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प्रवृति-मुक्त अभिकल्पनाओं पर एक अध्ययन

A Study on Trend-Free Designs

कृष्ण लाल
राजेन्द्र प्रसाद
विनोद कुमार गुप्ता

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FOREWORD

Trend-free block designs are quite useful in the experimental situations that may have trend effect in the experimental units. These designs have wide applications when experiments are conducted in Green house where the source of heat is located on sides of the house and the experimental units (pots) are kept in lines; in poultry experiments where the source of heat is at the centre of the cage and chicks of early age are in the different tiers and experiments in hilly areas where the land is undulated. Keeping in view the importance of trend-free designs, the Institute undertook a project **A Study on Trend-Free Designs**.

The results available on trend-free block designs in the literature are for the experimental situations with blocks of equal sizes. There, however, do occur experimental situations where block designs are to be used with unequal block sizes. When the block sizes are unequal, then intra-block variances are not constant. Therefore, the present investigation is an attempt to obtain trend-free block designs under heteroscedastic model for non-proper block designs. A ***trend-free block design*** is an arrangement of treatments to plots within blocks such that the known properties of ordinary analysis of variance for treatment and block sum of squares are preserved and variation due to trend effect is removed from the error sum of square.

It is known that binary variance balanced block designs (BBB) designs of Type α, where α can take any non-negative real value, are the most efficient designs in the competing class of designs. Therefore, a necessary and sufficient condition for a block design to be trend-free block design under heteroscedastic set up when intra-block variances are proportional to non-negative real power of block sizes is obtained. Using the condition catalogues of trend-free BBB designs of Type α, both under homoscedastic ($\alpha = 0$) and heteroscedastic model (for $\alpha = 1, 2, 3$), is prepared. Heteroscedasticity of the model increases as value of α increases. Catalogue of trend-free balanced incomplete block (BIB) designs and two associate class partially balanced incomplete block (PBIB) designs are also prepared. Sometimes it may not be possible to convert every design to trend-free design then linear trend-free designs are given. Further, nearly trend-free designs are identified when it is not possible to obtain even linear trend-free designs.

Nested block designs are required when there is need of making sub-blocks within a larger block. For such situations nested balanced incomplete block (NBIB) designs are quite useful. The condition for a NBIB design to be trend-free NBIB design at sub-block level is obtained. NBIB designs that are trend-free both at sub-block and block level have also been identified. Catalogues of trend-free and linear trend-free NBIB designs at sub-block levels are also prepared. NBIB designs with sub-block size 2 and designs for complete diallel cross experiments have a one-to-one correspondence. Utilizing this relationship a catalogue of trend-free optimal block designs for a diallel cross experiments with number of inbred lines, $p \leq 30$ is prepared. The catalogues of the trend-free/ linear trend-free designs, prepared in this project, will serve as a ready reckoner to the practicing statisticians and the experimenters. The scientists associated with the project deserve appreciation for carrying out this work.

S.D. SHARMA
DIRECTOR

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जिन परीक्षणों के परीक्षणात्मक इकाई में प्रवृति प्रभाव होता है उनमें प्रवृति–मुक्त ब्लॉक अभिकल्पनायें (trend free block designs) बहुत उपयोगी होती हैं । इन अभिकल्पनाओं का उपयोग अधिक तब होता है जब परीक्षण ग्रीन हाऊस में संचालित किए जाते हैं, जहाँ पर उष्मा का स्रोत ग्रीन हाऊस के किनारों पर होता है और परीक्षणात्मक इकाईयाँ पंक्ति में होती हैंय पोल्ट्री परीक्षणों में, जहाँ उष्मा का स्रोत पिंजरे के मध्य में होता हैं तथा कम उम्र के चूजे विभिन्न स्तरों में रखे जाते हैं एवं पहाड़ी क्षेत्रों के परीक्षणों में जहाँ भूमि समतल नहीं होती है । प्रवृति–मुक्त अभिकल्पनाओं की महत्ता देखते हुये संस्थान द्वारा भप्रवृति–मुक्त अभिकल्पनाओं पर एक अध्ययन ” नामक परियोजना शुरू की गई ।

साहित्य में प्रवृति–मुक्त अभिकल्पनाओं के परिणाम समान आकार के ब्लॉकों के परीक्षणों के लिए हैं। लेकिन ऐसी परीक्षणात्मक परिस्थितियाँ उत्पन्न होती हैं जिनमें ब्लॉक अभिकल्पनाएँ असमान आकार के ब्लॉकों के लिए प्रयोग में लाई जाती हैं । जब ब्लॉकों के आकार असमान होते हैं तब अतःब्लॉक प्रसरण स्थिर (constant) नहीं होता है । अतः प्रस्तुत अण्वेषण में नॉन–प्रॉपर ब्लॉक अभिकल्पनाओं के लिए विषमांगीय (heteroscedastic) मॉडल के तहत प्रवृति–मुक्त ब्लॉक अभिकल्पनाएँ प्राप्त करने का प्रयास किया गया है । एक प्रवृति–मुक्त ब्लॉक अभिकल्पना में, ट्रीटमेंटस (treatments) ब्लॉक के अन्दर के प्लाटों पर इस प्रकार सजोंए जाते हैं कि प्रसरण विश्लेषण (analysis of variance) में ट्रीटमेंटस तथा ब्लॉक के वर्गों का जोड़ वही रहता है तथा प्रवृति प्रभाव को सीधे त्राुटि के वर्गों के जोड़ ;मततवत`नउ वf֊नंतमद्ध में से घटा देते हैं ।

यह विदित है कि टाईप α की द्विआधारी प्रसरण संतुलित ब्लॉक अभिकल्पनाएँ (BBB), जहां α कोई भी नॉन–नेगेटिव वास्तविक मान हो सकता है, अभिकल्पनाओं की प्रतियोगी श्रेणी में सबसे अधिक दक्ष अभिकल्पनाएँ हैं । अतः एक ब्लॉक अभिकल्पना के विषमांगीय सैटअप के तहत, जब अन्तःब्लॉक प्रसरण ब्लॉक आकारों की नॉन–नेगेटिव वास्तविक घात के प्रति आनुपातिक हैं, तब प्रवृति–मुक्त ब्लॉक अभिकल्पना होने की अपेक्षित एवं पर्याप्त प्रतिबंध (necessary and sufficient condition) प्राप्त की गई है । इस प्रतिबंध का उपयोग करते हुए, समांगीक (α = 0) एवं विषमांगीक मॉडल (α = 1, 2, 3 के लिए) दोनों के तहत टाईप α की प्रवृति–मुक्त ठठठ अभिकल्पनाओं का कैटालॉग तैयार किया गया है । जैसे–जैसे α के मान में वृद्धि होती है, वैसे–वैसे मॉडल की विषमांगीयता में वृद्धि होती है । प्रवृति–मुक्त संतुलित अपूर्ण ब्लॉक ;ठप्ठद्ध अभिकल्पनाओं तथा दो साहचर्य आंशिक संतुलित अपूर्ण ब्लॉक ;च्ठप्ठद्ध अभिकल्पनाओं के कैटालॉग तैयार किए गए हैं । कभी–कभी सभी अभिकल्पनाओं को प्रवृति–मुक्त अभिकल्पना में परिवर्तित करना सम्भव नहीं होता, ऐसी स्थिति में रैखिक प्रवृति–मुक्त अभिकल्पनाएँ दी गई हैं । इसके

अतिरिक्त, जब रैखिक प्रवृति–मुक्त अभिकल्पनाएँ भी प्राप्त करना सम्भव नहीं होता तब लगभग रैखिक प्रवृति–मुक्त अभिकल्पनाओं की पहचान की गई है ।

जब बड़े ब्लॉक में उप–ब्लॉक बनाने की आवश्यकता होती है तब नेस्टड ब्लॉक अभिकल्पनाओं की आवश्यकता होती है । ऐसी स्थितियों में नेस्टड संतुलित अपूर्ण ब्लॉक ;छठप्ठद्ध अभिकल्नाएं बहुत उपयोगी होती हैं । एक नेस्टेड संतुलित अपूर्ण ब्लॉक अभिकल्पना को उप–ब्लॉक स्तर पर प्रवृति–मुक्त नेस्टेड संतुलित अपूर्ण ब्लॉक अभिकल्पना बनाने के लिये प्रतिबंध (condition) प्राप्त किया गया है । ऐसी नेस्टेड संतुलित अपूर्ण ब्लॉक अभिकल्पनायें, जो उप–ब्लॉक तथा ब्लॉक स्तर, दोनों स्तरों पर प्रवृति–मुक्त हैंए पहचान की गई है । उप–ब्लॉक स्तर पर प्रवृति–मुक्त तथा रैखिक प्रवृति–मुक्त नेस्टेड संतुलित अपूर्ण ब्लॉक ;छठप्ठद्ध अभिकल्पनाओं के कैटलॉग तैयार किए गए हैं । नेस्टेड संतुलित अपूर्ण ब्लॉक अभिकल्पनाओं (NBIB), जिनका उप–ब्लॉक आकार दो (2) है, का पूर्ण डायलल क्रास परीक्षणों की अभिकल्पनाओं से सीधा संबंध है । इस संबंध का उपयोग करते हुए, अंतःप्रजात क्रम (inbred lines), $p \leq 30$ के डायलल क्रॉस परीक्षणों के लिए प्रवृति–मुक्त इष्टतम ;वचजपउंसद्ध ब्लॉक अभिकल्पनाओं का कैटलॉग तैयार किया गया है । इस परियोजना में तैयार किए गए प्रवृति–मुक्त /रैखिक प्रवृति–मुक्त अभिकल्पनाओं के कैटलॉग सांख्यिकीविदों एवं परीक्षणकर्ताओं के लिए परिकलित्राा (ready reckoner) के रूप में मदद करेंगे । इन परियोजना से जुड़े वैज्ञानिक इस कार्य के लिए प्रशंसा के पात्र हैं ।

सुखदेव शर्मा

निदेशक

PREFACE

The data generated from designed experiments are used to draw valid inferences about the population. In several experimental situations, the response is dependent on the spatial or temporal position of the experimental units within a block and thus trend in the experimental units become another important nuisance factor. In such situations, a common polynomial trend of a specified degree over units within blocks may be appropriately assumed. One may think of suitable designs, in which treatment effects are orthogonal to trend effects, in the sense that analysis of the design could be done in usual manner, as if no trend effects were present. Such designs are called ***trend-free block designs***.

Most of the work on trend-free block designs is for the experimental situations having blocks of equal sizes. There, however do occur experimental situations where block designs with unequal block sizes and/ or with unequal replications are to be used. For example, non-proper block design setting occurs while experimenting with natural blocks such as littermates (animal experiments), trusses per blossom (horticultural experiments), family sizes as blocks (psychological experiments), batches of test material (industrial experiments), etc. Experimenting on hilly areas, wastelands or salinity in field experiments may also force the experimenter to have blocks of unequal sizes. When the block sizes are unequal, the intra block variances may not be constant. Through uniformity trial data, it has been shown in the literature that intra-block variances are proportional to non-negative real power of block sizes.

Therefore, a necessary and sufficient condition for a block design to be trend-free block design under heteroscedastic set up when intra-block variances are proportional to non-negative real power of block sizes is obtained. This condition is simplified for homoscedastic model. It is known that binary variance balanced block (BBB) designs of Type α, where α can take any non-negative real value and heteroscedasticity increases as value of α increases, are the most efficient designs in the competing class of designs. Therefore, catalogues of trend-free, linear trend-free and nearly linear trend-free BBB designs of Type α both under homoscedastic (α = 0) and heteroscedastic model (for α = 1, 2, 3) is prepared. Catalogues of trend-free, linear trend-free and nearly linear trend-free balanced incomplete block (BIB) designs and two associate class partially balanced incomplete block (PBIB) designs are also prepared.

In some experimental situations, the sources causing heterogeneity in the experimental material are nested within each other. To deal such situations nested block designs can be usefully employed. Seeing the usefulness of nested block designs in real life situations, a necessary and sufficient condition for a nested balanced incomplete block (NBIB) design to be trend-free NBIB design at sub-block level is obtained. Some NBIB designs that are trend-free/ linear trend-free/ nearly linear trend-free both at sub-block and block level have also been identified. NBIB designs with sub-block size 2 and designs for complete diallel cross experiments have a one-to-one correspondence. Utilizing this relationship a catalogue of trend-free optimal block designs for a diallel cross experiments with $p \leq 30$ is prepared.

It is generally said that there is no randomization in trend-free designs. It is shown through examples that randomization in trend-free designs is restricted to some extent but not vanished. It is similar to that as we go from designs complete randomized design to randomized complete block design and then to Latin square design, the randomization goes on restricted.

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आमुख

अभिकल्पित परीक्षणों से उत्पन्न आंकडे समष्टि (population) के बारे में वैद्य निष्कर्ष निकालने के लिए उपयोग में लाये जाते हैं । अधिकतर प्रयोगात्मक परिस्थितियों में अनुकिया (resonse), परीक्षण इकाईयों की ब्लॉक के अंदर स्पेशियल (spatial) या सामयिक (temporal) स्थिति पर निर्भर करता है अतः परीक्षण इकाईयों में प्रवृति एक अन्य प्रमुख विषमांगी घटक हो जाता है । इन परिस्थितियों में, एक स्पेसिफाइड़ ;चमबपपिमकद्ध घात वाली बहुपदी प्रवृति (polynomial trend) मानी जा सकती है । इस स्थिति में, ऐसी उपयुक्त अभिकल्पना सोची जा सकती है जिसमें ट्रीटमेंटस के प्रभाव, प्रवृति प्रभाव के लाम्बिक (orthogonal) हो तथा आंकडों का विशलेषण बिल्कुल उसी तरह किया जा सकता हो जैसे कोई प्रवृति प्रभाव नहीं है । इन अभिकल्पनाओं को प्रवृति मुक्त ब्लॉक अभिकल्पनायें कहते हैं ।

प्रवृति मुक्त ब्लॉक अभिकल्पनाओं में अधिकतर शोधकार्य समान आकार वाले ब्लॉकों की प्रयोगात्मक परिस्थितियों के लिए हुआ है । परन्तु ऐसी प्रयोगात्मक परिस्थितियां उत्पन्न होती हैं जिनमें असमान आकार के ब्लॉक तथा/या ट्रीटमेंटस की असमान पुनरावृति {रिपलिकेशन्स}, प्रयोग में लाये जाते हैं । उदाहरण के लिये, असमान ब्लॉक अभिकल्पनायें तब होती हैं जबकि परीक्षण प्राकृतिक ब्लॉक में जैसे कि लिटरमेटस {पशु परीक्षण} एक ट्रूसस (trusses) में बौर खिलना {बागवानी परीक्षण}, परिवार का आकार {मनोवैज्ञानिक परीक्षण}, गुणवता के जांच के लिए उत्पाद का समूह (batches of test material) {औद्योगिक परीक्षण}, इत्यादि होते हैं। पहाड़ी क्षेत्रों में, वेस्टलैंड़ (wasteland) में या लवणता वाले खेतों में, असमान आकार के ब्लॉक इस्तेमाल करने पड़ते हैं । जब ब्लॉकों के आकार असमान होते हैं तब अन्तः ब्लॉक प्रसरण ब्लॉक के आकार के नॉन–नेगेटिव वास्तविक घात के बराबरं होते हैं ।

अतः विषमांगीय सैट अप में एक ब्लॉक अभिकल्पना को प्रवृति–मुक्त ब्लॉक अभिकल्पना बनाने के लिये, जबकि अन्तः ब्लॉक प्रसरण ब्लॉक आकार की नॉन–नेगेटिव वास्तविक मान की घात के अनुपातिक है, अपेक्षित एवं प्रयाप्त प्रतिबंध (necessary and sufficient condition) प्राप्त की गयी है । इस प्रतिबंध को समांगीक {होमोसिडास्टिक} मॉडल के लिये सरलीकरण किया गया है । टाईप α की द्विआधारी प्रसरण संतुलित ब्लॉक अभिकल्पनायं (BBB designs of type α) जहां कि α कोई भी नॉन–नेगेटिव मान हो सकता है तथा विषमांगीयता α के मान बढ़ने से बढ़ती है, कम्पीटिंग क्लास की अभिकल्पनाओं में बहुत दक्ष जानी जाती है । अतः टाईप α की द्विआधारी प्रसरण संतुलित ब्लॉक अभिकल्पनाओं को, समांगीय ($\alpha = 0$)

तथा विषमांगीय माडल (α = 1,2,3) के लिये, प्रवृति–मुक्त, रैखिक प्रवृति–मुक्त तथा लगभग रैखिक प्रवृति–मुक्त अभिकल्पनाओं का कैटालॉग बनाया है । प्रवृति–मुक्त, रैखिक प्रवृति–मुक्त तथा लगभग रैखिक प्रवृति–मुक्त संतुलित अपूर्ण ब्लॉक ;ठप्ठद्ध तथा दो साहचर्य आंशिक संतुलित अपूर्ण ब्लॉक (PBIB) अभिकल्पनाओं का कैटालॉग बनाया गया है ।

कुछ प्रयोगात्मक स्थितियों में परीक्षण सामग्री के विषमांगीय के स्त्राोत एक दूसरे के अंदर नेस्टड हाते हैं । इन प्रयोगात्मक स्थितियों में नेस्टड ब्लॉक अभिकल्पनायें बहुत उपयोगी होती है । नेस्टड ब्लॉक अभिकल्पनाओं की यथार्थ प्रयोगात्मक स्थितियों में उपयोगिता को देखते हुये, नेस्टड संतुलित अपूर्ण ब्लॉक (NBIB) अभिकल्पनाओं को सब–ब्लॉक स्तर पर प्रवृति मुक्त ब्लॉक अभिकल्पनाओं में बदलने के लिये, एक अपेक्षित एवं पर्याप्त प्रतिबंध प्राप्त की गई है । कुछ नेस्टड संतुलित अपूर्ण ब्लॉक अभिकल्पनायें जो कि ब्लॉक तथा सब–ब्लॉक स्तर पर प्रवृति मुक्त/लगभग प्रवृति मुक्त हैं, प्राप्त की गई है । नेस्टड संतुलित अपूर्ण ब्लॉक अभिकल्पनाओं का, जिनका सब–ब्लॉक आकार दो है, पूर्ण डायलल कास परीक्षणों की अभिकल्पनाओं के साथ सीधा संबंध है । इस संबंध को इस्तेमाल करते हुये डायलल कास इष्टतम परीक्षणों के लिए {आप्टीमल} प्रवृति मुक्त ब्लॉक अभिकल्पनाओं के लिये का एक कैटलाग तैयार किया गया है ।

आमतौर पर ऐसा कहा जाता है कि प्रवृति–मुक्त अभिकल्पनाओं में यादृच्छिकरण (randomization) नहीं होता है । इस अध्ययन में सोदाहरण यह समझाया गया है कि प्रवृति मुक्त अभिकल्पनाओं में यादृच्छिकरण किसी स्तर तक सीमित तो हो जाता है, परन्तु पूरी तरह समाप्त नहीं होता है । यह उसी प्रकार है जैसे कि जब हम पूर्णतया यादृच्दिक (complete randomized) अभिकल्पना से यादृच्छिक पूर्ण ब्लॉक (randomized complete block) अभिकल्पना में तथा फिर लैटिन वर्ग (Latin square) अभिकल्पनाओं में जाते हैं तो यादृच्छिकरण सीमित हो जाता है।

इस परियोजना में, प्रवृति–मुक्त/रैखिक प्रवृति–मुक्त संतुलित अपूर्ण ब्लॉक अभिकल्पनाओं, आंशिक संतुलित अपूर्ण ब्लॉक अभिकल्पनाओं, द्विआधारी प्रसरण संतुलित ब्लॉक अभिकल्पनाओं, नेस्टेड संतुलित अपूर्ण ब्लॉक अभिकल्पनाओं एवं डायलल कास परीक्षणों के लिए इष्टतम ब्लॉक अभिकल्पनाओं के तैयार किये गए कैटालॉग सांख्यिकीविदों एवं परीक्षणकर्ताओं के लिए परिकलित्राा (ready reckoner) का कार्य करेगी ।

अनुसंधान के दौरान डा़ सुखदेव शर्मा, निदेशक, भारतीय कृषि सांख्यिकी अनुसंधान संस्थान, नई दिल्ली द्वारा दिये गये प्रोत्साहन के लिये हम उनके प्रति अपना हार्दिक आभार प्रकट करते है । हम डा़ वी.के. शर्मा, प्रभागाध्यक्ष, परीक्षण अभिकल्पना द्वारा इस अनुसंधान कार्य के लिये आवश्यकए दी गई सुविधायें प्रदान करने के लिए उनका हार्दिक धन्यवाद करते हैं । हम डॉ. लाल मोहन भर का अनुसंधान के दौरान किये गये लाभदायक विचार विमर्श के लिये, उनके आभारी हैं । हम डॉ.ए.बी. मण्डल, प्रमुख वैज्ञानिक, न्यूट्रीशन एवं फीड टेक्नोलोजी विभाग

(Division of Nutrition and Feed Technology)सी.ए.आर.आई., इज्जतनगर के साथ, प्रवृति–मुक्त अभिकल्पनाओं का ऐवियरी शोध (aviary research) में इस्तेमाल के लिये किये गये विचार विमर्श के लिए, विशेष रुप से आभारी हैं ।

हम आंतरिक विशेषज्ञ, डॉ. रवीन्द्र श्रीवास्तव एवं अज्ञात बाह्य विशेषज्ञ के आभारी हैं जिनके सुझावों से इस परियोजना रिपोर्ट की प्रस्तावना में सुधार हुआ है । श्रीमती रेणुका आहुजा का इस परियोजना रिपोर्ट को टाइपिंग करने में मदद के लिए धन्यवाद ।

दिसंबर, 2005

कृष्ण लाल
राजेन्द्र प्रसाद
वी.के. गुप्ता

CONTENTS

CHAPTER I

INTRODUCTION

1.1 Introduction

In several designed experiments, the experimental units exhibit a smooth trend over time or space and thus random allocation of treatments to the experimental units may no longer be appropriate for obtaining the efficient estimates of the parameters. Instead systematic run orders or designs, in which the treatments are to be allotted to experimental units in some order, may have to be used to eliminate the effects of such trend. The resulting designs are called as ***trend-free designs***.

Consider a sensory experiment where the quality of the product (say chicken, custard etc.) depends on temperature and thus treatments are different temperatures. In such experiments flavour of the product varies with temperature and the experimenter has to go from lower temperature to higher temperature stepwise and changing of temperature is technically difficult. Thus, there it is not possible to adopt the procedure of randomization.

For better understanding, we shall illustrate some experimental situations where an experimenter has to use systematic (trend-free) designs in different experimental settings.

1.2 Completely randomized designs

Consider the following experiment described by Cox (1951). An experimenter wants to compare the effect in processing of a number of treatments applied to wool. The wool is divided into lots as alike as possible and the lots are numbered in random order. In each week, only one lot is processed with a certain treatment. The experimenter believes that age of the wool affects the process so that there will be a smooth trend due to aging. In this situation a systematic order of assignment of treatments or treatment combination to experimental units, instead of randomized order may be reasonable to do in order to reduce or eliminate the effect of the trend. The intention is to find the systematic run order so that the properties of the ordinary analysis of variance are preserved. He gave some systematic designs in which trend effect is eliminated in the designs of zero-way elimination of heterogeneity setting.

Cox (1958) described an experimental situation in which the experimental units have a trend effect. This is described below:

Experimental Situation 1.1: "[Cox (1958)]. Consider an experiment to investigate the effect on textile process of changing the relative humidity. Suppose that three relative humidities 50, 60 and 70% are to be used. To obtain uniform experimental units a suitable quantity of raw material was taken and thoroughly mixed and then divided into, say, nine experimental units. The first batch was processed at one relative humidity in the first period, the second batch at different relative humidity in the second period, and so on. Superimposed on any treatment effects and on random variations remaining, is likely to be a smooth trend due to aging of the material. It would often be of interest to estimate this trend explicitly, as well as to set up the experiment so that the trend has little or no influence on the estimates of treatment effects."

Consider the following assignment or run order of treatments

T_{60}	T_{50}	T_{70}	T_{70}	T_{60}	T_{50}	T_{50}	T_{70}	T_{60}

The mean influence on T_{50} is $\frac{1}{3}(2+6+7)=5,$ the mean influence on T_{60} is $\frac{1}{3}(1+5+9)=5,$ and the mean influence on T_{70} is also $\frac{1}{3}(2+4+8)=5.$ Thus any contrast among these treatments is not affected by the linear trend.

Now we will illustrate an example how restricted randomization is possible in systematic designs.

