NESTED BLOCK DESIGNS FOR COMPARING TEST TREATMENTS WITH A CONTROL

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ABSTRACT: Nested balanced treatment incomplete block (NBTIB) designs are introduced for comparing test treatments with a control treatment. Several methods of constructing such designs are presented. The methods of construction use balanced treatment incomplete block (BTIB) designs and nested balanced incomplete block (NBIB) designs. A catalogue is included of efficient NBTIB designs with $v \leq 16$ and $r \leq 30$.

Key Words and Phrases: Nested balanced incomplete block design, nested partially balanced incomplete block design, nested balanced treatment incomplete block design, balanced treatment incomplete block design.

1. Introduction

A nested block design is a block design with two systems of blocks in which the second system of blocks is nested within the first system. These designs are useful for experimental situations in which a nuisance factor is nested within a blocking factor. For example, consider a field experiment on some crop conducted using a block design in which harvesting is done block-wise. The harvested samples are to be analyzed for their contents on quality indicators such as protein content, etc. in the laboratory by different technicians at the same time or by a technician over different periods of time. The variation arising due to technicians or due to different time periods within each block may be controlled by another system of blocks called sub-blocks that are nested within blocks. For such situations, Preece [18] introduced nested balanced incomplete block (NBIB) designs. Jimbo and Kuriki [10]; Dey, Das and Banerjee [4]; Parsad, Gupta and Srivastava [16] and Morgan, Preece and Rees [14] gave several methods of construction of NBIB designs. Morgan, Preece and Rees [14] also presented an exhaustive catalogue of NBIB designs with $v \leq 16$ and $r \leq 30$. NBIB designs are variance balanced in the sense that each elementary treatment contrast is estimated with the same variance. A NBIB design may not exist for a particular parameter set; even if it exists, it may require a large number of replications - which the experimenter may not be able to afford. To deal with such situations, Homel and Robinson [8] introduced nested partially balanced incomplete block (NPBIB) designs. Several methods of construction of NPBIB designs are available in the literature {see e.g.

Banerjee and Kageyama [1,2]; Kageyama, Philip and Banerjee [11]; Philip, Kageyama and Banerjee [17]; Saha, Dey and Midha [19] and Satpati and Parsad [20]). Satpati and Parsad [20] presented catalogues of two and three associate class NPBIB designs for $v \leq 30$ and $r \leq 15$. The nested block (NBIB and NPBIB) designs are useful for experimental situations where the experimenter is interested in making all possible paired comparisons with as high a precision as possible. However, there do occur experimental situations where the experimenter is interested in comparing several new treatments (called test treatments) with existing practice (a control treatment) with high precision and the comparisons among the test treatments are not of much importance. In the general block design setting, a lot of literature is available for obtaining efficient designs for such experimental situations; details were provided by Bechhofer and Tamhane [3]; Hedayat, Jacroux and Majumdar [6]; Parsad, Gupta and Prasad [15]; Majumdar [12] and Gupta and Parsad [5]. No work seems to have been done for obtaining nested block designs for making comparisons between test treatments and a control treatment. Therefore, in this investigation we deal with the combinatorial aspects of nested block designs for making comparisons between test treatments and a control.

It is well known that for a nested block design set up, the coefficient matrix of reduced normal equations for estimating the linear functions of treatment effects is the same as that obtained if the blocks are ignored in the analysis. For more details on this one may refer to Morgan [14] and Satpati and Parsad [20]. The properties of the coefficient matrix of reduced normal equations are completely determined by the treatments *vs* sub-blocks incidence matrix. From this, it follows that the arrangement of treatments in blocks is of no consequence. Therefore, a nested balanced treatment sub-bock (NBTSB) design that estimates all test treatments *vs* control treatment contrasts with the same variance can always be constructed if there exists a balanced treatment block (BTB) design of Jacroux and Majumdar [9]. To be clearer, consider a BTB design in v tests and a control arranged in $b_2 = qb_1$ blocks of size k_2 each. Let each of the test treatments be replicated r times and the control treatment be replicated r_0 times. Regroup b_2 blocks in b_1 sets such that there are q blocks of the BTB design in each set. Take the sets as blocks and the blocks of the original design as sub-blocks. The above procedure yields a NBTSB design in which v test treatments and a control treatment are arranged in b_1 blocks of size k_1 each, there being q sub-blocks of size $k_2 = k_1/q$ within each block. The other parameters of the NBTSB design are r, r_0, $\lambda_2 = \sum_{j'=1}^{b_2} n_{2ij'} n_{2i'j'}$; $\forall i \neq i' = 1, 2, \ldots, v$ and $\lambda_{20} = \sum_{j'=1}^{b_2} n_{20j'} n_{2ij'}$ $\forall i = 1, 2, \ldots, v$, where $n_{tj'}$ is the number of times treatment t occurs in sub-block j'; $t = 0, 1, \ldots, v$; $j' = 1, 2, \ldots, b_2$. However, in this arrangement, the characterization of the coefficient matrix of the reduced normal equations for estimating treatment effects using the block classification ignoring sub-blocks is of no consequence. The property of variance balance may also be desirable on the block classification ignoring sub-blocks, particularly when inference is required on the characters that are observed on the blocks. More details of such experimental situations were given in Satpati and Parsad [20]. Therefore, in this investigation we concentrate on combinatorial aspects of nested block designs in which the block classification ignoring sub-blocks leaves a BTB design and the sub-block classification also forms a BTB design. Such designs have been termed nested balanced treatment block (NBTB) designs. An NBTB design will be called a nested balanced treatment incomplete block (NBTIB) design if block classifications as well as sub-block classifications ignoring the other classification give a balanced treatment incomplete block (BTIB) design. The parameters of the NBTB designs will be denoted by v, b_1, b_2, r, r_0, k_1, k_2, λ_1, λ_{10}, λ_2, λ_{20}, where r and r_0 are respectively the replications of the test treatments and control treatment and $\lambda_1 = \sum_{j=1}^{b_1} n_{2ij} n_{2i'j}$; $\forall i \neq i' = 1, 2, \ldots, v$ and $\lambda_{10} = \sum_{j=1}^{b_1} n_{20j} n_{2ij}$ $\forall i = 1, 2, \ldots, v$, where n_{tj} is the number of times treatment t occurs in block j; $t = 0, 1, \ldots, v$; $j = 1, 2, \ldots, b_1$.

