

परियोजना रिपोर्ट

Project Report

दो पंक्तियों में बहुउपादानी परीक्षणों के लिए
पंक्ति-स्तम्भ अभिकल्पनाएं

**Row-column Designs
for Factorial Experiments in Two Rows**



सुकान्त दाश

Sukanta Dash

राजेन्द्र प्रसाद

Rajender Parsad

विनोद कुमार गुप्ता

V. K. Gupta

परीक्षण अभिकल्पना प्रभाग

Division of Design of Experiments



भारतीय कृषि सांख्यिकी अनुसन्धान संस्थान (भा. कृ. अनु. प.)

लाइब्रेरी एवेन्यू, पूसा, नई दिल्ली - 110012

Indian Agricultural Statistics Research Institute (I.C.A.R)

Library Avenue, Pusa, New Delhi – 110 012



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आमुख

पंक्ति स्तम्भ अभिकल्पना जिन परीक्षणात्मक परिस्थियों, परीक्षणात्मक सामग्री में विविधता के दो संकर वर्गीकृत स्रोत होते हैं के लिए उपयोगी है। व्यवहारिक कारणों से एक स्तम्भ में पंक्ति-स्तम्भ अभिकल्पना के एक स्तम्भ में दो से अधिक परीक्षणात्मक ईकाई रखे जाना सम्भव नहीं है। उदाहरणार्थ, द्वि-रंग माइक्रो अरे में दो पंक्तियों वाली पंक्ति-स्तम्भ अभिकल्पना, अत्यन्त उपयोगी है। दो पंक्तियों एवं बहु-उपादानी ट्रीटमेंट संरचना वाली पंक्ति स्तम्भ अभिकल्पना अनेकों कृषि परीक्षणों में भी उपयोगी पाई गई है। जब पंक्ति-स्तम्भ सैटअप में अभिकल्पना गैर-लाम्बिक होती है, तब सभी बहु-उपादानी प्रभावों का लाम्बिक आकलन अधिक दक्ष होता है। इसके लिए बड़ी संख्या में स्तम्भों की आवश्यकता होगी। मूल्य एवं समय को ध्यान में रखते हुए, सभी बहु-उपादानी प्रभावों हेतु अभिकल्पना में उतने परीक्षणात्मक ईकाई उपलब्ध नहीं होती है जितनी की सभी बहु-उपादानी प्रभावों के आकलन हेतु आवश्यक है। परीक्षणकर्ता की रुचि सभी मुख्य प्रभावों एवं द्वि-कारक अन्वोन्यक्रियाओं के लाम्बिक आकलन में हो सकती है। बहु-उपादानीय परीक्षण हेतु अभिकल्पना निर्माण में दूसरा मुख्य मुद्दा यह है कि सामान्य तौर पर बहु-उपादानी प्रभावों को लाम्बिक प्राचलीकरण के द्वारा परिभाषित किया जाता है। कृषि परीक्षणों के लिए जहां शून्य स्थिति या बेस-लाईन हो सकती है वहां परीक्षणकर्ता की रुचि लाम्बिक प्राचलीकरण में होगी। इन्हीं परीक्षणात्मक परिस्थितियों में दो पंक्तियों वाली दक्ष पंक्ति-स्तम्भ अभिकल्पनाओं की संरचना महत्वपूर्ण है। ऐसी परिस्थितियों से निपटने के लिए वर्तमान अध्ययन इस प्रकार है (अ) रनों की कम संख्या में सभी मुख्य प्रभावों एवं द्वि-कारक अन्वोन्यक्रियाओं के लाम्बिक आकलन हेतु द्वि-स्तरीय बहु-उपादानी परीक्षणात्मक पंक्ति स्तम्भ अभिकल्पनाओं की संरचना एवं (ब) बेस-लाईन प्राचलीकरण पर आधारित बहु-उपादानी परीक्षणात्मक पंक्ति-स्तम्भ अभिकल्पनाओं की संरचना की सामान्य विधि विकसित करना।

सभी मुख्य प्रभावों एवं द्वि-कारक अन्वोन्यक्रियाओं के लाम्बिक आकलन हेतु दो पंक्तियों वाले पंक्ति स्तम्भ अभिकल्पनाओं की संरचना की एक सामान्य विधि दी गयी है। $2^n (2 \leq n \leq 9)$ बहु-उपादानी परीक्षणों के लिए दक्ष पंक्ति स्तम्भ अभिकल्पनाओं की एक सूची तैयार की गई है। उसी प्रकार बेस-लाईन प्राचलीकरण स्थितियों से निपटने के लिए n -कारक मिश्रित स्तर बहु-उपादानीय परीक्षणात्मक दो पंक्तियों वाली पंक्ति-स्तम्भ अभिकल्पनाओं को प्राप्त करने की एक सामान्य विधि विकसित की गई है। लाम्बिक प्राचलीकरण एवं बेस-लाईन प्राचलीकरण पर आधारित बहु-उपादानी परीक्षणात्मक दो पंक्तियों वाले पंक्ति स्तम्भ अभिकल्पनाओं की संरचना के लिए एक वैब एप्लिकेशन भी विकसित की गई।

लेखकगण, भारतीय कृषि सांख्यिकी अनुसंधान संस्थान, नई दिल्ली के निदेशक महोदय के अत्यन्त आभारी हैं कि उन्होंने इस परियोजना में हमारे उत्साहवर्धन के साथ-साथ समय समय पर सुझाव एवं सुविधाएं भी उपलब्ध कराई हैं। हम परीक्षण अभिकल्पना प्रभाग के प्रभागाध्यक्ष एवं अन्य वैज्ञानिकों से प्राप्त सहयोग का हार्दिक आभार व्यक्त करते हैं। परीक्षण अभिकल्पना प्रभाग के श्री देवेन्द्र कुमार एवं श्रीमती सुनीता द्वारा की गई सहायता के लिए भी आभार प्रकट करते हैं।

मार्च 2014

सुकान्त दाश
राजेन्द्र प्रसाद
विनोद कुमार गुप्ता

PREFACE

Row-column designs are useful for the experimental situations in which there are two cross classified sources of heterogeneity in the experimental material. Due to practical considerations it may not be possible to accommodate more than two experimental units in a column of a row-column design. For example, row-column designs with two rows are very useful in two-colour microarray experiments. Row-column designs with two rows and with factorial treatment structure have also been found useful in many other agricultural experiments. When the design is non-orthogonal in a row-column set up, it would be desirable that it permits orthogonal estimation of all factorial effects with high efficiency. This may require a large number of columns. Due to cost and time considerations, it may not be possible to run a design in number of runs that are required for orthogonal estimation of all the factorial effects. The experimenter may, however, be interested in orthogonal estimation of all the main effects and two factor interactions. Another main issue in constructing designs for multifactor experiments is that the factorial effects are commonly defined via an orthogonal parameterization. In agricultural experiments, situations may also occur where experimenter is interested in baseline parameterization rather than orthogonal parameterization where null state or baseline may exist. In such experimental situations as well, it is important to obtain efficient row-column designs in two rows. To deal with the above experimental situations, the present study was undertaken (a) to obtain row-column designs for 2-level factorial experiments for estimating the main effects and two factor interactions in fewer number of runs based on orthogonal parameterization, and (b) to develop general methods of construction of row column designs for factorial experiments based on baseline parameterization.

A general method of construction of row-column designs with two rows has been given for orthogonal estimation of all main effects and two factor interactions under orthogonal parameterization. A catalogue of row-column designs for 2^n ($2 \leq n \leq 9$) factorial experiments in fewer number of replications has been prepared. Similarly, to deal with the situations of baseline parameterization a general procedure of obtaining row-column designs in two rows for n -factor mixed level factorial experiments has been developed. Web application for generation of row-column designs in two rows for orthogonal and baseline parametrization has also been developed.

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Sukanta Dash
Rajender Parsad
V.K. Gupta

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विषय
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INTRODUCTION AND REVIEW OF LITERATURE

1.1 Introduction

Row-column designs are useful for the experimental situations in which there are two cross classified sources of heterogeneity in the experimental material. In some of these experimental situations, one of the two factors causing heterogeneity in the experimental units has only two levels and as a consequence, it is not possible to allocate more than two units in a single column/row. For example, consider an experiment conducted for improving quality of products; experimental processes in the laboratory may require use of an oven divided into smaller sections in a linear fashion. In each section temperature or other conditions may vary. Further in each section, there are two positions on which treatments can be applied. Therefore, considering sections as columns and positions as rows, the experiment can be conducted using a row-column design in two rows. Another experimental situation is 2-colour multifactor microarray experiments conducted to study the effect of more than one factor (different types of tissues, drug treatments or time points of a biological process) simultaneously. Consider an example given by Glonek and Solomon (2004), in which it is desired to study and compare the two mutants at times zero hour and 24 hours. The interest is in measuring the changes over time. Therefore, there are two factors *viz.* varieties (two mutants) and time (0 hour and 24 hours). Row-column designs with two rows with columns representing arrays, rows representing dyes and treatments representing varieties are useful for such experimental situations. Another main issue in constructing designs for multifactor experiments is that the factorial effects are commonly defined via an orthogonal parameterization. In agricultural /laboratory experiments, situations may also occur where experimenter is interested in baseline parameterization rather than orthogonal parameterization. For example, in a toxicological study with binary factors, each representing the presence or absence of a particular toxin, the state of absence can be regarded as a natural baseline level of each factor. Null state or baseline of a factor need not strictly mean zero level on some scale, but may as well refer to a standard or control level like the one currently being used in practice. Such experimental situations involving the control or standard treatment (natural baseline) do occur in many real life situations. For example, suppose that the researchers are interested in studying the gene expression response of maize plants to a virus infection under drought conditions (Nettleton, 2012). Here, the state of absence (controlled condition) can be regarded as a natural baseline level of each factor. Null state or baseline of a

factor need not strictly mean zero level on some scale, but may as well refer to a standard or control level like the one currently being used in practice. Such experimental situations involving the control or standard treatment (natural baseline) do occur in many real life situations.

The main distinction between these two kinds of parameterizations is that in case of orthogonal parameterization the factorial effects are defined via mutually orthogonal treatment contrasts whereas in the baseline parameterization these effects are defined with natural baseline levels of the factors and, hence, entail non-orthogonality. Thus the interest in the present investigation is to obtain efficient designs for multifactor experiments based on both orthogonal as well as baseline parameterization.

1.2 Review of row-column designs for orthogonal parameterization

Considering the experimental situations described in Section 1.1, a lot of efforts have been made in the literature to obtain block/ row-column designs that enable orthogonal estimation of main effects and two factor interactions that are discussed in the sequel.

Yang and Draper (2003) developed an approach to obtain block designs for 2^n ($n \leq 5$) factorial experiments with blocks of size 2, which provide orthogonal estimates of main effects and two factor interactions by searching from all confounding patterns, which is a tedious process. It becomes difficult to find all confounding patterns as the number of factors increases.

Wang (2004) studied designing 2^{n-p} fractional factorial plans in blocks of size two and suggested that the number of runs to estimate all the available effects, as is possible in experiments without blocking, is $(n-p) 2^{n-p}$ for 2^{n-p} fractional factorial plans.

Kerr (2006) obtained block designs for 2^n factorial experiments in blocks of size 2 for estimation of all main effects and two factor interactions. The upper bound on minimum number of replications required for orthogonal estimation of all main effects and two factor interactions is $[\log_2 n] + 1$, here $[\cdot]$ denotes greatest integer function. The upper bound on minimum replications for orthogonal estimation of all main effects and two factor interactions for 2^n factorial experiments with $n = 2, 3, 4, 5, 6, 7, 8$ are respectively 2, 2, 3, 3, 3, 3, 4. Kerr (2006) has also given the procedure of obtaining block designs for 2, 3, 4 and 8 factors. For obtaining a design for 5, 6 or 7 factors it has been suggested that by making a computer aided search of all possible

blocked factorials in 3 replications, solution can be attained that may provide orthogonal estimation of all main effects and two factor interactions. For example, 2^4 factorial experiment requires minimum of three replications *i.e.* 48 runs to estimate all main effects and two factor interaction in block set up.

	1	2	3	4	5	6	7	8	9	10	11	12
R1	0000	0001	0010	0011	0100	0101	0110	0111	0000	0001	0010	0011
R2	1110	1111	1100	1101	1010	1011	1000	1001	1101	1100	1111	1110
	13	14	15	16	17	18	19	20	21	22	23	24
R1	0000	0001	0010	0011	0100	0101	0110	0111	0000	0001	0010	0011
R2	1001	1000	1011	1010	1011	1010	1001	1000	1111	1110	1101	1100

R# denotes Row Number

Kerr (2006) suggested finding out a factorial effect that is not confounded with blocks (represented as columns in a row-column set up) and then confound it with rows to get a row column design. Kerr (2006), however, did not provide any list of factorial effects that are not confounded with column effects.

Considering the above situations Dash *et al.* (2013) gave a method of construction of row-column designs for estimation of main effects and two factor interaction effects in 2^n factorial microarray experiments based on orthogonal parameterization. A catalogue of designs for $2 \leq n \leq 9$ using the method of construction was also provided. The catalogue gives the main effects and two-factor interactions confounded in different replications and the factorial effects that are not confounded in a replication. The efficiency factor of estimable main effects and two-factor interactions was also given. For each 2^n factorial, two designs were presented, one in which main effects are estimated with more efficiency and another in which two-factor interactions are estimated with more efficiency.

Dash *et al.* (2013) also developed a procedure of construction of row-column designs for estimation of all main effects and odd order factorial effects. Dash *et al.* (2013) also suggested a procedure for obtaining row-column designs having unequal replication of different treatment combinations for orthogonal estimation of all main effects and two factor interactions in fewer number of columns than the number of columns required for row-column designs with minimum number of replications for orthogonal estimation of all main effects and two factor interactions with equal replication for all treatment combinations. For example, by using the method of

construction in, 2^4 factorial experiment requires 40 runs to estimate all main effects and two factor interaction in row-column set up which is shown below

	1	2	3	4	5	6	7	8	9	10
R1	0000	0001	0111	0110	1011	1010	1100	1101	0101	0100
R2	0011	0010	0100	0101	1000	1001	1111	1110	0000	0001
	11	12	13	14	15	16	17	18	19	20
R1	0010	0011	1000	1001	1111	1110	0000	1111	0111	1000
R2	0111	0110	1101	1100	1010	1011	1111	0000	1000	0111

R# denotes Row Number

It may, however, be possible to achieve this in smaller number of runs in row-column set up as discussed in Chapter 2.

1.3 Review of row-column designs for baseline parameterization

In constructing efficient designs for multifactor experiments, the factorial effects are commonly defined via an orthogonal parameterization because orthogonal parameters are easy to deal with in the sense that their estimates are independent and can be calculated separately. In some experimental situations as described in Section 1.1, the experimenter is interested in treatment contrasts based on baseline parameterization rather than orthogonal parameterization. Although the baseline parameterization looks simpler than the orthogonal parameterization, it renders the task of finding optimal or efficient designs somewhat more challenging due to lack of orthogonality. Glonek and Solomon (2004) were the first to study designs for multi-factor microarray experiments under baseline parameterization. They introduced a criterion of statistical efficiency in terms of variances of the estimated parameters of interest. For given number of columns, b , a design is said to be admissible if the variance of each of the estimated parameters of interest is less than or equal to the variance of the estimated parameters of interest through any other design in same number of columns and strict inequality holds for at least one parameter. This criterion was illustrated in obtaining efficient designs for 2^2 factorial experiments for given number of columns.

