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# Analysis of rainfall in Patna Main Canal Command employing two parameter gamma probability distribution model

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Rainfall is one of the most important inputs for enhancing agricultural production. In arid and semi-arid region, conservation of every drop of rainwater and its proper utilization in production system is emphasized whereas in humid region and particularly in irrigated ecosystem the assessment and management of rainfall becomes more important from irrigation and drainage planning point of view. In a canal command if rainfall is assessed at different probability levels and crop water requirement is estimated using meteorological data, planning to meet irrigation requirement through canal or ground water and drainage requirement from the fields can be done in advance. Proper assessment and utilization of rainfall can reduce the dependency for irrigation on other water sources, which may increase the area under irrigation through canal or ground water.

The command of Patna Main Canal (PMC) under Sone Canal Project (one of the oldest irrigation systems in the country completed in 1873-74) was selected as study area. The total length of PMC is 124.8 Km. It has 12 reaches, 35 distributaries taking off from it and 12 sub-distributaries taking off from it's distributaries. As per records of Irrigation/ Water resources Department the total Canal Command Area (CCA) of PMC is 1.13 lakh ha. Detailed information about the Patna Main Canal is given in the report by Upadhyaya (2005).

In order to study the variability of rainfall in Patna Canal command, daily rainfall data for 32 years (1974-2005) at three representative raingauge stations i.e. (Daudnagar, Pauliganj and Naubatpur located in head, middle and tail reach of Patna Main Canal, respectively) were collected from Department of Statistics and Evaluation, Govt. of Bihar. Daily rainfall data was converted into weekly, monthly and annual rainfall and series of 32 years were analyzed for three raingauge stations. Some probability distribution models like Normal, Pearson, Extreme, Lognormal, Log pearson, Log extreme and Gamma distributions were evaluated to determine the best fit distribution for weekly, monthly and annual rainfall series at these stations but no single distribution except two parameter Gamma probability distribution was observed to be fitting well in weekly, monthly and annual rainfall. Upadhyaya and Singh (1994) reported that two parameter Gamma probability distribution was the best fit distribution for 42 years rainfall series of 52 weeks at Bhubaneswar.

Two parameter Gamma probability function is defined as (Hogg, V.R. and Craig, A.T., 1969)

$$f(y) = \frac{y^{\alpha-1}\, e^{-\frac{y}{\beta}}}{\beta^{\alpha}\,\Gamma(\alpha)} \tag{1}$$

and the two parameter Gamma distribution function is defined as (Abramowitz and Stegun, 1972)

$$P(\alpha, \beta, y) = \left(\frac{y}{\beta}\right)^{\alpha} \frac{e^{\left(-\frac{y}{\beta}\right)}}{\Gamma(\alpha+1)} M(1,1+\alpha,\frac{y}{\beta}) \tag{2}$$

where $M(1,1+\alpha, y/\beta)$ is Kummer function and is defined as given below. (Abramowitz and Stegun, 1972)

$$M(1,1+\alpha,\frac{y}{\beta}) = 1 + \frac{1.\left(\frac{y}{\beta}\right)}{(1+\alpha)} + \frac{1.2.\left(\frac{y}{\beta}\right)^{2}}{(1+\alpha).(2+\alpha).2!} + \ldots + \frac{1.2.3\ldots n.\left(\frac{y}{\beta}\right)^{n}}{(1+\alpha).(2+\alpha)\ldots(n+\alpha).n!} \tag{3}$$

In the two parameter Gamma distribution function $\alpha$ and $\beta$ are shape and scale parameters, respectively.

While fitting the two parameter Gamma distribution to

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weekly or monthly rainfall data zero and non-zero values were separated from the series of each week or month. The shape parameter, α, and scale parameter, β, were determined only for the non-zero values. Thom (1958) estimated the shape parameter (α) and scale parameter (β) based on truncation of series expansion of the maximum likelihood estimator. These parameters are defined as below.

$$\alpha=\frac{1}{4.A}\left(1+\sqrt{\left(1+4.\frac{A}{3}\right)}\right) \tag{4}$$

in which

$$A=\ln\bar{y}-\frac{\sum_{i=1}^{n}\ln y}{n} \tag{5}$$

Here n is the number of values in data series greater than zero, and y is arithmetic mean of non-zero values. Again α can be corrected for biasness and thus estimated value of α becomes

$$E(\alpha)=(n-3)\frac{\alpha}{n} \quad \text{for } n\geq 4 \tag{6}$$

and estimated value of β becomes

$$\beta=\frac{\bar{y}}{\alpha} \tag{7}$$

Variance ratio test suggested by Mooley (1974) was applied to test the fitting of Gamma distribution to the non-zero rainfall data. This test can be applied for all distributions for which the theoretical variance can be determined independently from the parameters estimated by a method other than the method of moments. If α and β are maximum likelihood estimates then the theoretical variance is $\beta^2\alpha$.