Suppose we have 2 varieties (treatments), each replicated 4 times in an experimental design of 8 units and the treatments are allotted to plots in the following order

T_2	T_1	T_1	T_2	T_1	T_2	T_2	T_1
-7	-5	-3	-1	1	3	5	7

The same experimental material can be arranged in the following manner

T_1	T_2	T_1	T_2	T_2	T_1	T_2	T_1
-7	-5	-3	-1	1	3	5	7

In both these arrangements sum of the positions is same or $\sum_i \xi'_p$ = 0, i = 1, 2 where ξ'_p is the orthogonal polynomial of order p (here p=1) and $\sum_i$ denotes summation over all plots receiving treatments T_i. This makes the treatment differences orthogonal to the linear trend effect and the design is ***trend-free design for linear effect***. Other arrangements can also be made by interchanging the treatments. Thus in trend-free designs estimate of treatment effects are improved by eliminating the trend effects at some cost of randomization.

Now we illustrate with an example on a set of live data the role of trend in an experiment.

Experimental situation 1.2: In a nutritional avian research experiment, four feeds were the four treatments; say T_1, T_2, T_3 and T_4. The chicks were kept in Cage-tier system. In a cage there are four tiers one below the other. The experimenter knows from previous experiences that if chicks are kept from top tier to bottom tier there are chances of trend due to sun light, fresh air etc. But he is not aware of trend-free designs. He applied the same treatment to all the tiers in a cage to avoid the effect of any trend. In each tier there were 10 birds. He measured the body weight of the birds from birth to sixth week of age.

To confirm whether there is effect of trend among the tiers within the cage, the obtained data were analyzed by two methods; one by usual analysis of variance (ANOVA) and the other by using analysis of covariance (ANCOVA) assuming linear trend within each cage. The results revealed that the efficiency was obtained up to 21 per cent.

Further, Sachdev *et al.* (1989) made a study to see the cage-tier effect on feed consumption, egg production and egg-quality traits of Japanese quails. The data used were on female adult quails, 200 each from line A and B, up to 50 weeks of age. Significant effect of cage-tier locations was observed in line A on feed consumption during 6 to 10th and 15 to 18 weeks of age, as well as on egg quality traits. Despite insignificant changes on total feed consumption and egg production, superior numerical values were recorded from the birds located in the top tier. Better quality eggs were produced by fourth (from top) quails. In line B, the feed consumption during 23rd and 26th weeks of age, total egg production, feed efficiency, yolk index and shell weight were significant influenced by cage-tier locations. Better feed efficiency and egg quality were recorded in the birds placed in the first and fourth respectively.

1.3 Designs for factorial experiments

Factorial designs have been widely used in agricultural/ industrial experiments. The application of systematic designs in factorial settings has been investigated in industrial experiments.Athough these designs can also be used where the experimental resources are scarce/costly and the experimental units may exhibit trend-effect such as agricultural experiments conducted on hills, in animal science experiments where the experimental cost is much and the animals (experimental units) may exhibit a trend-effect due to change in birth weight from animal to animal. Box and Hay (1953) gave the method of construction of a certain class of designs with quantity factors by means of which trend occurring during a comparative experiment may be eliminated without loss of efficiency. The design and analysis is illustrated with an example of bio-assay. Philips (1964, 1968a, 1968b) developed magic squares, magic rectangles of even orders for the balancing of linear, and occasionally quadratic or cubic trends in general class of factorial designs. To be clearer, we illustrate an example of linear trend-free design for factorial experiment given by Philips (1968b) by using magic square.

Experimental Situation 1.3: Consider the $2 \times 2 \times 2 \times 2$ factorial design shown in Table 2.1, in which A, B, C and D represent the four treatments and their subscripts 1 and 2 are the two levels of each of them. The numbers of the occasions of measurement have been entered in the body of the table as a symmetric magic square.

Table 1.1

		C_1		C_2	
		D_1	D_2	D_2	D_1
A_1	B_1	16	2	3	13
	B_2	5	11	10	8
A_2	B_2	9	7	6	12
	B_1	4	14	15	1

In above square table of order $n \times n$, sum of each row, column and diagonal is $\frac{1}{2}n(n^2+1)$ *i.e.* 34. The main effects A, B, C and D are linear trend-free as the average for each of the factor is $\frac{68}{8} = 8.5$. Also for the interaction $A \times B$, $A \times C$, $B \times D$ and $C \times D$ the mean effect is same for each of its level *e.g.* in interaction $A \times B$ for each level A_iB_j i = 1,2; j = 1,2 mean effect is $\frac{34}{4} = 8.5$. But it is not true for interactions $A \times D$ and $B \times C$. So these two interactions are not linearly trend-free. Also averages of numbers of higher order interactions are not balanced (for example the numbers for third order interactions $A \times B \times C \times D$ are inevitably unbalanced).

1.4 Block Designs

Heterogeneity in the experimental material is the most important problem to be reckoned within the statistical designing of scientific experiments. Occasionally, one can find a certain factor (called nuisance factor) that though not of interest to the experimenter, does contribute significantly to the variability in the experimental material. Various levels of this factor are used for blocking. In experimental situations with only one nuisance factor, block designs are used. These designs are useful in controlling the heterogeneity of the experimental units and it is ascribed to between blocks variability. There, however, do occur situations where the experimental units within blocks may be subjected to trend effects in one or more spatial or temporal dimensions. Some commonly encountered experimental situations in agricultural sciences are described in the sequel.

We now illustrate with an example on live data given by Federer and Schlottfeldt (1954) for completely randomized block design, how trend has affected the units within blocks after the experiment is established and data is analyzed by analysis of covariance (ANCOVA).

Experimental situation 1.4: "[Federer and Schlottfeldt (1954)]. An experiment was devised in the spring of 1951, to determine whether the exposure of tobacco seeds to different dosages of cathode rays would affect the growth of the resulting plants. The seeds were from a strain of tobacco that had been under controlled pollination since 1909, and, hence, the material used in the experiment was highly uniform with respect to its genetical background. The seven different treatments (the different doses of cathode rays) were laid out in a randomized complete block (RCB) design with eight replicates. The plot size was 2 rows by 10 plants with 3 feet between rows and 1.5 feet between the plants. The following measurements were made

i) plant height on 13-07-1951 and 14-08-1951,
ii) length of longest leaf on 13-07-1951 and 14-08-1951 and
iii) width of widest leaf on 13-07-1951.

Shortly after the plants were transplanted to the field it became apparent that an environmental gradient existed from the center of the replicates outward. This was confirmed when the data were obtained. The data were analyzed using: a) usual analysis of variance (ANOVA), b) analysis of covariance (ANCOVA) with position of the treatments within a replicate as covariates. Upon fitting curvilinear covariance of second degree a considerable reduction in mean square error (MSE) is obtained. In fact, the MSE by ANCOVA was little more than half that obtained by ANOVA.

In the above example the experimenter does not know in the beginning that some trend would occur in the experimental units, in later time. If the experimenter would have judged in the beginning before the start of the experiment by visualizing the experimental condition, it would better use the trend-free block design.

To have precise estimates of treatment contrasts it is necessary to eliminate these trend effects. For this the treatments have to be assigned to the plots within the blocks in such a way that known properties of analysis of variance for the treatment and block sum of squares are preserved and variation due to trend is also removed from error sum of squares. Such designs are called ***trend-free block designs***.

Now we discuss, how trend-free designs are better to ANCOVA, once we have a trend-free design. It is because in ANCOVA, the position of the plots are taken as auxiliary variable and the treatments are adjusted to the values that would have been obtained had there been no variation in the auxiliary variable. Here we adjust the treatments for auxiliary variable. But in trend-free block designs, the designs are constructed such that the treatments are orthogonal to treatments. It means that the treatments are assigned to the plots within the blocks in such a way that adjusted treatment sum of squares and unadjusted block sum of squares do not change and the sum of squares due to trends is directly subtracted from the error and thus the error is reduced to the extent (degree) trend is present in the experimental units. Also contrasts do not change in contrast analysis when analyzing the data using trend-free designs. Moreover, the analysis of ANCOVA gets complicated as the degree of trend increases while in trend-free designs only we have to work out the sum of squares separately for each degree of trend.

Bradley and Yeh (1980) first gave the rigorous treatment to the theory of Trend-free block designs. They have considered the situations where a common polynomial trend in one or more dimensions is assumed to exist over the plots in each block of a classical experimental design. They defined a trend-free block design as a block design in which the adjusted sum of squares due to treatments in a model with the trend effects remains the same as in the model without the trend effects. In other words, the presence of trend effects does not affect the adjusted sum of squares due to treatments. The error sum of squares, however, gets reduced when the trend effects are present in the model. They developed a necessary and sufficient condition for a block (complete or incomplete) design to be trend-free block design under a homoscedastic model. Yeh and Bradley (1983) discussed the existence of trend-free block (TFB) design for specified trends under a homoscedastic model when each treatment is equally replicated. Some results for linear and odd degree polynomial TFB designs are given. Bradley and Odeh (1988) gave an algorithm in FORTRAN77 for the construction of linear trend-free block (LTFB) design. Stufken (1988) gave a weak point of Yeh and Bradley (1983), " Every binary block design having $r(k+1) \equiv 0$ (mod 2) can be converted into Linear trend-free block design" with an example. Lin and Dean (1991) gave some general results on the existence of trend-free and partially trend-free designs for both varietals and factorial experiments. They also studied trend-free properties of cyclic, GC/n incomplete and complete block designs. Chai and Majumdar (1993) made a correction to Yeh and Bradley (1983) and proved that a binary block design can be converted into linear trend-free block (LTFB) design when i) the design is BIB design, or ii) k, the block size is an even number, or c) the design is balanced block design with $b \geq 3$. They also gave a distinct definition of LTFB and strongly linear trend-free block (SLTFB) design.

Let a design d be represented by a $k \times b$ array of symbols $1, ..., v$, with columns denoting blocks and rows periods. Thus, if the entry of a cell (i, j) of d is i, it means that under d, treatment i has to be applied in period l of block j. Also let $D(v, b, k)$ denotes the class of all connected block designs in b blocks, k periods based on v treatments. Let $d \in D(v, b, k)$ and s_{dil} denote the number of times treatment i appears in row (period)l. Then a design is LTFB design iff

$$\sum_{l=1}^{k} s_{dil}\Phi_1(l) = 0, \; i=1, ..., v \qquad (1.1)$$

where $\Phi_1(l)$ is the orthogonal polynomials of degree $1, l = 1, ..., k$ and s_{dil} denotes the number of times treatment i appears in row (period) l.

Condition (1.1) holds for binary as well as non-binary designs, and also irrespective of whether k is large, equal or smaller than v (Lin and Dean (1991).

The polynomials $\Phi_1(l)$ satisfy the condition $\Phi_1(l) = -\Phi_1(k-l+1)$. In addition, $\Phi_1((k+1)/2) = 0$, when k is odd. It follows that (1.1) is true whenever

$$s_{dil} = s_{di(k-l+1)}\ ,\ l = 1, ..., [(k+1)/2],\ i = 1, ..., v \tag{1.2}$$

with [.] denotes the largest integer function. It is to note here that, when k is odd condition (1.2) does not impose any restriction on $s_{di(k+1)/2}$. It can be easily seen that condition (1.1) does not in general implies condition (1.2) while the condition (1.2) always implies condition (1.1). Condition (1.2) is in fact necessary and sufficient condition for a design *d* to be "odd degree trend-free". Hence the condition (1.2) is known to be condition for a design to be strongly linear trend-free (SLTF) design.

Jacroux, Majumdar and Shah (1995, 1997) developed some methods for identifying efficient designs when different blocks may have linear trend effect of different slope. Majumdar and Martin (2002) extended the above study for quadratic cubic trend. Lin and Stufken (2002) considered the problem of strongly linear-trend free block design through the use of graph theory and gave algorithm for such designs.

Sometimes it may not be possible to convert every design to trend-free design then we go for linear trend-free design because much of the trend effect is reduced by using linear trend-free designs. But sometimes it is not possible to make the design even linear trend-free and this provides a motivation to go for nearly trend-free designs. Yeh, Bradley and Notz (1985) introduced the concept of nearly trend-free block (NTFB) designs. Let the usual additive model for a block design with polynomial trend terms added is written in terms of plot position *t* and block designation *j* as

$$y_{jt} = \mu + \sum_{i=1}^{v} \delta_{jt}^{i} \tau_i + \beta_j + \sum_{\alpha=1}^{p} \theta_\alpha \Phi_\alpha(t) + e_{jt} \tag{1.3}$$

$j=1, ..., b$; $t=1, ..., k$, where y_{jt} is the observation on plot t of block j; μ, τ_i and β_j are respectively, the usual mean, treatment and block parameters; e_{jt} are random errors assumed to be *iid* with zero mean; $\delta^{i}_{jt} = 1$ or 0 as the treatment i is or is not on the plot (j, t), $i = 1, . . . , v$. A block design under model (1.3) is TFB design iff each trend component is orthogonal to the treatment allocation through the experiment i.e.

$\sum_{i=1}^{v}\sum_{t=1}^{k} \delta_{jt}^{i} \Phi_\alpha(t) = 0$, $\alpha = 1, ..., p$; $i=1, ..., v$. Then Yeh *et al.* (1985) gave two definitions of NTFB designs as below:

Definition 1.1: A block design under model (1.3) is said to be NTFB design of Type A if

$$\sum_{\alpha=1}^{p}\sum_{i=1}^{v}\{\sum_{j=1}^{b}\sum_{t=1}^{k} \delta_{jt}^{i} \Phi_\alpha(t)\}^2 \tag{1.4}$$

is minimum among class of connected designs with the same (treatment-block) incidence matrix.

Definition 1.2: A block design under model (2) is said to be NTFB design of Type B if, among the class of connected designs with same incidence matrix

$$\sum_{\alpha=1}^{p-1}\sum_{i=1}^{v}\{\sum_{j=1}^{b}\sum_{t=1}^{k}\delta_{jt}^{i}\Phi_{\alpha}(t)\}^{2} \tag{1.5}$$

is minimum and

$$\sum_{i=1}^{v}\{\sum_{j=1}^{b}\sum_{t=1}^{k}\delta_{jt}^{i}\Phi_{\alpha}(t)\}^{2} \tag{1.6}$$

is minimum among the designs satisfying (1.4).

NTFB design of type A is natural. It can be interpreted as requiring the overall treatment arrangement to be as orthogonal to all specified trend components possible. The criterion for NTFB design of Type B focuses first on the lower degree components of the trend. It may be particularly useful when there exist designs completely free of trend effect up to degree *p-1*. Bradley and Odeh (1988) developed the algorithm for the construction of LTFB and NTFB design.

Chai (1995) suggested that when the condition $r\,(k+1) \equiv 0 \pmod 2$ for a proper block design to be TFB design does not hold (when k is even and r is odd) the best way, we can do in this case is to have a nearly linear trend-free version of design by permitting the treatment symbols (within block). Chai simplified and brought clarity for the condition of linear NTFB design as

$$(\sum_{l=1}^{k} s_{il}\Phi_1(l))^2 = 1 \qquad \text{for} \quad i = 1, \ldots, v.$$

Ultimately we can say that a block design is NLTFB design if

$$\sum_{i=1}^{v}\left[(\sum_{l=1}^{k} s_{il}\Phi_1(l))^2\right] \le v\,. \tag{1.7}$$

Chai (1995) gave some methods for construction of NTF version of BIB designs to be NTFB designs. He also discussed A-, D- and E- optimality of BIB designs for the model that include trend effects.

In the sequel we illustrate the situation where TFB design does not exist and we have to go for NTFB design.

Consider a BIB design with usual parameters v=4, b=6, r=3, k=2, λ=1. The six blocks are

Block 1	**Block 2**	**Block 3**	**Block 4**	**Block 5**	**Block 6**
1	1	1	2	2	3
2	3	4	3	4	4

In this design the condition $r(k+1) \equiv 0$ (mod 2) does not hold. So the best way is to go for NLTFB design that is given below:

Block 1	**Block 2**	**Block 3**	**Block 4**	**Block 5**	**Block 6**
1	3	1	2	4	3
2	1	4	3	2	4

Here $\Phi_1(1) = -1$ and $\Phi_1(2) = 1$ for all the treatments, as there are only two positions within each block. For the treatment 1, $s_{11} = 2$ and $s_{12} = 1$, as treatment 1 takes position 1 two times and position 2 one time only, in the NTF design. Now $\sum_{l=1}^{2} s_{il}\Phi_1(l) = -1$ and $(\sum_{l=1}^{2} s_{il}\Phi_1(l))^2 = 1$ as $k = 2$. This is true for the treatments $i = 2$, 3, 4. Thus $\sum_{i=1}^{4}\left[(\sum_{l=1}^{2} s_{il}\Phi_1(l))^2\right] = 4$. Thus condition (1.7) is satisfied and we can say that the said design is NLTFB design.

Further, in the class of binary incomplete block designs under proper settings balanced incomplete block (BIB) designs are the best (universal optimal) designs for estimating all possible elementary treatment contrasts. However, there do occur situations in which block designs with unequal block sizes and/ or with unequal replications may be required. For example, non-proper block design setting occurs while experimenting with natural blocks such as littermates (animal experiments), trusses per blossom (horticultural experiments), family sizes as blocks (psychological experiments), batches of test material (industrial experiments), etc. Experimenting on hilly areas, wastelands or salinity in field experiments may also force the experimenter to have blocks of unequal sizes. It is also known that in the class of binary block designs with unequal replications under non-proper setting variance balanced block designs are the most efficient designs for estimating all possible elementary treatment contrasts. In variance balanced block designs, generally it is assumed that intra-block variances are constant. Through empirical investigations, however, it has been shown that intra-block variances are proportional to non-negative real power of block sizes (see e.g. Sardana, Sreenath and Malhotra (1967), Bist, Malhotra and Sreenath (1975), Handa, Sreenath, Sastry, Rajpal and Shukla (1982)). This enforces one to extend from homoscedastic model set up to a heteroscedastic model, where intra block variances are proportional to non-negative real power of block sizes. This model under block design set up has been studied by Das, Gupta and Das (1992), Gupta (1995), Gupta, Das and Dey (1991), Lee and Jacroux (1987), Parsad and Gupta (1994), Parsad, Gupta and Singh (1996), among others. Chai (2002) has established a link between trend-free block designs and block designs for parallel line assays. Using this relation he obtained necessary and sufficient condition for the existence of a ψ-design with unequal block sizes and gave a method of construction of ψ-designs. It seems that no work has been done to obtain trend-free block designs under heteroscedastic model.

Above discussion relates to the experimental situations where there is only one nuisance factor. However, there do occur experimental situations in which one or more factors are nested within the blocking factor. Nested block designs, therefore, have been developed to deal with experimental situations where one nuisance factor is nested within blocking factor. Examples of such experiments are illustrated in Experimental situation 3.1 to Experimental situation 3.3 in Chapter III.

A nested block design is defined as a design with two systems of blocks where the second system is nested within the first. We could not find any literature on obtaining trend-free NBIB designs.

This investigation, therefore, aims to develop trend-free variance balanced, and trend-free nested balanced incomplete block designs. If trend-free designs for variance balanced and NBIB designs are not possible then we shall go for nearly trend-free variance balanced and NBIB designs.

The broad objectives of the proposed study are:

1.5 Objectives

1. To obtain trend-free and nearly trend-free non-proper variance balanced block designs under a homoscedastic and a hetroscedastic model.
2. To obtain trend-free and nearly trend-free nested balanced incomplete block (NBIB) designs.
3. To prepare a catalogue of trend-free/ nearly trend-free designs obtained in objectives 1 and 2.

1.6 Practical Utility

The designs obtained from this study are useful for research workers in adopting the suitable designs when trend of specified degree is expected to be present in the experimental units e.g. green house where source of heat is located on sides of the house, experiments conducted on hilly areas, poultry science experiments where a bulb is fixed in the centre of the shed to give heat to the birds in all directions, animal science experiments in which litter mates having different birth weight in a litter, orchard and vineyard experiments on undulated topography, experiments in which yields are affected by slowly migrating insects entering the area from one side, laboratory experiments where the responses to the experimental units may be affected within time periods by instrument drift or analyst fatigue, etc. The catalogues of the trend-free designs prepared will serve as a ready reckoner to the plasticizing statisticians and the experimenters.

CHAPTER II

TREND-FREE BLOCK DESIGNS UNDER HETEROSCEDASTIC SET UP

2.1 Introduction

In this chapter, we shall present some practical situations where the experimenter is forced to use unequal block designs and the experimental units within the blocks are subject to trend-effect over time or space. A necessary and sufficient condition is derived for a connected block design to be trend-free under heteroscedastic set up, when there is a common trend in experimental units within blocks. Using these results and results given in literature, a catalogue of balanced incomplete block (BIB), partially balanced incomplete block (PBIB) and variance balanced block designs with unequal replications in non-proper settings is also be given.

The meaning of trend-free block design is to assign treatments to plots within blocks so that the known properties of ordinary analysis of variance for treatment and block sum of squares are preserved and variation due to trend effect is removed from the error sum of square. Such an arrangement is called as ***trend-free block design***. We begin with some definitions:

Definition 2.1: A connected proper block design d (v, b, k) with v treatments in b blocks each of size $k \leq v$ is said to be trend-free block (TFB) design if this design trend free of the order up to p $\leq$ $(k\text{-}1)$. The block design is said to be linear trend-free block (LTFB) if it is trend free of order one.

Definition 2.2: A connected block design d (v, b, $k_1, \ldots, k_b$) with v treatments in b blocks of size $(k_1, \ldots, k_b) \leq v$ is said to be trend-free block (TFB) design if this design trend-free of order up to $p \leq \{\min(k_1, \ldots, k_s) - 1\}$. This design is also said to be linear trend-free block (LTFB) if it is trend-free of order one.

So when a TFB design exists for a specified order trend component, it is also a TFB for any subset of these components. If a TFB design does not exist for a specified order, a TFB design does not exist for any larger set of components.

We consider some experimental situations where trend may be present in the experimental units within a block.

Experimental situation 2.1: Tea-garden experiments are generally conducted in hilly areas. In hilly areas, usually the land is undulated. Different dulation of land are taken as blocks and the number of plots in a level is the block size. The number of plots in each block may or may not be equal because of limited available land in a particular dulation. Moreover different levels may be little sloppy due to hilly area and this may cause trend effect due to movement of nutrient in one direction or different soil depths, etc.

Experimental situation 2.2: A nutritional experiment was conducted with four feeds. The experimental units are piglets, the litters are blocks and the number of piglets in a litter is block size. For this, we use block design. As litter size is varying from litter to litter and so generally the block sizes are not equal. The response variable is weekly body weight. This response variable depends upon birth weight of piglets. Birth weights are not equal for all the piglets within a litter (block). This increasing/ decreasing order of birth weight in each litter may cause a trend effect in the experimental units. It is thus required to eliminate the trend effect, which may exist due to varying birth weights of piglets in a litter.

In such experiments usually one has to opt for designs in which block sizes are not equal. But the studies available in literature deal with obtaining trend-free block designs for equal block sizes under homoscedastic set up. No result seems to be available for non-proper block designs under a homoscedastic or heteroscedastic model. Hence, trend-free block designs for blocks of unequal sizes, needs to be investigated. Moreover when block sizes are unequal it is unrealistic to assume that the intra-block variances are equal. Keeping in view, following two-way classified, additive, fixed effects, heteroscedastic model is

$$y_{iju} = \mu + \tau_i + \beta_j + e_{iju}\ ,\ i = 1, \ldots, v;\ j = 1, \ldots, b;\ u = 1, \ldots, n_{ij} \qquad (2.1)$$

where y_{iju} is the observation pertaining to u^{th} experimental unit receiving the i^{th} treatment in the j^{th} block, μ is general mean, τ_i is effect of treatment i, β_j is the effect of block j and e_{iju} are random errors with

$$E(e_{iju}) = 0\ ,$$
$$\operatorname{cov}(e_{iju}, e_{i'j'u'}) = \sigma^2 k_j^{\alpha} \qquad \forall\ i = i', j = j', u = u'$$
$$= 0 \qquad \text{otherwise.}$$

Here $\alpha \in [0, \infty)$. This model is in fact a generalization of Fairfield Smith's variance law {see e.g. Sardana, Sreenath and Malhotra (1967), Bist, Malhotra and Sreenath (1975), Handa, Sreenath, Sastry, Rajpal and Shukla (1982)}. In these investigations it was found that the intra-block variances are proportional to non-negative real power of block sizes. The value of α was estimated by making use of uniformity trial data. This model was earlier been studied by Das, Gupta and Das (1992), Parsad and Gupta (1994a, 1994b), Gupta (1995), Parsad, Gupta and Singh (1996), Gupta and Parsad (2001), among others. A design is said to be binary balanced block design of type α, if information matrix (**C**-matrix) for the block design under model (1.1) is completely symmetric. A special case of $\alpha = 1$ has been investigated by Lee and Jacroux (1987) and Gupta, Das and Dey (1991). For $\alpha = 0$ we get the usual homoscedastic model.