The key reference for obtaining optimal/efficient designs for baseline parameterization is Banerjee and Mukerjee (2007). They have studied n -factor factorial experiments with factors as $F_1, F_2, \dots, F_j, \dots, F_n$ with factor $F_j, 1 \leq j \leq n$ at $s_j \geq 2$ levels represented by $0, 1, \dots, s_j - 1$. The

total number of treatment combinations is $v = \prod_{j=1}^n s_j$. The treatment combinations in

lexicographic order are given by $\mathbf{a}_1 \times \mathbf{a}_2 \times \dots \times \mathbf{a}_n$ where \times denotes the symbolic direct product and $\mathbf{a}'_j = (0, 1, \dots, s_{j-1})$, $j=1, 2, \dots, n$. Corresponding to the treatment combination $i_1 \dots i_n$, $0 \leq i_j \leq s_j - 1$, $1 \leq j \leq n$, $\tau_{i_1 \dots i_n}$ defines the response treatment combination $i_1 \dots i_n$ and transformed to log scale), *i.e.*, the effect, of the treatment combination $i_1 \dots i_n$. As before, the baseline level of each factor is denoted by 0. Hence, $\theta_{00 \dots 0} = \tau_{00 \dots 0}$ stands for the baseline effect. Also, baseline parameterization for main effect, F_1 , which is represented by the $s_1 - 1$ parameters

$$\theta_{i_1 0 \dots 0} = \tau_{i_1 0 \dots 0} - \tau_{00 \dots 0} \quad (1 \leq i_1 \leq s_1 - 1).$$

The baseline parameterization for a two-factor interaction, $F_1 F_2$, which is represented by $(s_1 - 1)(s_2 - 1)$ parameters

$$\theta_{i_1 i_2 0 \dots 0} = \tau_{i_1 i_2 0 \dots 0} - \tau_{i_1 0 \dots 0} - \tau_{0 i_2 0 \dots 0} + \tau_{000 \dots 0} \quad (1 \leq i_1 \leq s_1 - 1, 1 \leq i_2 \leq s_2 - 1).$$

Similarly, one can define $\theta_{i_1 \dots i_n}$ for every $i_1 \dots i_n \neq 0 \dots 0$ ($0 \leq i_j \leq s_j - 1, 1 \leq j \leq n$) so that any such $\theta_{i_1 \dots i_n}$ represents a factorial effect as determined by its non-zero subscripts. The total number of parameters $\{\theta_{i_1 \dots i_n} (i_1 \dots i_n \neq 0 \dots 0)\}$ to be estimated are $v - 1$ and these are collectively referred to as the θ 's for ease in presentation. Banerjee and Mukerjee (2007) have obtained lower bound to the variance of the BLUE of $\theta_{i_1 \dots i_n}$ when number of arrays are equal to $v - 1$. A design which attains these lower bounds for each of θ 's, is called an optimal design. Banerjee and Mukerjee (2007) have also given a method of construction of w-optimal designs in $v - 1$ columns. If all main effects and interaction effects are of interest, then a design in $v - 1$ columns is a saturated design and leaves no error degree of freedom for estimation of experimental error or testing of hypothesis regarding parameters of interest. Therefore, it is required to generate a design in number of arrays $b > v - 1$. For $b > v - 1$, a new criterion of optimality viz. w-optimality was introduced for $s_1 \times s_2$ factorial. A design $d \in D(s_1 \times s_2, b, 2)$ for $s_1 \times s_2$ factorial in given number of arrays, b , is said to be w-optimal, if it minimizes

$$T1 = \sum_{i_1=1}^{s_1-1} \text{var}(\hat{\theta}_{i_1 0}) + \sum_{i_2=1}^{s_2-1} \text{var}(\hat{\theta}_{0 i_2}) + w \sum_{i_1=1}^{s_1-1} \sum_{i_2=1}^{s_2-1} \text{var}(\hat{\theta}_{i_1 i_2}) \quad (1.3.1)$$

One approach to get a w-optimal design is to generate all possible $\binom{v(v-1)/2}{b}$ designs and select the design with minimum value of $T1$. The optimal design may not be unique. Another approach suggested by Banerjee and Mukerjee (2007) is to (i) generate all optimal saturated design in

$b = v - 1$ columns, (ii) given b , augment each optimal design in (i) in all possible ways to generate design with b columns, and (iii) select one design as per chosen optimality criterion $T1$ in (1.3.1). Using this approach, they have suggested the procedure of augmenting up to $b = (v - 1) + (s_1 - 1)(s_2 - 1)$ columns, *i.e.* adding any number of columns from 1 to $(s_1 - 1)(s_2 - 1)$ conjectured that the design obtained is w -optimal any $w \geq 1$.

Mukerjee and Tang (2011) have discussed two level fractional factorial plans under baseline parametrization using the criterion of minimum aberration.

Dash (2012) gave a procedure of obtaining efficient block designs in block size 2 for 3-factor mixed level factorial microarray experiments based on baseline parameterization for $v - 1 \leq b \leq (v - 1) + (s_1 - 1)(s_2 - 1) + (s_1 - 1)(s_3 - 1) + (s_2 - 1)(s_3 - 1) + 2((s_1 - 1)(s_2 - 1)(s_3 - 1))$. A software module was also developed using C# programming language with ASP.NET platform for generation of efficient block designs in block size 2 for $s_1 \times s_2 \times \dots \times s_n$ factorial experiments in $v - 1$ columns, where s_j denotes the number of levels of j^{th} factor and n denotes the number of factors and $v = \prod_{j=1}^n s_j$, the total number of treatment combinations. Web application is made available at Design Resources Server (www.iasri.res.in/dbp). For $n = 2$, the software developed can also generate efficient block designs for $v - 1 \leq b \leq (v - 1) + (s_1 - 1)(s_2 - 1)$, where b is the number of columns.

1.4 Motivation

As mentioned in Section 1.2 that Dash *et al.* (2013) suggested a procedure for obtaining row-column designs having unequal replication of different treatment combinations for orthogonal estimation of all main effects and two factor interactions in fewer number of columns than the number of columns required for row-column designs with minimum number of replications for orthogonal estimation of all main effects and two factor interactions with equal replication for all treatment combinations through an example of 2^4 factorial experiment. No general procedure of obtaining such designs for any number of factors was given. Therefore, more efforts are required to develop a general procedure of obtaining row-column designs in two rows having unequal replications than suggested by Kerr(2006) for equireplicated designs and prepare a catalogue of designs to serve as ready reckoner for the experimenters and make them available online.

Further, the problem of obtaining block designs with block size two for factorial experiments, in which one level of each factor represent natural baseline, has been studied in the literature. Dash (2012) developed a software module using C# programming language with ASP.NET platform for generation of optimal block designs (the design which attains the lower bound to the variance of each of the estimated factorial effects) for mixed factorial experiments in number of arrays equal to one less than the number of treatment combinations. For two-factor factorial experiments, the software developed can also generate efficient block designs with number of arrays equal to the pairs of treatment combinations with their corresponding baseline treatment combination. Here efficiency is defined as the ratio of sum of the variances of all factorial effects of interest of the given design to that of the design with minimum value of sum of the variances of all factorial effects in the class of row-column designs with given number of levels of two factors, given number of columns and two rows. A procedure of obtaining efficient block designs for 3-factor mixed level factorial microarray experiments based on baseline parameterization has been given by Dash (2012). Further, whether it is possible to develop a method of construction of efficient designs in number of arrays equal to or more than the number of treatment combinations for n - factor mixed level factorial experiments ($n \geq 4$) needs to be explored. Further, these designs with baseline parameterization in literature have been obtained under the assumption of absence of row effects. In the presence of row effects, it is required to obtain row-column designs. Therefore, it is required to see whether it is possible to get a row-column design in same or few extra columns as that of a block design and still the treatment combinations are orthogonal with respect to rows. Further, a catalogue of designs obtained need to be made available online through web application. Keeping in view the above, the following objectives have been framed.

1.5 Objectives

1. To obtain row-column designs for 2-level factorial experiments for estimating the main effects and two factor interactions in fewer number of runs based on orthogonal parameterization.
2. To develop general methods of construction of block/row column designs for factorial experiments based on baseline parameterization.

1.6 Scope of the present study

The results obtained in the present investigation have been presented in Chapter II and Chapter III. In the Chapter II, a general method of construction of row-column designs with two rows in

unequal replication of treatment combinations, which permit orthogonal estimation of all main effects and two factor interactions in factorial experiments and at the same time has fewer number of runs (or design points) than required by a row-column design with equal replication given by Kerr (2006) has been given. A catalogue of efficient row-column designs in two rows for 2^n ($2 \leq n \leq 9$) factorial experiments in fewer number of columns is also presented in Appendix I. Here in all the designs main effects and two factor interaction are estimated orthogonally and the treatment combinations have unequal replications. A web application of generation of these designs has also been developed and presented in Chapter 2.

The whole discussion revolves around two-level factorial experiments, the methods of construction for generation of confounded row-column designs in two rows for asymmetrical factorial experiments for estimation of all main effects and two factor interactions may be taken care of for further study.

The general procedure developed of obtaining w -efficient designs in two rows for n -factors mixed level factorial experiments based on baseline parameterization is presented in Chapter 3. The maximum number of blocks that can be accommodated for an n -factor mixed level factorial

experiment are $v - 1 \leq b \leq (v - 1) + \sum_{j=2}^n \left\{ (j - 1) \sum_{i_1 \neq i_2 \neq \dots \neq i_j = 1}^n (s_{i_1} - 1)(s_{i_2} - 1) \dots (s_{i_j} - 1) \right\}$. Here v is the total

number of treatment combinations, b is the number of blocks/columns. A catalogue of w -efficient block/row-column designs in two rows for n -factors mixed level factorial experiments based on baseline parameterization has also been prepared and presented in appendix II and appendix III. To make these designs available through online a web application is also developed.

**EFFICIENT ROW COLUMN DESIGNS FOR FACTORIAL
EXPERIMENTS WITH ORTHOGONAL PARAMETRIZATION**

2.1 Introduction

The purpose of this chapter is to propose a procedure of construction of row-column designs in two rows for estimation of main effects and two factor interaction effects in 2^n factorial experiments based on orthogonal parameterization in fewer number of columns than the number of columns required for row-column designs with minimum number of replications for orthogonal estimation of all main effects and two factor interactions with equal replication given by Kerr (2006) and Dash et al. (2013). The designs obtained have unequal number of replications for treatment combinations. The method of construction developed is given in Section 2.2. In Section 2.3, the procedure of construction of designs has been illustrated through an example. A catalogue of row-column designs in two rows for 2^n ($2 \leq n \leq 9$) factorial experiments is given in Appendix I. Web application developed for online generation of row-column designs with unequal replication of treatment combinations has been described in Section 2.4. A general discussion and future scope is given in Section 2.5.

2.2 Construction of row-column designs for factorial experiments with two rows

In this section, a method of construction of row-column designs with two rows for orthogonal estimation of main effects and two factor interaction effects in 2^n factorial experiments has been developed. In this method, we propose to add some extra columns, in the row-column design in two rows for 2^n factorial experiments in $[\log_2 n]$ replications using the procedure of Yang and Draper (2003) and Kerr(2006){block designs in block size 2} and Dash et al. (2013) {for row-column designs in two rows}. Therefore, the number of replications of some of the treatment combinations would be $[\log_2 n]$ and the replications for treatment combinations added to the design in $[\log_2 n]$ replications would be $[\log_2 n]+2$ but the total number of columns would always be less than the number of columns required for $[\log_2 n]+1$ replications as required by the block designs obtained by Kerr (2006) and row-column designs obtained by Dash et al. (2013). Here $[\cdot]$ denotes greatest integer function and n denotes the number of factors each at

level 2. The advantage in saving the number of experimental is purely due to the use of unequal replications. The steps of method of construction of are given in the sequel.

Step 1: Obtain a block design with block size 2 for a 2^n factorial experiment represented as $(2^n, 2)$ as given by Yang and Draper (2003) and Dash et al. (2013). For obtaining this design, total number of treatment combinations = 2^n , number of blocks of size two per replication = 2^{n-1} , total number of factorial effects confounded = $2^{n-1} - 1$, number of independent factorial effects confounded = $n-1$.

These $n-1$ independent factorial effects to be confounded may have all main effects, all two factor interactions or some main effects and some two factor interactions. Depending upon the number of main effects and two factor interactions in $n-1$ independent factorial effects to be confounded give rise to n different blocking types. Consider the blocking types confounding any of $n - 2$ main effects out of all n main effects in a single replication and combining them with a two factor interaction involving remaining 2 factors (Dash et al. (2013)). The total number of replications required is $\lceil \log_2 n \rceil$ and for each replication a different combination of $n - 2$ main effects and a two factor interaction involving remaining 2 factors should be selected.

Step 2: Following the procedure of Dash et al. (2013), convert the block design in block size 2 obtained in Step 1 into a row-column design in two rows by considering columns as blocks and units in columns as rows by rearranging the treatment combinations in such a way that the treatment combinations become most balanced with respect to rows. For achieving this, one may make use of Lemma 2.1 of Choi and Gupta (2008). Lemma is discussed in the sequel. Consider a symmetrical factorial experiment conducted using a row-column design with row and column sizes less than the number of treatment combinations and the confounding being done using classical method of confounding. Let $D_R[D_C]$ respectively, denote the block designs obtained ignoring column [row] classification and the confounding done in such a way that the factorial effect which is confounded in D_R is unconfounded in D_C and vice-versa. Then the factorial effects which are unconfounded in both D_R and D_C remain unconfounded in row-column design as well. Further, the factorial effects which are confounded separately for D_R and D_C , are also confounded in row-column design.

Following the above, Dash et al. (2013) suggested identifying factorial effects (possibly higher order interactions) which are unconfounded in all the replications of block design obtained in

Step 1. Now confound this factorial effect say a g -factor interaction with row-component design ($1 \leq g \leq n$). To achieve this, let the design obtained in Steps 1 be D_{CU} , the column component design before rearranging into row-column set up. Now arrange the treatment combinations in columns of D_{CU} in each replication in such a way that the treatment combinations in two rows represent the two blocks in which the identified g -factor interaction is confounded. Now rearrange the rows in replication number s [t] of D_{CU} , in such a way that in row 1, the sum of the levels of the factors involved in g -factor interaction is 0 [1] and in row 2 this sum is 1 [0] respectively, where $s = 1, 3, \dots, r$ (if r is odd) and $s = 1, 3, \dots, r-1$ (if r is even) and $t = 2, 4, \dots, r-1$ (if r is odd) and $t = 2, 4, \dots, r$ (if r is even). Now juxtaposing the columns of replications of D_{CU} obtained after the above rearrangement, we get a row-column design $D(v, b, k)$, where $v = 2^n$, b (number of columns) $= r2^{n-1}$ and k (number of rows) $= 2$. In the row-column design so obtained, the factorial effect confounded with rows of each replication of D_{CU} also becomes unconfounded. If r is even, the factorial effect confounded with rows in each replication of D_{CU} can be estimated free from row-effects in D and if r is odd the factorial effect confounded with rows in each replication of D_{CU} can be estimated after adjustment of row-effects in D . The factorial effects that can be confounded with rows are given in boldface type in Appendix I.