Some methods of construction of NBTB designs are given in Section 2. Once the designs are obtained, the next question arises "How efficient are these designs with respect to treatment contrasts of interest?". For studying the efficiencies of these designs, we make use of the results of A-optimality of general block designs for comparing test treatments with a control. The block [sub-block] classification is ignored for studying the efficiency of the design with the sub-block [block] classification. For this purpose, we consider D(v, b_1, b_2, k_1, k_2) as the class of all connected nested block designs in which v tests and a control are arranged in b_1 blocks of size k_1 each, there being k_1 / k_2 sub-blocks of size k_2 nested within each block. We make use of the sufficient condition for establishing the A-optimality of BTB designs obtained by Jacroux and Majumdar [9]. The sufficient condition gives the lower bound to the trace of the variance-covariance matrix of all the test treatments *vs* control treatment contrasts. A design that attains the lower bound is termed A-optimal. The condition is given in result 1.1.

Result 1.1: An NBTB design is A-optimal in the class of all designs with the same values of v, b_1, b_2, k_1, k_2 if

$$g(t_h, s_h) = \min\{g(x_h, z_h); (x_h, z_h) \in \Delta_h\} \quad \forall\, h = 1, 2 \tag{1.1}$$

where $\Delta_h = \{(x_h, z_h): x_h = 0, 1, \ldots, \text{int}[k_h/2] - 1;\ z_h = 0, 1, \ldots, b_h$ with $z_h > 0$, when $x_h = 0\}$ $\forall\, h = 1, 2$, and where

$$g(x_h, z_h) = v\,A(x_h, z_h) + v(v-1)^2/B(x_h, z_h),$$

with $A(x_h, z_h) = (k_h e_h - g_h)/k_h$; $B(x_h, z_h) = (c_h - p_h e_h + g_h)/k_h$ $\forall\, h = 1, 2$ and $e_h = b_h x_h + z_h$; $g_h = b_h x_h^2 + 2x_h z_h + z_h$; $c_h = v b_h k_h(k_h - 1) + v b_h V_h(v - 2k_h + vV_h)$; $p_h = v(k_h - 1) + k_h - 2vV_h$; $V_h = \text{int}[(b_h k_h)/v b_h]$. Here int[.] denotes the greatest integer function.

It is not possible to give a general method of construction which yields an A-optimal design in D (v, b_1, b_2, k_1, k_2) by satisfying the condition given in (1.1). Hence, we adopt the indirect approach of using the A-efficiency criterion, considered by Stufken [21], to obtain A-optimal NBTB designs. The A-efficiency is the ratio of the A-value of a hypothetical A-optimal design whose criterion value given in (1.1) is minimum for making test treatments-control treatment comparisons in a given class of designs, to the A-value of the design whose A-efficiency is to be obtained in the same class of designs. Here, A-value is the trace of the variance-covariance matrix of the estimated treatment contrasts of interest. A-efficiencies for the block designs are obtained by taking $h = 1$ in result (1.1) and by taking $h = 2$ to get the same for the sub-block designs. Further, a design {either block or sub-block design or both} is A-optimal if the A-efficiency is 1.0000. A-efficiencies of the designs for both block and sub-block structures obtained through the methods of construction given in Section 2 are computed and presented in the catalogues of NBTB designs with $v \le 16$ and $r \le 30$ in the appendix. In these catalogues, E_1 [E_2] denotes the A-efficiencies of the block [sub-block] design ignoring the other classification. It may be noted here that the term A-efficiency used here is different from the efficiency factor. The efficiency factor is computed as the ratio of the A-value of the complete block design with the same (v, r) to that of the design under consideration.

2. Methods of construction of NBTB designs

In this section, we give some methods of construction of NBTB designs based on BTIB designs, NBIB designs and initial block solutions. In most of these methods, sub-blocks give a BTIB design. The block classification ignoring sub-blocks may be a BTIB design or a BTB design.

Method 2.1: Let there exist an NBIB design with parameters v', b_1', b_2', r', k_1', k_2', λ_1', λ_2' such that $k_1'/k_2' = q$. Adding the control treatment once to each of the sub-blocks of the NBIB design, we get an NBTIB design with parameters

$v = v'$, $b_1 = b_1'$, $b_2 = b_2'$, $r = r'$, $r_0 = b_2'$, $k_1 = k_1' + q$, $k_2 = k_2' + 1$, $\lambda_1 = \lambda_1'$, $\lambda_{10} = qr$, $\lambda_2 = \lambda_2'$, $\lambda_{20} = r$.

Example 2.1: An NBIB design with parameters $v = 7$, $b_1 = 7$, $b_2 = 14$, $r = 6$, $k_1 = 6$, $k_2 = 3$, $\lambda_1 = 5$, $\lambda_2 = 2$, $q = 2$ exists and is obtained by developing the initial block

[(1, 2, 4); (6, 5, 3)] mod 7.

On adding control treatment 0 once to each of the sub-blocks, we get an NBTIB design with parameters $v = 7$, $b_1 = 7$, $b_2 = 14$, $r = 6$, $r_0 = 14$, $k_1 = 8$, $k_2 = 4$, $\lambda_1 = 5$, $\lambda_{10} = 12$, $\lambda_2 = 2$, $\lambda_{20} = 6$. The design is A-optimal for both block and sub-block structures.

A total of 68 NBTIB designs with $v \le 16$ and $r \le 30$ obtainable from Method 2.1 along with their A-efficiencies are given in Table 1. All the designs in Table 1 have A-efficiencies greater than 0.9000 for both the block and sub-block designs. 9 designs are A-optimal for both the block structures. The total number of A-optimal designs with sub-block structure is 14. 13 sub-block designs and 17 block designs have A-efficiencies greater than 0.9900, 23 sub-block designs and 19 block designs have A-efficiencies greater than 0.9500 but less than or equal to 0.9900 and 18 sub-block designs and 23 block designs have A-efficiencies greater than 0.9000 but less than or equal to 0.9500.

Method 2.2: Suppose there exists a BTIB design with parameters v', b', r', r_0', k', λ', λ_0', where the symbols have their usual meaning. For details on BTIB designs one may refer to Majumdar [12] and Gupta and Parsad [5]. Let there also exist an NB(I)B design with parameters k', b_1^*, b_2^*, r^*, k_1^*, k_2^*, λ_1^*, λ_2^*. Then writing each of the block contents of the BTIB design as an NB(I)B design, we get an NBTIB design with parameters $v = v'$, $b_1 = b'b_1^*$, $b_2 = b'b_2^*$, $r = r'r^*$, $r_0 = r_0'r^*$, $k_1 = k_1^*$, $k_2 = k_2^*$, $\lambda_1 = \lambda'\lambda_1^*$, $\lambda_{10} = \lambda_0'\lambda_1^*$, $\lambda_2 = \lambda'\lambda_2^*$, $\lambda_{20} = \lambda_0'\lambda_2^*$.