2.2 Condition for a block design to be trend-free block design

In this section we obtain the necessary and sufficient condition for a general block design (2.1) to be trend-free block design. Throughout this chapter we shall deal with only real matrices and vectors. Denote an n-component vector of all unities by $\mathbf{1}_n$, an identity matrix of order n by $\mathbf{I}_n$, $\mathbf{A}^-$ and $\mathbf{A}^{-1}$ will respectively denote a-generalized and true inverse of matrix $\mathbf{A}$. E(.) and D(.) are respectively the expectation and dispersion. $\mathbf{M}'$ is the transpose of matrix $\mathbf{M}$. $\sum_{i=1}^{n}{}^{+}\mathbf{P_i}$ denotes the direct sum of matrices $\mathbf{P}_i$. When the order of matrix is clear, it is not mentioned.

Consider model (2.1) in matrix notations, for the general block design with trend terms added

$$\mathbf{y} = \mu\mathbf{1} + \Delta'\boldsymbol{\tau} + \mathbf{D}'\boldsymbol{\beta} + \mathbf{Z}'\boldsymbol{\phi} + \mathbf{e} \tag{2.2}$$

$$E(\mathbf{e}) = \mathbf{0},\ D(\mathbf{e}) = \sigma^2\mathbf{V}, \qquad \mathbf{V} = \sum_{t=1}^{s}{}^{+}\{\mathbf{I}_{b_t} \otimes k_t^{\alpha}\mathbf{I}_{k_t}\},$$

where $\mathbf{y}$ is an $n\times 1$ observable random vector and let there are b_1 blocks of size k_1, b_2 blocks of size k_2, so on and b_s blocks are of size k_s such that $\sum_{t=1}^{s} b_t = b$. $\mathbf{\Delta}'$ is the $n\times v$ design matrix for parameters of interest, $\boldsymbol{\tau} = (\tau_1,\ldots,\tau_v)'$, the vector of treatment effects, $\mathbf{D}'$ is $n\times b$ design matrix for nuisance parameters $\boldsymbol{\beta} = (\beta_1,\ldots,\beta_b)'$, the vector of block effects. Also, $\mathbf{\Delta}'\mathbf{1}_v = \mathbf{1}_n = \mathbf{D}'\mathbf{1}_b$, $\mathbf{\Delta}\ \mathbf{1}_n = \mathbf{r}$, $\mathbf{D}\ \mathbf{1}_n = \mathbf{k}$ where $\mathbf{r} = (r_1,\ldots,r_v)'$ and $\mathbf{k} = (k_1,\ldots,k_b)'$ are vectors of replications and block sizes, respectively. Further, $\mathbf{\Delta\Delta}' = \mathbf{R} = diag(r_1,\cdots,r_v)$, $\mathbf{DD}' = \mathbf{K} = diag(k_1,\cdots,k_b)$ and $\alpha \in [0,\infty)$ is a known constant.

Let d be the connected block design under model (2.2) when $\boldsymbol{\phi} = \mathbf{0}$. We assume a common trend effect in all the blocks of degree $p = \{\min(k_1,\ldots,k_s) - 1\}$. Thus $\boldsymbol{\phi}$ is a $p\times 1$ vector of trend effects and the matrix $\mathbf{Z}'$ of order $n\times p$, is matrix of coefficients given by $\mathbf{Z} = (\mathbf{Z}_1\ \mathbf{Z}_2\ \cdots\ \mathbf{Z}_s)$ where matrix $\mathbf{Z}_t' = \mathbf{1}_{\mathbf{b}_t} \otimes \mathbf{F}_t'$ is of order $b_t k_t \times p$ and $\mathbf{F}_t'$ is $k_t\times p$, $t = 1, \ldots,$ s matrix with columns consisting of equi-spaced normalized orthogonal polynomials. Also $\mathbf{1}'\mathbf{F}_t' = 0, \mathbf{F}_t\mathbf{F}_t' = \mathbf{I}_t$, $\forall\ t = 1,\ldots,s$ and $\mathbf{ZZ}' = b\,\mathbf{I}_p$. Here problem is to assign treatments to plots within blocks so that the known properties of ordinary analysis of variance for treatments and for block sum of squares are preserved when variation due to the trend may be removed from error sum of squares in model (2.1). A design possessing above property is called a trend-free block design by Bradley and Yeh (1980). A formal definition of a trend-free block design is given as:

Definition 2.3: A connected block design d is said to be a trend-free block design if and only if the additional reduction in sum of squares due to fitting of parameters of interest over and above fitting of other parameters, for the two models one containing the trend effect and the other without trend effects, is equal i.e.

$$R(\tau/\ \mu, \beta, \phi) = R(\tau/\ \mu, \beta) \tag{2.3}$$

where $R(\tau/\ \mu, \beta)$ is the additional reduction in sum of squares due to fitting of $\boldsymbol{\tau}$, μ, β over and above fitting just μ, β (Searle, 1971) . Similarly $R(\tau/\ \mu, \beta, \phi)$ is the additional reduction in sum of squares due to fitting of $\boldsymbol{\tau}$, μ, β and ϕ over and above fitting just μ, β, ϕ.

Thus for deriving necessary and sufficient condition for a block design *d* to be TFB design, we consider the following two situations:

Case I When there is no trend effect *i. e.* $\phi = 0$ under model (2.2)

The usual **C**-matrix (information matrix) is

$$\mathbf{C}_1 = \mathbf{X}_1'\mathbf{V}^{-1}\mathbf{X}_1 - \mathbf{X}_1'\mathbf{V}^{-1}\mathbf{X}_2(\mathbf{X}_2'\mathbf{V}^{-1}\mathbf{X}_2)^{-}\mathbf{X}_2'\mathbf{V}^{-1}\mathbf{X}_1,$$

where $\mathbf{X}_1 = \boldsymbol{\Delta}'$, $\mathbf{X}_2 = (\mathbf{1} \quad \mathbf{D}')$,

$$(\mathbf{X}_2'\mathbf{V}^{-1}\mathbf{X}_2)^{-} = \begin{bmatrix} \mathbf{1}'\mathbf{V}^{-1}\mathbf{1} & \mathbf{1}'\mathbf{V}^{-1}\mathbf{D}' \\ \mathbf{D}\mathbf{V}^{-1}\mathbf{1} & \mathbf{D}\mathbf{V}^{-1}\mathbf{D}' \end{bmatrix}^{-} = \begin{bmatrix} 0 & \mathbf{0}' \\ \mathbf{0} & \mathbf{K}^{-1+\alpha} \end{bmatrix} \text{ and}$$

$$\mathbf{X}_2(\mathbf{X}_2'\mathbf{V}^{-1}\mathbf{X}_2)^{-}\mathbf{X}_2' = \mathbf{D}'\mathbf{K}^{-1+\alpha}\mathbf{D}.$$

So the **C**-matrix simplifies to

$$\mathbf{C}_1 = \boldsymbol{\Delta}\mathbf{V}^{-1}\boldsymbol{\Delta}' - \boldsymbol{\Delta}\mathbf{V}^{-1}\mathbf{D}'\mathbf{K}^{-1+\alpha}\mathbf{D}\mathbf{V}^{-1}\boldsymbol{\Delta}' = \mathbf{U}_1\mathbf{H}_1\mathbf{U}_1' \tag{2.4}$$

where $\mathbf{H}_1 = \mathbf{I} - \mathbf{U}_2'\mathbf{K}^{-1+\alpha}\mathbf{U}_2$, (2.5)

$\mathbf{U}_1 = \boldsymbol{\Delta}\mathbf{V}^{-1/2}$ $\mathbf{U}_2 = \mathbf{D}\mathbf{V}^{-1/2}$

and $R(\boldsymbol{\tau}/\boldsymbol{\mu},\boldsymbol{\beta}) = \mathbf{v}'\mathbf{H}_1\mathbf{U}_1'\mathbf{C}_1^{-}\mathbf{U}_1\mathbf{H}_1\mathbf{v}$ (2.6)

where $\mathbf{v} = \mathbf{V}^{-1/2}\mathbf{y}$.

Case II When trend effect is present *i. e.* $\phi \neq 0$ under model (2.2)

The **C**-matrix in presence of trend effect is,

$$\mathbf{C}_2 = \mathbf{X}_1'\mathbf{V}^{-1}\mathbf{X}_1 - \mathbf{X}_1'\mathbf{V}^{-1}\mathbf{X}_3(\mathbf{X}_3'\mathbf{V}^{-1}\mathbf{X}_3)^{-}\mathbf{X}_3'\mathbf{V}^{-1}\mathbf{X}_1$$

where $\mathbf{X}_3 = (\mathbf{1} \quad \mathbf{D}' \quad \mathbf{Z}')$, $(\mathbf{X}_3'\mathbf{V}^{-1}\mathbf{X}_3)^{-} = \begin{bmatrix} 0 & \mathbf{0}' & \mathbf{0}' \\ \mathbf{0} & \mathbf{K}^{-1+\alpha} & \mathbf{0} \\ \mathbf{0} & \mathbf{0} & m^{-1}\mathbf{I}_p \end{bmatrix}$ and

$$\mathbf{X}_3(\mathbf{X}_3'\mathbf{V}^{-1}\mathbf{X}_3)^{-}\mathbf{X}_3' = \mathbf{D}'\mathbf{K}^{-1+\alpha}\mathbf{D} + m^{-1}\mathbf{Z}'\mathbf{Z}, \qquad m^{-1} = \sum_{j=1}^{b} k_j^{-\alpha} .$$

Thus the information matrix simplifies to $\mathbf{C}_2 = \mathbf{U}_1\mathbf{H}_2\mathbf{U}_1'$, (2.7)

where $\mathbf{H}_2 = \mathbf{I} - \mathbf{U}_2'\mathbf{K}^{-1+\alpha}\mathbf{U}_2 - m^{-1}\mathbf{V}^{-1/2}\mathbf{Z}'\mathbf{Z}\mathbf{V}^{-1/2}$, (2.8)

and $R(\boldsymbol{\tau}/\boldsymbol{\mu},\boldsymbol{\beta},\boldsymbol{\phi}) = \mathbf{v}'\mathbf{H}_2\mathbf{U}_1'\mathbf{C}_2^{-}\mathbf{U}_1\mathbf{H}_2\mathbf{v}$. (2.9)

Now we have following result:

Theorem 2.1 A necessary and sufficient condition for a connected block design d to be trend-free design is $\mathbf{\Delta V}^{-1}\mathbf{Z}' = \mathbf{0}$.

Proof: Necessary Part For design d to be trend-free design, from (2.3), (2.6) and (2.9) we have

$$\mathbf{v}'\mathbf{H}_2\mathbf{U}_1'\mathbf{C}_2^{-}\mathbf{U}_1\mathbf{H}_2\mathbf{v} = \mathbf{v}'\mathbf{H}_1\mathbf{U}_1'\mathbf{C}_1^{-}\mathbf{U}_1\mathbf{H}_1\mathbf{v} \quad or \quad \mathbf{H}_2\mathbf{U}_1'\mathbf{C}_2^{-}\mathbf{U}_1\mathbf{H}_2 = \mathbf{H}_1\mathbf{U}_1'\mathbf{C}_1^{-}\mathbf{U}_1\mathbf{H}_1 \tag{2.10}$$

Pre- and post multiplying both sides of (2.10) by $\mathbf{U}_1$ and $\mathbf{U}_1'$ respectively, we have

$$\mathbf{U}_1\mathbf{H}_2\mathbf{U}_1'\mathbf{C}_2^{-}\mathbf{U}_1\mathbf{H}_2\mathbf{U}_1' = \mathbf{U}_1\mathbf{H}_1\mathbf{U}_1'\mathbf{C}_1^{-}\mathbf{U}_1\mathbf{H}_1\mathbf{U}_1',$$

or $\mathbf{C}_2\mathbf{C}_2^{-}\mathbf{C}_2 = \mathbf{C}_1\mathbf{C}_1^{-}\mathbf{C}_1$ or $\mathbf{C}_2 = \mathbf{C}_1$

or $\mathbf{U}_1(\mathbf{H}_2 - \mathbf{H}_1)\mathbf{U}_1' = \mathbf{0}$ or $\mathbf{U}_1\mathbf{V}^{-1/2}\mathbf{Z}'\mathbf{Z}\mathbf{V}^{-1/2}\mathbf{U}_1' = \mathbf{0}$

or $\mathbf{\Delta V}^{-1}\mathbf{Z}'\mathbf{Z}\mathbf{V}^{-1}\mathbf{\Delta}' = \mathbf{0}$ or $\mathbf{\Delta V}^{-1}\mathbf{Z}' = \mathbf{0}$. (2.11)

Sufficient Part To prove sufficiency, (2.11) is used with (2.5) and (2.8) to show that $\mathbf{C}_1 = \mathbf{C}_2$ and $\mathbf{U}_1\mathbf{H}_1 = \mathbf{U}_1\mathbf{H}_2$ and the equality of (2.6) and (2.9) follows.

Now if n observations under model (2.2) are arranged block wise such that b_1 blocks of size k_1 come first, b_2 blocks of size k_2 come next, so on, and last b_s blocks of size k_s appear in last, $\sum_{t=1}^{s} b_t k_t = n$. Also, the design matrix $\mathbf{\Delta}$ is also written block wise so that $v \times n$ matrix $\mathbf{\Delta}' = (\mathbf{\Delta}_1' \quad \mathbf{\Delta}_2' \quad \cdots \quad \mathbf{\Delta}_s')'$ where $\mathbf{\Delta}_t'$ is matrix of order $b_t\, k_t \times v$, $t = 1, \ldots, s$. Then result of Theorem 2.1 simplifies as

Corollary 2.1: A necessary and sufficient condition for a connected block design d, under heteroscedastic model, to be trend-free block design is

$$\mathrm{k}_1^{-\alpha}\mathbf{\Delta}_1\mathbf{Z}_1' + \mathrm{k}_2^{-\alpha}\mathbf{\Delta}_2\mathbf{Z}_2' + \ldots + \mathrm{k}_s^{-\alpha}\mathbf{\Delta}_s\mathbf{Z}_s' = \mathbf{0}\,. \tag{2.12}$$

This result under homoscedastic set up ($\alpha = 0$) is

$$\sum_{t=1}^{s}\mathbf{\Delta_t Z_t'} = \mathbf{0} \tag{2.13}$$

Corollary 2.2: If the model (2.2) is a homoscedastic model i.e. $\mathbf{V} = \sigma^2\,\mathbf{I}_n$ then the condition for a block design (proper or non-proper) to be TFB design is

$$\mathbf{\Delta Z}' = \mathbf{0}\,. \tag{2.14}$$

For proper block design settings under homoscedastic model, this condition is same as obtained by Bradley and Yeh (1980).

Remark 2.1 There are some trivial results in which a block design is a trend-free block design. A Latin square design of order k is always a trend-free block design of degree (k-1) when rows are regarded as complete blocks, column effects become common effects of plot position within blocks. A symmetric balanced incomplete block (BIB) design can always be converted into trend-free block design of degree k-1, where k is the block size of the design. All the cyclic designs and full n-cyclic designs are trend-free designs when the treatment labels are systematically ordered within blocks. These designs have the property that each treatment is replicated equal number of times (once) in each block and because of this property, these designs are trend-free designs.

Now we consider a simple example of a BIB design and show how the condition (2.14) is satisfied.

Example 2.1 Consider a trend-free balanced incomplete block (BIB) design with parameters $v = 5$, $b = 5$, $r = 4$, $k = 4$ and $\lambda = 3$ as

Block 1	4	2	3	1
Block 2	*3*	*1*	*4*	*5*
Block 3	*2*	*3*	*5*	*4*
Block 4	*5*	*4*	*1*	*2*
Block 5	*1*	*5*	*2*	*3*

The design matrix of the above BIB design and matrix coefficients of orthogonal polynomials is

$$\mathbf{\Delta}' = \begin{bmatrix} 0 & 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 \\ 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 1 & 0 \\ 1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 \\ 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 \end{bmatrix} \qquad \mathbf{Z}' = \begin{bmatrix} -3 & +1 & -1 \\ -1 & -1 & +3 \\ +1 & -1 & +1 \\ +3 & +1 & -1 \\ -3 & +1 & -1 \\ -1 & -1 & +3 \\ +1 & -1 & +1 \\ +3 & +1 & -1 \\ -3 & +1 & -1 \\ -1 & -1 & +3 \\ +1 & -1 & +1 \\ +3 & +1 & -1 \\ -3 & +1 & -1 \\ -1 & -1 & +3 \\ +1 & -1 & +1 \\ +3 & +1 & -1 \\ -3 & +1 & -1 \\ -1 & -1 & +3 \\ +1 & -1 & +1 \\ +3 & +1 & -1 \end{bmatrix}$$

In the above, it is seen that $\mathbf{\Delta Z}' = \mathbf{0}$ and hence this BIB design is trend-free.

2.3 Catalogue of trend-free balanced incomplete block designs

For completeness a BIB design is defined as

Definition 2.4: A BIB (v, b, r, k, λ) is an arrangement of v treatments in b blocks each of size k (<v) such that

i) each treatment occurs at most once in each block,

ii) each treatment occurs in exactly in r blocks,

iii) each pair of treatments occur together in λ blocks.

The integers v, b, r, k, λ are known as parameters of the BIB design. These parameters are not independent and satisfy the following parametric relations:

i) $vr = bk$

ii) $r(k-1) = \lambda(v-1)$

iii) $b \geq v$, Fisher's inequality.

A BIB design is said to be symmetric if $v = r$ and $b = k$.

Several attempts have been made in the literature to prepare a catalogue of BIB designs [see e.g. Fisher and Yates (1956), Takeuchi (1961), Raghavarao (1971), Kageyama (1972), Hall Jr. (1986) and Street and Street (1987)]. But these catalogues are not exhaustive. Parsad, Gupta and Khanduri (2000) prepared an exhaustive catalogue of BIB designs containing replication number up to 30 for symmetric BIB designs and up to 20 for asymmetric BIB designs and thus total 494 BIB designs are listed.

Dhall (1986) has given a catalogue of trend-free BIB designs for $3 \leq k \leq 6$ and linear trend-free BIB designs for $7 \leq k \leq 15$.

Here we give a catalogue of BIB designs for replications, $r \leq 20$ which can be converted trend-free and linear trend-free BIB designs. Some times it is neither possible to convert a design to trend-free nor to a linear trend-free design. Then the best option is to convert the design to a nearly linear trend-free (NLTF) design, by permutation of treatments, as defined by Chai (1995) and is shown in (1.7). Thus trend-free, linear trend-free and NLTF balanced incomplete block designs are presented in Table 2.1.1, Table 2.1.2 and Table 2.1.3, respectively. Symmetric BIB designs and their copies are not included in the catalogue as these designs are trivially trend-free designs. Further if a BIB design is obtained that can be converted into trend-free BIB design and let another BIB design that is constructed by taking the copies of said trend-free BIB design then this BIB design is not included in the catalogue because copies of a trend-free BIB designs is also trend-free block design.

Table 2.1.1: BIB designs for $r \leq 20$ that can be converted into trend-free BIB designs

Sr. No.	v	b	r	k	λ	n
1	4	12	6	2	2	24
2	5	10	4	2	1	20
3	5	10	6	3	3	30
4	6	30	10	2	2	60
5	6	30	15	3	6	90
6	6	30	20	4	12	120
7	7	21	6	2	1	42
8	7	21	15	5	10	105
9	8	56	14	2	2	112
10	9	36	8	2	1	72
11	9	36	12	3	3	108
12	9	18	8	4	3	72
13	9	18	10	5	5	90
14	10	90	18	2	2	180
15	10	30	9	3	2	90
16	10	30	12	4	4	120
17	10	30	18	6	10	180
18	11	55	10	2	1	110
19	11	55	15	3	3	165
20	11	55	20	4	6	220
21	13	78	12	2	1	156
22	13	26	6	3	1	78
23	13	39	15	5	5	195
24	13	26	12	6	5	156
25	13	26	14	7	7	182
26	15	105	14	2	1	210
27	16	80	20	4	4	320
28	16	48	15	5	4	240
29	17	136	16	2	1	272
30	17	68	20	5	5	340
31	17	34	16	8	7	272
32	17	34	18	9	9	306
33	19	171	18	2	1	342
34	19	57	9	3	1	171
35	19	57	12	4	2	228
36	19	57	18	6	5	342
37	21	210	20	2	1	420
38	21	105	20	4	3	420
39	21	42	12	6	3	252
40	21	42	20	10	9	420
41	25	50	8	4	1	200
42	29	406	28	2	1	812

Sr. No.	v	b	r	k	λ	n
43	31	155	20	4	2	620
44	31	93	15	5	2	465
45	37	222	18	3	1	666
46	37	111	12	4	1	444
47	41	164	20	5	2	820
48	46	92	20	10	4	920
49	49	196	16	4	1	784
50	49	98	18	9	3	882
51	57	114	16	8	2	912
52	61	305	20	4	1	1220
53	61	183	15	5	1	915
54	61	122	12	6	1	732
55	61	122	20	10	3	1220
56	81	324	20	5	1	1620
57	85	170	14	7	1	1190
58	91	273	18	6	1	1638
59	145	290	18	9	1	2610
60	181	362	20	10	1	3620

Table 2.1.2: BIB designs for $r \leq 20$ that can be converted into linear trend-free BIB designs

Sr. No.	v	b	r	k	λ	n
1	6	10	5	3	2	30
2	6	15	10	4	6	60
3	8	28	14	4	6	112
4	9	12	4	3	1	36
5	9	12	8	6	5	72
6	10	15	6	4	2	60
7	10	18	9	5	4	90
8	12	44	11	3	2	132
9	14	26	13	7	6	182
10	15	35	7	3	1	105
11	15	21	7	5	2	105
12	15	35	14	6	5	210
13	16	80	15	3	2	240
14	16	40	10	4	2	160
15	17	68	16	4	3	272
16	18	102	17	3	2	306
17	18	34	17	9	8	306
18	20	76	19	5	4	380
19	21	70	10	3	1	210
20	21	28	8	6	2	168
22	21	30	10	7	3	210

Sr. No.	v	b	r	k	λ	n
23	21	35	15	9	6	315
24	21	30	20	14	13	420
25	21	28	20	15	14	420
26	22	77	14	4	2	308
27	25	30	6	5	1	150
28	25	40	16	10	6	400
29	27	117	13	3	1	351
30	27	39	13	9	4	351
31	28	126	18	4	2	504
32	28	36	9	7	2	252
33	31	155	15	3	1	465
34	33	176	16	3	1	528
35	33	44	12	9	3	396
36	33	48	16	11	5	528
37	34	51	18	12	6	612
38	35	119	17	5	2	595
39	35	85	17	7	3	595
40	36	84	14	6	2	504
41	36	45	10	8	2	360
42	36	48	20	15	8	720
43	37	74	20	10	5	740
44	39	247	19	3	1	741
45	41	82	10	5	1	410
46	45	99	11	5	1	495
47	45	55	11	9	2	495
48	45	60	16	12	4	720
49	45	75	20	12	5	900
50	46	138	18	6	2	828
51	49	56	8	7	1	392
52	51	85	10	6	1	510
53	52	68	17	13	4	884
54	55	66	12	10	2	660
55	55	99	18	10	3	990
56	57	76	20	15	5	1140
57	64	144	18	8	2	1152
58	65	208	16	5	1	1040
59	65	80	16	13	3	1040
60	66	78	13	11	2	858
61	76	95	20	16	4	1520
62	78	91	14	12	2	1092
63	81	216	16	6	1	1296
64	81	90	10	9	1	810
65	85	102	18	15	3	1530
66	91	195	15	7	1	1365
68	100	150	18	12	2	1800

Sr. No.	v	b	r	k	λ	n
69	105	120	16	14	2	1680
70	111	185	20	12	2	2220
71	113	226	16	8	1	1808
72	120	136	17	15	2	2040
73	121	132	12	11	1	1452
74	136	153	18	16	2	2448
75	141	188	20	15	2	2820
76	145	232	16	10	1	2320
77	153	323	19	9	1	2907
78	153	171	19	17	2	2907
79	169	182	14	13	1	2366
80	171	190	20	18	2	3420
81	177	236	16	12	1	2832
82	225	240	16	15	1	3600
83	289	306	18	17	1	5202
84	361	380	20	19	1	7220

Table 2.1.3: BIB designs for $r \leq 20$ that can be converted into nearly linear trend-free BIB designs

Sr. No.	v	b	r	k	λ	n
1	4	6	3	2	1	12
2	6	15	5	2	1	30
3	8	28	7	2	1	56
4	10	45	9	2	1	90
5	10	15	9	6	5	90
6	12	66	11	2	1	132
7	12	22	11	6	5	132
8	14	91	13	2	1	182
9	16	120	15	2	1	240
10	16	24	9	6	3	144
11	16	40	15	6	5	240
12	16	24	15	10	9	240
13	18	153	17	2	1	306
14	18	51	17	6	5	306
15	20	190	19	2	1	380
16	20	38	19	10	9	380
17	26	65	15	6	3	390
18	28	378	27	2	1	756
19	28	42	15	10	5	420
20	32	496	31	2	1	992
21	36	42	7	6	1	252
23	46	69	9	6	1	414
24	46	69	15	10	3	690

Sr. No.	v	b	r	k	λ	n
25	66	143	13	6	1	858
26	76	190	15	6	1	1140
27	96	304	19	6	1	1824
28	136	204	15	10	1	2040
29	196	210	15	14	1	2940
30	324	342	19	18	1	6156

2.4 Catalogue of trend-free partial balanced incomplete block designs

Two-associate partially balanced incomplete block (PBIB) designs have extensively been used for the construction of binary balanced block designs. So we have studied the two-associate class PBIB designs given by Clatworthy (1973). Here we give a catalogue of PBIB designs which can be converted trend-free, linear trend-free and nearly linear trend free PBIB designs.