The above two steps were similar as those given by Dash et al. (2013). Now in nexts we describe the procedure of identifications of columns to be added to ensure orthogonal estimation of all main effects and two factor interactions.

Step 3: Identify the main effects and two factor interactions confounded in all $[\log_2 n]$ replications of the row-column design obtained in Step 2. Then to estimate these main effects and two factor interaction effects, one is required to add two blocks and then swap the pairs for balancing with to rows, i.e. for each factorial effect, one has to add 4 columns. If there are k main effects and interactions confounded in the design obtained in Step 2, then one has to add $4k$ columns in the design for the orthogonal estimation of all main effects and two factor interactions. It can easily be seen that $k < 2^{n-3}$. Now the question is which columns to be added for estimation of a given confounded factorial effect. To answer, this question, use the following procedure:

- i) Generate a block design in block size 2, using blocking arrangement by confounding all two factor interactions of $n-1$ factors with n^{th} factor, i.e. n^{th} is common in all $n-1$ two factor interactions. It gives 2^{n-1} blocks each of size 2. n^{th} factor need not be last factor and

may be chosen in such a way that two factor interaction with this factor to be confounded is not the confounded two factor interaction as identified above.

- ii) Group 2^{n-1} blocks in 2^{n-2} sets of blocks such that within a set, keeping the levels of factors involved in the factorial effect to be estimated, there is change in the levels of maximum number of factors.
- iii) Select any one set of two blocks out of 2^{n-2} sets of blocks and add these as columns and swap the positions of the treatment combinations in rows for each of the two columns added. In this way, four new columns have been added for estimating, one factorial effect (main effect or two factor interaction) which was confounded in row-column design obtained in step 2.
- iv) Repeat this (ii) and (iii), for each of the k confounded factorial effects in row-column design obtained in step 2.

We get a design in $[\log_2 n] \cdot 2^{n-1} + 4k$ columns. Hence, we can obtain the row column design in two rows for orthogonal estimation of all main effects and two factor interactions after adding the required number of column obtained by the step 4 in the design obtained in step 2.

2.3 Illustration

In this section, we illustrate, the method of construction given in Section 2.2 for constructing a row-column design in two rows for a 2^5 factorial experiment for orthogonal estimation of all main effects and two factor interactions.

Step 1: Obtain a block design with block size 2 in $[\log_2 5] = 2$ replications. Select any $[\log_2 5] = 2$ combinations of $n - 2 = 3$ main effects and a two factor interaction involving remaining 2 factors, out of a total of $\binom{5}{3} = 10$ combinations. The total of 10 such combinations are:

- Combination 1: 1,2,3,4,5; Combination 2: 1,2,4,3,5; Combination 3: 1,2,5,3,4;
- Combination 4: 1,3,4,2,5; Combination 5: 1,3,5,2,4; Combination 6: 1,4,5,2,3;
- Combination 7: 2,3,4,1,5; Combination 8: 2,3,5,1,4; Combination 9: 2,4,5,1,3;
- Combination 10: 3,4,5,1,2;

Select any two of these combinations, say Combination 1: 1,2,3,4,5 and Combination 2: 1,2,4,3,5. Using these combinations, generate, two replications of the block design in block size 2 as given below:

Replication 1:

Block	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16
	000 00	000 01	001 00	001 10	010 00	010 10	011 00	011 01	100 00	100 01	101 00	101 01	110 00	110 01	111 00	111 01
	000 11	000 10	001 11	001 01	010 11	010 01	011 11	011 10	100 11	100 10	101 11	101 10	110 11	110 10	111 11	111 10

Replication 2:

Column	17	18	19	20	21	22	23	24	25	26	27	28	29	30	31	32
Row1	000 00	000 01	000 10	001 10	010 00	011 00	010 10	010 11	100 00	100 01	100 10	100 11	110 00	110 01	110 10	110 11
Row2	001 01	001 00	001 11	000 11	011 01	010 01	011 11	011 10	101 01	101 00	101 11	101 10	111 01	111 00	111 11	111 10

Step 2: Identify the factorial effect that is unconfounded in both the replications. It can easily be seen that all four factor interactions are unconfounded with blocks in both the replications. Choose one of unconfounded factorial effects, say 1234. Now, rearrange the block contents in the above two replications of the block design by confounding 1234 in each of the replications. The row-column design in two rows obtained is

Replication 1:

Column	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16
Row1	000 00	000 01	001 11	001 10	010 11	010 10	011 00	011 01	100 11	100 10	101 00	101 01	110 00	110 01	111 11	111 10
Row2	000 11	000 10	001 00	001 01	010 00	010 01	011 11	011 10	100 00	100 01	101 11	101 10	110 11	110 10	111 00	111 01

Replication 2:

Column	17	18	19	20	21	22	23	24	25	26	27	28	29	30	31	32
Row1	000 00	000 01	001 11	001 10	011 01	011 00	010 10	010 11	101 01	101 00	100 10	100 11	110 00	110 01	111 11	111 10
Row2	001 01	001 00	000 10	000 11	010 00	010 01	011 11	011 10	100 00	100 01	101 11	101 10	111 01	111 00	110 10	110 11

Step 3: In the above design, main effect of factors 1, 2 and two factor interaction 12 is confounded in column effects, so these are not estimable. Therefore, we need to add 12 (4×3) columns in the above design in 32 columns to get a design in 44 columns whereas row-column

design in two rows with equal replication for orthogonal estimation of main effects and two factor interactions require 48 columns.

- i) Generate a block design in block size 2, using blocking arrangement by confounding 15, 25, 35, 45. The block contents of the design obtained are:

Block 1: (0 0 0 0 0, 1 1 1 1 1); Block 2: (0 0 0 1 0, 1 1 1 0 1);
 Block 3: (0 0 1 0 0, 1 1 0 1 1); Block 4: (0 0 1 1 0, 1 1 0 0 1);
 Block 5: (0 1 0 0 0, 1 0 1 1 1); Block 6: (0 1 0 1 0, 1 0 1 0 1);
 Block 7: (0 1 1 0 0, 1 0 0 1 1); Block 8: (0 1 1 1 0, 1 0 0 0 1);
 Block 9: (0 1 1 1 1, 1 0 0 0 0); Block 10: (0 1 1 0 1, 1 0 0 1 0);
 Block 11: (0 1 0 1 1, 1 0 1 0 0); Block 12: (0 1 0 0 1, 1 0 1 1 0);
 Block 13: (0 0 1 1 1, 1 1 0 0 0); Block 14: (0 0 1 0 1, 1 1 0 1 0);
 Block 15: (0 0 0 1 1, 1 1 1 0 0); Block 16: (0 0 0 0 1, 1 1 1 1 0).

- ii) Group these 16 blocks in 8 sets of blocks such that within a set, keeping the levels of factors involved in the factorial effect to be estimated, there is change in the levels of maximum number of factors. By keeping the levels of Factor 1 as fixed, the 8 sets of blocks are

For Factor 1: A

Set 1: (0 0 0 0 0, 1 1 1 1 1); (0 1 1 1 1, 1 0 0 0 0);
 Set 2: (0 0 0 1 0, 1 1 1 0 1); (0 1 1 0 1, 1 0 0 1 0);
 Set 3: (0 0 1 0 0, 1 1 0 1 1); (0 1 0 1 1, 1 0 1 0 0);
 Set 4: (0 0 1 1 0, 1 1 0 0 1); (0 1 0 0 1, 1 0 1 1 0);
 Set 5: (0 1 0 0 0, 1 0 1 1 1); (0 0 1 1 1, 1 1 0 0 0);
 Set 6: (0 1 0 1 0, 1 0 1 0 1); (0 0 1 0 1, 1 1 0 1 0);
 Set 7: (0 1 1 0 0, 1 0 0 1 1); (0 0 0 1 1, 1 1 1 0 0);
 Set 8: (0 1 1 1 0, 1 0 0 0 1); (0 0 0 0 1, 1 1 1 1 0).

Similarly 8 sets for factor 2 and Interaction 12 are:

For Factor 2: B

Set 1: (0 0 0 0 0, 1 1 1 1 1); (1 0 1 1 1, 0 1 0 0 0);
 Set 2: (0 0 0 1 0, 1 1 1 0 1); (1 0 1 0 1, 0 1 0 1 0);
 Set 3: (0 0 1 0 0, 1 1 0 1 1); (1 0 0 1 1, 0 1 1 0 0);
 Set 4: (0 0 1 1 0, 1 1 0 0 1); (1 0 0 0 1, 0 1 1 1 0);
 Set 5: (1 0 0 0 0, 0 1 1 1 1); (0 0 1 1 1, 1 1 0 0 0);
 Set 6: (1 0 0 1 0, 0 1 1 0 1); (0 0 1 0 1, 1 1 0 1 0);
 Set 7: (1 0 1 0 0, 0 1 0 1 1); (0 0 0 1 1, 1 1 1 0 0);
 Set 8: (1 0 1 1 0, 0 1 0 0 1); (0 0 0 0 1, 1 1 1 1 0).

For Interaction 12: C

Set 1: (0 0 0 0 0, 1 1 1 1 1); (0 0 1 1 1, 1 1 0 0 0);
 Set 2: (0 0 0 1 0, 1 1 1 0 1); (0 0 1 0 1, 1 1 0 1 0);
 Set 3: (0 0 1 0 0, 1 1 0 1 1); (0 0 0 1 1, 1 1 1 0 0);
 Set 4: (0 0 1 1 0, 1 1 0 0 1); (0 0 0 0 1, 1 1 1 1 0);
 Set 5: (0 1 0 0 0, 1 0 1 1 1); (0 1 1 1 1, 1 0 0 0 0);
 Set 6: (0 1 0 1 0, 1 0 1 0 1); (0 1 1 0 1, 1 0 0 1 0);
 Set 7: (0 1 1 0 0, 1 0 0 1 1); (0 1 0 1 1, 1 0 1 0 0);

Set 8: (0 1 1 1 0, 1 0 0 0 1); (0 1 0 0 1, 1 0 1 1 0);

iii) Select any one set of two blocks out of 8 sets of blocks for each of the factorial effects as main effect of factor 1 and factor 2 and Interaction of factors 1 and 2. Let us select Set 1 from each of the three sets A, B and C.

For Factor 1: {00000, 11111}, {01111, 10000};

For Factor 2: {00000, 11111}, {10111, 01000};

For Interaction 12: {00000, 11111}, {00111, 11000};

Now using these sets 1, 2, 3 and swapping the pairs of treatment combinations, in blocks, we get twelve new columns as

Columns→											
33	34	35	36	37	38	39	40	41	42	43	44
00000	01111	11111	10000	00000	10111	11111	01000	00000	00111	11111	11000
01111	00000	10000	11111	10111	00000	01000	11111	00111	00000	11000	11111

Affixing these 12 columns to the 32 columns of row-column design obtained in step 2, we get a design in 44 columns for orthogonal estimation of all main effects and two factor interactions.

Renumber 32 treatment combinations written in lexicographic order to 1, 2, 3,..., 31, 32 respectively. Following the usual procedure of block designs in factorial treatment structure, the variances of all main effects (1, 2, 3, 4, 5) and two factor interactions (12, 13, 14, 15, 23, 24, 25, 34, 35, 45) are $148.33\sigma^2$, $148.33\sigma^2$, $31.33\sigma^2$, $34.00\sigma^2$, $17.33\sigma^2$, $177.33\sigma^2$, $31.33\sigma^2$, $34.00\sigma^2$, $17.33\sigma^2$, $31.33\sigma^2$, $34.00\sigma^2$, $17.33\sigma^2$, $17.33\sigma^2$, $34.00\sigma^2$ and $31.33\sigma^2$. These values are obtained by writing a SAS Code as given in Appendix IV.

The method of construction given in Section 2.2 is general in nature and can be used for obtaining row-column design in two rows for any 2^n factorial experiments. Using this method, a catalogue of row-column designs in two rows for 2^n ($2 \leq n \leq 9$) factorial experiments has been prepared to serve as a ready reckoner and is given in Appendix I.

2.4 Web Application

In this section, we describe a web application developed for online generation of row-column designs in two rows with unequal replication of treatment combinations for orthogonal estimation of main effects and two factor interactions for wider dissemination of the results

obtained. The application has been developed using C# and asp.net. Some screen shots are given below:

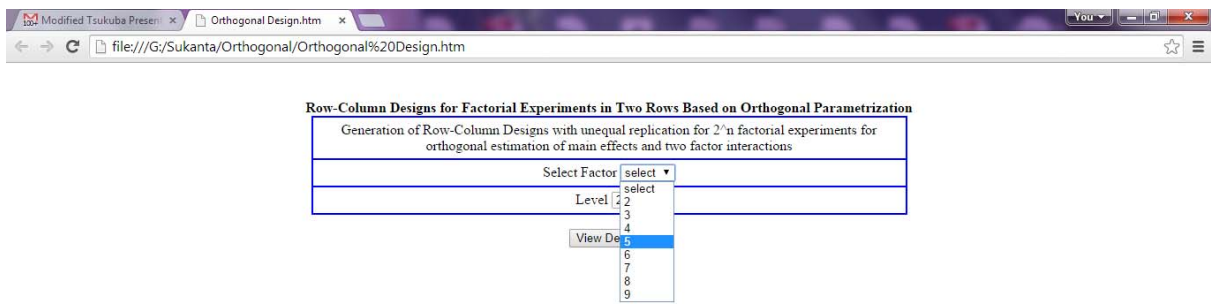


Figure 2.4.1: First Screen depicting Selection of number of factors

Column/Row	1	2	3	4	5	6	7	8	9
Row1	00000	00001	00111	00110	01011	01010	01100	01101	10011
Row2	00011	00010	00100	00101	01000	01001	01111	01110	10000

10	11	12	13	14	15	16	17	18	19
10010	10100	10101	11000	11001	11111	11110	00000	00001	00111
10001	10111	10110	11011	11010	11100	11101	00101	00100	00010

20	21	22	23	24	25	26	27	28	29
00110	01101	01100	01010	01011	10101	10100	10010	10011	11000
00011	01000	01001	01111	01110	10000	10001	10111	10110	11101

30	31	32	33	34	35	36	37	38	39
11001	11111	11110	00000	01111	11111	10000	00000	10111	11111
11100	11010	11011	01111	00000	10000	11111	10111	00000	01000

40	41	42	43	44
01000	00000	00111	11111	11000
11111	00111	00000	11000	11111

Figure 2.4.2: Row column designs with unequal replication for 2^5 factorial experiments for orthogonal estimation of main effects and two factor interactions

2.5 Discussion

In the present investigation, as per defined scope of the project, a general method of construction to obtain row-column designs for 2^n factorial experiments in two rows for the orthogonal estimation of main effects and two factor interactions has been developed. For further research investigation, the research work on methods of construction/computer algorithms for generation of confounded row-column designs in two rows for multi-level factorial experiments and mixed level factorial experiments for estimation of all main effects and two factor interaction needs to be taken up.