Example 2.2: Consider a BTIB design with parameters $v' = 9$, $b' = 12$, $r' = 4$, $r_0' = 12$, $k' = 4$, $\lambda' = 1$, $\lambda_0' = 4$ with block contents (column wise) as

1	2	3	1	4	7	1	2	3	3	2	1
4	5	6	2	5	8	5	6	4	5	4	6
7	6	9	3	6	9	9	7	8	7	9	8
0	0	0	0	0	0	0	0	0	0	0	0

There also exists an NBB design with parameters $v^* = k' = 4$, $b_1^* = 3$, $b_2^* = 6$, $r^* = 3$, $k_1^* = 4$, $k_2^* = 2$, $\lambda_1^* = 3$, $\lambda_2^* = 1$ with block contents as

[(A, B); (C, D)]; [(A, C); (B, D)]; [(A, D); (B, C)]

Then following the procedure of Method 2.2, one gets an NBTIB design with parameters $v = 9$, $b_1 = 36$, $b_2 = 72$, $r = 12$, $r_0 = 36$, $k_1 = 4$, $k_2 = 2$, $\lambda_1 = 3$, $\lambda_{10} = 12$, $\lambda_2 = 1$, $\lambda_{20} = 4$ with A-efficiencies 1.0000 and 0.9999 for block and sub-block design, respectively.

This is a fairly general method of construction and the existence of any BTIB design and an NBIB design satisfying the conditions mentioned in Method 2.2, implies the existence of an NBTIB design.

Using NBIB designs of different parametric combinations, we get the following families of NBTIB designs.

Family 2.2.1: Existence of a BTIB design with parameters v', b', r', r_0', $k' = 2t + 1$, λ', λ_0' and an NBIB design with parameters $v = 2t + 1 = b_1$, $b_2 = t(2t + 1)$, $r = 2t$, $k_1 = 2t$, $k_2 = 2$, $\lambda_1 = 2t - 1$, $\lambda_2 = 1$ implies the existence of an NBTIB design with parameters $v = v'$, $b_1 = b'(2t+1)$, $b_2 = tb'(2t+1)$, $r = 2r't$, $r_0 = 2tr_0'$, $k_1 = 2t$, $k_2 = 2$, $\lambda_1 = \lambda'(2t - 1)$, $\lambda_{10} = (2t - 1)\lambda_0'$, $\lambda_2 = \lambda'$, $\lambda_{20} = \lambda_0'$.

Family 2.2.2: Existence of a BTIB design with parameters v', b', r', r_0', $k' = mt + 1$, λ', λ_0' and an NBIB design with parameters $v = mt + 1$, $b_1 = t(mt + 1)$, $b_2 = ut(mt - 1)$, $r = mt$, $k_1 = m = 2u$, $k_2 = 2$, $\lambda_1 = m - 1$, $\lambda_2 = 1$ implies the existence of an NBTIB design with parameters $v = v'$, $b_1 = tb'(mt + 1)$, $b_2 = utb'(mt + 1)/2$, $r = mr't$, $r_0 = mtr_0'$, $k_1 = m$, $k_2 = 2$, $\lambda_1 = \lambda'(m - 1)$, $\lambda_{10} = (m - 1)\lambda_0'$, $\lambda_2 = \lambda'$, $\lambda_{20} = \lambda_0'$

Family 2.2.3: Existence of a BTIB design with parameters v', b', r', r_0', $k' = 12t - 8$, λ', λ_0' and an NBIB design with parameters $v = 12t + 8$, $b_1 = (3t + 2)(12t + 7)$, $b_2 = 2(3t + 2)(12t + 7)$, $r = 12t + 7$, $k_1 = 4$, $k_2 = 2$, $\lambda_1 = 3$, $\lambda_2 = 1$ implies the existence of an NBTIB design with parameters $v = v'$, $b_1 = b'(3t + 2)(12t + 7)$, $b_2 = 2b'(3t + 2)(12t + 7)$, $r = (12t + 7)r'$, $r_0 = (12t + 7)r_0'$, $k_1 = 4$, $k_2 = 2$, $\lambda_1 = 3\lambda'$, $\lambda_{10} = 3\lambda_0'$, $\lambda_2 = \lambda'$, $\lambda_{20} = \lambda_0'$.

Family 2.2.4: Existence of a BTIB design with parameters v', b', r', r_0', $k' = 2t - 1$, λ', λ_0' and an NBIB design with parameters $v = 2t + 1$, $b_1 = t(2t + 1)$, $b_2 = 2t(2t + 1)$, $r = 4t$, $k_1 = 4$, $k_2 = 2$, $\lambda_1 = 6$, $\lambda_2 = 2$ implies the existence of an NBTIB design with parameters $v = v'$, $b_1 = tb'(2t + 1)$, $b_2 = 2tb'(2t + 1)$, $r = 4r't$, $r_0 = 4tr_0'$, $k_1 = 4$, $k_2 = 2$, $\lambda_1 = 6\lambda'$, $\lambda_{10} = 6\lambda_0'$, $\lambda_2 = 2\lambda'$, $\lambda_{20} = 2\lambda_0'$.

Family 2.2.5: Existence of a BTIB design with parameters v', b', r', r_0', $k' = 4t + 1$, λ', λ_0' and an NBIB design with parameters $v = 4t + 1$, $b_1 = 4t + 1$, $b_2 = 2(4t + 1)$, $r = 4t$, $k_1 = 4t$, $k_2 = 2t$, $\lambda_1 = 4t - 1$, $\lambda_2 = 2t - 1$ implies the existence of

an NBTIB design with parameters $v = v'$, $b_1 = b'(4t+1)$, $b_2 = 2b'(4t + 1)$, $r = 4r't$, $r_0 = 4tr_0'$, $k_1 = 4t$, $k_2 = 2t$, $\lambda_1 = \lambda'(4t - 1)$, $\lambda_{10} = (4t - 1)\lambda_0'$, $\lambda_2 = (2t - 1)\lambda'$, $\lambda_{20} = (2t - 1)\lambda_0'$.