Chi-square test statistics is computed employing the equation given below.

$$\chi_v^{\,2}=\frac{\sum_{i=1}^{n}(y_i-\bar{y})^2}{\beta^2\alpha} \tag{8}$$

Computed Chi-square values are compared with tabulated Chi-square values for (n-1) degree of freedom at 95% confidence level. If computed Chi-square value at 95% confidence level is less than Chi-square value given in the table then the Gamma distribution fits the series of non-zero rainfall.

Probability, P, as defined by equation (2) can be determined for different values of non-zero rainfall using shape and scale parameters. Probability, Q, for zero rainfall is calculated by dividing the number of zero values by total number of rainfall values in a series, i.e.

$$Q=\frac{M}{N} \tag{9}$$

where M is number of zero values and N is total number of zero and non-zero rainfall values in a series.

The total probability of rain being less than or equal to a particular value is given by:

$$T=Q+(1-Q)P \tag{10}$$

where T is total probability of rain being less than or equal to a particular value in a mixed Gamma distribution; Q is probability of rain being zero in a series; (1-Q) is probability of rain being non-zero in a series; and P is the probability of rain being less than or equal to a particular value, y.

In order to determine the weekly, monthly, annual rainfall values at desired probability levels total probabilities were calculated for each week, month and year for the rainfall values starting from 2 mm with an increment of 2 mm and ending to the values at which probability becomes 99%. A table having rainfall values and probability corresponding to these values was generated from above analysis. In order to compute rainfall values at different probability levels the technique of linear interpolation as defined below was employed.

If at level of probability, T, value of rainfall, R, has to be determined and corresponding to $T_i$ and $T_{i+1}$ probability levels rainfall values Ri and Ri+1 are known, then interpolated value of rainfall, R, was computed as below.

$$R=\frac{(R_{i+1}-R_i)}{(T_{i+1}-T_i)}(T-T_i)+R_i \tag{11}$$

A computer program was written in Fortran 90 to create tables for each week containing rainfall and probabilities and technique of linear interpolation to compute rainfall at desired probability levels.

Average and probabilistic annual, monthly and weekly rainfall at Daudnagar raingauge station in head reach, Paliganj raingauge station in the middle reach, and Naubatpur raingauge station in the tail reach of Patna Main Canal were computed. Analysis reveals that average annual rainfall at these raingauge stations is 997.4, 888.9, and 974.1 mm, respectively. Maximum and minimum rainfall values in head, middle and tail reaches are 1555.0 mm in 1997 and 478.2 mm during 1982, 1342.4 mm in 1997 and 490.2 mm during 1998, and 1452.3 mm in 1997 and 537.6 mm during 1982. In head, middle and tail reaches at representative raingauge sites, during monsoon months (June to September), 87.6 %, 90.3% and 88.8% rainfall and during non-monsoon months, 12.4%, 9.7% and 11.2% rainfall occurred. At Daudnagar, average monthly rainfall of July is the highest i.e. 319.8 mm followed by 234.9 mm in August, 188.7 mm in September and 130.8 mm in June; whereas at Paliganj, the highest of 292.6 mm in July followed by 214.8 mm in August, 183.7 mm in September and 111.3 mm in June; and at Naubatpur the highest of 348 mm in July followed by 224.7 mm in August, 185.7 mm in September and 106.6 mm in June is observed. Average weekly rainfall is maximum of 82.2 mm, 75.4 mm, and 89.5 mm in 28th week and varies in the range of 426.6 to 0.0 mm, 534.8 to 0.0 mm, and 522.0 to 0.0 mm, respectively at Daudnagar, Paliganj and Naubatpur raingauge stations.

