange of treatments. Pearce (1948) cited two situations when the experiments were conducted in randomized complete block design. In the first case, he considered the case when treatment 1(say) appeared twice in block 1 and not at all in block 2 and treatment 2 occured twice in block 2 and not at all in block 1. This is termed as interchange of treatments. Pearce had suggested that analysis to be carried out as if no mistake was done and some adjustment done thereon. Pearce (1948) in second case considered another example (fictitious) wherein one variety was substituted by another variety due to scarcity of material. This has been termed as exchange of treatments. For this case also some analysis has been suggested. Though such errors do occur but are rarely reported for obvious reasons. This apprehension can be overcome if designs that are insensitive to such kind of mechanical errors i.e., they are robust to such kind of disturbances, are made available to the experimenter. With this end in view Batra (1993), Batra *et al.* (1997) have studied robustness of block designs against interchange or exchange of treatments. Batra and Parsad (2001) have studied robustness of standard reinforced incomplete block designs against interchange of a pair of treatments.

In many experimental situations the experimenter is interested in comparing newly developed varieties (called test treatments) with a standard one (called control treatment). For such situations Standard Reinforced Balanced Incomplete Block (SR-BIB) designs are appropriate. In this communication we study the robustness of SR-BIB designs against exchange of test treatments. Two situations have been considered viz., (i) when exchange of test treatment occurs with a test treatment belonging to same block resulting in non-binary design and (ii) when exchange of test treatment takes place with a test treatment not belonging to the same block. As a particular case efficiencies of SR-BIB designs obtainable by adding a control treatment to each block of BIB designs with $r \leq 20$ for asymmetric BIB designs and $r \leq 30$ for symmetric BIB designs have been worked out when such an accident has taken place.

1.1 Preliminaries

(a) Interchange of treatments in a block design

We say that interchange of treatment has taken place if at least a pair of treatments each belonging to different blocks in the design get interchanged in the layout. In order to make ideas clear and to show the implications of such a disturbance, consider following block design.

Blocks

1	2	3	5	6	1	5	3	6	7
2	3	5→	←4	7	2	6	1	5	8
3	4	4	6	8	4	7	4	8	5

Now, suppose treatment 5 in block 3 gets interchanged with treatment 4 in block 4. The resulting design after interchange is given below. It can be seen from preliminary considerations that the resulting design is no more connected and therefore this design is not robust against this kind of disturbance.

Blocks after interchange

1	2	3	5	6	1	5	3	6	7
2	3	4	5	7	2	6	1	5	8
3	4	4	6	8	4	7	4	8	5

(b) Exchange of treatments in a block design

Exchange of treatment is said to have taken place if in a block design at least one of the treatment in some of the blocks get substituted by some other treatment included in the trial. Consider the following design and suppose that treatment 5 in block 5 gets replaced (exchanged) by any of the treatments 1, 2, 3, 4 included in the trial. It can be seen that the resulting design is disconnected.

Blocks

1	1	1	2	1	5	5	5	6
2	2	3	3	2	6	6	7	7
3	4	4	4	5	7	8	8	8

1.2 Block Designs for Making Test Treatments-Control Comparisons

In many practical situations, interest of experimenter lies in making comparisons between a control treatment with test treatments. In such situations the designs that are efficient for all paired comparisons are generally not efficient for test treatments-control comparisons. For example balanced incomplete block (BIB) design which is universally optimal for pair-wise comparisons is not efficient for making test treatments-control comparisons. For such a situation one requires block designs that are efficient for making such kinds of comparisons.

Consider a block design d with treatments labeled as 0, 1,..., v with 0 denoting the control treatment and 1, 2,..., v denoting the test treatments. These treatments are arranged in b blocks of sizes $k_1, k_2, \ldots, k_b$. Let N_d be the incidence matrix of treatments vs. blocks in d. $r = (r_1, r_2, \ldots, r_v, r_0)$ is the replication vector with r_0 denoting the replication number of control treatment and r_i denoting the replication number of i^{th} test treatment. We can write $N_d = [\tilde{N}'_d \quad n_d]'$, where $\tilde{N}_d$ is the $v \times b$ incidence matrix of test treatments vs blocks and $\tilde{n}_d$ is the $b \times 1$ incidence vector of control vs blocks. Then, under the usual homoscedastic, two-way classified, fixed effects, additive linear model, the coefficient matrix C_d of the reduced normal equations for estimating treatment effects, obtained by using ordinary least square is given by

$$C_d = \begin{pmatrix} M_d & -\tilde{N}_d K_d^{-1} \tilde{n}_d \\ -\tilde{n}'_d K_d^{-1} \tilde{N}'_d & r_0 - \tilde{n}'_d K_d^{-1} \tilde{n}_d \end{pmatrix} \qquad (1.2.1)$$

where

$$M_d = R_d - \tilde{N}_d K_d^{-1} \tilde{N}'_d; \; R_d = \text{diag}(r_1, r_2, \ldots, r_v)$$

$$K_d = \text{diag}(k_1, k_2, \ldots, k_b)$$

Since M_d is non-negative definite and non-singular matrix of order v, a generalized inverse of C_d is given by

$$\overline{C}_d = \begin{pmatrix} M_d^{-1} & 0 \\ 0 & 0 \end{pmatrix} \qquad (1.2.2)$$