CHAPTER III

EFFICIENT ROW COLUMN DESIGNS FOR FACTORIAL EXPERIMENTS WITH BASELINE PARAMETERIZATION

3.1 Introduction

In factorial experiments discussed in Chapter II, main effects of factors and interactions among them are defined via orthogonal parameterization involving mutually orthogonal treatment contrasts. But there do occur experimental situations, wherein natural baseline or null state may exist and the experimenter is interested in baseline parametrization rather than orthogonal parametrization. For example, there may be tissues from two mutants, one of which proliferates a particular disease and other does not. Therefore, the mutant that does not proliferate into disease is baseline. In a toxicological study with binary factors, each representing the presence or absence of a particular toxin, the state of absence can be regarded as a natural baseline level of each factor. Null state or baseline of a factor need not strictly mean zero level on some scale, but may as well refer to a standard or control level like the one currently being used in practice. Such experimental situations involving the control or standard treatment (natural baseline) do occur even beyond the domain of agricultural experiments. In multi-factor experiments in which one of the levels of each factor is a natural baseline, the comparisons with the baseline are of importance rather than the usual main effects and interactions. To make the exposition clearer, consider a 2-colour microarray experiment in which it is desired to compare two cell lines FIA and V449E at times zero hour and 24 hours {see e.g. Glonek and Solomon (2004)}. The cell line V449E proliferates into leukaemia while FIA is non-leukaemic. Therefore, there are two factors dictating the cell populations. The first factor (F_1), namely, mutant has two levels FIA and V449E of which FIA, being non-leukaemic, is baseline. The two levels of mutants may be coded as 0 and 1 respectively. The second factor (F_2) is time with two levels as 0 hours and 24 hours and first of these levels (0 hours) is baseline. These two levels are also coded as 0 and 1 respectively. Thus considering the two factors together, there are four treatment combinations, 00, 01, 10, 11 representing the cell population. Let $\tau_{00}, \tau_{01}, \tau_{10}, \tau_{11}$ denote the expected log intensities, that is, the effects of these treatment combinations.

Now the question “Are there any genes specific to V449E that result into leukaemic effects?” can be answered from the treatment contrast $\tau_{10} - \tau_{00}$. Further, the change in intensity of FIA (natural baseline of mutant) between zero and 24 hours can be estimated from the treatment contrast $\tau_{01} - \tau_{00}$. Further, the difference in FIA and V449E at time 24 hours can be estimated from the treatment contrast $\tau_{11} - \tau_{01}$. The difference between these two lines at 0 hours was estimated using $\tau_{10} - \tau_{00}$. The difference of the two $\{(\tau_{11} - \tau_{01}) - (\tau_{10} - \tau_{00})\}$ represents the

differential expression between the two cell lines that exists at 24 hours beyond what was present at time zero. Similarly this difference can also be estimated as $(\tau_{11} - \tau_{10}) - (\tau_{01} - \tau_{00})$.

Therefore, the inference is required on the three treatment contrasts viz.

$$\theta_{10} = \tau_{10} - \tau_{00};$$

$$\theta_{01} = \tau_{01} - \tau_{00} \text{ and}$$

$$\theta_{11} = \tau_{11} - \tau_{01} - \tau_{10} + \tau_{00}.$$

Here θ_{10} , θ_{01} and θ_{11} are baseline parameterization of parameters of main effect of F_1 , F_2 and interaction F_1F_2 respectively. However, if at least one factor, like gender, lacks a natural baseline, then the baseline parameterization is inappropriate because this will arbitrarily single out one level of such a factor. In such situations, it is advisable to use the orthogonal parameterization.

The corresponding treatment contrasts of main effect F_1 , main effect F_2 and interaction effect F_1F_2 are

$$\theta_{10}^* = (\tau_{11} - \tau_{01} + \tau_{10} - \tau_{00}) / 2,$$

$$\theta_{01}^* = (\tau_{11} + \tau_{01} - \tau_{10} - \tau_{00}) / 2 \text{ and}$$

$$\theta_{11}^* = (\tau_{11} - \tau_{01} - \tau_{10} + \tau_{00}) / 2$$

From the above, it is clear that the definitions of main effects under the two parameterizations are entirely different. While θ_{11} is proportional to θ_{11}^* , this equivalence for two factor interaction also disappears in case of experiments involving more than two factors.

The main distinction between these two kinds of parameterization is that while the orthogonal parameterization defines the factorial effects via mutually orthogonal treatment contrasts, the baseline parameterization defines these effects with reference to natural baseline levels of the factors and, hence, entails non-orthogonality.

Glonek and Solomon (2004) were the first to study designs for multi-factor experiments under baseline parameterization. They have introduced a criterion of statistical efficiency in terms of variances of the estimated parameters of interest. For given number of columns (arrays in two colour microarray experiments), b , a design is said to be admissible if the variance of each of the estimated parameters of interest is less than or equal to the variance of the estimated parameters of interest through any other design in same number of columns and strict inequality holds for at

least one parameter. This criterion was illustrated in obtaining efficient designs for 2^2 factorial experiments for given number of columns. They have also illustrated the utility of admissible criterion for 2×3 factorial experiments

As discussed in Section 1.4 of Chapter I, Dash (2012) developed a software module using C# programming language with ASP.NET platform for generation of optimal block designs (the design which attains the lower bound to the variance of each of the estimated factorial effects) for mixed factorial experiments in number of columns equal to one less than the number of treatment combinations. A procedure of obtaining efficient block designs for 3-factor mixed level factorial experiments based on baseline parameterization has been given by Dash (2012). The w -optimality criteria given by Banerjee and Mukerjee (2007) as extended for 3-factors by Dash (2012) is given as: a design $d \in D(s_1 \times s_2 \times s_3, b, 2)$ for $s_1 \times s_2 \times s_3$ factorial in given number of columns, b , is said to be ω -optimal for main effects and two factor interactions, if it minimizes

$$T2 = \sum_{i_1=1}^{s_1-1} \text{var}(\hat{\theta}_{i_1 00}) + \sum_{i_2=1}^{s_2-1} \text{var}(\hat{\theta}_{0i_2 0}) + \sum_{i_3=1}^{s_3-1} \text{var}(\hat{\theta}_{00i_3}) \\ + \omega \left\{ \sum_{i_1=1}^{s_1-1} \sum_{i_2=1}^{s_2-1} \text{var}(\hat{\theta}_{i_1 i_2 0}) + \sum_{i_1=1}^{s_1-1} \sum_{i_3=1}^{s_3-1} \text{var}(\hat{\theta}_{i_1 0 i_3}) + \sum_{i_2=1}^{s_2-1} \sum_{i_3=1}^{s_3-1} \text{var}(\hat{\theta}_{0 i_2 i_3}) \right\} \quad (3.1.1)$$

The efficiency of the block designs with $b > v - 1$, can be obtained by the ratio of criterion T2 of the design obtained to that of the design with minimum value of T2 obtained through generating all possible $\binom{v(v-1)/2}{b}$ designs.

In the present investigation a general method of construction of obtaining w -efficient block design in block size 2 for mixed factorial experiments for number of columns has been developed as discussed in Section 3.2. This procedure can be used to obtain a block design in any number of blocks satisfying the inequality

$$v - 1 \leq b \leq (v - 1) + \sum_{j=2}^n \left\{ (j - 1) \sum_{i_1 \neq i_2 \neq \dots \neq i_j = 1}^n (s_{i_1} - 1)(s_{i_2} - 1) \dots (s_{i_j} - 1) \right\}. \quad \text{Here } v \text{ is the total number of}$$

treatment combinations, b is the number of blocks/columns.

Further, such designs may also be required for experimental situations requiring elimination of two-way heterogeneity settings as explained in Section 1.1 of Chapter I. Therefore, efforts have been made to convert the block designs obtainable from the procedure given in Section 3.2, into row-column designs in two rows with number of columns equal to or more than the number of treatment combinations for n - factor mixed level factorial experiments. The procedure has been

given in Section 3.3. Web application developed for online generation of block designs with block size 2 and row column designs in two rows for mixed level factorial experiments has been described in Section 3.4. A general discussion and future scope is given in Section 3.5.

3.2 Construction of generation of w -efficient block designs

In this section, we have used the w -optimality criteria extended for n -factor mixed level factorial experiments has been used. As per this criterion, a design is said to be w -optimal if it minimizes the sum of variances of all the parameters related to main effects and weighted variance of all the parameters related to two factor interactions in a given class of designs. A general procedure of obtaining a w -efficient block designs with number of blocks satisfying the inequality

$$v-1 \leq b \leq (v-1) + \sum_{j=2}^n \left\{ (j-1) \sum_{i_1 \neq i_2 \neq \dots \neq i_j=1}^n (s_{i_1}-1)(s_{i_2}-1) \dots (s_{i_j}-1) \right\},$$

where v , b , and n are number of treatment combinations, columns and factors respectively has been given in this section. Following, Banerjee and Mukerjee (2007) it can easily be seen that the block designs in block size 2 with number of blocks as $v-1$ are w -optimal and further, it is conjectured that the block designs with number of blocks $b \geq v$ are w -efficient. This has been illustrated through two examples for 2×3 and $2 \times 2 \times 3$.

We begin with some preliminaries required for generation of designs with baseline parameterization.

In a block design with block size 2, only two treatment combinations can be accommodated on each block. Let the treatment combinations $i_1 \dots i_n$ and $j_1 \dots j_n$ be associated with two positions in a block. Without loss of generality, let treatment combinations $i_1 \dots i_n$ is given to experimental unit 1 and $j_1 \dots j_n$ to experimental unit 2. Then a block design is denoted by a pair $(i_1 \dots i_n, j_1 \dots j_n)$. A design is represented by a collection of such pairs. Baseline of treatment combination $i_1 \dots i_n \neq 0 \dots 0$ $\rho(i_1 \dots i_n) = j_1 \dots j_n$ is obtained by replacing non-zero level of any factor by zero level and leaving the level of other factors unchanged. The procedure of obtaining the design is as follows:

Steps of method of Construction:

Case I: When $b = v - 1$

Step 1: Write all possible treatment combinations excluding the control treatment 00...0 in lexicographic order.

Step 2: Obtain baseline of each treatment combination by replacing the first non-zero level by zero and keeping levels of other factors unchanged.

Step 3: Keep all treatment combinations obtained in step 1 in lexicographic order in experimental unit 1 and corresponding baseline treatment combination in experimental unit 2 in a block.

Following, Banerjee and Mukerjee (2007), it yields a w -optimal saturated design in $b = v - 1$ blocks.

Example 1: For a 2×3 factorial experiment, block design in 5 blocks is

Block	1	2	3	4	5
Unit 1	01	02	10	11	12
Unit 2	00	00	00	01	02

Case II: When $(v - 1) < b \leq (v - 1) + \sum_{j=2}^n \left\{ (j - 1) \prod_{i_1 \neq i_2 \neq \dots \neq i_j = 1}^n (s_{i_1} - 1)(s_{i_2} - 1) \dots (s_{i_j} - 1) \right\}$

Step 1: Search for the first treatment combination having two non-zero levels of n factors from the first unit in the blocks of the block design obtained in Step 1 of Case I.

Step 2: Now identify its baseline treatment combination by replacing the second non-zero level by zero and keeping the levels of other factor unchanged.

Step 3: Add a block containing the treatment combination in Step 2 of Case II in experimental unit 1 and its baseline treatment combination in experimental unit 2 to the block design with $b = v - 1$ obtained in Case I.

This yields a block design in $b = v$ blocks.

Block	1	2	3	4	5	6
Unit 1	01	02	10	11	12	11
Unit 2	00	00	00	01	02	10

Step 4: For getting a design in $b \geq v+1$ blocks, repeat steps 1 to 3 of case II till the search complete for the factor with two non-zero level.

Block	1	2	3	4	5	6	7
Unit 1	01	02	10	11	12	11	12
Unit 2	00	00	00	01	02	10	10

Step 5: Then search the treatment combination having three non-zero levels of n factors in first row of the design obtained in Case I.

Step 6: Keep those treatment combinations in the experimental unit 1 and its baseline i.e by changing its second nonzero level by zero level and then third nonzero level by zero level in experimental unit 2 of the added block of the design obtained after Step 3 of Case II.

Step 7: Similarly repeat the process for all the treatment combination having three non-zero levels of n factors in first row of the design obtained in Case I. Now repeat the process of Step 5 and 6 for all other treatment combinations having non-zero levels of $n \geq 4$ factors. Completion of

this process, yield a block design in $b = (v-1) + \sum_{j=2}^n \left\{ (j-1) \sum_{i_1 \neq i_2 \neq \dots \neq i_j=1}^n (s_{i_1} - 1)(s_{i_2} - 1) \dots (s_{i_j} - 1) \right\}$ blocks.

As mentioned earlier, one approach to get a w -optimal design is to generate all possible

$\left((v-1) + \sum_{j=2}^n \left\{ (j-1) \sum_{i_1 \neq i_2 \neq \dots \neq i_j=1}^n (s_{i_1} - 1)(s_{i_2} - 1) \dots (s_{i_j} - 1) \right\} \right)$ designs and select the design with minimum

value of T_2 in (3.1.1). Another approach is to generate optimal saturated design in $b = v-1$ columns, and for given b , either augment each design in all possible ways to generate design with b columns, and select one design as per chosen optimality criterion T_2 in (3.1.1). We have followed the second approach.

In case of 2×3 factorial experiment in example 1, to get a design in $b=6$ blocks, we augmented the the design in $b=5$ blocks added with the block contents (11, 10) and (12, 10) one by one. It is observed that the minimum value of $T_2 = 6$ was obtained when the block with contents (11, 10) was added. For getting the design for $b=7$, the only block remaining is (12, 10) and the

design was obtained by augmenting this block to the design in 6 blocks and value of T_2 for this design for $w=1$ is 4.9333.

Similarly a w -optimal design with $b=11$ blocks for a $2 \times 2 \times 3$ factorial experiment is obtained as per procedure of Case I. For getting the design in $b= 12,13,14,15,16,17,18,19,20$ blocks, the value of T_2 is minimum when the added block is per procedure of Case II. The minimum values of T_2 for $b= 12,13,14,15,16,17,18,19,20$ are 10.000, 8.522, 7.529, 7.066, 6.264, 6.812, 5.325, 4.871 and 4.703 respectively.

Using the above, it is conjectured that the designs obtained using Steps of Case I and case II are w -efficient.

Using the above procedure, a catalogue of w -efficient block designs for mixed level (number of levels: 2, 3 and 4; number of factors: 2 and 3) factorial experiments based on baseline parameterization has been developed and given in Appendix II.