Table 2.2.1: Singular Group Divisible designs that can be converted into trend-free PBIB designs

S-2	S-4	S-9	S-10	S-15	S-19	S-23	S-26
S-29	S-33	S-42	S-44	S-52	S-56	S-60	S-65
S-68	S-72	S-77	S-80	S-84	S-90	S-93	S-96
S-99	S-104	S-105	S-111	S-115	S-119	S-123	S-124

Table 2.2.2: Singular Group Divisible designs that can be converted into linear trend-free PBIB designs

S-1	S-3	S-5	S-7	S-12	S-13	S-17	S-21
S-22	S-24	S-25	S-31	S-35	S-36	S-37	S-39
S-40	S-45	S-48	S-50	S-51	S-53	S-54	S-55
S-57	S-58	S-59	S-62	S-66	S-67	S-70	S-71
S-73	S-76	S-79	S-82	S-83	S-85	S-86	S-87
S-88	S-89	S-91	S-92	S-94	S-95	S-97	S-98
S-100	S-101	S-102	S-103	S-107	S-112	S-113	S-116
S-120	S-121						

Table 2.2.3: Singular Group Divisible designs that can be converted into nearly linear trend-free PBIB designs

S-18	S-20	S-27	S-28	S-30	S-32	S-34	S-38
S-41	S-43	S-46	S-47	S-49	S-106	S-108	S-109
S-110	S-114	S-117	S-118	S-122			

Table 2.3.1: Semi-regular Group Divisible designs that can be converted into trend-free PBIB designs

SR-1	SR-2	SR-3	SR-4	SR-5	SR-7	SR-9	SR-10
SR-11	SR-12	SR-13	SR-15	SR-17	SR-20	SR-23	SR-24
SR-25	SR-30	SR-33	SR-36	SR-39	SR-44	SR-45	SR-49
SR-60	SR-61	SR-65	SR-67	SR-68	SR-70	SR-72	SR-87
SR-92	SR-95	SR-97	SR-102	SR-105	SR-108		

Table 2.3.2: Semi-regular Group Divisible designs that can be converted into linear trend-free PBIB designs

SR-18	SR-19	SR-21	SR-22	SR-26	SR-27	SR-28	SR-29
SR-31	SR-32	SR-34	SR-35	SR-37	SR-38	SR-40	SR-42
SR-47	SR-51	SR-52	SR-53	SR-54	SR-55	SR-56	SR-57
SR-58	SR-59	SR-62	SR-63	SR-64	SR-66	SR-69	SR-71
SR-74	SR-76	SR-78	SR-80	SR-81	SR-82	SR-83	SR-84
SR-85	SR-86	SR-88	SR-89	SR-90	SR-91	SR-93	SR-99
SR-100	SR-101	SR-103	SR-104	SR-106	SR-107		

Table 2.3.3: Semi-regular Group Divisible designs that can be converted into nearly linear trend-free PBIB designs

SR-6	SR-8	SR-14	SR-16	SR-73	SR-75	SR-77	SR-79
SR-109	SR-110						

Table 2.4.1: Regular Group Divisible designs that can be converted into trend-free PBIB designs

R-1	R-4	R-8	R-9	R-10	R-14	R-15	R-16
R-17	R-18	R-19	R-22	R-23	R-24	R-28	R-29
R-30	R-32	R-33	R-34	R-35	R-36	R-37	R-38
R-40	R-41	R-42	R-43	R-44	R-49	R-50	R-51
R-52	R-54	R-55	R-56	R-57	R-58	R-60	R-64
R-65	R-69	R-71	R-75	R-79	R-80	R-81	R-83
R-84	R-86	R-87	R-89	R-90	R-91	R-92	R-94
R-95	R-96	R-98	R-99	R-104	R-105	R-106	R-109
R-110	R-112	R-113	R-114	R-115	R-116	R-117	R-120
R-128	R-129	R-130	R-133	R-134	R-135	R-136	R-137
R-138	R-139	R-141	R-142	R-143	R-144	R-145	R-146
R-147	R-148	R-149	R-150	R-151	R-152	R-153	R-154
R-158	R-159	R-160	R-162	R-163	R-166	R-168	R-170
R-171	R-172	R-173	R-174	R-175	R-176	R-177	R-178
R-179	R-180	R-182	R-183	R-186	R-187	R-188	R-189
R-190	R-191	R-193	R-194	R-195	R-196	R-197	R-198
R-199	R-200	R-201	R-202	R-203	R-204	R-205	R-206
R-207	R-208	R-209					

Table 2.4.2: Regular Group Divisible designs that can be converted into linear trend-free PBIB designs

R-45	R-46	R-47	R-48	R-53	R-59	R-61	R-62
R-63	R-66	R-67	R-68	R-70	R-72	R-73	R-74
R-76	R-77	R-78	R-82	R-85	R-88	R-93	R-102
R-103	R-107	R-108	R-111	R-118	R-123	R-127	R-131
R-132	R-140	R-155	R-156	R-157	R-161	R-165	R-169
R-181	R-184	R-185	R-192				

Table 2.4.3: Regular Group Divisible designs that can be converted into nearly linear trend-free PBIB designs

R-2	R-3	R-5	R-6	R-7	R-11	R-12	R-13
R-20	R-21	R-25	R-26	R-27	R-31	R-39	R-164
R-167							

Table 2.5.1: Triangular Group Divisible designs that can be converted into trend-free PBIB designs

T-1	T-3	T-5	T-6	T-7	T-8	T-	T-9
T-10	T-11	T-12	T-13	T-16	T-17	T-19	T-23
T-29	T-31	T-33	T-34	T-36	T-38	T-39	T-42
T-52	T-53	T-55	T-56	T-58	T-60	T-61	T-67
T-71	T-77	T-81	T-84	T-91	T-94	T-95	T-100

Table 2.5.2: Triangular Group Divisible designs that can be converted into linear trend-free PBIB designs

T-14	T-15	T-18	T-20	T-21	T-22	T-25	T-26
T-27	T-28	T-30	T-32	T-35	T-37	T-40	T-43
T-44	T-45	T-46	T-47	T-48	T-49	T-50	T-51
T-54	T-62	T-63	T-64	T-65	T-66	T-68	T-69
T-70	T-72	T-73	T-74	T-75	T-76	T-78	T-79
T-80	T-82	T-83	T-85	T-86	T-87	T-88	T-89
T-90	T-92	T-93	T-96	T-97	T-98	T-99	

Table 2.5.3: Triangular Group Divisible designs that can be converted into nearly linear trend-free PBIB designs

T-2	T-4	T-57	T-59

Table 2.6.1: Latin square type Group Divisible designs that can be converted into trend-free PBIB designs

LS-1	LS-2	LS-3	LS-5	LS-6	LS-9	LS-12	LS-16
LS-17	LS-18	LS-19	LS-20	LS-22	LS-24	LS-26	LS-27
LS-30	LS-34	LS-38	LS-42	LS-45	LS-46	LS-47	LS-49
LS-50	LS-60	LS-63	LS-66	LS-67	LS-68	LS-69	LS-70
LS-71	LS-77	LS-78	LS-82	LS-83	LS-101	LS-104	LS-110
LS-114	LS-116	LS-117	LS-118	LS-131	LS-134	LS-136	LS-146

Table 2.6.2: Latin square type Group Divisible designs that can be converted into linear trend-free PBIB designs

LS-7	LS-8	LS-10	LS-11	LS-13	LS-14	LS-15	LS-21
LS-23	LS-25	LS-28	LS-31	LS-32	LS-35	LS-40	LS-48
LS-51	LS-52	LS-53	LS-54	LS-55	LS-56	LS-57	LS-58
LS-59	LS-61	LS-62	LS-64	LS-65	LS-72	LS-73	LS-74
LS-75	LS-76	LS-80	LS-81	LS-84	LS-85	LS-86	LS-87
LS-88	LS-89	LS-90	LS-91	LS-92	LS-93	LS-94	LS-95
LS-96	LS-97	LS-98	LS-102	LS-106	LS-108	LS-109	LS-111
LS-112	LS-115	LS-119	LS-120	LS-121	LS-122	LS-123	LS-124
LS-125	LS-126	LS-127	LS-128	LS-129	LS-130	LS-132	LS-133
LS-135	LS-137	LS-139	LS-140	LS-141	LS-142	LS-143	LS-145

Table 2.6.3: Latin square type Group Divisible designs that can be converted into nearly linear trend-free PBIB designs

LS-4	LS-79	LS-138	LS-144

2.5 Methods of Construction of block design under heteroscedastic model

As discussed earlier, non-proper block designs are quite useful in many experimental situations. When block sizes are unequal it is unrealistic to assume that the intra-block variances are equal. Thus, two-way classified, additive, fixed effects, heteroscedastic model (2.1) is considered for the present study. The information matrix (**C**-matrix) of a block design under the heteroscedastic model (2.1), obtained by using the principle of generalized least squares is given by

$$\mathbf{C} = \sum_{j=1}^{b} k_j^{-\alpha}[\mathbf{R}_j - k_j^{-1}\mathbf{N}_j\mathbf{N}'_j] \qquad (2.15)$$

where $\mathbf{N}_j$ is the jth column of $v \times b$ treatment vs blocks incidence matrix, **N** and $\mathbf{R}_j$ = diag($n_{1j}, \ldots, n_{vj}$), where n_{ij} is the number of times treatment i is applied to block j. A design is said to be a binary variance balanced block design of type α, if **C**-matrix given in (2.15) is complete symmetric. For $\alpha = 0$, all results reduce to that of usual homoscedastic setup. For detail, one may refer to Parsad, Gupta and Khanduri (2000). Now, we give some results for trend-free block designs under heteroscedastic model.

Most of the binary variance balanced block (BBB) designs under heteroscedastic set up of Type α ($\alpha \neq 0$) are constructed by using component BBB designs under homoscedastic model. Let BBB design under homoscedastic model be connected and have v treatments, b_1 blocks of size k_1, b_2 blocks of size k_2, so on, and b_s blocks of size k_s. By taking copies of block sizes $k_1, \ldots, k_s$ in the ratio of $k_1^{\alpha} : \ldots : k_s^{\alpha}$ respectively, we have a BBB design of Type α ($\alpha \neq 0$) which also remains connected. Now we have the following result:

Theorem 2.2: If there exists a trend-free binary balanced block (TFBBB) design with blocks of unequal sizes under homoscedastic model, then it can be expanded into TFBBB design of Type α ($\alpha \neq 0$) under heteroscedastic model.

Proof For a TFBBB design under homoscedastic set up (2.13) holds. If we take $\frac{k_1^{\alpha}}{c}$ copies of b_1 blocks of size k_1, $\frac{k_2^{\alpha}}{c}$ copies of b_2 blocks of size k_2, so on, and $\frac{k_s^{\alpha}}{c}$ copies of b_s blocks of size k_s then the design obtained is BBB of type α ($\alpha \neq 0$), c being the highest common factor of $k_1^{\alpha}, \ldots, k_s^{\alpha}$. With this each factor $\mathbf{\Delta}_t \mathbf{Z}_t'$ in condition (2.12) is multiplied by $\frac{k_t^{\alpha}}{c}$ $\forall$ $t = 1, \cdots, s$ and thus the condition (2.12) reduces to (2.13) which clearly holds to start with.

Binary balanced block designs are known to be efficient in $D(v, b, n)$, the class of connected block designs in which v treatments are arranged in b blocks and total number of experimental units is n. Here we give a method to develop trend-free binary balanced block (TFBBB) designs obtained by the method of Khatri (1982). Consider an unreduced BIB design with parameters v, $b = \binom{v}{k}$, $r = \binom{v-1}{k-1}$, k, $\lambda = \binom{v-2}{k-2}$, where v, b, r and k are the number of treatments, blocks, replication number of each treatment and block size respectively and λ is the number of blocks in which every pair of treatments occur together. Now select any p disjoint blocks out of b blocks such that kp < v. Put these kp treatments in one block and repeat this block $\lambda_0 p$ times where $\lambda_0 = \binom{kp-2}{k-2}$. Now from the b blocks delete $b_0 = \binom{kp}{k}$ blocks in which all replicates of the selected kp treatments occur. This process yields a binary balanced block (BBB) design with parameters
$v^* = v$, $b_1^* = \lambda_0 p$, $k_1^* = kp$, $b_2 = b - b_0$, $k_2^* = k$, $r^* = [(\lambda_0 p + r - r_0)\ \mathbf{1}'_{kp}, \mathrm{r}\ \mathbf{1}'_{v-kp}]$

where $r_0 = \begin{pmatrix} kp-1 \\ k-1 \end{pmatrix}$. (2.16)

For such type of BBB designs we have the following different cases for $k \leq 3$ under homoscedastic set up *i. e.* $\alpha = \boldsymbol{0}$.

Case A. For k=2, when v and p both are odd, we get a linear trend free block (LTFB) design

For k=2, the parameters (3.1) simplify to
$v^* = v,\ b_1^* = p,\ k_1^* = 2p,\ b_2^* = b - b_0,\ k_2^* = 2,$
$r_i^* = p + r - r_0 = v\text{-}p$ for $i = 1, \ldots, 2p$
$\quad = r = v\text{-}1,\ \forall\ i = 2p+1, \cdots, v.$

First we consider the treatment numbers $2p+1, 2p+2, \ldots, v$ which are replicated v-1 (even number) times. We, therefore, can permute these treatments in a block of size 2 such that half of treatments get first position and the other half gets the second position. Further first 2p treatments are replicated p times in blocks of size 2p and v-2p times in blocks of size 2. Now these 2p treatments can easily be permuted in such a way that the design is LTFB design. We illustrate this procedure by an example.

Example 2.2: Let the unreduced BIB design be v = 7, b = 21, r = 6, k = 2, λ = 1. Let p = 3, then we have a variance balanced block design with parameters v^* = 7, b_1^* = 3, k_1^* = 6, b_2^* = 6, k_2^* = 2, r_i^* = 4 for i =1, …, 6 and r_7^* = 6. This variance balanced block design, under homoscedastic set up (α = 0), can be converted into TFB design of degree {min (k_1, k_2) – 1} = 1 which is given below with rows as blocks.

Block No.						
1	4	1	2	5	6	3
2	6	3	1	4	5	2
3	5	2	3	6	4	1
(Polynomial coefficients	*-5*	*-3*	*-1*	*1*	*3*	*5)*
4	1	7				
5	2	7				
6	3	7				
7	7	4				
8	7	5				
9	7	6				
(Polynomial coefficients	*-1*	*1)*				

For variance balanced block design, under heteroscedastic set up ($\alpha \neq 0$), we take the copies of blocks of size, k_1^*, k_2^* in the ratio of $(k_1^*)^\alpha$: $(k_2^*)^\alpha$. So from Theorem 2.2, above design under heteroscedastic set up of type α, for any value of α, will also be TFBBB design.

We can obtain a series of BBB designs which can be converted into TFB designs by the method given in Case A. Thus TFBBB designs for $\{\max(r_1, r_2)\} \leq 10$ are given below.

Table 2.7: Variance balanced block designs which can be converted into TFB designs for $\{\max(r_1, r_2)\} \leq 10$

p	v	b_1^*	k_1^*	b_2^*	k_2^*	r_1^*	r_2^*
3	7	$3^{\alpha+1}$	6	6	2	4	6
3	9	$3^{\alpha+1}$	6	21	2	6	8
3	11	$3^{\alpha+1}$	6	40	2	8	10
5	11	$5^{\alpha+1}$	10	10	2	6	10

Note: Above designs are TFB designs for any value of α.

Case B For k=2, when v is even LTFB design is not possible

When v is even and $k = 2$, $v\text{-}kp=u$ (say) is even. These u treatments will be replicated $r = v\text{-}1$ (an odd number) times in block of size 2. So, LTFB design is not possible. [Theorem 3.1, Yeh and Bradley (1983)].

Case C For k=2, when v is odd and p is even LTFB design is not possible

When v is odd and p is even, $2p$ is even. First $2p$ treatments will appear p times in block size $2p$ and $r\text{-} r_0$ or v - $2p$ times in block of size 2. But v - $2p$ is an odd number. So LTFB design is not possible.

Case D For k=3, when p is even LTFB design is possible

If p is even kp treatments will appear in $\lambda_0\, p$ blocks i.e. an even number of blocks. So these kp treatments can be permuted to LTFB design. These kp treatments and the remaining v - kp treatments will appear in block of size 3 with different replications. Now $m \leq (v - kp)$ treatments will appear in each of the blocks of size 3. Thus these blocks of size 3 can easily be permuted to LTFB design. (Theorem 3.2, Chai and Majumdar (1993)).

Example 2.3: Let the unreduced BIB design be $v = 7$, $b = 35$, $r = 15$, $k = 3$, $\lambda = 5$. Let $p = 2$, then for $kp = 6$, and $\lambda_0 p = 8$, we have a variance balanced block design with parameters $v^*=7$, $b_1^* = 8$, $k_1^* = 6$, $b_2^* = 15$, $k_2^* = 3$, $r_i^* = 13$ for $i = 1, \ldots, 6$ and $r_7^* = 15$. This variance balanced block design can be converted into TFB design with rows as blocks for any value of α.

Block No.						
1	2	3	1	6	4	5
2	4	5	3	2	6	1
3	6	1	5	4	2	3
4	1	2	6	5	3	4
5	6	5	4	3	2	1
6	5	6	4	3	1	2
7	3	4	2	1	5	6
8	1	2	3	4	5	6
(Polynomial coefficients	*-5*	*-3*	*-1*	*1*	*3*	*5)*
9	1	7	2			
10	7	4	1			
11	7	5	1			
12	7	1	6			
13	2	3	7			
14	2	4	7			
15	7	5	2			
16	7	2	6			
17	7	6	3			
18	7	5	4			
19	6	4	7			
20	6	5	7			
21	1	3	7			
22	4	3	7			
23	3	5	7			
(Polynomial coefficients	*-1*	*0*	*1)*			

Theorem 2.3: If a BBB design is obtained as a union of two block designs, the BBB design thus obtained will be trend-free block design if the two block designs can be converted into trend-free block designs, individually.

Proof simplifies from Corollary (2.1).

2.6 Analysis of data from trend-free block designs

In a trend-free block design we assign treatments to plots within blocks so that the known properties of ordinary analysis of variance for treatment and block sum of squares are preserved and variation due to trend effect is removed from the error sum of square. So the analysis of a trend-free block design is same as that the analysis of block design without trend-effect, the only difference is that sum of squares due trend effects are worked out separately and is subtracted from the error sum of squares. Now the steps for the analysis of a trend-free block design are given below.

Under the model (2.2), let the block design *d* is a connected design. Then the reduced normal equations for estimating linear functions of treatment effects, using the design *d*, are

$$\mathbf{C}_d \boldsymbol{\tau}_d = \mathbf{Q}_d,$$

where $\mathbf{C}_d = \mathbf{R}_d - \mathbf{N}_d \mathbf{K}_d^{-1} \mathbf{N}'_d$,

and $\mathbf{Q}_d = \mathbf{T}_d - \mathbf{N}_d \mathbf{K}_d^{-1} \mathbf{B}_d$.

Here, $\mathbf{T}_d$ is the vector corresponding to treatment totals and $\mathbf{B}_d$ is the vector of block totals. $\mathbf{Q}_d$ is known as the vector of adjusted treatment totals. The i^{th} element of $\mathbf{Q}_d$ is $Q_i = T_i - \sum_{j=1}^{b} n_{ij} B_j / k_j$.

For design *d*, the adjusted sum of squares for treatment effects is $\mathbf{Q}'_d \mathbf{C}_d^{-} \mathbf{Q}_d$, where $\mathbf{C}_d^{-}$ is a g-inverse of $\mathbf{C}_d$. Also, the unadjusted block sum of squares is $\mathbf{B}'_d \mathbf{K}_d^{-1} \mathbf{B}_d - \dfrac{(\mathbf{B}'_d \mathbf{1}_b)^2}{n} = \sum_{j=1}^{b} \dfrac{B_j^2}{k_j} - \dfrac{G^2}{n}$, where *G* is the grand total.

Now sum of square due to i^{th} component of trend Wi is the i^{th} component of the vector **W** = **Z' y**

Thus the analysis of variance table for a trend-free design is as follows:

ANOVA

Source	d.f.	SS
Treatment effects	$v - 1$	$\mathbf{Q}'_d \mathbf{C}_d \mathbf{Q}_d$
Block effects	$b-1$	$\mathbf{B}'_d \mathbf{K}_d^{-1} \mathbf{B}_d \quad \dfrac{(\mathbf{B}'_d \mathbf{1}_b)^2}{n} = \sum_{j=1}^{b} \dfrac{B_j^2}{k_j} - \dfrac{G^2}{n}$
Trend term 1	1	W^2_1 / b
Trend term 2	1	W^2_2 / b
.	.	.
.	.	.
.	.	.
trend term p	1	W^2_P / b
Error	$n - v - b - p + 1$	By subtraction
Total	$n-1$	$\mathbf{Y}'\mathbf{Y} - \dfrac{(\mathbf{B}'_d \mathbf{1}_b)^2}{n} = \sum_{i-1}^{v} \sum_{j=1}^{b} y_{ij}^2 - \dfrac{G^2}{n}$

2.7 Catalogue of trend-free binary variance balanced block designs

On the basis of above results, a catalogue of BBB designs of type α with replication number of treatments $r \leq 30$ and for values of α = 0, 1, 2 and 3 with two distinct block sizes, which can be converted trend-free, linear trend-free and nearly linear trend free BBB designs are given in Table 2.8 to Table 2.11.

Table 2.8.1: BBB designs with two distinct block sizes for $\alpha = 0$ that can be converted into trend-free BIB designs.