A comparison of average annual rainfall at the raingauge stations representing head, middle and tail reaches of PMC shows that it lies in the range of 888.9 mm to 999.4 mm. Average annual rainfall is the highest in the head reach followed by tail and middle reaches. In the month of July average monthly rainfall is the highest at Naubatpur followed by Daudnagar and Paliganj, respectively.

Two parameter Gamma probability distribution model was tried to fit in annual, monthly and weekly rainfall at Daudnagar, Paliganj and Naubatpur. It is observed that this model fits well in weekly, monthly and annual rainfall of all the three locations. The model was used to predict annual, monthly and weekly rainfall at different probability levels at these locations, since in planning, decision making, and canal operation rainfall at different probability levels play an important role. So, predicted annual, monthly, and weekly rainfall values at probability of exceedance (0.50, 0.75 and 0.90) are given below in Table 1-3 at Daudnagar, Paliganj and Naubatpur raingauge stations.

Table 1. *Predicted annual rainfall at three different probabilities using two parameter Gamma probability distribution model at Daudnagar, Paliganj and Naubatpur (Head, Middle and Tail Reach of Patna Main Canal)*

| Place | Predicted annual rainfall at different probabilities | | |
|---|---|---|---|
| | 0.50 | 0.75 | 0.90 |
| Daudnagar | 970.8 | 793.8 | 654.5 |
| Paliganj | 869.2 | 724.2 | 608.6 |
| Naubatpur | 953.8 | 798.6 | 674.4 |

It may be observed from Table 1 that annual rainfall at Paliganj (middle reach) at 50%, 75% and 90% probability levels is the lowest compared to annual rainfall at Daudnagar (head reach) and Naubatpur (tail reach). At 50% probability level annual rainfall at Naubatpur (tail reach) is less than annual rainfall at Daudnagar (head reach) but at 75% and 90% probability levels annual rainfall at Naubatpur (tail reach) is more than the annual rainfall at Daudnagar (head reach). Annual rainfall at Naubatpur at 50%, 75% and 90% probability levels are 953.8 mm, 798.6 mm and 674.4 mm, respectively, whereas at Daudnagar and Paliganj, these values are 970.8 mm, 793.8 mm, and 654.5 mm and 869.2 mm, 724.2 mm, and 608.6 mm, respectively.

Predicted monthly and weekly rainfall values at different probability levels (0.50, 0.75 and 0.90) computed at Daudnagar, Paliganj and Naubatpur raingauge stations are reported above in Tables 2 and 3, respectively. It may be observed from Table 2 that in the month of July rainfall at all these locations is maximum and reduces gradually in August, September and June. Table 3 shows significant variation in weekly rainfall at all the three locations and suggests that studies should be conducted using rainfall of these raingauge stations representing head, middle and tail reach of Patna Main Canal.

Two parameter Gamma probability distribution model was observed to fit well in weekly, monthly and annual rainfall data of 32 years (1974-2005) at all the three raingauge stations located in head, middle and tail reaches. Employing this model probable weekly, monthly and annual rainfall values at probability levels of 0.50, 0.75 and 0.90 were computed. Weekly and monthly variation in rainfall at the three raingauge stations representing head, middle and tail reaches of Patna Main Canal is significant so for better planning about release of water

Table 2. *Predicted monthly rainfall at different probabilities using two parameter Gamma probability distribution model at Daudnagar, Paliganj and Naubatpur (Head, Middle and Tail Reach of Patna Main Canal)*

| Month | Predicted monthly rainfall in Head, Middle and Tail reaches at three different probability levels | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|
| | 0.50 | | | 0.75 | | | 0.90 | | |
| | Head Reach | Middle Reach | Tail Reach | Head Reach | Middle Reach | Tail Reach | Head Reach | Middle Reach | Tail Reach |
| January | 5.6 | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 |
| February | 3.2 | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 |
| March | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 |
| April | 0.0 | 4.8 | 2.7 | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 |
| May | 16.2 | 7.9 | 21.1 | 3.7 | 0.0 | 7.3 | 0.0 | 0.0 | 0.0 |
| June | 100.5 | 95.2 | 95.1 | 49.2 | 56.3 | 61.4 | 22.4 | 32.4 | 39.2 |
| July | 293.4 | 261.6 | 312.2 | 202.1 | 169.8 | 204.0 | 138.6 | 109.0 | 132.0 |
| August | 216.7 | 201.4 | 201.8 | 151.1 | 146.5 | 132.1 | 105.1 | 106.8 | 85.6 |
| September | 157.2 | 159.4 | 158.4 | 84.8 | 97.3 | 93.2 | 39.7 | 58.1 | 53.4 |
| October | 26.5 | 19.3 | 24.8 | 5.7 | 4.7 | 3.7 | 0.0 | 0.0 | 0.0 |
| November | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 |
| December | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 |