3.3 Row-column design for baseline parametrization

In section 3.2, we have obtained efficient block designs in blocks of size 2 for estimation of parameters of main effects and two factor interactions under baseline parameterization. As in two-colour microarray experiments and other experimental situations described in Section 3.1, designs for multi-factor experiments under baseline parametrization are also required for two-way elimination of heterogeneity settings. Therefore, in this section, we have made an attempt to obtain a row-column design in two rows from the block designs obtained in Section 3.2 by rearranging the treatment combinations in two positions such that the each treatment combination appears equally frequently in each of the two rows of the design. Efforts have been made to make this arrangement in the same number of columns as the number of blocks in block design. The design so obtained possesses the same characterization properties and efficiency as that of block designs as treatment versus rows classification is orthogonal. However, for all parameteric combinations, it may not be possible to obtain a row-column design which is balanced with respect to rows in the same number of columns as that of block design. For obtaining a row-column design for such situations, the efforts have been made to get the balance with respect to rows for maximum number of columns and for the remaining columns, the simplest approach of swapping the positions of treatment combinations. As a consequence from a block consisting of the treatment combinations in the ordered pair as $(i_1 \cdots i_n, j_1 \cdots j_n)$. This ordered pair is taken as one column of row-column design and add one more column with

the order pair $(j_1 \cdots j_n, i_1 \cdots i_n)$. Following this procedure, the number of columns in the row-column design are more than the number of blocks in the block design obtained in Section 3.2. This is illustrated through some examples:

Example 3.3.1: For 2×2 factorial experiment, after swapping the positions of treatment combinations in the block design the row-column design in $b=4$ blocks is

Column	1	2	3	4
	01	00	11	10
	00	10	01	11

Example 3.3.2: For 3×2 factorial experiment, after swapping the positions of treatment combinations in the block design with 7 block we get:

Column	1	2	3	4	5	6	7
Row 1	01	00	11	00	21	10	20
Row 2	00	10	01	20	01	11	21

In this design 01 is appearing once in row 1 and twice in row 2 where as 00 is appearing twice in row 1 and once in row 2. Rest of treatment combinations are appearing equally frequently in two rows. Therefore, by adding a column with 01 in row 1 and 00 in column 2, we get a design in 8 columns. The final design is

Column	1	2	3	4	5	6	7	8
Row 1	01	00	11	00	21	10	20	01
Row 2	00	10	01	20	01	11	21	00

Example 3.3.3: As per procedure of Section 3.2, the block design for $2 \times 2 \times 3$ factorial experiment with baseline parameterization with $b = 20$ is

Block	1	2	3	4	5	6	7	8	9	10	11
Unit1	001	002	010	011	012	100	101	102	110	111	112
Unit2	000	000	000	001	002	000	001	002	010	011	012

Block	12	13	14	15	16	17	18	19	20
Unit1	011	012	101	102	110	111	111	112	112
Unit2	010	010	100	100	100	101	110	102	110

By rearranging the position of treatment combinations and swapping the positions for some of the blocks, the row-column design which is balanced with respect to rows is:

Column	1	2	3	4	5	6	7	8	9	10
Row 1	001	000	010	011	012	000	101	002	110	111
Row 2	000	002	000	001	002	100	001	102	010	011
Column	11	12	13	14	15	16	17	18	19	20
Row 1	112	011	010	100	100	110	101	102	111	112
Row 2	012	010	012	101	102	100	111	112	110	110
Column	21	22	23	24	25	26	27	28	29	
Row 1	002	001	102	011	012	010	100	101	001	
Row 2	012	101	100	111	010	011	101	001	011	

Using the above procedure, a catalogue of w -efficient row-column designs in two rows for mixed level (number of levels: 2 and 3; number of factors: 2 and 3) factorial experiments based on baseline parameterization has been developed and given in Appendix III.

3.4 Web application

In this section, we describe a web application developed for online generation of block designs with block size 2 and row column designs in two rows for mixed level factorial experiments based on baseline parameterization for wider dissemination of the results obtained. The application has been developed using C# and asp.net. Some screen shots are given below:

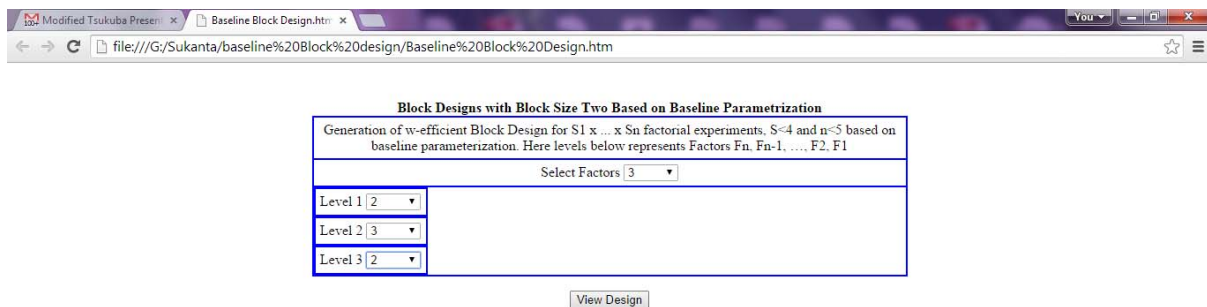


Figure 3.4.1: First Screen depicting Selection of number of factors and their levels

Column	1	2	3	4	5	6	7	8	9	10	11
	001	010	011	020	021	100	101	110	111	120	121
	000	000	001	000	001	000	001	010	011	020	021
	12	13	14	15	16	17	18	19	20		
	011	021	101	110	120	111	111	121	121		
	010	020	100	100	100	101	110	101	120		

Figure 3.4.2: Block designs with block size two for $2 \times 3 \times 2$ factorial experiments in 11, 12, ... 20 blocks for estimation of factorial effects based on baseline parameterization

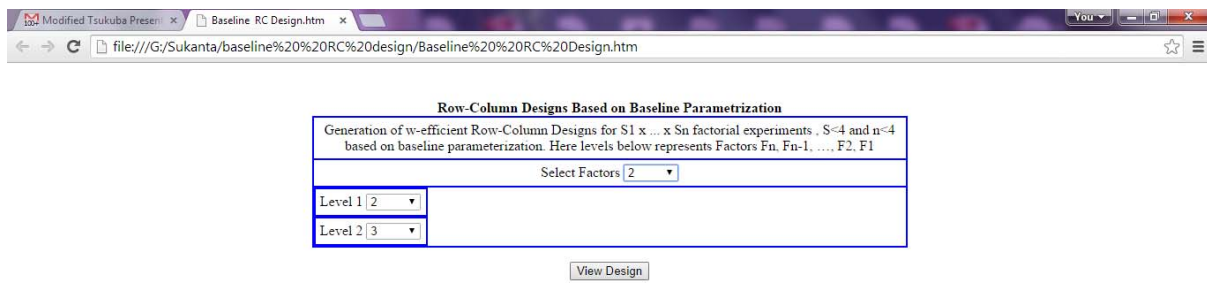


Fig 3.4.3: Selection of number of factors and their levels

Factorial experiments: 2x3

Column	1	2	3	4	5	6	7	8
	01	00	10	01	02	11	12	00
	00	02	00	11	12	10	10	01

Fig 3.4.4: Row-Column designs with two rows for 2×3 factorial experiments for estimation of factorial effects based on baseline parameterization

3.5 Discussion

In this Chapter, we have discussed a general procedure of obtaining efficient w -efficient block designs with block size two for mixed level factorial experiments with baseline parametrization. It has been conjectured that the block designs with $b \geq v$ blocks are w -efficient. However, a theoretical proof/sufficient conditions for w -optimality needs to be obtained further. A heuristic approach of obtaining row-column designs in two rows for mixed level factorial experiments based on baseline parameterization using the block designs. Efforts need to be made to convert this heuristic approach in a general procedure for obtaining efficient row-column designs in two rows for mixed level factorial experiments based on baseline parameterization.

SUMMARY

In a row-column design set up, because of practical considerations it may not be possible to accommodate more than two experimental units in a column. One application of row-column designs with two rows is in factorial experiments where the treatment structure is factorial in nature. Due to cost and time considerations, it may not be possible to run a design for estimation of all the factorial effects. The experimenter may, however, be interested in orthogonal estimation of all the main effects and two factor interactions. Thus it is required to obtain a general method of construction of row-column designs with two rows, which permit orthogonal estimation of all main effects and two factor interactions in factorial experiments and at the same time minimize the number of runs (or design points). To deal with such situations, a general method of construction of row-column designs with two rows for orthogonal estimation of main effects and two factor interactions in factorial experiments in minimum number of runs has been given for orthogonal parameterization. A catalogue of efficient row-column designs for 2^n ($2 \leq n \leq 9$) factorial experiments in minimum number of replications has been prepared. Here in all the designs main effects and two factor interaction are estimated orthogonally. A SAS program for checking the orthogonal estimation of main effects and two factor interactions has been prepared. A web application of generation of these designs has also been developed.

The above discussion relates to the factorial experiments run in block design or row-column design, where the interest of the experimenter is in orthogonal parameterization of the factorial effects. However, in some experimental situations, like designs for 2-colour micro-array experiments, where null state or baseline may exist, the experimenter would be interested in baseline parameterization rather than orthogonal parameterization. Since the designs obtained are in incomplete columns, it is important to study the efficiency of designs obtained. In other words, there is a need to obtain a general procedure of generating w-efficient row-column designs in two rows for n -factors mixed level factorial experiments based on baseline parameterization. To deal with such situations, a general procedure of obtaining w-efficient row-column designs in two rows for n -factors mixed level factorial experiments based on baseline parameterization has also been developed. A catalogue of w-efficient row-column designs in two rows for n -factors mixed level factorial experiments based on baseline parameterization has been prepared. To make these designs available through online a web application has also been developed.

सारांश

पंक्ति-स्तम्भ अभिकल्पना सेटअप में, व्यावहारिक कारणों से, हो सकता है कि एक कॉलम में दो से अधिक परीक्षणात्मक इकाइयों को समायोजित करना सम्भव न हो। दो पंक्तियों वाली पंक्ति-स्तम्भ अभिकल्पनाओं का एक उपयोग बहुउपादानी परीक्षणों में है जहाँ ट्रीटमेन्ट संरचना बहुउपादानी प्रकृति की होती हैं। मूल्य एवं समय के कारण हो सकता है कि सभी बहुउपादानी प्रभावों के आकलन के लिए अभिकल्पना संचालित करना सम्भव न हो एवं परीक्षणकर्ता की रुचि सभी मुख्य प्रभावों एवं द्वि-कारक अन्योन्य क्रियाओं के आर्थोगोनल आकलन में हो। अतः, दो पंक्ति वाली पंक्ति-स्तम्भ अभिकल्पनाओं के संरचना की एक सामान्य विधि प्राप्त करने की आवश्यकता है, जो बहुउपादानी परीक्षणों में समस्त मुख्य प्रभावों एवं द्वि-कारक अन्योन्य क्रिया के आर्थोगोनल आकलन करता हो, साथ ही, रन की संख्या (अथवा अभिकल्पना अंक) कम हो। ऐसी स्थितियों से निपटने के लिए, रनों की कम संख्या में बहुउपादानी परीक्षणों में मुख्य प्रभावों एवं द्वि-कारक अन्योन्य क्रिया के आर्थोगोनल आकलन के लिए दो पंक्तियों वाली पंक्ति-स्तम्भ अभिकल्पनाओं की संरचना एक सामान्य विधि आर्थोगोनल प्राचलीकरण के लिए उपलब्ध करायी गयी है। 2^n ($2 \leq n \leq 9$) बहुउपादानी परीक्षणों के लिए दक्ष पंक्ति-स्तम्भ अभिकल्पनाओं की एक सूची तैयार की गयी है। यहाँ सभी अभिकल्पनाओं में मुख्य प्रभाव तथा द्वि-कारक अन्योन्य क्रियाओं आर्थोगोनल आकलन किया जा सकता है। मुख्य प्रभावों एवं द्वि-कारक अन्योन्य क्रियाओं के आर्थोगोनल आकलन की जाँच के लिए एक एस.ए.एस. प्रोग्राम तैयार किया गया है। लाम्बिक प्राचलीकरण पर आधारित बहु-उपादानी परीक्षणात्मक दो पंक्तियों वाले पंक्ति स्तम्भ अभिकल्पनाओं की संरचना के लिए एक वैब एप्लिकेशन भी विकसित की गई।

उपरोक्त विवरण ब्लॉक अभिकल्पना एवं पंक्ति-स्तम्भ अभिकल्पना में बहुउपादानी परीक्षणों के रन से सम्बन्धित है, जहाँ परीक्षणकर्ता की रुचि बहुउपादानी प्रभावों के आर्थोगोनल प्राचलीकरण में है। हालाँकि, कुछ परीक्षणात्मक परिस्थितियों में, जैसे 2-कलर माइक्रो-एरे परीक्षणों के लिए अभिकल्पनाएँ, जहाँ शून्य स्टेट अथवा बेसलाइन मौजूद हो सकती है, परीक्षणकर्ता की रुचि आर्थोगोनल प्राचलीकरण के स्थान पर बेसलाइन प्राचलीकरण में होगी। चूँकि प्राप्त अभिकल्पनाएँ अपूर्ण स्तम्भ हैं, अतः प्राप्त अभिकल्पनाओं की दक्षता का अध्ययन करना महत्वपूर्ण है। दूसरे शब्दों में, बेसलाइन प्राचलीकरण पर आधारित n -कारकों के मिश्रित स्तर हेतु बहुउपादानी परीक्षणों के लिए दो पंक्तियों में w -दक्ष पंक्ति स्तम्भ अभिकल्पनाएँ उत्पन्न करने के लिए एक आम प्रक्रिया प्राप्त करने की आवश्यकता है। ऐसी परिस्थितियों से निपटने के लिए बेसलाइन प्राचलीकरण पर आधारित n -कारकों के मिश्रित स्तर बहुउपादानी परीक्षणों के लिए दो पंक्तियों में w -दक्ष पंक्ति-स्तम्भ अभिकल्पनाएँ प्राप्त करने के लिए एक आम प्रक्रिया भी विकसित की गयी है। बेसलाइन प्राचलीकरण पर आधारित n -कारकों के मिश्रित स्तर बहुउपादानी परीक्षणों के लिए दो पंक्तियों में w -दक्ष पंक्ति-स्तम्भ अभिकल्पनाओं का एक सूची तैयार की गयी है। बेसलाइन प्राचलीकरण पर आधारित n -कारकों के मिश्रित स्तर बहुउपादानी परीक्षणात्मक दो पंक्तियों वाले पंक्ति स्तम्भ अभिकल्पनाओं की संरचना के लिए एक वैब एप्लिकेशन भी विकसित की गई।

Appendix

Appendix I: Catalogue of row-column designs for 2^n ($2 \leq n \leq 9$) factorial experiments in two rows for the orthogonal estimation of main effects and two factor interactions.