Sl. No.	v	r	b_1	k_1	b_2	k_2	n	Reference Design			
1	6	8	6	4	12	2	48	2	S1 :	1	R18
2	6	8	12	3	6	2	48	3	SR18 :	2	(3 2)
3	6	9	18	2	6	3	54	2	SR6 :	3	(2 3)
4	6	14	12	4	12	3	84	4	S1 :	3	SR18
5	6	16	12	4	24	2	96	4	S1 :	1	R23
6	6	16	24	3	12	2	96	3	SR19 :	4	(3 2)
7	6	17	24	2	18	3	102	2	R18 :	3	R42
8	6	18	12	3	36	2	108	3	SR18 :	2	R19
9	6	19	48	2	6	3	114	2	R24 :	3	(2 3)
10	6	22	18	2	24	4	132	1	SR7 :	4	R94
11	6	23	42	3	6	2	138	3	R46 :	2	(3 2)
12	6	24	18	4	36	2	144	2	S3 :	3	R18
13	6	24	18	2	36	3	144	2	SR6 :	3	R43
14	6	26	12	4	54	2	156	1	S4 :	2	R27
15	6	26	24	3	42	2	156	3	SR19 :	2	R21
16	6	27	54	2	18	3	162	2	SR8 :	9	(2 3)
17	6	27	36	4	6	3	162	4	SR35 :	3	(2 3)
18	6	28	24	4	24	3	168	8	S1 :	3	SR19
19	6	29	12	4	42	3	174	4	S1 :	3	R46
20	8	9	8	6	8	3	72	2	S18 :	1	R54
21	8	10	16	4	8	2	80	1	SR39 :	2	(4 2)
22	8	11	24	3	8	2	88	1	R57 :	2	(4 2)
23	8	12	8	4	32	2	96	1	SR36 :	1	R30
24	8	18	16	6	16	3	144	4	S18 :	1	R55
25	8	20	16	2	32	4	160	1	SR9 :	2	R98
26	8	22	48	3	16	2	176	3	R55 :	4	(4 2)
27	8	22	24	4	40	2	176	3	SR36 :	1	R33
28	8	24	16	4	64	2	192	1	SR39 :	2	R30
29	8	25	64	2	24	3	200	2	R30 :	3	R54
30	8	26	24	6	16	4	208	6	S18 :	1	SR39
31	8	27	40	5	8	2	216	5	R134 :	2	(4 2)
32	8	27	24	3	24	6	216	1	R58 :	2	R164
33	8	29	24	3	40	4	232	3	R54 :	4	R97
34	8	30	24	4	48	3	240	4	S6 :	3	R55
35	8	30	24	6	48	2	240	3	S19 :	2	R29
36	9	10	27	2	9	4	90	1	R34 :	1	R104

Sl. No.	v	r	b_1	k_1	b_2	k_2	n	Reference			Design
37	9	12	9	6	27	2	108	3	S21 :	1	R34
38	9	12	18	2	18	4	108	1	LS1 :	2	LS26
39	9	15	54	2	9	3	135	2	R34 :	3	(3 3)
40	9	17	27	3	18	4	153	3	SR23 :	2	R104
41	9	20	54	2	18	4	180	2	R34 :	1	R105
42	9	20	18	4	18	6	180	2	LS26 :	3	LS72
43	9	21	18	6	27	3	189	6	S21 :	1	SR25
44	9	22	27	6	9	4	198	3	SR65 :	1	R104
45	9	22	18	3	36	4	198	3	LS7 :	4	LS26
46	9	24	36	2	36	4	216	1	LS2 :	4	LS26
47	9	24	18	6	54	2	216	3	S22 :	2	R34
48	9	26	36	2	54	3	234	2	LS1 :	3	LS12
49	9	27	54	2	45	3	243	2	R34 :	3	R59
50	9	28	27	6	45	2	252	3	SR65 :	1	R35
51	9	29	27	3	90	2	261	3	SR23 :	2	R35
52	9	29	63	3	18	4	261	3	R62 :	2	R104
53	9	30	36	6	18	3	270	4	SR65 :	1	R60
54	10	12	20	5	10	2	120	1	SR55 :	2	(5 2)
55	10	12	20	3	10	6	120	1	T12 :	2	T57
56	10	14	30	2	20	4	140	2	T2 :	1	T31
57	10	14	10	6	40	2	140	1	S26 :	1	R36
58	10	15	50	2	10	5	150	2	SR11 :	5	(2 5)
59	10	15	30	2	30	3	150	2	T2 :	3	T9
60	10	15	10	3	20	6	150	1	T9 :	2	T60
61	10	16	20	5	20	3	160	1	SR55 :	1	R69
62	10	17	30	3	20	4	170	1	T13 :	4	T28
63	10	21	60	2	30	3	210	2	T1 :	1	T13
64	10	22	50	2	20	6	220	1	SR12 :	2	R166
65	10	22	10	4	30	6	220	2	T28 :	3	T60
66	10	22	80	2	20	3	220	2	R36 :	1	R69
67	10	22	20	6	20	5	220	2	S26 :	1	SR55
68	10	23	20	4	30	5	230	4	T28 :	5	T44
69	10	24	20	8	40	2	240	4	S51 :	1	R36
70	10	24	40	5	20	2	240	5	SR52 :	4	(5 2)
71	10	24	30	2	30	6	240	1	T1 :	3	T60
72	10	24	30	3	30	5	240	3	T9 :	5	T44
73	10	25	30	3	40	4	250	3	T9 :	4	T33
74	10	26	60	3	20	4	260	3	T12 :	4	T28
75	10	26	30	2	50	4	260	1	T1 :	2	T37
76	10	27	60	2	30	5	270	2	T1 :	5	T44
77	10	28	60	2	40	4	280	2	T1 :	4	T33
78	10	27	30	3	30	6	270	1	T13 :	6	T57
79	10	29	30	3	50	4	290	1	T13 :	2	T35
80	10	30	20	5	100	2	300	1	SR55 :	2	R37
81	10	30	60	2	60	3	300	2	T1 :	3	T12

Sl. No.	v	r	b_1	k_1	b_2	k_2	n		Reference		Design
82	10	30	20	3	40	6	300	1	T10 :	4	T60
83	12	14	24	6	12	2	168	3	SR66 :	2	(6 2)
84	12	14	12	6	24	4	168	1	SR67 :	2	R109
85	12	15	12	6	36	3	180	2	S27 :	1	R75
86	12	15	36	4	12	3	180	4	SR41 :	3	(4 3)
87	12	16	48	3	12	4	192	3	SR26 :	4	(3 4)
88	12	16	12	8	48	2	192	4	S53 :	1	R38
89	12	17	60	3	12	2	204	3	R70 :	2	(6 2)
90	12	18	60	2	24	4	216	1	R40 :	2	R109
91	12	21	108	2	12	3	252	2	R39 :	3	(4 3)
92	12	25	12	5	24	10	300	1	R145 :	2	R203
93	12	26	12	8	36	6	312	4	S53 :	3	SR68
94	12	27	24	9	36	3	324	6	S82 :	1	R75
95	12	28	24	8	48	3	336	8	S53 :	3	SR26
96	12	28	12	8	60	4	336	1	S56 :	2	R111
97	12	28	60	5	12	3	336	5	R145 :	3	(4 3)
98	12	28	24	6	48	4	336	3	SR66 :	4	R109
99	12	28	48	6	24	2	336	3	SR69 :	4	(6 2)
100	12	29	60	5	24	2	348	5	R144 :	4	(6 2)
101	12	30	24	6	72	3	360	2	SR67 :	3	R71
102	13	14	39	2	26	4	182	1	C10 :	1	C21
103	13	21	39	3	26	6	273	1	C19 :	2	C23
104	13	25	39	3	52	4	325	3	C16 :	2	C21
105	13	26	26	4	39	6	338	1	C21 :	3	C23
106	14	16	14	8	28	4	224	2	S59 :	1	R113
107	14	18	14	6	56	3	252	1	S33 :	2	R79
108	14	20	84	3	14	2	280	3	R79 :	2	(7 2)
109	14	21	98	2	14	7	294	2	SR14 :	7	(2 7)
110	14	30	28	10	28	5	420	2	R205 :	1	OTR16
111	15	9	15	3	15	6	135	1	T16 :	1	T61
112	15	18	30	3	30	6	270	1	T17 :	2	T61
113	15	19	45	3	30	5	285	3	T16 :	5	T48
114	15	19	60	4	15	3	285	4	R114 :	3	(5 3)
115	15	20	60	2	30	6	300	1	T5 :	3	T62
116	15	20	15	10	75	2	300	5	S100 :	1	R41
117	15	20	75	3	15	5	300	3	SR28 :	5	(3 5)
118	15	22	90	2	30	5	330	2	T6 :	5	T48
119	15	24	90	2	60	3	360	2	T6 :	3	T14
120	15	24	15	6	30	9	360	1	T61 :	3	T83
121	15	25	150	2	15	5	375	2	R41 :	5	(3 5)
122	15	25	120	2	45	3	375	2	T5 :	3	T16
123	15	26	15	10	60	4	390	5	S100 :	2	R117
124	15	26	45	2	30	10	390	1	T6 :	5	T92
125	15	27	45	3	45	6	405	1	T19 :	3	T61
126	15	30	90	2	45	6	450	2	T6 :	3	T61

Sl. No.	v	r	b_1	k_1	b_2	k_2	n		Reference			Design
127	16	21	48	3	48	4	336	3	LS18	:	4	LS29
128	16	22	48	2	64	4	352	1	LS3	:	2	LS42
129	16	22	96	3	16	4	352	3	R86	:	4	(4 4)
130	16	25	32	8	48	3	400	2	SR92	:	1	OTR01
131	16	26	144	2	32	4	416	2	LS4	:	1	LS34
132	17	24	34	4	34	8	408	1	C22	:	2	C26
133	17	24	68	2	68	4	408	1	C11	:	2	C22
134	18	20	36	9	18	2	360	3	SR99	:	2	(9 2)
135	18	21	54	6	18	3	378	2	SR73	:	3	(6 3)
136	18	23	90	4	18	3	414	2	OTR11	:	3	(6 3)
137	18	26	144	3	18	2	468	3	R88	:	2	(9 2)
138	18	27	162	2	18	9	486	2	SR16	:	9	(2 9)
139	20	24	80	5	20	4	480	5	SR58	:	4	(5 4)
140	20	25	100	4	20	5	500	4	SR46	:	5	(4 5)
141	20	29	180	3	20	2	580	3	R90	:	2	(10 2)
142	21	16	105	2	21	6	336	1	T8	:	3	T65
143	21	27	21	9	126	3	567	3	S88	:	2	R91
144	21	28	147	3	21	7	588	3	SR31	:	7	(3 7)
145	24	28	96	6	24	4	672	3	SR74	:	4	(6 4)
146	24	29	120	5	24	4	696	5	R153	:	4	(6 4)
147	26	29	78	9	26	2	754	3	R199	:	2	(13 2)
148	27	15	27	9	27	6	405	1	SR102	:	1	R170

Table 2.8.2: BBB designs with two distinct block sizes that can be converted into linear trend-free BIB designs for $\alpha = 0$.

Sl. No.	v	r	b_1	k_1	b_2	k_2	n		Reference Design			
1	6	14	18	2	16	3	84	2	SR6	:	1	R47
2	6	24	48	2	16	3	144	2	R24	:	1	R47
3	8	6	16	2	4	4	48	1	SR9	:	2	(2 4)
4	8	15	12	4	24	3	120	2	S6	:	3	R54
5	8	16	24	2	20	4	128	1	R29	:	2	R97
6	8	18	12	4	48	2	144	1	S7	:	2	R29
7	8	26	32	2	36	4	208	1	R30	:	2	R101
8	9	7	6	6	9	3	63	2	S21	:	1	SR23
9	9	14	18	3	12	6	126	1	LS12	:	2	LS72
10	9	17	18	6	15	3	153	2	SR65	:	1	R59
11	9	18	36	2	30	3	162	2	LS1	:	1	LS15
12	9	18	30	3	12	6	162	1	LS14	:	2	LS72
13	9	19	54	2	21	3	171	2	R34	:	1	R61
14	9	22	12	6	42	3	198	1	S24	:	2	R62
15	9	23	9	3	30	6	207	1	SR23	:	2	R165
16	9	24	24	3	24	6	216	1	LS13	:	4	LS72
17	9	27	21	3	30	6	243	1	R62	:	2	R165

Sl. No.	v	r	b_1	k_1	b_2	k_2	n		Reference		Design
18	10	28	25	4	30	6	280	1	T35 :	3	T60
19	10	29	50	4	15	6	290	2	T37 :	1	T59
20	12	10	8	6	24	3	120	1	SR66 :	1	R71
21	12	10	48	2	6	4	120	1	R38 :	2	(3 4)
22	12	14	16	3	20	6	168	1	SR26 :	1	OTR24
23	12	14	6	8	30	4	168	2	S53 :	1	R111
24	12	16	15	8	12	6	192	1	S58 :	1	SR67
25	12	19	12	9	40	3	228	3	S82 :	1	R78
26	12	19	24	6	28	3	228	3	SR66 :	1	R73
27	12	20	15	8	60	2	240	1	S58 :	1	R40
28	12	20	36	6	6	4	240	3	SR68 :	2	(3 4)
29	12	20	16	6	48	3	240	1	SR69 :	2	R71
30	12	20	18	8	24	4	240	2	SR90 :	1	OTR05
31	12	23	20	3	36	6	276	1	R70 :	2	OTR22
32	12	24	36	2	54	4	288	1	SR13 :	2	OTR06
33	12	24	15	8	42	4	288	1	S58 :	2	OTR03
34	12	24	40	3	28	6	288	1	R77 :	2	OTR21
35	12	25	20	6	60	3	300	2	S28 :	3	R70
36	12	26	12	6	80	3	312	1	S29 :	2	R78
37	12	26	96	2	40	3	312	2	R38 :	1	R76
38	12	27	120	2	28	3	324	2	R40 :	1	R73
39	12	28	32	3	40	6	336	1	SR27 :	2	OTR24
40	12	30	20	6	120	2	360	1	S31 :	2	R40
41	14	8	7	8	14	4	112	1	S59 :	1	R112
42	14	28	84	3	35	4	392	3	R79 :	1	OTR09
43	15	12	10	6	30	4	180	1	S35 :	2	R114
44	15	12	10	9	30	3	180	1	S85 :	1	R81
45	15	12	20	3	20	6	180	1	T14 :	2	T62
46	15	16	50	3	15	6	240	1	T18 :	1	T61
47	15	22	20	3	30	9	330	1	T14 :	3	T83
48	15	23	15	3	50	6	345	1	T16 :	2	T63
49	15	24	40	3	40	6	360	1	T15 :	4	T62
50	15	24	20	6	60	4	360	1	S36 :	4	R114
51	15	26	20	6	90	3	390	2	S35 :	3	R81
52	15	26	50	3	60	4	390	1	R85 :	4	R114
53	15	30	50	3	50	6	450	1	T18 :	2	T63
54	16	12	48	2	24	4	192	1	LS3 :	2	LS36
55	16	12	64	2	8	8	192	1	SR15 :	4	(2 8)
56	16	14	12	8	32	4	224	1	S62 :	2	SR44
57	16	18	24	4	24	8	288	1	LS40 :	2	LS98
58	16	22	16	4	36	8	352	1	LS30 :	2	LS100
59	16	24	12	8	72	4	384	1	S62 :	2	R122
60	16	24	48	2	36	8	384	1	LS3 :	2	LS100
61	18	8	36	3	6	6	144	1	SR30 :	2	(3 6)
62	18	15	12	9	54	3	270	1	SR99 :	1	R89

Sl. No.	v	r	b_1	k_1	b_2	k_2	n		Reference			Design
63	18	18	24	6	45	4	324	1	R169	:	1	OTR11
64	18	22	20	9	36	6	396	1	S87	:	2	SR72
65	18	26	24	6	36	9	468	2	S37	:	3	SR99
66	18	28	36	9	60	3	504	3	SR99	:	1	OTR02
67	20	16	15	8	20	10	320	1	S67	:	1	SR108
68	21	18	70	3	28	6	378	1	T22	:	4	T65
69	21	20	105	2	35	6	420	1	T8	:	1	T70
70	24	11	72	3	8	6	264	1	R92	:	2	(4 6)
71	25	23	20	10	75	5	575	2	S112	:	3	SR60
72	25	28	60	5	40	10	700	3	LS61	:	2	LS135
73	27	12	81	3	9	9	324	1	SR33	:	3	(3 9)
74	32	10	64	4	8	8	320	1	SR49	:	2	(4 8)
75	36	22	126	4	36	8	792	2	T41	:	1	T81
76	36	28	84	3	84	9	1008	1	T24	:	3	T85
77	48	28	192	6	24	8	1344	3	SR78	:	4	(6 8)
78	54	21	162	6	18	9	1134	2	SR79	:	3	(6 9)
79	78	29	234	9	26	6	2262	3	R201	:	2	(13 6)

Table 2.8.3: BBB designs with two distinct block sizes that can be converted into nearly linear trend-free BIB designs for $\alpha = 0$.

Sl. No.	v	r	b_1	k_1	b_2	k_2	n	Reference Design				
1	6	13	12	3	21	2	78	3	SR18	:	1	R21
2	6	21	24	3	27	2	126	6	SR18	:	1	R26
3	6	28	21	2	42	3	168	1	R21	:	3	R46
4	8	15	12	6	24	2	120	3	S18	:	1	R29
5	8	18	36	2	24	3	144	1	R31	:	3	R54
6	9	11	6	6	21	3	99	2	S21	:	1	R62
7	9	13	18	6	3	3	117	2	SR65	:	1	(3 3)
8	10	11	25	2	10	6	110	1	SR11	:	1	R166
9	10	12	15	2	15	6	120	1	T2	:	3	T57
10	12	7	12	6	6	2	84	1	SR67	:	1	(6 2)
11	12	9	36	2	6	6	108	1	SR13	:	3	(2 6)
12	12	12	20	3	14	6	144	1	R70	:	1	OTR21
13	12	13	6	6	40	3	156	1	S27	:	1	R78
14	12	15	10	6	60	2	180	1	S28	:	1	R40
15	12	17	32	3	18	6	204	2	SR26	:	1	R167
16	12	27	18	6	108	2	324	3	S27	:	2	R39
17	14	9	7	6	28	3	126	1	S32	:	1	R79
18	14	27	21	6	84	3	378	1	S34	:	3	R79
19	18	7	18	6	6	3	126	1	SR72	:	1	(6 3)
20	18	11	10	9	18	6	198	1	S86	:	1	SR72
21	20	11	20	10	10	2	220	1	SR108	:	1	(10 2)
22	20	15	100	2	10	10	300	1	SR17	:	5	(2 10)
23	27	10	27	9	9	3	270	1	SR102	:	1	(9 3)

Table 2.9.1: BBB designs with two distinct block sizes for $\alpha = 1$ that can be converted into trend-free BIB designs.

Sr. No.	v	r	b_1	k_1	b_2	k_2	n	Reference Design				
1	6	12	12	4	12	2	72	4	S1	:	1	R18
2	6	21	36	2	18	3	126	4	SR6	:	9	(2 3)
3	6	22	36	3	12	2	132	9	SR18	:	4	(3 2)
4	6	24	24	2	24	4	144	1	R24	:	4	R94
5	8	8	16	2	8	4	64	1	SR9	:	4	(2 4)
6	8	15	16	6	8	3	120	4	S18	:	1	R54
7	8	16	32	2	16	4	128	1	SR10	:	8	(2 4)
8	8	26	24	2	40	4	208	1	R29	:	4	R97
9	8	30	32	6	16	3	240	8	S18	:	1	R55
10	9	14	27	2	18	4	126	1	R34	:	2	R104
11	9	20	18	2	36	4	180	1	LS1	:	4	LS26
12	9	24	27	6	27	2	216	9	S21	:	1	R34
13	10	18	20	3	20	6	180	1	T12	:	4	T57
14	10	22	30	2	40	4	220	1	T3	:	8	T28
15	10	26	30	6	40	2	260	3	S26	:	1	R36
16	10	27	10	3	40	6	270	1	T9	:	4	T60
17	12	12	48	2	12	4	144	1	R38	:	4	(3 4)
18	12	21	24	6	36	3	252	4	S27	:	1	R75
19	12	26	60	2	48	4	312	1	R40	:	4	R109
20	13	22	39	2	52	4	286	1	C10	:	2	C21
21	14	12	14	6	28	3	168	2	S32	:	1	R79
22	14	12	14	8	14	4	168	2	S59	:	1	R112
23	14	24	28	8	28	4	336	4	S59	:	1	R113
24	15	15	15	3	30	6	225	1	T16	:	2	T61
25	15	24	30	9	30	3	360	3	S85	:	1	R81
26	15	28	30	6	60	4	420	3	S35	:	4	R114
27	15	30	30	3	60	6	450	1	T17	:	4	T61
28	16	18	48	2	48	4	288	1	LS3	:	4	LS36
29	16	24	64	2	32	8	384	1	SR15	:	1	6 (2 8)
30	18	27	36	9	54	3	486	3	SR99	:	1	R89
31	21	28	105	2	63	6	588	1	T8	:	9	T65
32	27	18	81	3	27	9	486	1	SR33	:	9	(3 9)

Table 2.9.2: BBB designs with two distinct block sizes for α = *1* that can be converted into linear trend-free BIB designs.

Sr. No.	v	r	b_1	k_1	b_2	k_2	n	Reference Design				
1	8	12	12	4	24	2	96	2	S6	:	1	R29
2	8	22	16	2	36	4	176	1	SR9	:	2	R100
3	9	11	12	6	9	3	99	4	S21	:	1	SR23
4	9	22	24	6	18	3	198	8	S21	:	1	SR24
5	12	14	16	6	24	3	168	2	SR66	:	1	R71
6	12	16	12	6	40	3	192	2	S27	:	1	R78
7	12	18	12	8	30	4	216	4	S53	:	1	R111
8	12	20	36	2	42	4	240	1	SR13	:	2	OTR04
9	12	26	32	3	36	6	312	1	SR27	:	2	R167
10	12	30	30	4	120	2	360	1	S12	:	2	R40
11	15	11	25	3	15	6	165	1	SR28	:	1	R168
12	15	22	50	3	30	6	330	1	SR29	:	2	R168
13	16	10	12	8	16	4	160	2	S61	:	1	SR44
14	16	20	24	8	32	4	320	4	S61	:	1	SR45
15	16	30	24	4	48	8	480	1	LS40	:	4	LS98
16	16	30	36	8	48	4	480	2	S64	:	3	SR44
17	18	10	36	3	12	6	180	1	SR30	:	4	(3 6)
18	18	13	36	6	6	3	234	2	SR72	:	1	(6 3)
19	18	27	30	9	36	6	486	3	S86	:	2	SR72
20	24	13	72	3	16	6	312	1	R92	:	4	(4 6)
21	27	28	81	9	9	3	756	3	SR102	:	1	(9 3)
22	32	12	64	4	16	8	384	1	SR49	:	4	(4 8)

Table 2.9.3: BBB designs with two distinct block sizes for α =*1* that can be converted into nearly linear trend-free BIB designs

Sr. No.	v	r	b_1	k_1	b_2	k_2	n	Reference Design				
1	6	27	36	3	27	2	162	9	SR18	:	1	R26
2	9	15	12	6	21	3	135	4	S21	:	1	R62
3	9	25	36	6	3	3	225	4	SR65	:	1	(3 3)
4	9	26	30	3	24	6	234	1	LS14	:	4	LS72
5	9	29	36	6	15	3	261	4	SR65	:	1	R59
6	10	23	25	2	30	6	230	1	SR11	:	3	R166
7	10	30	15	2	45	6	300	1	T2	:	9	T57
8	12	13	16	3	18	6	156	1	SR26	:	1	R167
9	12	15	15	4	60	2	180	1	S11	:	1	R40
10	12	15	36	2	18	6	180	1	SR13	:	9	(2 6)
11	12	19	36	6	6	2	228	3	SR67	:	1	(6 2)
12	12	19	20	3	28	6	228	1	R70	:	2	OTR21
13	12	24	16	3	40	6	288	1	SR26	:	2	OTR24
14	12	25	30	6	60	2	300	3	S28	:	1	R40
15	15	20	20	3	40	6	300	1	T14	:	4	T62
16	21	26	70	3	56	6	546	1	T22	:	8	T65

Table 2.10.1: BBB designs with two distinct block sizes for $\alpha = 2$ that can be converted into trend-free BIB designs

Sr. No.	v	r	b_1	k_1	b_2	k_2	n	Reference Design
1	6	20	24	4	12	2	120	8 S1 : 1 R18
2	8	12	16	2	16	4	96	1 SR9 : 8 (2 4)
3	8	18	24	4	24	2	144	4 S6 : 1 R29
4	8	24	32	2	32	4	192	1 SR10 : 1 6 (2 4)
5	8	27	32	6	8	3	216	8 S18 : 1 R54
6	9	22	27	2	36	4	198	1 R34 : 4 R104
7	10	30	20	3	40	6	300	1 T12 : 8 T57
8	12	16	48	2	24	4	192	1 R38 : 8 (3 4)
9	14	18	28	6	28	3	252	4 S32 : 1 R79
10	14	20	28	8	14	4	280	4 S59 : 1 R112
11	15	27	15	3	60	6	405	1 T16 : 4 T61
12	16	30	48	2	96	4	480	1 LS3 : 8 LS36
13	32	16	64	4	32	8	512	1 SR49 : 8 (4 8)

Table 2.10.2: BBB designs with two distinct block sizes for $\alpha = 2$ that can be converted into linear trend-free BIB designs

Sr. No.	v	r	b_1	k_1	b_2	k_2	n	Reference Design
1	9	19	24	6	9	3	171	8 S21 : 1 SR23
2	12	20	30	4	60	2	240	2 S11 : 1 R40
3	12	22	32	6	24	3	264	4 SR66 : 1 R71
4	12	22	16	3	36	6	264	1 SR26 : 2 R167
5	12	22	24	6	40	3	264	4 S27 : 1 R78
6	12	26	24	8	30	4	312	8 S53 : 1 R111
7	15	17	25	3	30	6	255	1 SR28 : 2 R168
8	16	16	24	8	16	4	256	4 S61 : 1 SR44
9	18	14	36	3	24	6	252	1 SR30 : 8 (3 6)
10	18	25	72	6	6	3	450	4 SR72 : 1 (6 3)
11	24	17	72	3	32	6	408	1 R92 : 8 (4 6)

Table 2.10.3: BBB designs with two distinct block sizes for $\alpha = 2$ that can be converted into nearly linear trend-free BIB designs

Sr. No.	v	r	b_1	k_1	b_2	k_2	n	Reference Design
3	9	23	24	6	21	3	207	8 S21 : 1 R62

Table 2.11.1: BBB designs with two distinct block sizes for $\alpha = 3$ that can be converted into trend-free BIB designs

S.No.	v	r	b_1	k_1	b_2	k_2	n	Reference Design			
1	8	20	16	2	32	4	160	1	SR9	:	16 (2 4)
2	8	30	48	4	24	2	240	8	S6	:	1 R29
3	12	24	48	2	48	4	288	1	R38	:	16 (3 4)
4	12	30	60	4	60	2	360	4	S11	:	1 R40
5	14	30	56	6	28	3	420	8	S32	:	1 R79
6	16	28	48	8	16	4	448	8	S61	:	1 SR44
7	32	24	64	4	64	8	768	1	SR49	:	16 (4 8)

Table 2.11.2: BBB designs with two distinct block sizes for $\alpha = 3$ that can be converted into linear trend-free BIB designs

S.No.	v	r	b_1	k_1	b_2	k_2	n	Reference Design			
1	15	29	25	3	60	6	435	1	SR28	:	4 R168
2	18	22	36	3	48	6	396	1	SR30	:	16 (3 6)
3	24	25	72	3	64	6	600	1	R92	:	16 (4 6)

Remark 2.1 No BBB designs with two distinct block sizes for $\alpha = 3$ could be obtained that can be converted into nearly linear trend-free BIB designs

CHAPTER III

TREND-FREE NESTED BALANCED INCOMPLETE BLOCK DESIGNS AND DESIGNS FOR DIALLEL CROSS EXPERIMENTS

3.1 Introduction

In Chapter II we have discussed the experiments where there is only one nuisance factor. However, there do occur experimental situations in which one or more factors are nested within the blocking factor. Nested block designs have been developed to deal with experimental situations where one nuisance factor is nested within blocking factor. In this chapter, we shall present some practical situations where the experimenter has to use nested block designs. A nested block design is a design with two systems of blocks where the second system is nested within the first. To be clearer let us consider the following experimental situation:

Experimental situation 3.1: This example relates to a virological experiment, quoted by Preece (1967). Suppose the half-leaves of a plant form the experimental units, on which a number of treatments, say, inoculations with sap from tobacco plants infected with *tobacco necrosis virus*, are to be applied. Suppose the number of treatments is more than the number of suitable half-leaves per plant. Now, there is one source of variation present due to variability among plants. Further, leaves within a plant may exhibit variation among themselves due to their being located on the upper branch, middle branch or on the lower branch of the same plant. Thus leaves within plants form a nested 'nuisance' factor, the nesting being within the plants The half-leaves being experimental units, we then have two systems of 'blocks', leaves (which may be called sub-blocks) being nested within plants (which may be called blocks).