Our interest is in estimating the contrasts of the type $l'\tau$ where $l' = [I_v \ \ -1_v]$, and $\tau = (\tau_1, ..., \tau_v, \tau_0)$, τ_i denoting the effect of i^{th} test treatment and τ_0 the effect of control treatment. Then the variance of $l'\hat{\tau}$ the best linear unbiased estimator of $l'\tau$ is given by

$$\text{Var}(l'\tau) = \sigma^2 l'\bar{C}_d l = \sigma^2 M_d^{-1} \qquad (1.2.3)$$

M_d matrix can therefore be utilized to draw inference on the comparisons of the type $l'\tau$.

Balanced treatment incomplete block (BTIB) design is generally employed when comparisons of the type $l'\tau$ described above are involved. For completeness we therefore give the definition of BTIB design.

Definition 1.2.1 (Bechoffer & Tamahane 1981); An arrangement of v test treatments and a control in b blocks each of size k ($k \le v$) is said to be BTIB design if

(i) Every test treatment appears with control treatment in equal number of blocks $\{\lambda_{0t} = \lambda_0 \text{ (constant)} \forall\ t = 1,2,..., v\}$.

(ii) Every pair of test treatments appears together in equal number of blocks over the entire design $\{\lambda_{tt'} = \lambda_1 \text{(constant)} \forall\ t \ne t' = 1,2,..., v\}$.

SR-BIB design is a particular case of BTIB design obtained by augmenting each block of BIB design by a control.

1.2.2 Robustness criteria

Generally following two criteria are used to study robustness of designs.

Criterion 1: A connected block design d (v, b, r, k) is said to be robust against the disturbance η if the residual design d^* obtained after the disturbance η has occurred, remains connected.

Criterion 2 : A connected block design d (v, b, r, k) is said to be robust against the disturbance η if the design is robust under criterion 1 and the efficiency of the residual design d^* is not too small as compared to the original design d.

If $C_d(C_{d^*})$ denotes the coefficient matrix of the reduced normal equations for estimating treatment effects through design $d(d^*)$ then the relative efficiency E of design d^* compared to original design d is given by

$$E = \frac{\text{Harmonic mean of non-zero eigenvalues of } C_{d^*}}{\text{Harmonic mean of non-zero eigenvalues of } C_d} \qquad (1.2.4)$$

It may be observed that (1.2.4) is related to A-efficiency criterion.

2. ROBUSTNESS OF SR-BIB DESIGN

Consider a block design d for making test treatments-control comparisons via b blocks each of size k+1. The treatments are labeled as 0,1,...,v with 0 denoting the control treatment and 1,...,v denoting the test treatments. The test treatments are replicated r times each while r_0 denotes the replication number of the control. Let N_d be the $(v+1)\times b$ incidence matrix of the design d. Then, the coefficient matrix C_d of the reduced normal equations for estimating treatment effects, obtained by using ordinary least square is given by (1.2.1) with $K_d = (k+1) I_b$, The parameters of design d are $v^* = v+1$, b, r, r_0 and $k^* = k+1, \lambda_1$ and λ_0 with λ_1 denoting the number of blocks in which a pair of test treatments occurs together and λ_0 the concurrences of control treatment with each of the test treatment. Without loss of generality assume that first k entries in first column of N_d are unities. Then we can write N_d as

$$N_d = \begin{bmatrix} \tilde{N}'_d & \tilde{n}_d \end{bmatrix} = \begin{bmatrix} p & N_{d1} \\ n_0 & n'_{d1} \end{bmatrix} \qquad (2.1)$$

where

$p = (1'_k \ \ 0'_{v-k})'$

N_{d1} = $v\times(b-1)$ incidence matrix giving incidence of v test treatments in remaining (b – 1) blocks

n'_{d1} = $1\times(b-1)$ incidence vector of control treatment in remaining (b–1) blocks

$$n_0 = \begin{cases} 1 & \text{if control appears in first block} \\ 0 & \text{otherwise} \end{cases}$$