Table 3. *Predicted weekly rainfall at different probabilities at Daudnagar, Paliganj and Naubatpur (Head, Middle and Tail Reach of Patna Main Canal)*

| Month | Predicted weekly rainfall in Head, Middle and Tail reaches at three different probability levels | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|
| | 0.50 | | | 0.75 | | | 0.90 | | |
| | Head | Middle | Tail | Head | Middle | Tail | Head | Middle | Tail |
| 1 | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 |
| 2 | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 |
| 3 | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 |
| 4 | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 |
| 5 | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 |
| 6 | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 |
| 7 | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 |
| 8 | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 |
| 9 | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 |
| 10 | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 |
| 11 | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 |
| 12 | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 |
| 13 | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 |
| 14 | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 |
| 15 | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 |
| 16 | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 |
| 17 | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 |
| 18 | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 |
| 19 | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 |
| 20 | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 |
| 21 | 0.0 | 0.0 | 3.2 | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 |
| 22 | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 |
| 23 | 0.0 | 0.0 | 2.7 | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 |
| 24 | 0.0 | 6.5 | 6.8 | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 |
| 25 | 19.7 | 23.8 | 18.8 | 0.0 | 8.2 | 3.2 | 0.0 | 0.0 | 0.0 |
| 26 | 35.8 | 36.7 | 29.2 | 8.2 | 8.5 | 4.1 | 0.0 | 0.0 | 0.0 |
| 27 | 56.3 | 39.5 | 64.9 | 25.5 | 15.4 | 30.6 | 0.0 | 3.1 | 9.2 |
| 28 | 48.7 | 48.7 | 57.7 | 6.8 | 16.2 | 19.1 | 0.0 | 2.4 | 2.8 |
| 29 | 56.0 | 51.2 | 52.1 | 26.0 | 21.4 | 17.3 | 11.1 | 5.0 | 2.6 |
| 30 | 56.9 | 48.7 | 47.8 | 27.3 | 20.0 | 22.0 | 10.8 | 4.6 | 9.3 |

| | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|
| 31 | 34.8 | 38.6 | 36.0 | 18.3 | 17.6 | 17.5 | 8.2 | 6.5 | 7.9 |
| 32 | 44.8 | 40.2 | 38.7 | 19.9 | 9.8 | 9.8 | 5.3 | 0.0 | 0.0 |
| 33 | 37.5 | 38.8 | 32.7 | 17.7 | 17.8 | 12.3 | 6.8 | 5.0 | 0.0 |
| 34 | 43.0 | 29.8 | 48.2 | 14.7 | 7.2 | 17.6 | 0.0 | 0.0 | 3.2 |
| 35 | 39.8 | 31.1 | 26.9 | 14.1 | 10.2 | 7.0 | 2.4 | 0.0 | 0.0 |
| 36 | 39.8 | 38.6 | 38.2 | 13.8 | 17.4 | 12.5 | 0.0 | 4.8 | 0.0 |
| 37 | 37.8 | 36.9 | 42.1 | 10.8 | 13.2 | 17.6 | 0.0 | 3.3 | 5.7 |
| 38 | 8.5 | 13.5 | 18.1 | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 |
| 39 | 4.4 | 8.9 | 9.7 | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 |
| 40 | 0.0 | 9.7 | 12.9 | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 |
| 41 | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 |
| 42 | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 |
| 43 | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 |
| 44 | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 |
| 45 | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 |
| 46 | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 |
| 47 | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 |
| 48 | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 |
| 49 | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 |
| 50 | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 |
| 51 | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 |
| 52 | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 |

from Patna Main Canal for irrigation of crops in its command probable rainfall at head, middle and tail reach should be considered.

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