For this type of situations, Kleczkowski (1960) devised a form of nested incomplete block design with eight treatments for a series of experiments in which bean plants, in two primary leaves stage, were inoculated with the sap from tobacco plants infected with the tobacco necrosis virus. The treatments were eight different virus concentrations. Each leaf had two inoculations, one for each half-leaf; ignoring the leaf positions, plants and leaves were, respectively, the blocks (of size 4) and sub-blocks (of size 2) of a nested balanced incomplete block (NBIB) design (which shall be discussed later).

Experimental situation 3.2: In animal experiments, generally littermates (animals born in the same litter) are experimental units within a block *i.e.* litters are blocks. However, animals within the same litter may be varying in their initial body weight. If body weight is taken as another blocking factor, we have a system of nested blocks within a block.

Experimental situation 3.3: Consider a field experiment conducted using a block design and harvesting is done blockwise. To meet the objective of the experiment, the harvested samples are to be analyzed for their contents in the laboratory by different technicians over different periods of time. Therefore, to control the variation due to technicians it is taken as another blocking factor. Hence, we have a system of nested (sub) blocks *i.e.* technicians.

In the experimental situation 3.1, the position of the leaves is nested within plants. This position of leaves may exhibit trend effect in the order in which they are nested. In the experimental situation 2, units nested within the block do not have exactly same body weight and may give the trend effect in the experimental units. And in experimental situation 3.3 different technicians may exhibit the trend effect in the experimental units. In general we can say that nested block designs may have trend-effect at sub-block or at block level over space and time. In such situation, it is required to have trend-free block designs. The information matrix (**C**-matrix) of a nested block design is in terms of parameters of sub-block. Thus a necessary and sufficient condition, for a nested block design to be trend-free design at sub-block level, is derived. Using this condition a catalogue of nested balanced incomplete block (NBIB) designs will be given. A nested block design with sub-block size 2 has a one to one correspondence with designs for diallel crosses. The question whether these optimal block designs for diallel crosses can be converted into trend-free designs is investigated. A catalogue of optimal proper block designs for diallel cross experiments that can be converted into trend-free designs is also prepared.

We begin with some preliminaries of nested block designs.

3.2. Nested block designs

A nested block design is an arrangement of v treatments in a nested block design with b_1 blocks, there being q_j mutually exclusive sub-blocks nested within the j^{th} block, $j = 1,2,\ldots,b_1$ and $b_2 = \sum_{j=1}^{b_1} q_j$ be the total number of sub blocks. Let $\mathbf{N} = ((\, n_{ij}\,))$ be the $v \times b_1$ treatment-block incidence matrix, where n_{ij} is the number of times the i^{th} treatment appears in the j^{th} block, $i = 1,2,\ldots, v$. $\mathbf{N1} = \mathbf{r} = (r_1, \ldots, r_v)'$, $\mathbf{1'N} = \mathbf{k} = (k_1, \ldots, k_{b_1})'$, where r_i and k_j denote the replication of the i^{th} treatment and j^{th} block size respectively with $\sum_{i=1}^{v} r_i = \sum_{j=1}^{b_1} k_j = n$, total number of observations. Let $\mathbf{M} = ((\, m_{ij'(j)}\,))$ denote $v \times b_2$ treatment-sub block incidence matrix, where $m_{ij'(j)}$ denote the number of times the i^{th} treatment appears in the j'^{th} sub block nested within the j^{th} block, $j' = 1,\ldots, q_j$. $\mathbf{M1} = \mathbf{r}$ and $\mathbf{1'M} = \mathbf{h}_{b_1 \times 1} = (\mathbf{h}'_{(1)}, ,\ldots, \mathbf{h}'_{(b_1)})'$, where $\mathbf{h}'_{(\mathbf{j})} = (h_{1(j)}, \cdots, h_{q_j(j)})$. $\mathbf{R} = \text{diag}\,(r_1, \ldots, r_v)$, $\mathbf{K} = \text{diag}\,(k_1, \ldots, k_{b_1})$. Let $\mathbf{H}_j$ and $\mathbf{H}$ are the diagonal matrices whose diagonal elements are the successive elements of $\mathbf{h}_j$ and $\mathbf{h}$ and $\mathbf{W}$ is the $b_1 \times b_2$ block *vs* sub-block incidence matrix. The model under consideration can be written as

$$y_{ij'(j)u} = \mu + \tau_i + \beta_j + \eta_{j'(j)} + e_{ij'(j)u} \tag{3.1}$$

where $y_{ij'(j)u}$ is the u^{th} observation obtained from the i^{th} treatment in the j'^{th} sub block of the j^{th} block; $u = 1, 2, \ldots, m_{ij'(j)}$, μ is the general mean, τ_i is the i^{th} treatment effect, β_j is the j^{th} block effect, $\eta_{ij'(j)}$ is the effect of the j'^{th} sub block nested within the j^{th} block and $e_{ij'(j)u}$ is the uncorrelated errors with mean zero and common variance σ^2. The model (3.1) can be written in matrix notations as

$$\mathbf{y} = \mu\mathbf{1} + \boldsymbol{\Delta}'\boldsymbol{\tau} + \mathbf{D}'\boldsymbol{\beta} + \boldsymbol{\Phi}'\boldsymbol{\eta} + \mathbf{e} \tag{3.2}$$

where

$\mathbf{y} = (y_1, \ldots, y_n)$ is an $n \times 1$ observable random vector.

$\boldsymbol{\beta} = (\beta_1, \ldots, \beta_b)'$, is the $b_1 \times 1$ vector of block effects.

$\boldsymbol{\eta} = (\eta_{1((l)}, \eta_{2(l)}, \cdots, \eta_{q_{bj}(b_j)})'$, is the $b_2 \times 1$ vector of sub-block effects nested within the block effects.

$\boldsymbol{\Delta}'$ is the $n \times v$ observations *vs* treatments incidence matrix,

$\mathbf{D}'$ is the $n \times b_1$ observations *vs* blocks incidence matrix,

$\boldsymbol{\Phi}'$ is the $n \times b_2$ observations *vs* sub-blocks nested within the blocks incidence matrix,

$\mathbf{e}$ is the $n \times 1$ vector of random errors.

The following relations can easily be seen

$\boldsymbol{\Delta}\,\boldsymbol{\Delta}' = \mathbf{R}$, $\mathrm{diag}(r_1, \cdots, r_v)$, $\boldsymbol{\Delta}'\,\mathbf{1} = r$, $\boldsymbol{\Delta}\,\boldsymbol{\Phi} = \mathbf{M}$, $\boldsymbol{\Delta}\,\mathbf{D}' = \mathbf{N}$, $\boldsymbol{\Phi}\,\mathbf{1} = \mathbf{h}$, $\boldsymbol{\Phi}\,\boldsymbol{\Phi}' = \mathbf{H}$, $\boldsymbol{\Phi}\,\mathbf{D}' = \mathbf{W}$ $\mathbf{D}\mathbf{D}' = \mathbf{K}$, $\mathbf{D1} = \mathbf{k}$, $\mathbf{1}'\mathbf{1} = n$.

Without loss of generality, we assume that the observations are assumed to be arranged in the order of (j, j'). Therefore, we can write $\mathbf{W} = \sum_{j=1}^{b_j}{}^{+}\mathbf{h}_j$ and $\mathbf{H} = \sum_{j=1}^{b_1}{}^{+}\mathbf{H}_j$. $\mathbf{L} = \sum_{j=1}^{b_1}{}^{+}\mathbf{l}_j$. Then we have

$\mathbf{N}=\mathbf{ML}$, $\mathbf{W}= \mathbf{L}'\mathbf{H}$, $\mathbf{K} = \mathbf{L}'\mathbf{HL} = \mathbf{WL}$, $n = \mathbf{1}'_{b_1}\mathbf{H1}_{b_1}$, $\mathbf{k} = \mathbf{L}'\mathbf{H1}_{b_j}$ and $\mathbf{WH}^{-1}\mathbf{W}' = \mathbf{L}'\mathbf{W}' = \mathbf{k}$.

The reduced normal equation for estimating treatment contrasts is

$$\mathbf{C} = \mathbf{R} - \mathbf{MH}^{-1}\mathbf{M}'. \tag{3.3}$$

It can be seen that the coefficient matrix of the reduced normal equations for estimating the treatments effects is same as obtained if blocks are ignored and the design is analyzed as sub-blocks. Therefore, as far as the estimation of the treatment effects are considered it is the sub block structure that only matters. The properties of the **C**-matrix are completely determined by treatments versus sub-blocks incidence matrix. Therefore, for obtaining the trend-free designs for nested block designs, it is proper to consider the trend effect at the sub-block level.

Further, if the experimenter is interested in estimating all elementary treatment contrasts with same precision, then we require nested block designs that are variance balanced. In the sequel we shall give some definitions and results on nested variance balanced block designs.

Definition 3.1: A connected nested block design is said to be sub-block variance balanced if and only if all the non-zero eigenvalues of **C** matrrix are equal.

The information matrix $\mathbf{C} = \mathbf{R} - \mathbf{M}\mathbf{H}^{-1}\mathbf{M}'$ for a variance balanced design is given by

$$\mathbf{C} = \theta(\mathbf{I} - \frac{1}{v}\mathbf{1}\mathbf{1}') \tag{3.4}$$

where $\theta = \dfrac{1}{(v-1)}[n - \sum_{i=1}^{v}\sum_{j=1}^{b_1}\sum_{j'=1}^{q_j} m_{ij'(j)}^2 / t_{j'(j)}]$ is unique non-zero eigen value of **C** matrix. A nested block design is said to be binary if $m_{ij'(j)} = 0$ or 1. For a binary nested sub-block balanced design $\theta = \dfrac{(n-b_1)}{(v-1)}$. The above definition considered the variance balanced property with respect to sub-blocks only. However, one may consider that the block classification ignoring sub-blocks also forms a variance balanced block design. Most of the available literature relates to the combinatorial aspects of the nested block designs that are variance balanced with respect to block classification as well as sub-block classification ignoring other classification. Such designs have been called as nested variance balanced block designs. A binary, proper and equireplicated nested variance balanced block design is the nested balanced incomplete block (NBIB) design that was introduced by Preece (1967). A NBIB design is defined as follows:

Definition 3.2: A NBIB design with parameter (v, r, b_1, k_1, λ_1, b_2, k_2, λ_2, m) is an arrangement of v treatments, each replicated r times with two system of blocks such that:

a) The second system is nested within the first, with each block of the first system containing exactly m blocks from the second system.
b) Ignoring the second system leaves a balanced incomplete block (BIB) design with b_1 blocks each of k_1 units and with λ_1 concurrences and
c) Ignoring the first system leaves a BIB design with b_2 blocks each of k_2 units and, λ_2 concurrences.

The following parametric relations hold good in case of a NBIB design

1. $vr = b_1 k_1 = m\, b_1 k_2 = b_2 k_2$;
2. $(v-1)\lambda_1 = (k_1-1)r$; $(v-1)\lambda_2 = (k_2-1)r$ and
3. $(v-1)(\lambda_1 - m\lambda_2) = (m-1)r$.

The work on combinatorial aspects of NBIB designs was initiated by Preece (1967) who gave some trail and error solutions of NBIB designs and provided a list with r ≤ 15. Jimbo and Kuriki (1983) gave theorems in order to construct NBIB designs. Dey, Das and Banerjee (1986) have given some methods of construction of NBIB designs through initial solutions and resolvable BIB designs as special cases. Morgan, Preece and Rees (2000) presented an excellent review of NBIB designs and provided a catalogue of NBIB designs with v ≤ 16, r ≤ 30. Satpati (2001) has given an exhaustive review of nested balanced block, nested balanced incomplete block and nested partially balanced incomplete block designs alongwith generalization of methods of construction of NBIB designs.

3.3 Trend-free nested balanced incomplete block designs

In this section we obtain the necessary and sufficient condition for a nested balanced incomplete block to be trend-free block design when experimental units within the blocks are subject to trend effect over space or time.

Consider the NBIB design *d* with parameter ($v, r, b_1, k_1, \lambda_1, b_2, k_2, \lambda_2, m$) under model 3.2 with trend terms added

$$\mathbf{y} = \mu\mathbf{1} + \boldsymbol{\Delta}'\boldsymbol{\tau} + \mathbf{D}'\boldsymbol{\beta} + \boldsymbol{\varphi}'\boldsymbol{\eta} + \mathbf{Z}'\boldsymbol{\psi} + \mathbf{e} \tag{3.5}$$

$$\mathrm{E}(\mathbf{e}) = \mathbf{0},\ \mathrm{D}(\mathbf{e}) = \sigma^2\,\mathbf{I}.$$

Let *d* be the connected block design under model (3.5) when $\boldsymbol{\psi} = \mathbf{0}$. $\boldsymbol{\psi}$ is a $p\times 1$ vector of trend effects. We assume a common trend effect in all the blocks of degree p = (k_2 – 1). The matrix $\mathbf{Z}'$ is matrix of coefficients given by matrix $\mathbf{Z}' = \mathbf{1}_{\mathbf{b}_2} \otimes \mathbf{F}'$ is of order $n \times p$ and $\mathbf{F}'$ is $k_2 \times p$, matrix with columns consisting of equi-spaced normalized orthogonal polynomials. Also $\mathbf{1}'\mathbf{F}' = \mathbf{0}, \mathbf{F}\,\mathbf{F}' = \mathbf{I}_P$, and $\mathbf{Z}\mathbf{Z}' = \mathrm{b}_2\,\mathbf{I}_\mathrm{p}$. Here problem is to assign treatments to plots within blocks so that the known properties of ordinary analysis of variance for treatments and for block sum of squares are preserved when variation due to the trend may be removed from error sum of squares in model (3.2). A formal definition of a trend-free block design is given as:

Definition 3.3: A NBIB block design *d* is said to be a trend-free block design if and only if the additional reduction in sum of squares due to fitting of parameters of interest over and above fitting of other parameters, for the two models one containing the trend effect and the other without trend effects, is equal i.e.

$$\mathrm{R}(\tau/\mu,\boldsymbol{\beta},\boldsymbol{\eta},\boldsymbol{\psi}) \;=\; \mathrm{R}(\tau/\mu,\boldsymbol{\beta},\boldsymbol{\eta}) \tag{3.6}$$

where $\mathrm{R}(\tau/\mu,\boldsymbol{\beta},\boldsymbol{\eta},\boldsymbol{\psi})$ is the additional reduction in sum of squares due to fitting of $\tau,\boldsymbol{\beta},\boldsymbol{\eta}$ and $\boldsymbol{\psi}$ over and above fitting just $\tau,\boldsymbol{\beta},\boldsymbol{\eta}$. Similarly $\mathrm{R}(\tau/\mu,\boldsymbol{\beta},\boldsymbol{\eta})$ is the additional reduction in sum of squares due to fitting of $\tau,\boldsymbol{\beta}$ and $\boldsymbol{\eta}$ over and above fitting just $\tau,\boldsymbol{\beta}$.

Thus for deriving necessary and sufficient condition for a NBIB design to be trend-free design, we consider the following two situations:

Case I When there is no trend effect *i. e.* $\psi = 0$ under model (3.5)

The usual **C**-matrix (information matrix) is

$\mathbf{C}_1 = \mathbf{X}_1'[\mathbf{I} - \mathbf{X}_2(\mathbf{X}_2'\mathbf{X}_2)^{-}\mathbf{X}_2']\mathbf{X}_1$,

where $\mathbf{X}_1 = \boldsymbol{\Delta}'$, $\mathbf{X}_2 = [\ \mathbf{1} \quad \boldsymbol{\Phi}' \quad \mathbf{D}'\ ]$

$$\mathbf{X}_2'\mathbf{X}_2 = \begin{bmatrix} n & \mathbf{h}' & \mathbf{k}' \\ \mathbf{h} & \mathbf{H} & \mathbf{W}' \\ \mathbf{k} & \mathbf{W} & \mathbf{K} \end{bmatrix} \quad \text{and} \quad (\mathbf{X}_2'\mathbf{X}_2)^{-} = \begin{bmatrix} 0 & \mathbf{0}' & \mathbf{0}' \\ \mathbf{0} & \mathbf{H}^{-1} & \mathbf{0} \\ \mathbf{0} & \mathbf{0} & \mathbf{0} \end{bmatrix}$$

$\mathbf{X}_2(\mathbf{X}_2'\mathbf{X}_2)^{-}\mathbf{X}_2' = \boldsymbol{\Phi}'\mathbf{H}^{-1}\boldsymbol{\Phi}$.

So the **C**-matrix simplifies to

$$\mathbf{C}_1 = \boldsymbol{\Delta}(\mathbf{I} - \boldsymbol{\Phi}'\mathbf{H}^{-1}\boldsymbol{\Phi})\boldsymbol{\Delta}' = \boldsymbol{\Delta}\boldsymbol{\Omega}_1\boldsymbol{\Delta}' \tag{3.7}$$

where, $\boldsymbol{\Omega}_1 = \mathbf{I} - \boldsymbol{\Phi}'\mathbf{H}^{-1}\boldsymbol{\Phi}$ (3.8)

and $R(\tau/\mu, \boldsymbol{\beta}, \boldsymbol{\eta}) = \mathbf{y}'\boldsymbol{\Omega}_1\boldsymbol{\Delta}'\mathbf{C}_1^{-}\boldsymbol{\Delta}\boldsymbol{\Omega}_1\mathbf{y}$. (3.9)

Case II When trend effect is present *i. e.* $\psi \neq 0$ under model (3.5)

The **C**-matrix in presence of trend effect is,

$\mathbf{C}_2 = \mathbf{X}_1'[\mathbf{I} - \mathbf{X}_3(\mathbf{X}_3'\mathbf{X}_3)^{-}\mathbf{X}_3']\mathbf{X}_1$

where $\mathbf{X}_3 = [\ \mathbf{1} \quad \mathbf{D}' \quad \boldsymbol{\phi}' \quad \mathbf{Z}'\ ]$,

$$(\mathbf{X}_3'\mathbf{X}_3) = \begin{bmatrix} n & \mathbf{h}' & \mathbf{k}' & \mathbf{1}'\mathbf{Z} \\ \mathbf{h} & \mathbf{H} & \mathbf{W}' & \Phi\mathbf{Z} \\ \mathbf{k} & \mathbf{W} & \mathbf{K} & \mathbf{D}\mathbf{Z} \\ \mathbf{Z}'\mathbf{1} & \mathbf{Z}'\Phi & \mathbf{Z}'\mathbf{D} & \mathbf{Z}'\mathbf{Z} \end{bmatrix} = \begin{bmatrix} n & \mathbf{h}' & \mathbf{k}' & \mathbf{0} \\ \mathbf{h} & \mathbf{H} & \mathbf{W}' & \mathbf{0} \\ \mathbf{k} & \mathbf{W} & \mathbf{K} & \mathbf{0} \\ \mathbf{0} & \mathbf{0} & \mathbf{0} & b_2\mathbf{I_p} \end{bmatrix} \text{ and}$$

$$(\mathbf{X}_3'\mathbf{X}_3)^{-} = \begin{bmatrix} 0 & \mathbf{0}' & \mathbf{0}' & \mathbf{0}' \\ \mathbf{0} & \mathbf{H}^{-} & \mathbf{0} & \mathbf{0} \\ \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} \\ \mathbf{0} & \mathbf{0} & \mathbf{0} & b_2^{-1}\mathbf{I_p} \end{bmatrix}.$$

Thus the information matrix simplifies to

$$\mathbf{C}_2 = \Delta(\mathbf{I} - \Phi'\mathbf{H}^{-1}\Phi + b_2^{-1}\mathbf{Z}'\mathbf{Z})\Delta' = \boldsymbol{\Delta}\boldsymbol{\Omega}_2\boldsymbol{\Delta}' \tag{3.10}$$

where $\boldsymbol{\Omega}_2 = (\mathbf{I} - \Phi'\mathbf{H}^{-1}\Phi + b_2^{-1}\mathbf{Z}'\mathbf{Z})$, (3.11)

and $R(\tau/\mu, \boldsymbol{\beta}, \boldsymbol{\eta}, \boldsymbol{\psi}) = \mathbf{y}'\boldsymbol{\Omega}_2\boldsymbol{\Delta}'\mathbf{C}_2^{-}\boldsymbol{\Delta}\boldsymbol{\Omega}_2\mathbf{y}$ (3.12)

Now we have following result:

Theorem 3.1: A necessary and sufficient condition for a NBIB design *d* to be trend-free at sub-block level is $\Delta\mathbf{Z}' = \mathbf{0}$.

Proof: Necessary Part For design *d* to be trend-free design, from (3.6), (3.9) and (3.12) we have

$$\mathbf{y}'\boldsymbol{\Omega}_2\boldsymbol{\Delta}'\mathbf{C}_2^{-}\boldsymbol{\Delta}\boldsymbol{\Omega}_2\mathbf{y} = \mathbf{y}'\boldsymbol{\Omega}_1\boldsymbol{\Delta}'\mathbf{C}_1^{-}\boldsymbol{\Delta}\boldsymbol{\Omega}_1\mathbf{y}$$

$$\text{or} \quad \boldsymbol{\Omega}_2\boldsymbol{\Delta}'\mathbf{C}_2^{-}\boldsymbol{\Delta}\boldsymbol{\Omega}_2 = \boldsymbol{\Omega}_1\boldsymbol{\Delta}'\mathbf{C}_1^{-}\boldsymbol{\Delta}\boldsymbol{\Omega}_1 \tag{3.13}$$

Pre- and post multiplying both sides of (3.13) by $\boldsymbol{\Delta}$ and Δ' respectively, we have

$$\Delta\boldsymbol{\Omega}_2\boldsymbol{\Delta}'\mathbf{C}_2^{-}\boldsymbol{\Delta\Omega}_2\ \Delta' = \Delta\boldsymbol{\Omega}_1\boldsymbol{\Delta}'\mathbf{C}_1^{-}\boldsymbol{\Delta\Omega}_1\ \Delta'$$
or $\mathbf{C}_2\mathbf{C}_2^{-}\mathbf{C}_2 = \mathbf{C}_1\mathbf{C}_1^{-}\mathbf{C}_1$ or $\mathbf{C}_2 = \mathbf{C}_1$
or $\boldsymbol{\Delta}(\boldsymbol{\Omega}_2 - \boldsymbol{\Omega}_1)\boldsymbol{\Delta}' = \mathbf{0}$ or $b_2^{-1}\boldsymbol{\Delta}\mathbf{Z}'\mathbf{Z}\boldsymbol{\Delta}' = \mathbf{0}$.
or $\boldsymbol{\Delta}\mathbf{Z}' = \mathbf{0}$. (3.14)

Sufficient Part To prove sufficiency, (3.14) is used with (3.8) and (3.11) to show that $\mathbf{C}_1 = \mathbf{C}_2$ and $\boldsymbol{\Delta\Omega}_1 = \Delta\boldsymbol{\Omega}_2$ so the equality of (3.9) and (3.12) follows.