Under the above set up the coefficient matrix C_d of the reduced normal equations is given by

$$C_d = \begin{bmatrix} M_d & -\frac{1}{(k+1)}(N_{d1}n_{d1} + n_0 p) \\ -\frac{1}{(k+1)}(n'_{d1}N'_{d1} + n_0 p') & r_0 - \frac{1}{(k+1)}(n'_{d1}n_{d1} + n_0^2) \end{bmatrix}$$

where

$$M_d = R_d - \frac{1}{(k+1)}(N_{d1}N'_{d1} + pp')$$
$$R_d = rI_v$$

As stated earlier the matrix M_d can be utilized to infer on the test treatments-control comparisons. For SR-BIB design

$$M_d = \frac{1}{(k+1)}\left[(rk+\lambda_1)I_v - \lambda_1 1 1'_v\right]$$

Now suppose without loss of generality the test treatment 1 of first block gets exchanged with any of the v-1 remaining test treatments of the design. Call the residual design as d*. There arise two cases.

Case I : When test treatment 1 gets exchanged with one of the test treatments already occurring in first block say treatment 2. In this case vector p takes the following form

$$p_1^* = [0 \quad 2 \quad u_{v-2}]' ; \; u_{v-2} = [1'_{k-2} \quad 0'_{v-k}]' \qquad (2.2)$$

Clearly

$$p_1^* = p + [-1 \quad 1 \quad 0'_{(v-2)}]'$$

It can be seen that design d* is non-binary. The parameters of design d* are $v^* = v+1$, $b^* = b$, $r^* = (r-1 \quad r+1 \quad r1'_{v-2} \quad r_0)'$, $k^* = k+1$

Case II : When test treatment 1 gets exchanged with one of the test treatments not occurring in first block say $(k+1)^{th}$ treatment. In this case, vector p takes the following form

$$p_2^* = [0 \quad 1'_{k-1} \quad 1 \quad 0'_{v-k-1}]' \qquad (2.3)$$

It is easy to see that

$$p_2^* = p + [-1 \quad 0'_{k-1} \quad 1 \quad 0'_{v-k-1}]'$$

$$v^* = v+1, b^* = b$$

$$r^* = (r-1 \quad r1_{k-1} \quad r+1 \quad r1_{v-k-1} \quad r_0)'$$

$$k^* = k+1$$

Then M_{d^*} the M_d matrix of design d* is given by

$$M_{d^*} = M_d + \frac{1}{(k+1)}\begin{bmatrix} -k & 1 & u' \\ 1 & k-2 & -u' \\ u & -u & 0_{(v-2)\times(v-2)} \end{bmatrix} \qquad \text{(for case I)}$$

and

$$M_{d^*} = M_d + \frac{1}{(k+1)}T \qquad \text{(for case II)}$$

where

$$T = \begin{bmatrix} -k & 1'_{k-1} & 0 & 0'_{v-k-1} \\ 1_{k-1} & 0_{(k-1)\times(k-1)} & -1_{k-1} & 0_{(k-1)\times(v-k-1)} \\ 0 & -1'_{k-1} & k & 0'_{v-k-1} \\ 0_{v-k-1} & 0_{(v-k-1)\times(k-1)} & 0_{v-k-1} & 0_{(v-k-1)\times(v-k-1)} \end{bmatrix}$$

The relative efficiency E of design d* compared to the original design d can now be obtained by using the expression

$$E = \frac{\text{Harmonic mean of non-zero eigenvalues of } M_{d^*}}{\text{Harmonic mean of non-zero eigenvalues of } M_d}$$

As a special case we have studied robustness of SR-BIB designs against exchange of a test treatment. These designs are always robust against exchange of a test treatment under Criterion 1. We have therefore computed efficiencies of SR-BIB designs obtainable by adding a control treatment to each block of BIB designs with $r \le 20$ for asymmetric BIB designs and $r \le 30$ for symmetric BIB designs when such an accident has taken place. The designs in Parsad *et al.* (2000) have been utilized to obtain SR-BIB designs. Efficiency wise summary is given in the following tables.

Table 1. Number of designs in different ranges of efficiencies of SR-BIB design (obtained from asymmetric BIB designs) against exchange of one test treatment with $r \le 20$

E_1	All the 191 designs have efficiency more than 0.95			
E_2	0.80-0.85	0.85-0.90	0.90-0.95	0.95 & above
Number of designs	16	40	51	84

Table 2. Number of designs in different ranges of efficiencies of SR-BIB designs (obtained from symmetric BIB designs) against exchange of one test treatment with $r \leq 30$

E_1	All the 94 designs have efficiency more than 0.95			
E_2	0.80-0.85	0.85-0.90	0.90-0.95	0.95 & above
Number of designs	8	12	16	58

Here E_1 (E_2) denotes efficiency in first (second) case.

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