Number of factors=2, Number of columns=4

Column/rows	1	2	3	4
1	00	11	01	10
2	10	01	00	11

Number of factors	Replication number	Independent factorial effects confounded	Generalized factorial effects cofounded	Some Factorial Effects which are not confounded*	Some Factorial Effects which are confounded in all the replication
2	R ₁	1		2, 12	
	R ₂	2		1, 12	

Number of factors=3, Number of columns=8

Row/Col.	1	2	3	4	5	6	7	8
1	000	100	001	101	111	110	011	010
2	110	010	111	011	100	101	000	001

Number of factors	Replication number	Independent factorial effects confounded	Generalized factorial effects cofounded	Some Factorial Effects which are not confounded*	Some Factorial Effects which are confounded in all the replication
3	R ₁	1, 23	123	12, 13	
	R ₂	2, 13	123	12, 23	

Number of factors=4, Number of columns=20

Rows	1	2	3	4	5	6	7	8
1	0000	0001	0111	0110	1011	1010	1100	1101
2	0011	0010	0100	0101	1000	1001	1111	1110

9	10	11	12	13	14	15	16
0101	0100	0010	0011	1000	1001	1111	1110
0000	0001	0111	0110	1101	1100	1010	1011

17	18	19	20
0000	1111	0111	1000
1111	0000	1000	0111

Number of factors	Replication number	Independent factorial effects confounded	Generalized factorial effects cofounded	Some Factorial Effects(higher order interactions) which are not confounded*	Some main effects and two factor interactions which are confounded in all the replication
4	R ₁	1, 2, 34	1234	123, 124	1
	R ₂	1, 3, 24	1234	123, 134	

Number of factors=5, Number of columns=44

Column	1	2	3	4	5	6	7	8	9
Row1	00000	00001	00111	00110	01011	01010	01100	01101	10011
Row2	00011	00010	00100	00101	01000	01001	01111	01110	10000

10	11	12	13	14	15	16	17	18	19
10010	10100	10101	11000	11001	11111	11110	00000	00001	00111
10001	10111	10110	11011	11010	11100	11101	00101	00100	00010

20	21	22	23	24	25	26	27	28	29
00110	01101	01100	01010	01011	10101	10100	10010	10011	11000
00011	01000	01001	01111	01110	10000	10001	10111	10110	11101

30	31	32	33	34	35	36	37	38	39
11001	11111	11110	00000	01111	11111	10000	00000	10111	11111
11100	11010	11011	01111	00000	10000	11111	10111	00000	01000

40	41	42	43	44
01000	00000	00111	11111	11000
11111	00111	00000	11000	11111

Number of factors	Replication number	Independent factorial effects confounded	Generalized factorial effects cofounded	Some Factorial Effects(higher order interactions) which are not confounded*	Some main effects and two factor interactions which are confounded in all the replication
5	R ₁	1, 2, 3, 45	12345	1234, 1235	1, 2, 12
	R ₂	1, 2, 4, 35	12345	1234, 1235	

Number of factors=6, Number of columns=88

Column/Row	1	2	3	4	5	6	7	8	9
Row1	00000 0	00000 1	00010 0	00010 1	00100 0	001001	00110 0	001101	01000 0
Row2	00001 1	00001 0	00011 1	00011 0	00101 1	001010	00111 1	001110	01001 1
10	11	12	13	14	15	16	17	18	19
01000 1	01010 0	01010 1	01100 0	01100 1	01110 0	011101	10000 0	100001	10010 0
01001 0	01011 1	01011 0	01101 1	01101 0	01111 1	011110	10001 1	100010	10011 1
20	21	22	23	24	25	26	27	28	29
10010 1	10100 0	10100 1	10110 0	10110 1	11000 0	110001	11010 0	110101	11100 0
10011 0	10101 1	10101 0	10111 1	10111 0	11001 1	110010	11011 1	110110	11101 1
30	31	32	33	34	35	36	37	38	39
11100 1	11110 0	11110 1	00000 0	00000 1	00001 0	000011	00100 0	001001	00101 0
11101 0	11111 1	11111 0	00010 1	00010 0	00011 1	000110	00110 1	001100	00111 1
40	41	42	43	44	45	46	47	48	49
00101 1	01000 0	01000 1	01001 0	01001 1	01100 0	011001	01101 0	011011	10000 0
00111 0	01010 1	01010 0	01011 1	01011 0	01110 1	011100	01111 1	011110	10010 1
50	51	52	53	54	55	56	57	58	59
10000 1	10001 0	10001 1	10100 0	10100 1	10101 0	101011	11000 0	110001	11001 0
10010 0	10011 1	10011 0	10110 1	10110 0	10111 1	101110	11010 1	110100	11011 1
60	61	62	63	64	65	66	67	68	69
11001 1	11100 0	11100 1	11101 0	11101 1	00000 0	111111	01111 1	000001	00000 0
11011 0	11110 1	11110 0	11111 1	11111 0	11111 1	000000	00000 1	011111	11111 1
70	71	72	73	74	75	76	77	78	79
11111 1	10111 1	01000 0	00000 0	11111 1	11011 1	001000	00000 0	111111	00111 1
00000 0	01000 0	10111 1	11111 1	00000 0	00100 0	110111	11111 1	000000	11000 0
80	81	82	83	84	85	86	87	88	
11000 0	00000 0	11111 1	01011 1	10100 0	00000 0	11111 1	10011 1	01100 0	
00111 1	11111 1	00000 0	10100 0	01011 1	11111 1	00000 0	01100 0	10011 1	

Number of factors	Replication number	Independent factorial effects confounded	Generalized factorial effects cofounded	Some Factorial Effects(higher order interactions) which are not confounded*	Some main effects and two factor interactions which are confounded in all the replication
6	R ₁	1, 2, 3, 4, 56	123456	12345, 12346	1, 2, 12, 3, 13, 23
	R ₂	1, 2, 3, 5, 46	123456	12345, 12356	

Number of factors=7, Number of columns=168

Column/Row	1	2	3	4	5	6	7	8	9
Row1	0000 000	00000 01	00001 00	00001 01	00010 00	00010 01	00011 00	00011 01	00100 00
Row2	0000 011	00000 10	00001 11	00001 10	00010 11	00010 10	00011 11	00011 10	00100 11

10	11	12	13	14	15	16	17	18	19
00100 01	00101 00	00101 01	00110 00	00110 01	00111 00	00111 01	01000 00	01000 01	01001 00
00100 10	00101 11	00101 10	00110 11	00110 10	00111 11	00111 10	01000 11	01000 10	01001 11

20	21	22	23	24	25	26	27	28	29
01001 01	01010 00	01010 01	01011 00	01011 01	01100 00	01100 01	01101 00	01101 01	01110 00
01001 10	01010 11	01010 10	01011 11	01011 10	01100 11	01100 10	01101 11	01101 10	01110 11

30	31	32	33	34	35	36	37	38	39
01110 01	01111 00	01111 01	10000 00	10000 01	10001 00	10001 01	10010 00	10010 01	10011 00
01110 10	01111 11	01111 10	10000 11	10000 10	10001 11	10001 10	10010 11	10010 10	10011 11

40	41	42	43	44	45	46	47	48	49
10011 01	10100 00	10100 01	10101 00	10101 01	10110 00	10110 01	10111 00	10111 01	11000 00
10011 10	10100 11	10100 10	10101 11	10101 10	10110 11	10110 10	10111 11	10111 10	11000 11

50	51	52	53	54	55	56	57	58	59
11000 01	11001 00	11001 01	11010 00	11010 01	11011 00	11011 01	11100 00	11100 01	11101 00
11000 10	11001 11	11001 10	11010 11	11010 10	11011 11	11011 10	11100 11	11100 10	11101 11

60	61	62	63	64	65	66	67	68	69
11101 01	11110 00	11110 01	11111 00	11111 01	00000 00	00000 01	00000 10	00000 11	00010 00
11101 10	11110 11	11110 10	11111 11	11111 10	00001 01	00001 00	00001 11	00001 10	00011 01

70	71	72	73	74	75	76	77	78	79
00010 01	00010 10	00010 11	00100 00	00100 01	00100 10	00100 11	00110 00	00110 01	00110 10
00011 00	00011 11	00011 10	00101 01	00101 00	00101 11	00101 10	00111 01	00111 00	00111 11

80	81	82	83	84	85	86	87	88	89
00110 11	01000 00	01000 01	01000 10	01000 11	01010 00	01010 01	01010 10	01010 11	01100 00
00111 10	01001 01	01001 00	01001 11	01001 10	01011 01	01011 00	01011 11	01011 10	01101 01

90	91	92	93	94	95	96	97	98	99
01100 01	01100 10	01100 11	01110 00	01110 01	01110 10	01110 11	10000 00	10000 01	10000 10
01101 00	01101 11	01101 10	01111 01	01111 00	01111 11	01111 10	10001 01	10001 00	10001 11

100	101	102	103	104	105	106	107	108	109
10000 11	10010 00	10010 01	10010 10	10010 11	10100 00	10100 01	10100 10	10100 11	10110 00
10001 10	10011 01	10011 00	10011 11	10011 10	10101 01	10101 00	10101 11	10101 10	10111 01

110	111	112	113	114	115	116	117	118	119
10110 01	10110 10	10110 11	11000 00	11000 01	11000 10	11000 11	11010 00	11010 01	11010 10
10111 00	10111 11	10111 10	11001 01	11001 00	11001 11	11001 10	11011 01	11011 00	11011 11

120	121	122	123	124	125	126	127	128	129
11010 11	11100 00	11100 01	11100 10	11100 11	11110 00	11110 01	11110 10	11110 11	00000 00
11011 10	11101 01	11101 00	11101 11	11101 10	11111 01	11111 00	11111 11	11111 10	11111 11

130	131	132	133	134	135	136	137	138	139
11111 11	01111 11	10000 00	00000 00	11111 11	10111 11	01000 00	00000 00	11111 11	11011 11
00000 00	10000 00	01111 11	11111 11	00000 00	01000 00	10111 11	11111 11	00000 00	00100 00

140	141	142	143	144	145	146	147	148	149
00100 00	00000 00	11111 11	11101 11	00010 00	00000 00	11111 11	00111 11	11000 00	00000 00
11011 11	11111 11	00000 00	00010 00	11101 11	11111 11	00000 00	11000 00	00111 11	11111 11

150	151	152	153	154	155	156	157	158	159
11111 11	01011 11	10100 00	00000 00	11111 11	01101 11	10010 00	00000 00	11111 11	10011 11
00000 00	10100 00	01011 11	11111 11	00000 00	10010 00	01101 11	11111 11	00000 00	01100 00

160	161	162	163	164	165	166	167	168
10011 11	00000 00	11111 11	10101 11	01010 00	00000 00	11111 11	11001 11	00110 00
01100 00	11111 11	00000 00	01010 00	10101 11	11111 11	00000 00	00110 00	11001 11

Number of factors	Replication number	Independent factorial effects confounded	Generalized factorial effects cofounded	Some Factorial Effects(higher order interactions) which are not confounded*	Some main effects and two factor interactions which are confounded in all the replication
7	R ₁	1, 2, 3, 4, 5, 67	1234567	123456, 123457	1, 2, 12, 3, 13, 23, 4, 14, 24, 34
	R ₂	1, 2, 3, 4, 6, 57	1234567	123456, 123467	

Number of factors=8, Number of columns=424

Column/Row	1	2	3	4	5	6	7	8	9
Row1	0000 0000	00000 001	00000 100	00000 101	00001 000	00001 001	00001 100	00001 101	00010 000
Row2	0000 0011	00000 010	00000 111	00000 110	00001 011	00001 010	00001 111	00001 110	00010 011

10	11	12	13	14	15	16	17	18	19
00010 001	00010 100	00010 101	00011 000	00011 001	00011 100	00011 101	00100 000	00100 001	00100 100
00010 010	00010 111	00010 110	00011 011	00011 010	00011 111	00011 110	00100 011	00100 010	00100 111

20	21	22	23	24	25	26	27	28	29
00100 101	00101 000	00101 001	00101 100	00101 101	00110 000	00110 001	00110 100	00110 101	00111 000
00100 110	00101 011	00101 010	00101 111	00101 110	00110 011	00110 010	00110 111	00110 110	00111 011

30	31	32	33	34	35	36	37	38	39
00111 001	00111 100	00111 101	01000 000	01000 001	01000 100	01000 101	01001 000	01001 001	01001 100
00111 010	00111 111	00111 110	01000 011	01000 010	01000 111	01000 110	01001 011	01001 010	01001 111

40	41	42	43	44	45	46	47	48	49
01001 101	01010 000	01010 001	01010 100	01010 101	01011 000	01011 001	01011 100	01011 101	01100 000
01001 110	01010 011	01010 010	01010 111	01010 110	01011 011	01011 010	01011 111	01011 110	01100 011

50	51	52	53	54	55	56	57	58	59
01100 001	01100 100	01100 101	01101 000	01101 001	01101 100	01101 101	01110 000	01110 001	01110 100
01100 010	01100 111	01100 110	01101 011	01101 010	01101 111	01101 110	01110 011	01110 010	01110 111

60	61	62	63	64	65	66	67	68	69
01110 101	01111 000	01111 001	01111 100	01111 101	10000 000	10000 001	10000 100	10000 101	10001 000
01110 110	01111 011	01111 010	01111 111	01111 110	10000 011	10000 010	10000 111	10000 110	10001 011

70	71	72	73	74	75	76	77	78	79
10001 001	10001 100	10001 101	10010 000	10010 001	10010 100	10010 101	10011 000	10011 001	10011 100
10001 010	10001 111	10001 110	10010 011	10010 010	10010 111	10010 110	10011 011	10011 010	10011 111

80	81	82	83	84	85	86	87	88	89
10011 101	10100 000	10100 001	10100 100	10100 101	10101 000	10101 001	10101 100	10101 101	10110 000
10011 110	10100 011	10100 010	10100 111	10100 110	10101 011	10101 010	10101 111	10101 110	10110 011

90	91	92	93	94	95	96	97	98	99
10110 001	10110 100	10110 101	10111 000	10111 001	10111 100	10111 101	11000 000	11000 001	11000 100
10110 010	10110 111	10110 110	10111 011	10111 010	10111 111	10111 110	11000 011	11000 010	11000 111

100	101	102	103	104	105	106	107	108	109
11000 101	11001 000	11001 001	11001 100	11001 101	11010 000	11010 001	11010 100	11010 101	11011 000
11000 110	11001 011	11001 010	11001 111	11001 110	11010 011	11010 010	11010 111	11010 110	11011 011

110	111	112	113	114	115	116	117	118	119
11011 001	11011 100	11011 101	11100 000	11100 001	11100 100	11100 101	11101 000	11101 001	11101 100
11011 010	11011 111	11011 110	11100 011	11100 010	11100 111	11100 110	11101 011	11101 010	11101 111

120	121	122	123	124	125	126	127	128	129
11101 101	11110 000	11110 001	11110 100	11110 101	11111 000	11111 001	11111 100	11111 101	00000 000
11101 110	11110 011	11110 010	11110 111	11110 110	11111 011	11111 010	11111 111	11111 110	00000 101

130	131	132	133	134	135	136	137	138	139
00000 001	00000 010	00000 011	00001 000	00001 001	00001 010	00001 011	00010 000	00010 001	00010 010
00000 100	00000 111	00000 110	00001 101	00001 100	00001 111	00001 110	00010 101	00010 100	00010 111

140	141	142	143	144	145	146	147	148	149
00010 011	00011 000	00011 001	00011 010	00011 011	00100 000	00100 001	00100 010	00100 011	00101 000
00010 110	00011 101	00011 100	00011 111	00011 110	00100 101	00100 100	00100 111	00100 110	00101 101

150	151	152	153	154	155	156	157	158	159
00101 001	00101 010	00101 011	00110 000	00110 001	00110 010	00110 011	00111 000	00111 001	00111 010
00101 100	00101 111	00101 110	00110 101	00110 100	00110 111	00110 110	00111 101	00111 100	00111 111

160	161	162	163	164	165	166	167	168	169
00111 011	01000 000	01000 001	01000 010	01000 011	01001 000	01001 001	01001 010	01001 011	01010 000
00111 110	01000 101	01000 100	01000 111	01000 110	01001 101	01001 100	01001 111	01001 110	01010 101

170	171	172	173	174	175	176	177	178	179
01010 001	01010 010	01010 011	01011 000	01011 001	01011 010	01011 011	01100 000	01100 001	01100 010
01010 100	01010 111	01010 110	01011 101	01011 100	01011 111	01011 110	01100 101	01100 100	01100 111