The necessary and sufficient condition for a NBIB design to be trend-free designs at the sub-block level is given in (3.14). Here the matrix $\mathbf{Z}' = \mathbf{1}_{\mathbf{b}_2} \otimes \mathbf{F}'$ is of order $n \times p$, $p \le (k_2 - 1)$ and $\mathbf{F}'$ is $k_2 \times p$, matrix with columns consisting of equi-spaced normalized orthogonal polynomials. This condition is obtained in terms of sub-block because the **C**-matrix is in terms of sub-block.

Similarly when the trend effect is at block level, we can say that the condition for a NBIB design to be trend-free at block level if $\boldsymbol{\Delta}\mathbf{Z}'_* = \mathbf{0}$. (3.15)
where $\mathbf{Z}'_* = \mathbf{1}_{\mathbf{b}_1} \otimes \mathbf{F}'_*$ is of order $n \times q$ $q=(k_1 - 1)$, $q>p$ and $\mathbf{F}'_*$ is $k_1 \times q$, matrix with columns consisting of equi-spaced normalized orthogonal polynomials.
Now to be clearer, we define a NBIB design to be trend-free as

Definition 3.3: A NBIB design with parameter ($v, r, b_1, k_1, \lambda_1, b_2, k_2, \lambda_2, m$) is said to be completely trend-free design if it is trend-free at block and sub-block level. A NBIB design is said to be trend-free design at block level if it trend-free at block level only and it is said to be trend-free design at sub-block level if it is trend-free at sub-block level only.

Here we give an example of a complete trend-free NBIB design and show how the condition (3.14) and (3.15) are satisfied.

Example 3.1: Consider NBIB design with parameters $v = 5$, $b_1 = 5$, $b_2 = 10$, $r = 4$, $k_1 = 4$, $k_2 = 2$, $\lambda_1 = 3$, $\lambda_2 = 1$ *as* [(1,4),(2,3)]; [(2,5),(3,4)]; [(3,1),(4,5)]; [(4,2),(5,1)]; [(5,3),(1,2)].

For this NBIB design the matrices $\boldsymbol{\Delta}$, $\mathbf{Z}'$ and $\mathbf{Z}'_*$ of order 20×5, 20×1 and 20×3 respectively are

$$
\begin{array}{ccc}
\mathbf{\Delta}' & \mathbf{Z}' & \mathbf{Z}'_{*} \\
\begin{bmatrix}
1 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 1 & 0 \\
0 & 1 & 0 & 0 & 0 \\
0 & 0 & 1 & 0 & 0 \\
0 & 1 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 1 \\
0 & 0 & 1 & 0 & 0 \\
0 & 0 & 0 & 1 & 0 \\
0 & 0 & 1 & 0 & 0 \\
1 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 1 & 0 \\
0 & 0 & 0 & 0 & 1 \\
0 & 0 & 0 & 1 & 0 \\
0 & 1 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 1 \\
1 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 1 \\
0 & 0 & 1 & 0 & 0 \\
1 & 0 & 0 & 0 & 0 \\
0 & 1 & 0 & 0 & 0
\end{bmatrix}
&
\begin{vmatrix}
-1 \\ +1 \\ -1 \\ +1 \\ -1 \\ +1 \\ -1 \\ +1 \\ -1 \\ +1 \\ 1 \\ +1 \\ -1 \\ +1 \\ -1 \\ +1 \\ -1 \\ +1 \\ -1 \\ +1
\end{vmatrix}
&
\begin{bmatrix}
-3 & +1 & -1 \\
-1 & -1 & +3 \\
+1 & -1 & +1 \\
+3 & +1 & -1 \\
-3 & +1 & -1 \\
-1 & -1 & +3 \\
+1 & -1 & +1 \\
+3 & +1 & -1 \\
-3 & +1 & -1 \\
-1 & -1 & +3 \\
+1 & -1 & +1 \\
+3 & +1 & -1 \\
-3 & +1 & -1 \\
-1 & -1 & +3 \\
+1 & -1 & +1 \\
+3 & +1 & -1 \\
-3 & +1 & -1 \\
-1 & -1 & +3 \\
+1 & -1 & +1 \\
+3 & +1 & -1
\end{bmatrix}
\end{array}
$$

Here we see that $\mathbf{\Delta Z}' = \mathbf{0}$ and $\mathbf{\Delta Z}'_{*} = \mathbf{0}$. Thus in the above design trend effect is completely eliminated.

3.4 Construction of trend-free nested balanced incomplete block designs

In this section we shall study some families of NBIB designs that can be converted into trend-free design. A catalogue of NBIB designs that can be converted into completely trend-free design, trend-free designs at sub-block levels, trend-free designs at block level and nearly linear trend-free designs will be given.

Families of NBIB designs given by Dey, Das and Banerjee ((1986), Gupta and Kageyama (1994), Das and Gupta (1997), Das, Dey and Dean (1998), Parsad, Gupta and Srivastava (1999) and Parsad, Gupta and Gupta (2000) have been studied and families of NBIB designs (with parameters) that can be converted into trend-free NBIB designs at sub-block and block levels are given in Table 3.1.

Table 3.1: Families of NBIB designs that can be converted into trend-free NBIB designs at the block and at the sub-block level

S. No.	v	b_1	b_2	r	k_1	k_2	λ_1	λ_2	Remark
1.	2t+1	2t+1	t(2t+1)	2t	2t	2	2t-1	1	
2.	mt+1	t(mt+1)	ut(mt+1)	Mt	2u	2	m-1	1	m = 2u, $u \geq 2$
3.	2t+1	t(2t+1)	2t(2t+1)	4t	4	2	6	2	
4.	4t+1	4t+1	2(4t+1)	4t	4t	2t	4t-1	2t-1	
5.	6t+1	t(6t+1)	2t(6t+1)	6t	6	3	5	2	
6.	9t+1	t(9t+1)	3t(9t+1)	9t	9	3	8	2	
7.	12t+1	t(12t+1)	4t(12t+1)	12t	12	3	11	2	
8.	6t+1	6t+1	2t(6t+1)	6t	6t	3	6t-1	2	
9.	4t-1	4t-1	2(4t-1)	2(2t-1)	4t-2	2t-1	4t-3	2t-2	

Note: i) $t \geq 1$ for all the cases.

ii) v is prime or prime power for all the cases except at Sl. No. 1.

A general method of construction of NBIB (Parsad, Gupta and Srivastava (1999)) is given below:

Suppose there exists a BIB designs with parameters v', b', r', k', λ' and there also exists an NBIB designs with parameters $k', b_1^*, b_2^*, r^*, k_1^*, k_2^*, \lambda_1^*, \lambda_2^*$. Then writing each of the block contents of BIB design as NBIB design, we get an NBIB design with

parameters $v = v', b_1 = b'b_1^*, b_2 = b'b_2^*, r = r'r^*, k_1 = k_1^* k_2 =, k_2^*, \lambda_1 = \lambda'\lambda_1^*, \lambda_2 = \lambda'\lambda_2^*$.

Using this result we have families of NBIB that can be converted into trend-free NBIB designs at sub-block and block levels are given in Table 3.2.

Table 3.2 Some other families of NBIB designs that can be converted into trend-free NBIB designs at the block and at the sub-block level

Sr. No.	b_1	b_2	r	k_1	k_2	λ_1	λ_2
1.	b'(2t+1)	b't(2t+1)	2tr'	2t	2	(2t-1)λ'	λ'
2.	b't(mt+1)	b'mt(mt+1)	mtr'	M	2	(m-1)λ'	λ'
3.	b't(2t+1)	2b't(2t+1)	4tr'	4	2	6λ'	2λ'
4.	b'(4t+1)	2b'(4t+1)	4tr'	4t	2t	(4t-1)λ'	(2t-1) λ'
5.	tb'(6t+1)	2tb'(6t+1)	6tr'	6	3	5λ'	2λ'
6.	tb'(9t+1)	3tb'(9t+1)	9tr'	9	3	8λ'	2λ'
7.	tb'(12t+1)	4tb'(12t+1)	12tr'	12	3	11λ'	2λ'
8.	b'(6t+1)	2tb'(6t+1)	6tr'	6t	3	(6t-1)λ'	2λ'
9.	b'(4t-1)	2(4t-1)	2(2t-1) r'	4t-2	2t-1	(4t-3)λ'	(2t-2)λ'

Note: i) $v = v'$ for all the designs

ii) $t \geq 1$ for all the cases.

iii) v is prime or prime power for all the cases except at Sr. No. 1.

Now we have the following result:

Result 3.1: Let there exist a nested BIB design with parameters $v, b_1, b_2, r, k_1, k_2, \lambda_1, \lambda_2$ that is linear trend-free with respect to sub blocks. Then this design can be converted in linear trend-free with respect to blocks if

(i) either replication r is even, or

(ii) for r odd, k_2, and m both are odd, where $k_1 = m\,k_2$.

Out of 68 NBIB designs given by Morgan, Preece and Rees (2001), 44 designs satisfy condition (i) and 4 designs satisfy condition (ii)

Result 3.3: NBIB designs obtained by the method given in Theorem 3.1, Kageyama, Philip and Banerjee (1995) with parameters

$$v = v', b_1 = \binom{v'}{2}, b_2 = 2b_1, r = (v'-1)(k'-\lambda'), k_1 = 2(k'-\lambda'), k_2 = (k'-\lambda'),$$

$\lambda_1 = (k'-\lambda')^2 + \lambda'(v'-2k'+\lambda'), \lambda_2 = \lambda'(v'-2k'+\lambda'),$ where $v' = b', r' = k', \lambda'$ are the parameters of a symmetric BIB design, can be converted into complete trend-free NBIB designs if v is odd and trend-free NBIB design at sub-block level, otherwise.

3.5 Catalogue of trend-free Nested Balanced Incomplete Block Designs

A catalogue of NBIB designs with v ≤ 16, r ≤30 is prepared by Morgan, Preece and Rees (2000). In this catalogue, the NBIB design obtainable as copies of other NBIB designs have been excluded. Here we give a catalogue of above NBIB designs that can be converted into completely trend-free, linear trend-free at sub-block level and nearly linear trend-free at sub-block level NBIB designs.

Table 3.3.1 NBIB designs that can be converted into completely trend-free NBIB designs

Sr. No.	v	b_1	b_2	r	k_1	k_2	λ_1	λ_2	Source
1.	5	5	10	4	4	2	3	1	MPR 1
2.	7	7	21	6	6	2	5	1	MPR 2
3.	7	7	14	6	6	3	5	2	MPR 3
4.	7	21	42	12	4	2	6	2	MPR 19
5.	9	9	36	8	8	2	7	1	MPR 8
6.	9	9	18	8	8	4	7	3	MPR 6
7.	9	18	36	8	4	2	3	1	MPR 5
8.	10	10	30	9	9	3	8	2	MPR 12
9.	10	30	60	18	6	3	10	4	MPR 47
10.	11	11	55	10	10	2	9	1	MPR 14
11.	11	55	110	20	4	2	6	2	MPR 49
12.	11	55	165	30	6	2	15	3	MPR 66
13.	11	55	110	30	6	3	15	6	MPR 67
14.	11	11	22	10	10	5	9	4	MPR 15

Sr. No.	v	b_1	b_2	r	k_1	k_2	λ_1	λ_2	Source
15.	13	13	78	12	12	2	11	1	MPR 23
16.	13	26	78	12	6	2	5	1	MPR 21
17.	13	39	78	12	4	2	3	1	MPR 20
18.	13	39	156	24	8	2	14	2	MPR 55
19.	13	26	52	12	6	3	5	2	MPR 22
20.	13	13	52	12	12	3	11	2	MPR 24
21.	13	26	78	18	9	3	12	3	MPR 48
22.	13	13	39	12	12	4	11	3	MPR 25
23.	13	39	78	24	8	4	14	6	MPR 56
24.	13	13	26	12	12	6	11	5	MPR 26
25.	15	15	105	14	14	2	13	1	MPR 31
26.	15	15	30	14	14	7	13	6	MPR 32
27.	15	105	210	28	4	2	6	2	MPR 59
28.	16	16	80	15	15	3	14	2	MPR 44
29.	16	16	48	15	15	5	14	4	MPR 45
30.	16	48	96	30	10	5	18	8	MPR 68

Table 3.3.2 NBIB designs that can be converted into trend-free NBIB designs at the sub-block level

Sr. No.	v	b_1	b_2	r	k_1	k_2	λ_1	λ_2	Source
1.	6	15	30	10	4	2	6	2	MPR 13
2.	8	28	56	21	6	3	15	6	MPR 51
3.	9	12	36	8	6	2	5	1	MPR 9
4.	9	18	36	8	4	2	3	1	MPR 5
5.	10	15	30	9	6	3	5	2	MPR 11
6.	10	45	90	27	6	3	15	6	MPR 58
7.	10	45	90	18	4	2	6	2	MPR 46
8.	12	33	132	22	8	2	14	2	MPR 53
9.	14	91	182	26	4	2	6	2	MPR 57
10.	15	21	105	14	10	2	9	1	MPR 29
11.	15	35	105	14	6	2	5	1	MPR 27
12.	15	35	105	21	9	3	12	3	MPR 52
13.	15	35	210	28	12	2	22	2	MPR 62
14.	15	35	105	28	12	4	22	6	MPR 64
15.	15	42	210	28	10	2	18	2	MPR 60
16.	16	20	80	15	12	3	11	2	MPR 41
17.	16	24	48	15	10	5	9	4	MPR 39

Table 3.3.3 NBIB designs that can be converted into linear trend-free NBIB designs at the sub-block level

Sr. No.	v	b_1	b_2	r	k_1	k_2	λ_1	λ_2	Source
1.	9	18	36	8	4	2	3	1	MPR 5
2.	12	33	66	22	8	4	14	6	MPR 54
3.	15	21	42	14	10	5	9	4	MPR 30
4.	15	35	70	14	6	3	5	2	MPR 28
5.	15	35	140	28	12	3	22	4	MPR 63
6.	15	35	105	28	12	4	22	6	MPR 64
7.	15	35	70	28	12	6	22	10	MPR 65
8.	15	42	84	28	10	5	18	8	MPR 61

Table 3.3.4 NBIB designs that can be converted into nearly linear trend-free NBIB designs at the sub-block level

Sr. No.	v	b_1	b_2	r	k_1	k_2	λ_1	λ_2	Source
1.	8	14	28	7	4	2	3	1	MPR 4
2.	8	28	84	21	6	2	15	3	MPR 50
3.	10	15	45	9	6	2	5	1	MPR 10
4.	12	22	66	11	6	2	5	1	MPR 17
5.	12	33	66	11	4	2	3	1	MPR16
6.	12	22	44	11	6	3	5	2	MPR 18
7.	16	20	120	15	12	2	11	1	MPR 40
8.	16	20	40	15	12	6	11	5	MPR 43
9.	16	24	120	15	10	2	9	1	MPR 38
10.	16	30	120	15	8	2	7	1	MPR 36
11.	16	40	120	15	6	2	5	1	MPR 34
12.	16	40	80	15	6	2	5	1	MPR 35
13.	16	60	120	15	4	2	3	1	MPR 33

3.6 Designs for Diallel Cross Experiments

The diallel cross is a type of mating design used to study the genetic properties of a set of inbred lines. Suppose there are p inbred lines and it is desired to perform a diallel cross experiment involving $p(p-1)/2$ cross of type $(i \times j)$ for $i < j$, i, j = 1,2, ... , p. This is a type IV mating design of Griffing.

The problem of generating optimal mating designs for experiments with diallel crosses has been recently investigated by several authors [see *e.g.*, Gupta and Kageyama (1994), Dey and Midha (1996), Mukerjee (1997), Das, Dey and Dean (1998), Parsad, Gupta and Srivastava (1999), Chai and Mukerjee (1999)]. These authors used nested balanced incomplete block (NBIB) designs of Preece (1967) for this purpose.

Similar to block designs, experimental units in block design for a diallel cross experiments may be subject to trend-effect over space or time. So condition for a block design for a diallel cross experiments to be trend-free block design for a diallel cross experiment has been obtained. Further, a catalogue of trend-free block design for a diallel cross experiments will be prepared.

3.6.1 Necessary and Sufficient Condition

Let d be a block design for a diallel cross experiments of the type IV involving *p-inbred* lines, b blocks each of size k_1. This means that there are k_1 crosses and $2k_2$ lines, respectively in each block. It may be mentioned here that the designs for diallel crosses have two types of block sizes, $k_1^@$, the block sizes with respect to crosses and $k_2^@$, the block sizes with respect to the lines and $k_2^@ = 2k_1^@$. It, therefore, follows that the block designs for diallel crosses may also be viewed as nested block designs with sub blocks of size *2* each and the pair of treatments in each sub block form the crosses, the treatments being the lines. Further, let r_{dl} denote the number of times the l^{th} cross appears in d, $l = 1, 2,.., p(p-1)/2$ and similarly s_{di} denotes the number of times the i^{th} line occurs in the crosses in the whole design d, $i = 1,2,\ldots,p$. Then it is easy to see that $\sum_{l=1}^{p(p-1)/2} r_{dl} = \sum_{j=1}^{b} k_j = n$, the total number of observations, and $\sum_{i=1}^{p} s_{di} = 2\sum_{j=1}^{b} k_j$, (because in every cross there are two lines).

For the data obtained from the design d, we postulate the model

$$\mathbf{Y} = \mu \mathbf{1}_n + \Delta_1' \mathbf{g} + \Delta_2' \beta + \mathbf{e} \tag{3.16}$$

where $\mathbf{Y}$ is the $nx1$ vector of observed responses, μ is a general mean effect, $\mathbf{1}_n$ denotes an n - component column vector of all ones, $\mathbf{g}$ and $\boldsymbol{\beta}$ are vectors of p gca effects and b block effects, respectively. Δ_1' and Δ_2' are the corresponding $n \times p$ and $n \times b$ design matrices respectively, *i.e.*, the $(s, t)^{th}$ element of Δ_1' is *1* if the s^{th} observation pertains to the t^{th} line and is zero otherwise. Similarly $(s, t)^{th}$ element of Δ_2' is *1* if the s^{th} observation comes from the t^{th} block and is zero otherwise. $\mathbf{e}$ is the random error which follows a $N_n\left(\mathbf{0},\ \sigma^2 \mathbf{I}_n\right)$.

In the model (3.16) we have not included the specific combining ability effects. Under this model, it can be shown that the coefficient matrix for reduced normal equations for estimating linear functions of *gca* effects using a design d is

$$\mathbf{C_d} = \mathbf{G}_d - \mathbf{N_d}\, \mathbf{K}_d^{-1} \mathbf{N}_d'$$

where $\mathbf{G_d} = \Delta_1\Delta_1' = ((g_{dii'}))$, $\mathbf{N_d} = \Delta_1\Delta_2' = ((n_{dij}))$, $g_{dii} = s_{di}$ and for $i \neq i'$, $g_{dii'}$ is the number of times the cross $(i \, x \, i')$ appears in *d*; n_{dij} is the number of times line *i* occurs in the block *j* of *d*.

A design d is said to be connected if and only if Rank $(\mathbf{C}_d) = p-1$, or equivalently, if and only if all elementary contrasts among the *gca* effects are estimable using *d*. A connected design *d* is variance balanced if and only if all the diagonal elements of the matrix $\mathbf{C}_d$ are equal and all the off diagonal elements are also equal. In other words, the matrix $\mathbf{C}_d$ is completely symmetric.

For given positive integers *p, b, k, n,* $\mathbf{D}_0(p,b,k,n)$ will denote the class of all connected block designs *d* with *p* lines, *b* blocks each of size k and *n* experimental units. On the similar lines, as given in Chapter 2, the condition for a block design for a diallel cross experiment to be trend-free a block design for a diallel cross experiment is given below:

Theorem 3.1: A necessary and sufficient condition for a connected block design for diallel cross experiments to be trend-free block design for diallel cross experiments is $\Delta_1\mathbf{Z}' = \mathbf{0}$. (3.17)

Further, if the NBIB design with parameters $v = p$, b_1, k_1, $b_2 = b_1k_1 / 2$, $k_2 = 2$ is such that $\lambda_2 = 1$ or equivalently $b_1k_1 = p(p-1)$, then the optimal design d^* for diallel crosses derived from this design has each cross replicated just once and hence uses the minimal number of experimental units. Keeping in view the above, we can say that **t**he existence of a NBIB design *d* with parameters $v = p$, $b_1 = b$, $b_2 = bk$; $k_1 = 2k$, $k_2 = 2$ implies the existence of a universally optimal incomplete block design d^* for diallel crosses. Thus by using the families of NBIB designs that can be converted into trend-free NBIB designs (Table 3.1) and result in Theorem 3.1, a catalogue of optimal block designs for diallel cross experiments that can be converted into trend-free optimal block designs for diallel cross experiments with $p \leq 30$ is given in Table 3.4. First we consider an optimal block designs for diallel cross experiments and illustrate how the condition (3.17) holds good.

Example 3.2: An optimal design for diallel cross experiments with parameters *p=7, b=7, k =3* can be constructed into trend-free block design and is shown below:

[1x6, 2x5, 3x4]; [2x7,3x6, 4x5]; [1x3,4x7, 5x6]; [2x4, 1x5, 6x7];
[3x5, 2x6 1x7]; [4x6, 3x7, 1x2]; [5x7,1x4, 2x3].

We have design matrix and matrix of orthogonal polynomials for linear and quadratic trend as

Δ_1'						
1	**2**	**3**	**4**	**5**	**6**	**7**
1	0	0	0	0	1	0
0	1	0	0	1	0	0
0	0	1	1	0	0	0
0	1	0	0	0	0	1
0	0	1	0	0	1	0
0	0	0	1	1	0	0
1	0	1	0	0	0	0
0	0	0	1	0	0	1
0	0	0	0	1	1	0
0	1	0	1	0	0	0
1	0	0	0	1	0	0
0	0	0	0	0	1	1
0	0	1	0	1	0	0
0	1	0	0	0	1	0
1	0	0	0	0	0	1
0	0	0	1	0	1	0
0	0	1	0	0	0	1
1	1	0	0	0	0	0
0	0	0	0	1	0	1
1	0	0	1	0	0	0
0	1	1	0	0	0	0

$\mathbf{Z'}$	
L	**Q**
-1	1
0	-2
1	1
-1	1
0	-2
1	1
-1	1
0	-2
1	1
-1	1
0	-2
1	1
-1	1
0	-2
1	1
-1	1
0	-2
1	1
-1	1
0	-2
1	1

Here the condition for a block design for diallel cross experiments to be trend-free block design is $\Delta_1 \mathbf{Z'} = \mathbf{0}$ is satisfied.