Family 2.2.6: Existence of a BTIB design with parameters v', b', r', r_0', $k' = 6t + 1$, λ', λ_0' and an NBIB design with parameters $v = 6t+1$, $b_1 = t(6t + 1)$, $b_2 = 2t(6t + 1)$, $r = 6t$, $k_1 = 6$, $k_2 = 3$, $\lambda_1 = 5$, $\lambda_2 = 2$ implies the existence of an NBTIB design with parameters $v = v'$, $b_1 = tb'(6t + 1)$, $b_2 = 2tb'(6t + 1)$, $r = 6r't$, $r_0 = 6tr_0'$, $k_1 = 6$, $k_2 = 3$, $\lambda_1 = 5\lambda'$, $\lambda_{10} = 5\lambda_0'$, $\lambda_2 = 2\lambda'$, $\lambda_{20} = 2\lambda_0'$.

Family 2.2.7: Existence of a BTIB design with parameters v', b', r', r_0', $k' = 9t + 1$, λ', λ_0' and an NBIB design with parameters $v = 9t + 1$, $b_1 = t(9t + 1)$, $b_2 = 3t(9t + 1)$, $r = 9t$, $k_1 = 9$, $k_2 = 3$, $\lambda_1 = 8$, $\lambda_2 = 2$ implies the existence of an NBTIB design with parameters $v = v'$, $b_1 = tb'(9t + 1)$, $b_2 = 3tb'(6t + 1)$, $r = 9r't$, $r_0 = 9tr_0'$, $k_1 = 9$, $k_2 = 3$, $\lambda_1 = 8\lambda'$, $\lambda_{10} = 8\lambda_0'$, $\lambda_2 = 2\lambda'$, $\lambda_{20} = 2\lambda_0'$.

Family 2.2.8: Existence of a BTIB design with parameters v', b', r', r_0', $k' = 12t + 1$, λ', λ_0' and an NBIB design with parameters $v = 12t + 1$, $b_1 = t(12t + 1)$, $b_2 = 4t(12t + 1)$, $r = 12t$, $k_1 = 12$, $k_2 = 3$, $\lambda_1 = 11$, $\lambda_2 = 2$ implies the existence of an NBTIB design with parameters $v = v'$, $b_1 = tb'(12t + 1)$, $b_2 = 4tb'(12t + 1)$, $r = 12r't$, $r_0 = 12tr_0'$, $k_1 = 12$, $k_2 = 3$, $\lambda_1 = 11\lambda'$, $\lambda_{10} = 11\lambda_0'$, $\lambda_2 = 2\lambda'$, $\lambda_{20} = 2\lambda_0'$.

Family 2.2.9: Existence of a BTIB design with parameters v', b', r', r_0', $k' = 6t + 1$, λ', λ_0' and an NBIB design with parameters $v = 6t + 1 = b_1$, $b_2 = 2t(6t + 1)$, $r = 6t$, $k_1 = 6t$, $k_2 = 3$, $\lambda_1 = 6t - 1$, $\lambda_2 = 2$ implies the existence of an NBTIB design with parameters $v = v'$, $b_1 = b'(6t + 1)$, $b_2 = 2tb'(6t + 1)$, $r = 6r't$, $r_0 = 6tr_0'$, $k_1 = 6t$, $k_2 = 3$, $\lambda_1 = (6t - 1)\lambda'$, $\lambda_{10} = (6t - 1)\lambda_0'$, $\lambda_2 = 2\lambda'$, $\lambda_{20} = 2\lambda_0'$.

Family 2.2.10: Existence of a BTIB design with parameters v', b', r', r_0', $k' = 4t - 1$, λ', λ_0' and an NBIB design with parameters $v = 4t - 1$, $b_1 = 4t - 1$, $b_2 = 2(4t - 1)$, $r = 2(2t - 1)$, $k_1 = 4t - 2$, $k_2 = 2t - 1$, $\lambda_1 = 4t - 3$, $\lambda_2 = 2t - 2$ implies the existence of an NBTIB design with parameters $v = v'$, $b_1 = b'(4t - 1)$, $b_2 = 2b'(4t - 1)$, $r = 2(2t - 1)r'$, $r_0 = 2(2t - 1)r_0'$, $k_1 = 4t - 2$, $k_2 = 2t - 1$, $\lambda_1 = (4t - 3)\lambda'$, $\lambda_{10} = (4t - 3)\lambda_0'$, $\lambda_2 = (2t - 2)\lambda'$, $\lambda_{20} = (2t - 2)\lambda_0'$.

If in place of an NBIB design, we take a nested balanced block (NBB) design in which each of the treatments appears exactly once in each of the blocks, and the sub-blocks form a BIB design, then also we get an NBTIB design. One such family of NBTIB designs is given below:

Family 2.2.11: Let there exist a BTIB design with parameters $v', b', r', r_0', k' =$ [illegible] λ', λ_0' and there also exists a NBB design with parameters $v^* = 2t$, $b_1 = 2t - 1$, $b_2^* = t(2t - 1)$, $r^* = 2t - 1$, $k_1^* = 2t$, $k_2^* = 2$, $\lambda_1^* = 2t - 1$, $\lambda_2^* = 1$ obtained by developing the initial block

$$[(1, v^*);\ (2, v^* - 1); \ldots;\ (t, v^* - t + 1)] \bmod (2t - 1)$$

with the v treatment as invariant.

Then following the procedure of Method 2.2, we get an NBTIB design with parameters as $v = v'$, $b_1 = b'(2t - 1)$, $b_2 = tb'(2t - 1)$, $r = (2t - 1)r'$, $r_0 = (2t - 1)r_0'$, $k_1 = 2t$, $k_2 = 2$, $\lambda_1 = (2t - 1)\lambda'$, $\lambda_{10} = (2t - 1)\lambda_0'$, $\lambda_2 = \lambda'$, $\lambda_{20} = \lambda_0'$.

Remark 2.1: A BTIB design with parameters $v', b', r', r_0' = b', k', \lambda', \lambda_0' = r'$, can be obtained by adding a control treatment once to each of the blocks of a BIB design $v', b', r', k' - 1, \lambda'$. Now, let there exist a BIB design with parameters $v', b', r', k' = 3, \lambda'$ and j^{th} block contents as (x_{1j}, x_{2j}, x_{3j}), $j = 1, 2, \ldots, b'$. On adding the control treatment 0 once to each of the b' blocks, we get a BTIB design with parameters $v', b', r', r_0' = b', k' = 4, \lambda', \lambda_0' = r'$. Let $(x_{1j}, x_{2j}, x_{3j}, 0)$ denote the j^{th} block contents of the BTIB design, $j = 1, 2, \ldots, b'$. Now arrange $(x_{1j}, x_{2j}, x_{3j}, 0)$ in three blocks in the following manner:

$$[(x_{1j}, 0); (x_{3j}, x_{2j})];\ [(x_{2j}, 0); (x_{3j}, x_{1j})];\ [(x_{3j}, 0); (x_{1j}, x_{2j})]$$

Repeating this process for each of the b' blocks of the BTIB design, we get an NBTIB design with parameters $v = v'$, $b_1 = 3b'$, $b_2 = 6b'$, $r = 3r'$, $r_0 = 3b'$, $k_1 = 4$, $k_2 = 2$, $\lambda_1 = 3\lambda'$, $\lambda_{10} = 3r'$, $\lambda_2 = \lambda'$, $\lambda_{20} = r'$.