180	181	182	183	184	185	186	187	188	189
01100 011	01101 000	01101 001	01101 010	01101 011	01110 000	01110 001	01110 010	01110 011	01111 000
01100 110	01101 101	01101 100	01101 111	01101 110	01110 101	01110 100	01110 111	01110 110	01111 101

190	191	192	193	194	195	196	197	198	199
01111 001	01111 010	01111 011	10000 000	10000 001	10000 010	10000 011	10001 000	10001 001	10001 010
01111 100	01111 111	01111 110	10000 101	10000 100	10000 111	10000 110	10001 101	10001 100	10001 111

200	201	202	203	204	205	206	207	208	209
10001 011	10010 000	10010 001	10010 010	10010 011	10011 000	10011 001	10011 010	10011 011	10100 000
10001 110	10010 101	10010 100	10010 111	10010 110	10011 101	10011 100	10011 111	10011 110	10100 101
210	211	212	213	214	215	216	217	218	219

10100 001	10100 010	10100 011	10101 000	10101 001	10101 010	10101 011	10110 000	10110 001	10110 010
10100 100	10100 111	10100 110	10101 101	10101 100	10101 111	10101 110	10110 101	10110 100	10110 111

220	221	222	223	224	225	226	227	228	229
10110 011	10111 000	10111 001	10111 010	10111 011	11000 000	11000 001	11000 010	11000 011	11001 000
10110 110	10111 101	10111 100	10111 111	10111 110	11000 101	11000 100	11000 111	11000 110	11001 101

230	231	232	233	234	235	236	237	238	239
11001 001	11001 010	11001 011	11010 000	11010 001	11010 010	11010 011	11011 000	11011 001	11011 010
11001 100	11001 111	11001 110	11010 101	11010 100	11010 111	11010 110	11011 101	11011 100	11011 111

240	241	242	243	244	245	246	247	248	249
11011 011	11100 000	11100 001	11100 010	11100 011	11101 000	11101 001	11101 010	11101 011	11110 000
11011 110	11100 101	11100 100	11100 111	11100 110	11101 101	11101 100	11101 111	11101 110	11110 101

250	251	252	253	254	255	256	257	258	259
11110 001	11110 010	11110 011	11111 000	11111 001	11111 010	11111 011	00000 000	00000 001	00000 010
11110 100	11110 111	11110 110	11111 101	11111 100	11111 111	11111 110	00001 001	00001 000	00001 011

260	261	262	263	264	265	266	267	268	269
00000 011	00000 100	00000 101	00000 110	00000 111	00010 000	00010 001	00010 010	00010 011	00010 100
00001 010	00001 101	00001 100	00001 111	00001 110	00011 001	00011 000	00011 011	00011 010	00011 101

270	271	272	273	274	275	276	277	278	279
00010 101	00010 110	00010 111	00100 000	00100 001	00100 010	00100 011	00100 100	00100 101	00100 110
00011 100	00011 111	00011 110	00101 001	00101 000	00101 011	00101 010	00101 101	00101 100	00101 111

280	281	282	283	284	285	286	287	288	289
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00100 111	00110 000	00110 001	00110 010	00110 011	00110 100	00110 101	00110 110	00110 111	01000 000
00101 110	00111 001	00111 000	00111 011	00111 010	00111 101	00111 100	00111 111	00111 110	01001 001

290	291	292	293	294	295	296	297	298	299
01000 001	01000 010	01000 011	01000 100	01000 101	01000 110	01000 111	01010 000	01010 001	01010 010
01001 000	01001 011	01001 010	01001 101	01001 100	01001 111	01001 110	01011 001	01011 000	01011 011

300	301	302	303	304	305	306	307	308	309
01010 011	01010 100	01010 101	01010 110	01010 111	01100 000	01100 001	01100 010	01100 011	01100 100
01011 010	01011 101	01011 100	01011 111	01011 110	01101 001	01101 000	01101 011	01101 010	01101 101

310	311	312	313	314	315	316	317	318	319
01100 101	01100 110	01100 111	01110 000	01110 001	01110 010	01110 011	01110 100	01110 101	01110 110
01101 100	01101 111	01101 110	01111 001	01111 000	01111 011	01111 010	01111 101	01111 100	01111 111

320	321	322	323	324	325	326	327	328	329
01110 111	10000 000	10000 001	10000 010	10000 011	10000 100	10000 101	10000 110	10000 111	10010 000
01111 110	10001 001	10001 000	10001 011	10001 010	10001 101	10001 100	10001 111	10001 110	10011 001

330	331	332	333	334	335	336	337	338	339
10010 001	10010 010	10010 011	10010 100	10010 101	10010 110	10010 111	10100 000	10100 001	10100 010
10011 000	10011 011	10011 010	10011 101	10011 100	10011 111	10011 110	10101 001	10101 000	10101 011

340	341	342	343	344	345	346	347	348	349
10100 011	10100 100	10100 101	10100 110	10100 111	10110 000	10110 001	10110 010	10110 011	10110 100
10101 010	10101 101	10101 100	10101 111	10101 110	10111 001	10111 000	10111 011	10111 010	10111 101

350	351	352	353	354	355	356	357	358	359
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10110 101	10110 110	10110 111	11000 000	11000 001	11000 010	11000 011	11000 100	11000 101	11000 110
10111 100	10111 111	10111 110	11001 001	11001 000	11001 011	11001 010	11001 101	11001 100	11001 111

360	361	362	363	364	365	366	367	368	369
11000 111	11010 000	11010 001	11010 010	11010 011	11010 100	11010 101	11010 110	11010 111	11100 000
11001 110	11011 001	11011 000	11011 011	11011 010	11011 101	11011 100	11011 111	11011 110	11101 001

370	371	372	373	374	375	376	377	378	379
11100 001	11100 010	11100 011	11100 100	11100 101	11100 110	11100 111	11110 000	11110 001	11110 010
11101 000	11101 011	11101 010	11101 101	11101 100	11101 111	11101 110	11111 001	11111 000	11111 011

380	381	382	383	384	385	386	387	388	389
11110 011	11110 100	11110 101	11110 110	11110 111	00000 000	11111 111	01111 111	10000 000	00000 000
11111 010	11111 101	11111 100	11111 111	11111 110	11111 111	00000 000	10000 000	01111 111	11111 111

390	391	392	393	394	395	396	397	398	399
11111 111	10111 111	01000 000	00000 000	11111 111	11011 111	00100 000	00000 000	11111 111	11101 111
00000 000	01000 000	10111 111	11111 111	00000 000	00100 000	11011 111	11111 111	00000 000	00010 000

400	401	402	403	404	405	406	407	408	409
00010 000	00000 000	11111 111	00111 111	11000 000	00000 000	11111 111	01011 111	10100 00	00000 000
11101 111	11111 111	00000 000	11000 000	00111 111	11111 111	00000 000	10100 00	01011 111	11111 111

410	411	412	413	414	415	416	417	418	419
11111 111	01101 111	10010 000	00000 000	11111 111	10011 111	01100 000	00000 000	11111 111	10101 111
00000 000	10010 000	01101 111	11111 111	00000 000	01100 000	10011 111	11111 111	00000 000	01010 000

420	421	422	423	424
01010	00000	11111	11001	00110

000	000	111	111	000
10101	11111	00000	00110	11001
111	111	000	000	111

Number of factors	Replication number	Independent factorial effects confounded	Generalized factorial effects cofounded	Some Factorial Effects(higher order interactions) which are not confounded*	Some main effects and two factor interactions which are confounded in all the replication
8	R ₁	1, 2, 3, 4, 5, 6,7,8	12345678	1234567, 123467, 1234568	1, 2, 12, 3, 13, 23, 4, 14, 24, 34
	R ₂	1, 2, 3, 4, 5,7, 68	12345678	1234567, 123467, 1234578	
	R ₃	1,2,3,4,6, 7, 58	12345678	1234567, 123456, 123478	

Number of factors=9, Number of columns=828

Column/Row	1	2	3	4	5	6	7	8	9
Row1	00000 0000	00000 0001	00000 0100	00000 0101	00000 1000	00000 1001	00000 1100	00000 1101	00001 0000
Row2	00000 0011	00000 0010	00000 0111	00000 0110	00000 1011	00000 1010	00000 1111	00000 1110	00001 0011

10	11	12	13	14	15	16	17	18	19
00001 0001	00001 0100	00001 0101	00001 1000	00001 1001	00001 1100	00001 1101	00010 0000	00010 0001	00010 0100
00001 0010	00001 0111	00001 0110	00001 1011	00001 1010	00001 1111	00001 1110	00010 0011	00010 0010	00010 0111

20	21	22	23	24	25	26	27	28	29
00010 0101	00010 1000	00010 1001	00010 1100	00010 1101	00011 0000	00011 0001	00011 0100	00011 0101	00011 1000
00010 0110	00010 1011	00010 1010	00010 1111	00010 1110	00011 0011	00011 0010	00011 0111	00011 0110	00011 1011

30	31	32	33	34	35	36	37	38	39
00011 1001	00011 1100	00011 1101	00100 0000	00100 0001	00100 0100	00100 0101	00100 1000	00100 1001	00100 1100
00011 1010	00011 1111	00011 1110	00100 0011	00100 0010	00100 0111	00100 0110	00100 1011	00100 1010	00100 1111

40	41	42	43	44	45	46	47	48	49
00100 1101	00101 0000	00101 0001	00101 0100	00101 0101	00101 1000	00101 1001	00101 1100	00101 1101	00110 0000
00100	00101	00101	00101	00101	00101	00101	00101	00101	00110

1110	0011	0010	0111	0110	1011	1010	1111	1110	0011
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50	51	52	53	54	55	56	57	58	59
00110 0001	00110 0100	00110 0101	00110 1000	00110 1001	00110 1100	00110 1101	00111 0000	00111 0001	00111 0100
00110 0010	00110 0111	00110 0110	00110 1011	00110 1010	00110 1111	00110 1110	00111 0011	00111 0010	00111 0111

60	61	62	63	64	65	66	67	68	69
00111 0101	00111 1000	00111 1001	00111 1100	00111 1101	01000 0000	01000 0001	01000 0100	01000 0101	01000 1000
00111 0110	00111 1011	00111 1010	00111 1111	00111 1110	01000 0011	01000 0010	01000 0111	01000 0110	01000 1011
70	71	72	73	74	75	76	77	78	79
01000 1001	01000 1100	01000 1101	01001 0000	01001 0001	01001 0100	01001 0101	01001 1000	01001 1001	01001 1100
01000 1010	01000 1111	01000 1110	01001 0011	01001 0010	01001 0111	01001 0110	01001 1011	01001 1010	01001 1111

80	81	82	83	84	85	86	87	88	89
01001 1101	01010 0000	01010 0001	01010 0100	01010 0101	01010 1000	01010 1001	01010 1100	01010 1101	01011 0000
01001 1110	01010 0011	01010 0010	01010 0111	01010 0110	01010 1011	01010 1010	01010 1111	01010 1110	01011 0011

90	91	92	93	94	95	96	97	98	99
01011 0001	01011 0100	01011 0101	01011 1000	01011 1001	01011 1100	01011 1101	01100 0000	01100 0001	01100 0100
01011 0010	01011 0111	01011 0110	01011 1011	01011 1010	01011 1111	01011 1110	01100 0011	01100 0010	01100 0111

100	101	102	103	104	105	106	107	108	109
01100 0101	01100 1000	01100 1001	01100 1100	01100 1101	01101 0000	01101 0001	01101 0100	01101 0101	01101 1000
01100 0110	01100 1011	01100 1010	01100 1111	01100 1110	01101 0011	01101 0010	01101 0111	01101 0110	01101 1011

110	111	112	113	114	115	116	117	118	119
01101 1001	01101 1100	01101 1101	01110 0000	01110 0001	01110 0100	01110 0101	01110 1000	01110 1001	01110 1100
01101 1010	01101 1111	01101 1110	01110 0011	01110 0010	01110 0111	01110 0110	01110 1011	01110 1010	01110 1111

120	121	122	123	124	125	126	127	128	129
01110 1101	01111 0000	01111 0001	01111 0100	01111 0101	01111 1000	01111 1001	01111 1100	01111 1101	10000 0000
01110 1110	01111 0011	01111 0010	01111 0111	01111 0110	01111 1011	01111 1010	01111 1111	01111 1110	10000 0011

130	131	132	133	134	135	136	137	138	139
10000 0001	10000 0100	10000 0101	10000 1000	10000 1001	10000 1100	10000 1101	10001 0000	10001 0001	10001 0100
10000 0010	10000 0111	10000 0110	10000 1011	10000 1010	10000 1111	10000 1110	10001 0011	10001 0010	10001 0111

140	141	142	143	144	145	146	147	148	149
10001 0101	10001 1000	10001 1001	10001 1100	10001 1101	10010 0000	10010 0001	10010 0100	10010 0101	10010 1000
10001 0110	10001 1011	10001 1010	10001 1111	10001 1110	10010 0011	10010 0010	10010 0111	10010 0110	10010 1011

150	151	152	153	154	155	156	157	158	159
10010 1001	10010 1100	10010 1101	10011 0000	10011 0001	10011 0100	10011 0101	10011 1000	10011 1001	10011 1100
10010 1010	10010 1111	10010 1110	10011 0011	10011 0010	10011 0111	10011 0110	10011 1011	10011 1010	10011 1111

160	161	162	163	164	165	166	167	168	169
10011 1101	10100 0000	10100 0001	10100 0100	10100 0101	10100 1000	10100 1001	10100 1100	10100 1101	10101 0000
10011 1110	10100 0011	10100 0010	10100 0111	10100 0110	10100 1011	10100 1010	10100 1111	10100 1110	10101 0011

170	171	172	173	174	175	176	177	178	179
10101 0001	10101 0100	10101 0101	10101 1000	10101 1001	10101 1100	10101 1101	10110 0000	10110 0001	10110 0100
10101 0010	10101 0111	10101 0110	10101 1011	10101 1010	10101 1111	10101 1110	10110 0011	10110 0010	10110 0111

180	181	182	183	184	185	186	187	188	189
10110 0101	10110 1000	10110 1001	10110 1100	10110 1101	10111 0000	10111 0001	10111 0100	10111 0101	10111 1000
10110 0110	10110 1011	10110 1010	10110 1111	10110 1110	10111 0011	10111 0010	10111 0111	10111 0110	10111 1011
190	191	192	193	194	195	196	197	198	199

10111	10111	10111	11000	11000	11000	11000	11000	11000	11000
1001	1100	1101	0000	0001	0100	0101	1000	1001	1100
10111	10111	10111	11000	11000	11000	11000	11000	11000	11000
1010	1111	1110	0011	0010	0111	0110	1011	1010	1111

200	201	202	203	204	205	206	207	208	209
11000	11001	11001	11001	11001	11001	11001	11001	11001	11010
1101	0000	0001	0100	0101	1000	1001	1100	1101	0000
11000	11001	11001	11001	11001	11001	11001	11001	11001	11010
1110	0011	0010	0111	0110	1011	1010	1111	1110	0011