3.7 Catalogue of trend-free Universally Optimal Binary Balanced Block Designs for Diallel Cross Experiments

A catalogue of universally optimal binary balanced block designs for diallel cross Experiments with p $\leq$ 30 that can be converted into trend-free optimal binary balanced block designs for diallel cross experiments is given in Table 3.4below:

Table 3.4: Universally Optimal Binary Balanced Block Designs for Diallel Cross Experiments that can be converted into trend-free block designs

Sr. No.	p	b	k	n	Method of construction
1	4	3	2	6	Series 2: Gupta and Kageyama (1994)
2[a,b]	5	5	2	10	Family 1 : Parsad, Gupta and Srivastava(1999)
3	5	10	2	20	Family 4 : Das, Dey and Dean (1998)
4	6	5	3	15	Series 2 : Gupta and Kageyama(1994)
5	6	30	2	60	Family 2 : Parsad, Gupta and Srivastava(1999)

Sr. No.	p	b	k	n	Method of construction
6[a,c]	7	7	3	21	Family 1 : Parsad, Gupta and Srivastava(1999)
7	8	7	4	28	Series 2 : Gupta and Kageyama(1994)
8	8	14	2	28	Family 3: Das, Dey and Dean (1998)
9	8	56	3	168	Family 2 : Parsad, Gupta and Srivastava(1999)
10[a]	9	9	4	36	Family 1 : Parsad, Gupta and Srivastava(1999)
11[b]	9	18	2	36	Family 1 : Parsad, Gupta and Srivastava(1999)
12[d]	9	36	2	72	Family 4 : Das, Dey and Dean (1998)
13	10	9	5	45	Series 2 : Gupta and Kageyama(1994)
14	10	90	4	360	Family 2 : Parsad, Gupta and Srivastava(1999)
15[a]	11	11	5	55	Family 1 : Parsad, Gupta and Srivastava(1999)
16[d]	11	55	2	110	Family 2 : Parsad, Gupta and Srivastava(1999)
17	12	11	6	66	Series 2 : Gupta and Kageyama(1994)
18	12	132	5	660	Family 2 : Parsad, Gupta and Srivastava(1999)
19[a]	13	13	6	78	Family 1 : Parsad, Gupta and Srivastava(1999)
20[c]	13	26	3	78	Family 1 : Parsad, Gupta and Srivastava(1999)
21[b]	13	39	2	78	Family 1 : Parsad, Gupta and Srivastava(1999)
22[d]	13	78	2	156	Family 4 : Das, Dey and Dean (1998)
23	13	143	6	858	Family 2 : Parsad, Gupta and Srivastava(1999)
24	14	13	7	91	Series 2 : Gupta and Kageyama(1994)
25	14	182	3	546	Family 2 : Parsad, Gupta and Srivastava(1999)
26	15	15	7	105	Series 1 : Gupta and Kageyama (1994)
27	15	105	2	210	Family 2: Parsad, Gupta and Srivastava (1999)
28	16	15	8	120	Series 2 : Gupta and Kageyama(1994)
29[a]	17	17	8	136	Family 1 : Parsad, Gupta and Srivastava(1999)
30	17	34	4	136	Family 1 : Parsad, Gupta and Srivastava(1999)
31[b]	17	68	2	136	Family 1 : Parsad, Gupta and Srivastava(1999)
32[d]	17	136	2	272	Family 4 : Das, Dey and Dean (1998)
33	18	17	9	153	Series 2 : Gupta and Kageyama(1994)
34	19	95	2	190	Family 3 : Das, Dey and Dean (1998)
35[d]	19	171	2	342	Family 2 : Parsad, Gupta and Srivastava(1999)
36	19	171	4	684	Family 2 : Parsad, Gupta and Srivastava(1999)
37	20	19	10	190	Series 2 : Gupta and Kageyama(1994)
38	20	95	2	190	Family 3: Das, Dey and Dean (1998)
39	20	380	2	760	Family 2 : Parsad, Gupta and Srivastava(1999)
40	21	21	10	210	Series 1 : Gupta and Kageyama (1994)
41	21	105	2	210	Family 2 : Parsad, Gupta and Srivastava(1999)
42	22	154	3	462	Family 2 : Parsad, Gupta and Srivastava(1999)
43[a]	23	23	11	253	Family 1 : Parsad, Gupta and Srivastava(1999)
44[d]	23	253	2	506	Family 4 : Das, Dey and Dean (1998)
45	24	23	12	276	Series 2 : Gupta and Kageyama(1994)
46[a]	25	25	12	300	Family 1 : Parsad, Gupta and Srivastava(1999)
47	25	50	6	300	Family 1 : Parsad, Gupta and Srivastava(1999)
48	25	75	4	300	Family 1 : Parsad, Gupta and Srivastava(1999)
49[c]	25	100	3	300	Family 1 : Parsad, Gupta and Srivastava(1999)
50[b]	25	150	2	300	Family 1 : Parsad, Gupta and Srivastava(1999)
51	25	225	4	900	Family 2 : Parsad, Gupta and Srivastava(1999)
52	25	300	2	600	Family 4 : Das, Dey and Dean (1998)
53	26	25	13	325	Series 2 : Gupta and Kageyama(1994)
54[a]	27	27	13	351	Family 1 : Parsad, Gupta and Srivastava(1999)
55[d]	27	351	2	702	Family 4 : Das, Dey and Dean (1998)

Sr. No.	p	b	k	n	Method of construction
56	28	27	14	378	Series 2 : Gupta and Kageyama(1994)
57	28	252	3	756	Family 2 : Parsad, Gupta and Srivastava(1999)
58[a]	29	29	14	406	Family 1 : Parsad, Gupta and Srivastava(1999)
59	29	58	7	406	Family 1 : Parsad, Gupta and Srivastava(1999)
60	29	203	2	406	Family 1 : Parsad, Gupta and Srivastava(1999)
61	29	406	2	812	Family 4 : Das, Dey and Dean (1998)
62	30	29	15	435	Series 2 : Gupta and Kageyama(1994)

[a] denotes that the design can also be obtained from Series 1: Gupta and Kageyama (1994)
[b] denotes that the design can also be obtained from Family 1: Das, Dey and Dean (1998)
[c] denotes that the design can also be obtained from Family 2: Das, Dey and Dean (1998)
[d] denotes that the design can also be obtained from Family 4: Das, Dey and Dean (1998)

Discussion

Trend-free block designs are quite useful in the experimental situations that may have trend effect in the experimental units. As mentioned earlier such situations may occur in Green house experiments where the source of heat is located on sides of the house and the experimental units (pots) are kept in lines; in poultry experiments where the source of heat is at the centre of the shed and chicks of early age are in the cages; in animal experiments where littermates (animals born in the same litter) are experimental units within a block *i.e.* litters are blocks. Other such experiments are orchard and vineyard experiments on undulating topography, experiments in which response variable of interest is affected by slowly migrating insects entering the area from one side, laboratory experiments where the responses to the experimental units may be affected within time periods by instrument drift or analyst fatigue, etc.

A question is generally raised on the utility of trend-free block designs due to lack of randomization. It is not exactly true although randomization is restricted to some extent. Now we illustrate two examples, one each for completely randomized design and balanced incomplete block design, to see that extent to which randomization is restricted in trend-free design?

Suppose we have 2 treatments, each replicated 4 times in an experimental design of 8 units and the treatments are allotted to plots in the following order

T_2	T_1	T_1	T_2	T_1	T_2	T_2	T_1

The same experimental material can be arranged in the following manner

T_1	T_2	T_1	T_2	T_2	T_1	T_2	T_1

These both the arrangements are trend-free for linear effect. Other arrangements can also be made by interchanging the positions of the treatments.

In block designs, if we have a trend-free block design, the randomization can be done among the blocks as well as within first block *i e.* if we change the position of treatments in the first block we get other arrangements by permuting the treatments in other blocks accordingly. Therefore, one can generate all possible layouts that are trend-free and select one among them randomly for experimentation.

Consider the example of trend-free block design, given in example 2.1 with parameters *$v = 5$, $b = 5$, $r = 4$, $k = 4$ and $\lambda = 3$ and blocks contents as:*

Block 1	4	2	3	1
Block 2	*3*	*1*	*4*	*5*
Block 3	*2*	*3*	*5*	*4*
Block 4	*5*	*4*	*1*	*2*
Block 5	*1*	*5*	*2*	*3*

Another trend-free design can easily be obtained by permuting the treatments within the first block and permute the treatments in other blocks accordingly. Hence the trend-free design is

Block 1	3	2	1	4
Block 2	*1*	*3*	*4*	*5*
Block 3	*5*	*4*	*3*	*2*
Block 4	*4*	*5*	*2*	*1*
Block 5	*2*	*1*	*5*	*3*

Similarly other trend-free designs can be constructed as discussed above.

It is of interest to highlight one point here that as we impose restriction on the designs, the randomization is restricted step by step. For example, in the designs with zero-way of elimination of heterogeneity, randomization is maximum; in the designs with one-way of elimination of heterogeneity, randomization is reduced; in the designs with two-way of elimination of heterogeneity, randomization is again reduced to the significant extent *e g.* in Latin square design, there is no randomization within the row or within the column. In this case the randomization is at the selection of complete Latin square design among the available Latin square designs.

Thus we see that as the randomization is restricted in Latin square design, the randomization is restricted in trend-free designs in the similar way.

Now the question arises, to get a layout for trend-free design. For this, the procedures given in the text can be utilized. So, one needs computation algorithm for constricting the trend-free/ linear trend-free designs from the available design with given parameters and layout. However, efforts in this direction have been initiated and will be taken up in future study. Regarding the developing the software, efforts have been started.

Chai (2002) has shown a very good application of trend-free block designs in bioassays. He established a link between trend-free block designs and block designs for parallel line assays. Using this relation he obtained necessary and sufficient condition for the existence of a ψ-design with unequal block sizes and gave a method of construction of ψ-designs. The application of the designs obtained in the present investigation in obtaining efficient designs for bioassays needs further investigation.

SUMMARY

The data generated from designed experiments are used to draw valid inferences about the population. Heterogeneity in the experimental material is the major source of variability to be reckoned within the statistical designing of scientific experiments. Occasionally, one can find a certain factor (called nuisance factor) that though not of interest to the experimenter, does contribute significantly to the variability in the experimental material. Various levels of this factor are used for blocking. In experimental situations with only one nuisance factor, block designs are used. These designs are useful in controlling the heterogeneity of the experimental units and it is ascribed to between blocks variability. Much of the literature is available on block designs viz. randomized complete block designs, balanced incomplete block designs, partially incomplete block designs, variance balanced block designs, etc. But many times experimental situations arise in which the response is dependent on the spatial or temporal position of the experimental units within a block and thus trend in the experimental units become another important nuisance factor. In such situations, a common polynomial trend of a specified degree over units within experimental units may be appropriately assumed. One way to account for the presence of trends is to use the analysis of covariance, treating trend values as covariates. However, one may think of suitable designs, in which treatment effects are orthogonal to trend effects, in the sense that analysis of the design could be done in usual manner, as if no trend effects were present. Such designs may be called trend-free designs. When experimental units within blocks, in block designs, exhibit a trend we use trend-free block designs. The meaning of trend-free block design is to assign treatments to plots within blocks so that the known properties of ordinary analysis of variance for treatment and block sum of squares are preserved and variation due to trend effect is removed from the error sum of square. Such an arrangement is called as ***trend-free block design***.

The work on trend-free block designs in proper block settings under homoscedastic model is available in the literature. There, however do occur experimental situations where block designs with unequal block sizes and/ or with unequal replications are to be used. For example, non-proper block design setting occurs while experimenting with natural blocks such as littermates (animal experiments), trusses per blossom (horticultural experiments), family sizes as blocks (psychological experiments), batches of test material (industrial experiments), etc. Experimenting on hilly areas, wastelands or salinity in field experiments may also force the experimenter to have blocks of unequal sizes. It is also known that in the class of binary block designs with unequal replications under non-proper settings, binary variance balanced block (BBB) designs are the most efficient designs for estimating all possible elementary contrasts among treatments. In variance balanced block designs, generally it is assumed that intra-block variances are constant. Through empirical investigations, however, it has been shown that intra-block variances are proportional to non-negative real power of block sizes.

However, the work on trend-free block designs for heteroscedastic model under non-proper block design settings and for nested balanced incomplete block designs could not be traced from the available literature. This investigation, therefore, deals with the trend-free block designs under heteroscedastic set up when intra-block variances are proportional to non-negative real power of block sizes. Further, there do occur experimental situations in which one or more factors are nested within the blocking factor. In such situations nested block designs and nested balanced incomplete block designs are quite useful. Such designs may also have trend-effect at sub block or block level. Similar to block designs, experimental units in block design for a diallel cross experiments may be subject to trend-effect over space or time. Thus, trend-free nested balanced incomplete block designs and trend-free block design for a diallel cross experiments have been studied. Many times it may not be possible to convert every block design to trend-free block design, we go for linear trend-free design because using linear trend-free designs eliminates much of the trend. Sometimes, it is not possible to make the design linear trend-free or trend-free and this provides a motivation to go for nearly linear trend-free designs. Thus nearly linear trend-free designs have also been investigated.

In Chapter I, various experimental situations have been described in which the trend may exist in non-proper block designs and NBIB designs. Some examples have been illustrated for better understanding of trend effect in complete randomized, randomized block and in factorial designs.

In Chapter II, a necessary and sufficient condition for a block design to be trend-free block design under heteroscedastic set up when intra-block variances are proportional to non-negative real power of block sizes is obtained. Using the condition catalogues of trend-free BBB designs of Type α, both under homoscedastic ($\alpha = 0$) and heteroscedastic model (for $\alpha = 1, 2, 3$), is prepared. Heteroscedasticity of the model increases as value of α increases. Catalogues of trend-free balanced incomplete block (BIB) designs with replications, $r \leq 20$ and two associate class partially balanced incomplete block (PBIB) designs are also prepared. Sometimes it may not be possible to convert every design to trend-free design then linear trend-free designs are given. Further, nearly linear trend-free designs are identified when it is not possible to obtain even linear trend-free designs.

Chapter III deals with nested balanced incomplete block (NBIB) designs and block design for a diallel cross experiments. A necessary and sufficient condition for a NBIB design to be trend-free block design at sub-block level is obtained. Catalogues of trend-free/ linear trend-free NBIB designs at sub-block levels, of NBIB designs given by given by Morgan, Preece and Rees (2001), are prepared. NBIB designs with sub-block size 2 and designs for complete diallel cross experiments have a one-to-one correspondence. Utilizing this relationship a catalogue of trend-free optimal block designs for a diallel cross experiments with number of inbred lines, $p \leq 30$ is prepared.

सार

अभिकल्पित परीक्षणों से उत्पन्न आंकडे समष्टि (population) के बारे में वैद्य निष्कर्ष निकालने के लिए उपयोग में लाये जाते हैं । वैज्ञानिक परीक्षणों की सांख्यिकीय अभिकल्पनाओं द्वारा सामग्री की विषमांगीयता (heterogeneity) का निर्धारण किया जाता है । कभी–कभी कुछ घटक परीक्षण ईकाईयों में विषमांगीयता के कारक होते हैं । इनमें परीक्षणकर्ताओं की कोई रुचि नहीं होती हैए इन्हें न्यूसेंस घटक (nuisance factors) कहा जाता है । इस घटक के विभिन्न स्तरों के आधार पर परीक्षणात्मक ईकाईयों के समूह (ब्लॉक) बनाये जाते हैं । केवल एक न्यूसेंस घटक वाली प्रयोगात्मक परिस्थितियों में ब्लॉक अभिकल्पनायें प्रयोग की जाती हैं । ये अभिकल्पनायें परीक्षणात्मक इकाइयों में विषमांगीयता रोकने में प्रयोग होते हैं तथा इनको ब्लॉक के बीच के प्रसरण के रुप में जाना जाता है । ब्लॉक अभिकल्पनाओं जैस कि यादृाच्छिक पूर्ण ब्लॉक अभिकल्पनाओं (randomized complete block designs)] संतुलित अपूर्ण ब्लॉक अभिकल्पनाओं (balanced incomplete block designs)] आंशिक संतुलित अपूर्ण ब्लॉक अभिकल्पनाओं (partially balanced incomplete block designs), इत्यादि पर बहुत साहित्य उपलब्ध है । परन्तु बहुत बार ऐसी परीक्षणात्मक परिस्थितियाँ उत्पन्न होती हैं जिसमें अनुकिया (response) परीक्षणात्मक इकाई की ब्लॉक में स्पेशियल (spatial) या सामयिक (temporal) स्थिति पर निर्भर करता है तथा जिससे प्रवृति परीक्षणत्मक इकाई एक अन्य न्यूसेंस घटक हो जाता है । इन परिस्थितियों में, एक स्पेसिफाइड़ (specified) घात वाली बहुपदी प्रवृति (polynomial trend) मानी जा सकती है । प्रवृति से मुक्ति पाने का एक तरीका सह प्रसरण विश्लेषण (analysis of covariance) भी है जिसमें प्रवृति के मान को सहचर (covariate) माने । फिर भी इसके लिए, ऐसी उपयुक्त अभिकल्पना सोची जा सकती है जिसमें ट्रीटमेंटस के प्रभाव, प्रवृति प्रभाव के लाम्बिक (orthogonal) हो तथा आंकडों का विशलेषण बिल्कुल उसी तरह किया जा सकता हो जैसे कोई प्रवृति प्रभाव नहीं है । इन अभिकल्पनाओं को प्रवृति मुक्त ब्लॉक अभिकल्पनायें कहते हैं ।

जब ब्लॉक अभिकल्पनाओं में ब्लॉकों के अन्दर परीक्षणात्मक इकाइयों में प्रवृति होती है, हम प्रवृति–मुक्त ब्लॉक अभिकल्पनायें प्रयोग में लाते हैं । प्रवृति–मुक्त अभिकल्पना का तात्पर्य है कि ब्लॉक अभिकल्पना में ट्रीटमेंटस (treatments) ब्लॉक के अन्दर के प्लाटों पर इस प्रकार संजोये जायें कि प्रसरण विश्लेषण (analysis of variance) में ट्रीटमेंटस तथा ब्लॉक के वर्गों का जोड़ वही रहता है जैसे कोई प्रवृति प्रभाव नहीं है तथा प्रवृति प्रभाव को सीधे त्राुटि के वर्गों के जोड़ (error sum of square) में

घटा देंते है । इस प्रकार संजोयी गयी अभिकल्पना को प्रवृति–मुक्त ब्लॉक अभिकल्पना कहते हैं ।

प्रवृति मुक्त ब्लॉक अभिकल्पनाओं में अधिकतर शोधकार्य समान आकार वाले ब्लॉकों की प्रयोगात्मक परिस्थितियों के लिए हुआ है । परन्तु ऐसी प्रयोगात्मक परिस्थितियां उत्पन्न होती हैं जिनमें असमान आकार के ब्लॉक तथा/या ट्रीटमेंटस की असमान पुनरावृति {रिपलिकेशन्स}, प्रयोग में लाये जाते हैं । उदाहरण के लिये, असमान ब्लॉक अभिकल्पनायें तब होती हैं जबकि परीक्षण प्राकृतिक ब्लॉक में जैसे कि लिटरमेटस {पशु परीक्षण} एक टूसस ;जतनेमेद्ध में बौर खिलना {बागवानी परीक्षण}, परिवार का आकार {मनोवैज्ञानिक परीक्षण}, गुणवता के जांच के लिए उत्पाद का समूह (batches of test material) {औद्योगिक परीक्षण}, इत्यादि होते हैं । पहाड़ी क्षेत्राों में, वेस्टलैंड़ ॅंजमसंदकद्ध में या लवणता वाले खेतों में, असमान आकार के ब्लॉक इस्तेमाल करने पड़ते हैं । यह विदित है कि द्विआधारी ब्लॉक अभिकल्पनायें (binary block designs सभी के ऐलीमेप्ट्री ट्रीटमेंटस कन्ट्रास्टस (elementary treatments contrasts) का आकलन करने के लिये कम्पीटिंग क्लास की अभिकल्पनाओं में सर्वाधिक दक्ष अभिकल्पनायें हैं । द्विआधारी प्रसरण संतुलित खण्ड अभिकल्पनाओं में आमतौर पर अन्तः ब्लॉक प्रसरण (intra block variance) स्थिर माना जाता है । परन्तु एम्पीरिकल (empirical) परीक्षणों से यह दिखाया गया है कि अन्तः ब्लॉक प्रसरण ब्लॉक के आकारों के नॉन–नेगेटिव (non-negative) वास्तविक घात के प्रति अनुपातिक हैं । ऐसा प्रतीत होता है कि नॉन–प्रापर (non-proper) ब्लॉक अभिकल्पना सैटिंगस के हेट्रोसिडास्टिक (heteroscadastic) माडल तथा नेस्टड सन्तुलित अपूर्ण ब्लॉक (NBIB) अभिकल्पनाओं के लिये प्रवति–मुक्त ब्लॉक अभिकल्पनाओं पर कोई शोध कार्य नहीं हुआ है । अतः यह अन्वेषण विषमांगी सेट अप में, जब अन्तः ब्लॉक प्रसरण ब्लॉक आकारों की नॉन–नेगेटिव वास्तविक घात के प्रति अनुपातिक है, के लिये किया गया है । इसके अतिरिक्त, इस प्रकार की परीक्षणात्मक परिस्थितियां होती हैं जिसमं एक या अधिक घटकों बड़े ब्लाकिंग घटक में नेस्टड होते हैं । इन परिस्थितियों में नेस्टड ब्लॉक अभिकल्पनायें तथा नेस्टड संतुलित अपूर्ण ब्लॉक (NBIB) अभिकल्पनायें बहुत उपयोगी होती हैं । इन अभिकल्पनाओं में भी उप–ब्लॉक या ब्लॉक स्तर पर प्रवृति प्रभाव हो सकता है । ब्लॉक अभिकल्पपनाओं की तरह डायलल क्रास की ब्लॉक अभिकल्पनाओं की परीक्षणात्मक इकाइयों में, स्पेस (space) या समय के होते, प्रवृति प्रभाव हो सकता है । अतः प्रवृति–मुक्त नेस्टड संतुलित अपूर्ण ब्लॉक अभिकल्पनाओं तथा प्रवृति–मुक्त डायलल क्रास की ब्लॉक अभिकल्पनाओं का अध्ययन किया है । सभी ब्लॉक अभिकल्पनाओं को प्रवृति–मुक्त ब्लॉक अभिकल्पना में बदलना सम्भव नहीं होता है, तब हम रैखिक प्रवृति–मुकत अभिकल्पना को ढूंढते हैं

क्योंकि रैखिक–प्रवृति अभिकल्पनायें ज्यादातर प्रवृति को विलुप्त कर देती हैं । कभी–2 अभिकल्पना को रैखिक प्रवृति–मुक्त तथा प्रवृति–मुक्त अभिकल्पना में बदलना संभव नहीं होता है तब निकट रैखिक प्रवृति–मुक्त अभिकल्पना की ओर जाने के लिये प्रेरित होते हैं । अतः निकट रैखिक प्रवृति–मुक्त अभिकल्पनाओं की पहचानी गयी हैं ।

अध्याय-I में उन परीक्षणात्मक परिस्थितियों का नॉन–प्रापर (non-proper) ब्लॉक अभिकल्पनाओं तथा नेस्टड संतुलित अपूर्ण ब्लॉक (NBIB) अभिकल्पनाओं का वर्णन किया है जिनमें प्रवृति हो सकती है । प्रवृति के प्रभाव को अच्छी तरह समझने के लिये पूर्ण यादृच्छिक (complete randomized), यादृच्छिक ब्लॉक (randomized block) तथा बहुपादातीय (factorial) अभिकल्पनाओं के कुछ उदाहरण दिये गये हैं ।

अध्याय.II में एक ब्लॉक अभिकल्पना के विषमांगीय सैटअप के तहत, जब अन्तःब्लॉक प्रसरण ब्लॉक आकारों की नॉन–नेगेटिव वास्तविक घात के प्रति आनुपातिक हैं, तब प्रवृति–मुक्त ब्लाक अभिकल्पना होने के लिये अपेक्षित एवं पर्याप्त प्रतिबंध (necessary and sufficient condition) प्राप्त की गई है । इस प्रतिबंध का उपयोग करते हुए, समांगीक (α = 0) एवं विषमांगीक मॉडल ;α त्र 1, 2, 3 के लिए) दोनों के तहत टाईप α की प्रवृति–मुक्त ठठठ अभिकल्पनाओं का कैटालॉग तैयार किया गया। जैसे–जैसे α के मान में वृद्धि होती है, वैसे–वैसे मॉडल की विषमांगीयता (हिट्रोसिडाटिसिटी) में वृद्धि होती है । प्रवृति–मुक्त संतुलित अपूर्ण ब्लॉक (BIB) अभिकल्पनाओं के जिनके रेपलिकेशन्स $r \le 20$ हैं, तथा दो साहचर्य आंशिक संतुलित अपूर्ण ब्लॉक (2-associate PBIB) अभिकल्पनाओं के कैटालॉग भी तैयार किए गए । कभी–कभी सभी अभिकल्पनाओं को प्रवृति–मुक्त अभिकल्पना में परिवर्तित करना सम्भव नहीं होता है, ऐसी स्थिति में रैखिक प्रवृति–मुक्त अभिकल्पनाएँ दी गई हैं । इसके अतिरिक्त, जब रैखिक प्रवृति–मुक्त अभिकल्पनाएँ भी प्राप्त करना सम्भव नहीं होता तब निकट रैखिक प्रवृति–मुक्त अभिकल्पनाओं की पहचान की गई है ।

नेस्टड संतुलित अपूर्ण ब्लॉक (NBIB) अभिकल्पनाओं तथा डायलल कास की ब्लॉक अभिकल्पनाओं को अध्याय-III में दर्शाया है । एक नेस्टेड संतुलित अपूर्ण ब्लॉक (NBIB) अभिकल्पना को उप–ब्लॉक स्तर पर प्रवृति–मुक्त नेस्टेड संतुलित अपूर्ण ब्लॉक अभिकल्पना बनाने के लिये प्रतिबंध प्राप्त किया गया । नेस्टड संतुलित अपूर्ण ब्लॉक ;छठप्ठद्ध अभिकल्पनायें, जो कि मोर्गन, प्रीस एवं रीस {2000} नें दी हैं, के लिये उप ब्लॉक स्तर पर प्रवृति–मुक्त/रैखिक प्रवृति–मुक्त नेस्टड संतुलित

अपूर्ण ब्लॉक (NBIB) अभिकल्पनाओं के लिए कैटालॉग तैयार किये । नेस्टेड संतुलित अपूर्ण ब्लॉक अभिकल्पनाओं (NBIB), जिनका उप–ब्लॉक आकार दो (2) है, का पूर्ण डायलल क्रास परीक्षणों की अभिकल्पनाओं से सीधा संबंध है । इस संबंध का उपयोग करते हुए, अंतःप्रजात क्रम (इनब्रैड लाइन्स), $p \leq 30$ के डायलल क्रॉस परीक्षणों के लिए प्रवृति–मुक्त इष्टतम (ऑप्टीमल) ब्लॉक अभिकल्पनाओं का कैटालॉग तैयार किया गया है ।

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