Example 2.3: Consider a BIB design with parameters $v' = 4$, $b' = 4$, $r' = 3$, $k' = 3$, $\lambda' = 2$. The block contents of this BIB design can be obtained by developing the initial block (1, 2, 3) mod 4. Add a control treatment 0 to these blocks and arrange the array (1, 2, 3, 0) in three blocks in the following manner

[(1, 0); (3, 2)]; [(2, 0); (3, 1)]; and [(3, 0); (1, 2)].

Repeat this process for all the blocks of the BIB design and get an NBTIB design with parameters $v = 4$, $b_1 = 12$, $b_2 = 24$, $r = 9$, $r_0 = 12$, $k_1 = 4$, $k_2 = 2$, $\lambda_1 = 6$, $\lambda_{10} = 9$, $\lambda_2 = 2$, $\lambda_{20} = 3$ which has A-efficiencies 1.0000 and 0.9429 for the block and sub-block designs, respectively.

NBTIB designs with $v \leq 16$ and $r \leq 30$ obtainable from this method are given in Table 2. The designs included in Table 2 are those obtainable from Family 2.2.1 and Family 2.2.11 and Remark 2.1. Here, BTIB designs $v', b', r', r_0', k', \lambda', \lambda_0'$, are those obtainable by adding a control treatment once to each of

the blocks of a BIB design $v', b', r', k' - 1, \lambda'$. Table 2 consists of 10 NBTIB designs. All the block designs (ignoring the sub-block classification) and 7 sub-block designs (ignoring the block classification) have A-efficiencies greater than 0.9900. Only one sub-block design has A-efficiency less than 0.9500.

Method 2.3: Let there exist an NBIB design with parameters as $v + \alpha$, b_1, b_2, r, k_1, k_2, λ_1, λ_2. Let the treatments be denoted by $1, 2, \ldots, v, v + 1, v + 2, \ldots, v + \alpha$. On merging the treatments $v + 1, v + 2, \ldots$, and $v + \alpha$ to the $(v + 1)^{th}$ treatment and calling this $(v + 1)^{th}$ treatment a control treatment, we get an NBTIB design with parameters as v, b_1, b_2, r, $r_0 = \alpha r$, k_1, k_2, $\lambda_1 = \lambda_1$, $\lambda_{10} = \alpha\lambda_1$, $\lambda_2 = \lambda_2$, $\lambda_{20} = \alpha\lambda_2$.

Example 2.4: Consider an NBIB design with parameters $v = 7$, $b_1 = 7$, $b_2 = 21$, $r = 6$, $k_1 = 6$, $k_2 = 2$, $\lambda_1 = 5$, $\lambda_2 = 1$ with block contents as

[(1, 7); (2, 6); (3, 5)]; [(2, 1); (3, 7); (4, 6)]; [(3, 2); (4, 1); (5, 7)];
[(4, 3); (5, 2); (6, 1)]; [(5, 4); (6, 3); (7, 2)]; [(6, 5); (7, 4); (1, 3)];
[(7, 6); (1, 5); (2, 4)].

Now merge the treatments 6 and 7 and call the merged treatment the control treatment 0. The new layout is given as

[(1, 0); (2, 0); (3, 5)]; [(2, 1); (3, 0); (4, 0)]; [(3, 2); (4, 1); (5, 0)];
[(4, 3); (5, 2); (0, 1)]; [(5, 4); (0, 3); (0, 2)]; [(0, 5); (0, 4); (1, 3)];
[(0, 0); (1, 5); (2, 4)].

The above design is an NBTIB design with parameters $v = 5$, $b_1 = 7$, $b_2 = 21$, $r = 6$, $r_0 = 12$, $k_1 = 6$, $k_2 = 2$, $\lambda_1 = 5$, $\lambda_{10} = 10$, $\lambda_2 = 1$, $\lambda_{20} = 2$.

Note 2.1: This method can produce designs with useless sub-blocks, that is, sub-blocks containing only the control and which therefore provide no information for the experiment. This may lead to sacrificing the efficiency for balance. However, a small sacrifice in efficiency can be a worthwhile trade for the case of interpretation offered by balance. However, this method should not be used when it produces more than a very few useless blocks, nor when it produces useless blocks of large size.

NBTIB designs with $v \leq 16$ and $r \leq 30$ obtainable through this method for $\alpha = 2$ are given in Table 3. All the 67 designs in Table 3 have A-efficiencies more than 0.9000 for both block design and sub-block design. For the block classification, ignoring the sub-blocks, 29 designs have A-efficiencies greater than 0.9900 and 25 designs have A-efficiencies in the range of 0.9500-0.9900. The sub-block designs ignoring the block classification have smaller efficiencies. Only 2 designs have A-efficiencies greater than 0.9400, 2 designs have A-efficiencies in the range 0.9300-0.9400 and 21 designs have A-efficiencies in the range 0.9200-0.9300. The rest of the designs have A-efficiencies in the range 0.9000-0.9200.

Method 2.4: This method is based on trial and error solutions. An NBTIB design with parameters $v = 7$, $b_1 = 7$, $b_2 = 21$, $r = 6$, $r_0 = 21$, $k_1 = 9$, $k_2 = 3$, $\lambda_1 = 5$, $\lambda_{10} = 18$, $\lambda_2 = 1$, $\lambda_{20} = 6$ can be obtained by developing the initial block [(1, 4, c); (2, c, 3); (c, 0, 5)] mod 7; where c denotes the control treatment with A-efficiencies 0.9824 and 0.9848 respectively for the block design and sub-block design.

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References

[1] S.Banerjee and S.Kageyama. Existence of α-resolvable nested balanced incomplete block designs. *Utilitas Mathematica*, 38 (1990), 237-243.

[2] S.Banerjee and S.Kageyama. Methods of constructing nested partially balanced incomplete block designs. *Utilitas Mathematica*, 43 (1993), 3-6.

[3] R.E.Bechhofer, and A.C.Tamhane. Incomplete block designs for comparing treatments with a control (I): General Theory. *Technometrics*, 23 (1981), 45-57.

[4] A.Dey, U.S. Das, and A.K.Banerjee. On nested balanced incomplete block designs. *Calcutta Statist. Assoc. Bull.*, 35(1986) , 161-167.

[5] V.K.Gupta and R.Parsad. Block designs for comparing test treatments with control treatments - An overview. *Special issue of Statistics and Applications to felicitate the 80th Birthday of Dr. M.N.Das*. 3(1 & 2) (2001), 133-146.

[6] A.S.Hedayat, M.Jacroux and D.Majumdar. Optimal designs for comparing test treatments with a control. *Statist. Sc.*, 3 (1988), 462-491.

[7] A.S.Hedayat and D.Majumdar. A-optimal incomplete block designs for control-test treatment comparisons. *Technometrics*, 26 (1984), 363-370.

[8] R.J.Homel. and J.Robinson. Nested partially balanced incomplete block designs. *Sankhyā* , B37 (1975), 201-210.

[9] M.Jacroux and D.Majumdar. Optimal designs for comparing a set of test treatments with a control when $k > v$. *J. Statist. Plann. Inf.*, 23 (1989), 381-396.

[10] M.Jimbo and S.Kuriki. Constructions of nested designs. *Ars Comb.*, 16(1983), 275-285.

[11] S.Kageyama, J.Philip and S.Banerjee. Some constructions of nested BIB and 2-associate PBIB designs under restricted dualization. *Bull. Fac. Sch. Educ. Hiroshima Univ., Part II*, 17 (1995), 33-39.

[12] D.Majumdar. Optimal and efficient treatment-control designs. *Handbook of Statistics*, 13, Editors S. Ghosh and C.R. Rao, Elsevier Science. B.V. (1996), 1007-1053.

[13] J.P.Morgan. Nested designs. *Handbook of Statistics*, 13, Editors S. Ghosh and C.R. Rao, Elsevier Science, B.V. (1996), 939-976.

[14] J.P.Morgan, D.A. Preece and D.H.Rees. Nested balanced incomplete block designs. *Discrete Mathematics*, 231 (2001), 351-389.

[15] R.Parsad, V.K.Gupta and N.S.G.Prasad. On construction of A-efficient balanced treatment incomplete block designs. *Utilitas Mathematica*, 47 (1995), 185-190.

[16] R.Parsad, V.K.Gupta and R.Srivastava. Universally optimal block designs for diallel crosses. *Statistics and Applications*, 1(1999), 35-52.

[17] J.Philip, S.Banerjee and S.Kageyama. Construction of nested t-associate class PBIB designs under restricted dualisation. *Utilitas Mathematica*, 51 (1997), 27-32.

[18] D.A.Preece (1967).Nested balanced incomplete block designs. *Biometrika*, 54 (1967), 479-486.

[19] G.M.Saha, A.Dey and C.K.Midha. Construction of nested incomplete block designs. *Calcutta Statist. Assoc. Bull.*, 48 (1998), 195-205.

[20] S.K.Satpati and R.Parsad. Construction and cataloguing of nested partially balanced incomplete block designs. *Ars Combinatoria* (2003) In Press.

[21] J. Stufken. On bounds for the efficiency of block designs for comparing test treatments with a control. *J. Statist. Plann. Inf.*, 19 (1988), 361-372.

Table 1: NBTIB Designs with $v \leq 16$, $r \leq 30$ obtainable from Method 2.1.

Sl.No.	v	b_1	b_2	r	r_0	k_1	k_2	λ_1	λ_{10}	λ_2	λ_{20}	E_1	E_2	Reference Design
1	5	5	10	4	10	6	3	3	8	1	4	1.0000	1.0000	MPR 1
2	6	15	30	10	30	6	3	6	20	2	10	0.9932	0.9924	MPR 13
3	7	7	21	6	21	9	3	5	18	1	6	0.9824	0.9848	MPR 2
4	7	7	14	6	14	8	4	5	12	2	6	1.0000	1.0000	MPR 3
5	7	21	42	12	42	6	3	6	24	2	12	0.9846	0.9847	MPR 19
6	8	14	28	7	28	6	3	3	14	1	7	0.9752	0.9767	MPR 4
7	8	28	84	21	84	9	3	15	63	3	21	0.9719	0.9766	MPR 50
8	8	28	56	21	56	8	4	15	42	6	21	1.0000	1.0000	MPR 51
9	9	9	36	8	36	12	3	7	32	1	8	0.9623	0.9686	MPR 8
10	9	9	18	8	18	10	5	7	16	3	8	0.9933	1.0000	MPR 9
11	9	12	36	8	36	9	3	5	24	1	8	0.9614	0.9686	MPR 6
12	9	12	24	8	24	8	4	5	16	2	8	1.0000	1.0000	MPR 7
13	9	18	36	8	36	6	3	3	16	1	8	0.9657	0.9686	MPR 5
14	10	10	30	9	30	12	4	8	27	2	9	0.9998	0.9999	MPR 12
15	10	15	45	9	45	9	3	5	27	1	9	0.9519	0.9608	MPR10
16	10	15	30	9	30	8	4	5	18	2	9	0.9999	0.9999	MPR 11
17	10	45	90	18	90	6	3	6	36	2	18	0.9563	0.9608	MPR 46
18	10	30	60	18	60	8	4	10	36	4	18	0.9999	0.9998	MPR 47
19	10	45	90	27	90	8	4	15	54	6	27	0.9999	0.9998	MPR 58
20	11	11	55	10	55	15	3	9	50	1	10	0.9498	0.9534	MPR 14
21	11	55	110	20	110	6	3	6	40	2	20	0.9474	0.9534	MPR 49
22	11	55	165	30	165	9	3	15	90	3	30	0.9440	0.9534	MPR 66
23	11	55	110	30	110	8	4	15	60	6	30	0.9988	0.9988	MPR 67
24	11	11	22	10	22	12	6	9	20	4	10	0.9775	0.9911	MPR 15
25	12	22	66	11	66	9	3	5	33	1	11	0.9373	0.9465	MPR 17
26	12	33	66	11	66	6	3	3	22	1	11	0.9389	0.9465	MPR 16
27	12	33	132	22	132	12	3	14	88	2	22	0.9421	0.9465	MPR 53
28	12	22	44	11	44	8	4	5	22	2	11	0.9970	0.9971	MPR 18
29	12	33	66	22	66	10	5	14	44	6	22	1.0000	1.0000	MPR 54
30	13	13	78	12	78	18	3	11	72	1	12	0.9362	0.9399	MPR 23
31	13	26	78	12	78	9	3	5	36	1	12	0.9315	0.9399	MPR 21
32	13	39	78	12	78	6	3	3	24	1	12	0.9309	0.9399	MPR20
33	13	39	156	24	156	12	3	14	96	2	24	0.9357	0.9399	MPR 55
34	13	26	52	12	52	8	4	5	24	2	12	0.9946	0.9950	MPR 22
35	13	13	52	12	52	16	4	11	48	2	12	0.9935	0.9950	MPR 24
36	13	26	78	18	72	12	4	12	54	3	18	0.9939	0.9950	MPR 48
37	13	13	39	12	39	15	5	11	36	3	12	0.9999	1.0000	MPR 25
38	13	39	78	24	78	10	5	14	48	6	24	1.0000	1.0000	MPR 56
39	13	13	26	12	26	14	7	11	24	5	12	0.9616	0.9762	MPR 26
40	14	91	182	26	182	6	3	6	52	2	26	0.9234	0.9338	MPR 57
41	15	15	105	14	105	21	3	13	98	1	14	0.9251	0.9280	MPR 31
42	15	15	30	14	30	16	8	13	28	6	14	0.9496	0.9611	MPR 32
43	15	21	105	14	105	15	3	9	70	1	14	0.9228	0.9288	MPR 29
44	15	21	42	14	42	12	6	9	28	4	14	0.9950	0.9997	MPR 30
45	15	35	105	14	105	9	3	5	42	1	14	0.9210	0.9288	MPR 27
46	15	35	70	14	70	8	4	5	28	2	14	0.9890	0.9900	MPR 28
47	15	35	105	21	105	12	4	12	63	3	21	0.9879	0.9900	MPR 52
48	15	35	210	28	210	18	3	22	168	2	28	0.9249	0.9280	MPR 62
49	15	35	140	28	140	16	4	22	112	4	28	0.9872	0.9900	MPR 63
50	15	35	105	28	105	15	5	22	84	6	28	1.0000	1.0000	MPR 64
51	15	35	70	28	70	14	7	22	56	10	28	0.9744	0.9861	MPR 65
52	15	42	210	28	210	15	3	18	140	2	28	0.9228	0.9800	MPR 60
53	15	42	84	28	84	12	6	18	56	8	28	0.9950	0.9997	MPR 61
54	15	105	210	28	210	6	3	6	56	2	28	0.9162	0.9280	MPR 59
55	16	16	80	15	80	20	4	14	75	2	15	0.9842	0.9873	MPR 44
56	16	16	48	15	48	18	6	14	45	4	15	0.9949	1.0000	MPR 45
57	16	20	120	15	120	18	3	11	90	1	15	0.9195	0.9226	MPR 40
58	16	20	80	15	80	16	4	11	60	2	15	0.9839	0.9873	MPR 41
59	16	20	60	15	60	15	5	11	45	3	15	1.0000	1.0000	MPR 42
60	16	20	40	15	40	14	7	11	30	5	15	0.9793	0.9896	MPR 43
61	16	24	120	15	120	15	3	9	75	1	15	0.9175	0.9226	MPR 38
62	16	24	48	15	48	12	6	9	30	4	15	0.9970	1.0000	MPR 39
63	16	30	120	15	120	12	3	7	60	1	15	0.9179	0.9226	MPR 36
64	16	30	60	15	60	10	5	7	30	3	15	1.0000	1.0000	MPR 37
65	16	40	120	15	120	9	3	5	45	1	15	0.9160	0.9226	MPR 34
66	16	40	80	15	80	8	4	5	30	2	15	0.9860	0.9873	MPR 35
67	16	60	120	15	120	6	3	3	30	1	15	0.9096	0.9226	MPR 33
68	16	48	96	30	96	12	6	18	60	8	30	0.9970	1.0000	MPR 68

* MPR# denotes the design at serial number # in Morgan, Preece and Rees [14].

Table 2: NBTIB Designs with $v \le 16$, $r \le 30$ obtainable from Method 2.2.

Sl.No.	v	b_1	b_2	r	r_0	k_1	k_2	λ_1	λ_{10}	λ_2	λ_{20}	E_1	E_2	Reference BIB Design
1	4	12	24	9	12	4	2	6	9	2	3	1.0000	0.9429	(4,4,3,3,1)
2	5	30	60	18	30	4	2	9	18	3	6	1.0000	0.9706	(5,10,6,3,2)
3	6	30	60	15	30	4	2	6	15	2	5	1.0000	0.9860	(6,10,5,3,2)
4	7	21	42	9	21	4	2	3	9	1	3	1.0000	0.9943	(7,7,3,3,1)
5	9	36	72	12	36	4	2	3	12	1	4	1.0000	0.9999	(9,12,4,3,1)
6	10	90	180	27	90	4	2	6	27	2	9	0.9998	0.9998	(10,30,9,3,1)
7*	13	65	130	16	52	4	2	3	12	1	4	0.9971	0.9971	(13,13, 4,4, 1)
8	13	78	156	18	78	4	2	3	18	1	6	0.9950	0.9950	(13,26,6,3,1)
9	15	105	210	21	105	4	2	3	21	1	7	0.9900	0.9900	(15,35,7,3,1)
10*	16	100	200	20	80	4	2	3	15	1	5	0.9999	0.9999	(16,20,5,4,1)

* denotes that the designs are obtainable from Family 2.2.1 and rest of the designs are obtained from Family 3.2.11 and Remark 2.1.

Table 3: NBTIB Designs With $v \le 16$, $r \le 30$ obtainable from Method 3.3 for $\alpha = 2$ using NBIB Designs of Morgan, Preece and Rees [14]

Sl.No.	v	b_1	b_2	r	r_0	k_1	k_2	λ_1	λ_{10}	λ_2	λ_{20}	E_1	E_2	Reference Design
1	4	15	30	10	20	4	2	6	12	2	4	0.9697	0.9143	MPR 13
2	5	7	21	6	12	6	2	5	10	1	2	0.9958	0.9246	MPR 2
3	5	7	14	6	12	6	3	5	10	2	4	0.9958	0.9246	MPR 3
4	5	21	42	12	24	4	2	6	12	2	4	0.9524	0.9245	MPR 19
5	6	14	28	7	14	4	2	3	6	1	2	0.9412	0.9282	MPR 4
6	6	28	84	21	42	6	2	15	30	3	6	0.9881	0.9279	MPR 50
7	6	28	56	21	42	6	3	15	30	6	12	0.9881	0.9279	MPR 51
8	7	9	36	8	16	8	2	7	14	1	2	0.9846	0.9279	MPR 8
9	7	9	18	8	16	8	4	7	14	3	6	0.9846	0.9333	MPR 9
10	7	12	36	8	16	6	2	5	10	1	2	0.9792	0.9279	MPR 6
11	7	12	24	8	16	6	3	5	10	2	4	0.9792	0.9279	MPR 7
12	7	18	36	8	16	4	2	3	6	1	2	0.9333	0.9279	MPR 5
13	8	10	30	9	18	9	3	8	16	2	4	0.9820	0.9259	MPR 12
14	8	15	45	9	18	6	2	5	10	1	2	0.9596	0.9259	MPR10
15	8	15	30	9	18	6	3	5	10	2	4	0.9596	0.9259	MPR 11
16	8	30	60	18	36	6	3	10	20	4	8	0.9596	0.9259	MPR 47
17	8	45	90	27	54	6	3	15	30	6	12	0.9596	0.9259	MPR 58
18	8	45	90	18	36	4	2	6	12	2	4	0.9275	0.9259	MPR 46
19	9	11	55	10	20	10	2	9	18	1	2	0.9760	0.9229	MPR 14
20	9	55	110	20	40	4	2	6	12	2	4	0.9231	0.9229	MPR 49
21	9	55	165	30	60	6	2	15	30	3	6	0.9600	0.9229	MPR 66
22	9	55	110	30	60	6	3	15	30	6	12	0.9600	0.9229	MPR 67
23	9	11	22	10	20	10	5	9	18	4	8	0.9760	0.9429	MPR 15
24	10	22	66	11	22	6	2	5	10	1	2	0.9509	0.9194	MPR 17
25	10	33	66	11	22	4	2	3	6	1	2	0.9194	0.9194	MPR 16
26	10	33	132	22	44	8	2	14	28	2	4	0.9565	0.9194	MPR 53
27	10	22	44	11	22	6	3	5	10	2	4	0.9509	0.9194	MPR 18
28	10	33	66	22	44	8	4	14	28	6	12	0.9565	0.9194	MPR 54
29	11	13	78	12	24	12	2	11	22	1	2	0.9607	0.9156	MPR 23
30	11	26	78	12	24	6	2	5	10	1	2	0.9422	0.9156	MPR 21
31	11	39	78	12	24	4	2	3	6	1	2	0.9156	0.9156	MPR20
32	11	39	156	24	48	8	2	14	28	2	4	0.9495	0.9156	MPR 55
33	11	26	52	12	24	6	3	5	10	2	4	0.9422	0.9156	MPR 22
34	11	13	52	12	24	12	3	11	22	2	4	0.9607	0.9156	MPR 24

[illegible]	[illegible]	b_1	b_2	r	r_0	k_1	k_2	λ_1	λ_{10}	λ_2	λ_{20}	E_1	E_2	Reference Design
[illegible]	[illegible]	[illegible]	78	18	36	9	3	12	24	3	6	0.9502	0.9156	MPR 48
[illegible]	[illegible]	13	39	12	24	12	4	11	22	3	6	0.9607	0.9156	MPR 25
37	[illegible]	39	78	24	48	8	4	14	28	6	12	0.9495	0.9156	MPR 56
38	11	13	26	12	24	12	6	11	22	5	10	0.9607	0.9422	MPR 26
39	12	91	182	26	52	4	2	6	12	2	4	0.9116	0.9116	MPR 57
40	13	15	105	14	28	14	2	13	26	1	2	0.9461	0.9077	MPR 31
41	13	15	30	14	28	14	7	13	26	6	12	0.9461	0.9339	MPR 32
42	13	21	105	14	28	10	2	9	18	1	2	0.9387	0.9077	MPR 29
43	13	21	42	14	28	10	5	9	18	4	8	0.9387	0.9104	MPR 30
44	13	35	105	14	28	6	2	5	10	1	2	0.9261	0.9077	MPR 27
45	13	35	70	14	28	6	3	5	10	2	4	0.9261	0.9077	MPR 28
46	13	35	105	21	42	9	3	12	24	3	6	0.9364	0.9077	MPR 52
47	13	35	210	28	56	12	2	22	44	2	4	0.9454	0.9077	MPR 62
48	13	35	140	28	56	12	3	22	44	4	8	0.9454	0.9077	MPR 63
49	13	35	105	28	56	12	4	22	44	6	12	0.9454	0.9077	MPR 64
50	13	35	70	28	56	12	6	22	44	10	20	0.9454	0.9261	MPR 65
51	13	42	210	28	56	10	2	18	36	2	4	0.9387	0.9077	MPR 60
52	13	42	84	28	56	10	5	18	36	8	16	0.9387	0.9104	MPR 61
53	13	105	210	28	56	4	2	6	12	2	4	0.9077	0.9077	MPR 59
54	14	16	80	15	30	15	3	14	28	2	4	0.9403	0.9038	MPR 44
55	14	16	48	15	30	15	5	14	28	4	8	0.9403	0.9051	MPR 45
56	14	20	120	15	30	12	2	11	22	1	2	0.9383	0.9038	MPR 40
57	14	20	80	15	30	12	3	11	22	2	4	0.9383	0.9038	MPR 41
58	14	20	60	15	30	12	4	11	22	3	6	0.9383	0.9038	MPR 42
59	14	20	40	15	30	12	6	11	22	5	10	0.9383	0.9188	MPR 43
60	14	24	120	15	30	10	2	9	18	1	2	0.9315	0.9038	MPR 38
61	14	24	48	15	30	10	5	9	18	4	8	0.9315	0.9051	MPR 39
62	14	30	120	15	30	8	2	7	14	1	2	0.9300	0.9038	MPR 36
63	14	30	60	15	30	8	4	7	14	3	6	0.9300	0.9038	MPR 37
64	14	40	120	15	30	6	2	5	10	1	2	0.9188	0.9038	MPR 34
65	14	40	80	15	30	6	3	5	10	2	4	0.9188	0.9038	MPR 35
66	14	60	120	15	30	4	2	3	6	1	2	0.9038	0.9038	MPR 33
67	14	48	96	30	60	10	5	18	36	8	16	0.9315	0.9051	MPR 68

MPR# : denotes the NBIB design at serial number # in Morgan, Preece and Rees [14].
E_1: denotes the A-efficiency of the block design ignoring the sub-block classification.
E_2: denotes the A-efficiency of the sub-block design ignoring the block classification.

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