210	211	212	213	214	215	216	217	218	219
11010	11010	11010	11010	11010	11010	11010	11011	11011	11011
0001	0100	0101	1000	1001	1100	1101	0000	0001	0100
11010	11010	11010	11010	11010	11010	11010	11011	11011	11011
0010	0111	0110	1011	1010	1111	1110	0011	0010	0111

220	221	222	223	224	225	226	227	228	229
11011	11011	11011	11011	11011	11100	11100	11100	11100	11100
0101	1000	1001	1100	1101	0000	0001	0100	0101	1000
11011	11011	11011	11011	11011	11100	11100	11100	11100	11100
0110	1011	1010	1111	1110	0011	0010	0111	0110	1011

230	231	232	233	234	235	236	237	238	239
11100	11100	11100	11101	11101	11101	11101	11101	11101	11101
1001	1100	1101	0000	0001	0100	0101	1000	1001	1100
11100	11100	11100	11101	11101	11101	11101	11101	11101	11101
1010	1111	1110	0011	0010	0111	0110	1011	1010	1111

240	241	242	243	244	245	246	247	248	249
11101	11110	11110	11110	11110	11110	11110	11110	11110	11111
1101	0000	0001	0100	0101	1000	1001	1100	1101	0000
11101	11110	11110	11110	11110	11110	11110	11110	11110	11111
1110	0011	0010	0111	0110	1011	1010	1111	1110	0011

250	251	252	253	254	255	256	257	258	259
11111	11111	11111	11111	11111	11111	11111	00000	00000	00000
0001	0100	0101	1000	1001	1100	1101	0000	0001	0010
11111	11111	11111	11111	11111	11111	11111	00000	00000	00000
0010	0111	0110	1011	1010	1111	1110	0101	0100	0111

260	261	262	263	264	265	266	267	268	269
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00000	00000	00000	00000	00000	00001	00001	00001	00001	00001
0011	1000	1001	1010	1011	0000	0001	0010	0011	1000
00000	00000	00000	00000	00000	00001	00001	00001	00001	00001
0110	1101	1100	1111	1110	0101	0100	0111	0110	1101

270	271	272	273	274	275	276	277	278	279
00001	00001	00001	00010	00010	00010	00010	00010	00010	00010
1001	1010	1011	0000	0001	0010	0011	1000	1001	1010
00001	00001	00001	00010	00010	00010	00010	00010	00010	00010
1100	1111	1110	0101	0100	0111	0110	1101	1100	1111

280	281	282	283	284	285	286	287	288	289
00010	00011	00011	00011	00011	00011	00011	00011	00011	00100
1011	0000	0001	0010	0011	1000	1001	1010	1011	0000
00010	00011	00011	00011	00011	00011	00011	00011	00011	00100
1110	0101	0100	0111	0110	1101	1100	1111	1110	0101

290	291	292	293	294	295	296	297	298	299
00100	00100	00100	00100	00100	00100	00100	00101	00101	00101
0001	0010	0011	1000	1001	1010	1011	0000	0001	0010
00100	00100	00100	00100	00100	00100	00100	00101	00101	00101
0100	0111	0110	1101	1100	1111	1110	0101	0100	0111

300	301	302	303	304	305	306	307	308	309
00101	00101	00101	00101	00101	00110	00110	00110	00110	00110
0011	1000	1001	1010	1011	0000	0001	0010	0011	1000
00101	00101	00101	00101	00101	00110	00110	00110	00110	00110
0110	1101	1100	1111	1110	0101	0100	0111	0110	1101

310	311	312	313	314	315	316	317	318	319
00110	00110	00110	00111	00111	00111	00111	00111	00111	00111
1001	1010	1011	0000	0001	0010	0011	1000	1001	1010
00110	00110	00110	00111	00111	00111	00111	00111	00111	00111
1100	1111	1110	0101	0100	0111	0110	1101	1100	1111

320	321	322	323	324	325	326	327	328	329
00111	01000	01000	01000	01000	01000	01000	01000	01000	01001
1011	0000	0001	0010	0011	1000	1001	1010	1011	0000
00111	01000	01000	01000	01000	01000	01000	01000	01000	01001
1110	0101	0100	0111	0110	1101	1100	1111	1110	0101

330	331	332	333	334	335	336	337	338	339
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01001 0001	01001 0010	01001 0011	01001 1000	01001 1001	01001 1010	01001 1011	01010 0000	01010 0001	01010 0010
01001 0100	01001 0111	01001 0110	01001 1101	01001 1100	01001 1111	01001 1110	01010 0101	01010 0100	01010 0111

340	341	342	343	344	345	346	347	348	349
01010 0011	01010 1000	01010 1001	01010 1010	01010 1011	01011 0000	01011 0001	01011 0010	01011 0011	01011 1000
01010 0110	01010 1101	01010 1100	01010 1111	01010 1110	01011 0101	01011 0100	01011 0111	01011 0110	01011 1101

350	351	352	353	354	355	356	357	358	359
01011 1001	01011 1010	01011 1011	01100 0000	01100 0001	01100 0010	01100 0011	01100 1000	01100 1001	01100 1010
01011 1100	01011 1111	01011 1110	01100 0101	01100 0100	01100 0111	01100 0110	01100 1101	01100 1100	01100 1111

360	361	362	363	364	365	366	367	368	369
01100 1011	01101 0000	01101 0001	01101 0010	01101 0011	01101 1000	01101 1001	01101 1010	01101 1011	01110 0000
01100 1110	01101 0101	01101 0100	01101 0111	01101 0110	01101 1101	01101 1100	01101 1111	01101 1110	01110 0101

370	371	372	373	374	375	376	377	378	379
01110 0001	01110 0010	01110 0011	01110 1000	01110 1001	01110 1010	01110 1011	01111 0000	01111 0001	01111 0010
01110 0100	01110 0111	01110 0110	01110 1101	01110 1100	01110 1111	01110 1110	01111 0101	01111 0100	01111 0111

380	381	382	383	384	385	386	387	388	389
01111 0011	01111 1000	01111 1001	01111 1010	01111 1011	10000 0000	10000 0001	10000 0010	10000 0011	10000 1000
01111 0110	01111 1101	01111 1100	01111 1111	01111 1110	10000 0101	10000 0100	10000 0111	10000 0110	10000 1101

390	391	392	393	394	395	396	397	398	399
10000 1001	10000 1010	10000 1011	10001 0000	10001 0001	10001 0010	10001 0011	10001 1000	10001 1001	10001 1010
10000 1100	10000 1111	10000 1110	10001 0101	10001 0100	10001 0111	10001 0110	10001 1101	10001 1100	10001 1111

400	401	402	403	404	405	406	407	408	409
10001	10010	10010	10010	10010	10010	10010	10010	10010	10011

1011	0000	0001	0010	0011	1000	1001	1010	1011	0000
10001	10010	10010	10010	10010	10010	10010	10010	10010	10011
1110	0101	0100	0111	0110	1101	1100	1111	1110	0101

410	411	412	413	414	415	416	417	418	419
10011	10011	10011	10011	10011	10011	10011	10100	10100	10100
0001	0010	0011	1000	1001	1010	1011	0000	0001	0010
10011	10011	10011	10011	10011	10011	10011	10100	10100	10100
0100	0111	0110	1101	1100	1111	1110	0101	0100	0111

420	421	422	423	424	425	426	427	428	429
10100	10100	10100	10100	10100	10101	10101	10101	10101	10101
0011	1000	1001	1010	1011	0000	0001	0010	0011	1000
10100	10100	10100	10100	10100	10101	10101	10101	10101	10101
0110	1101	1100	1111	1110	0101	0100	0111	0110	1101

430	431	422	433	424	435	436	437	438	439
10101	10101	10101	10110	10110	10110	10110	10110	10110	10110
1001	1010	1011	0000	0001	0010	0011	1000	1001	1010
10101	10101	10101	10110	10110	10110	10110	10110	10110	10110
1100	1111	1110	0101	0100	0111	0110	1101	1100	1111

440	441	442	443	444	445	446	447	448	449
10110	10111	10111	10111	10111	10111	10111	10111	10111	11000
1011	0000	0001	0010	0011	1000	1001	1010	1011	0000
10110	10111	10111	10111	10111	10111	10111	10111	10111	11000
1110	0101	0100	0111	0110	1101	1100	1111	1110	0101

450	451	452	453	454	455	456	457	458	459
11000	11000	11000	11000	11000	11000	11000	11001	11001	11001
0001	0010	0011	1000	1001	1010	1011	0000	0001	0010
11000	11000	11000	11000	11000	11000	11000	11001	11001	11001
0100	0111	0110	1101	1100	1111	1110	0101	0100	0111

460	461	462	463	464	465	466	467	468	469
11001	11001	11001	11001	11001	11010	11010	11010	11010	11010
0011	1000	1001	1010	1011	0000	0001	0010	0011	1000
11001	11001	11001	11001	11001	11010	11010	11010	11010	11010
0110	1101	1100	1111	1110	0101	0100	0111	0110	1101

470	471	472	473	474	475	476	477	478	479
11010	11010	11010	11011	11011	11011	11011	11011	11011	11011
1001	1010	1011	0000	0001	0010	0011	1000	1001	1010

11010	11010	11010	11011	11011	11011	11011	11011	11011	11011
1100	1111	1110	0101	0100	0111	0110	1101	1100	1111

480	481	482	483	484	485	486	487	488	489
11011	11100	11100	11100	11100	11100	11100	11100	11100	11101
1011	0000	0001	0010	0011	1000	1001	1010	1011	0000
11011	11100	11100	11100	11100	11100	11100	11100	11100	11101
1110	0101	0100	0111	0110	1101	1100	1111	1110	0101

490	491	492	493	494	495	496	497	498	499
11101	11101	11101	11101	11101	11101	11101	11110	11110	11110
0001	0010	0011	1000	1001	1010	1011	0000	0001	0010
11101	11101	11101	11101	11101	11101	11101	11110	11110	11110
0100	0111	0110	1101	1100	1111	1110	0101	0100	0111

500	501	502	503	504	505	506	507	508	509
11110	11110	11110	11110	11110	11111	11111	11111	11111	11111
0011	1000	1001	1010	1011	0000	0001	0010	0011	1000
11110	11110	11110	11110	11110	11111	11111	11111	11111	11111
0110	1101	1100	1111	1110	0101	0100	0111	0110	1101

510	511	512	513	514	515	516	517	518	519
11111	11111	11111	00000	00000	00000	00000	00000	00000	00000
1001	1010	1011	0000	0001	0010	0011	0100	0101	0110
11111	11111	11111	00000	00000	00000	00000	00000	00000	00000
1100	1111	1110	1001	1000	1011	1010	1101	1100	1111

520	521	522	523	524	525	526	527	528	529
00000	00001	00001	00001	00001	00001	00001	00001	00001	00010
0111	0000	0001	0010	0011	0100	0101	0110	0111	0000
00000	00001	00001	00001	00001	00001	00001	00001	00001	00010
1110	1001	1000	1011	1010	1101	1100	1111	1110	1001

530	531	532	533	534	535	536	537	538	539
00010	00010	00010	00010	00010	00010	00010	00011	00011	00011
0001	0010	0011	0100	0101	0110	0111	0000	0001	0010
00010	00010	00010	00010	00010	00010	00010	00011	00011	00011
1000	1011	1010	1101	1100	1111	1110	1001	1000	1011

540	541	542	543	544	545	546	547	548	549
00011	00011	00011	00011	00011	00100	00100	00100	00100	00100
0011	0100	0101	0110	0111	0000	0001	0010	0011	0100
00011	00011	00011	00011	00011	00100	00100	00100	00100	00100

1010	1101	1100	1111	1110	1001	1000	1011	1010	1101
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550	551	552	553	554	555	556	557	558	559
00100 0101	00100 0110	00100 0111	00101 0000	00101 0001	00101 0010	00101 0011	00101 0100	00101 0101	00101 0110
00100 1100	00100 1111	00100 1110	00101 1001	00101 1000	00101 1011	00101 1010	00101 1101	00101 1100	00101 1111

560	561	562	563	564	565	566	567	568	569
00101 0111	00110 0000	00110 0001	00110 0010	00110 0011	00110 0100	00110 0101	00110 0110	00110 0111	00111 0000
00101 1110	00110 1001	00110 1000	00110 1011	00110 1010	00110 1101	00110 1100	00110 1111	00110 1110	00111 1001

570	571	572	573	574	575	576	577	578	579
00111 0001	00111 0010	00111 0011	00111 0100	00111 0101	00111 0110	00111 0111	01000 0000	01000 0001	01000 0010
00111 1000	00111 1011	00111 1010	00111 1101	00111 1100	00111 1111	00111 1110	01000 1001	01000 1000	01000 1011

580	581	582	583	584	585	586	587	588	589
01000 0011	01000 0100	01000 0101	01000 0110	01000 0111	01001 0000	01001 0001	01001 0010	01001 0011	01001 0100
01000 1010	01000 1101	01000 1100	01000 1111	01000 1110	01001 1001	01001 1000	01001 1011	01001 1010	01001 1101

590	591	592	593	594	595	596	597	598	599
01001 0101	01001 0110	01001 0111	01010 0000	01010 0001	01010 0010	01010 0011	01010 0100	01010 0101	01010 0110
01001 1100	01001 1111	01001 1110	01010 1001	01010 1000	01010 1011	01010 1010	01010 1101	01010 1100	01010 1111

600	601	602	603	604	605	606	607	608	609
01010 0111	01011 0000	01011 0001	01011 0010	01011 0011	01011 0100	01011 0101	01011 0110	01011 0111	01100 0000
01010 1110	01011 1001	01011 1000	01011 1011	01011 1010	01011 1101	01011 1100	01011 1111	01011 1110	01100 1001

610	611	612	613	614	615	616	617	618	619
01100 0001	01100 0010	01100 0011	01100 0100	01100 0101	01100 0110	01100 0111	01101 0000	01101 0001	01101 0010
01100 1000	01100 1011	01100 1010	01100 1101	01100 1100	01100 1111	01100 1110	01101 1001	01101 1000	01101 1011

620	621	622	623	624	625	626	627	628	629
01101 0011	01101 0100	01101 0101	01101 0110	01101 0111	01110 0000	01110 0001	01110 0010	01110 0011	01110 0100
01101 1010	01101 1101	01101 1100	01101 1111	01101 1110	01110 1001	01110 1000	01110 1011	01110 1010	01110 1101

630	631	632	633	634	635	636	637	638	639
01110 0101	01110 0110	01110 0111	01111 0000	01111 0001	01111 0010	01111 0011	01111 0100	01111 0101	01111 0110
01110 1100	01110 1111	01110 1110	01111 1001	01111 1000	01111 1011	01111 1010	01111 1101	01111 1100	01111 1111

640	641	642	643	644	645	646	647	648	649
01111 0111	10000 0000	10000 0001	10000 0010	10000 0011	10000 0100	10000 0101	10000 0110	10000 0111	10000 0000
01111 1110	10000 1001	10000 1000	10000 1011	10000 1010	10000 1101	10000 1100	10000 1111	10000 1110	10000 1001

650	651	652	653	654	655	656	657	658	659
10001 0001	10001 0010	10001 0011	10001 0100	10001 0101	10001 0110	10001 0111	10010 0000	10010 0001	10010 0010
10001 1000	10001 1011	10001 1010	10001 1101	10001 1100	10001 1111	10001 1110	10010 1001	10010 1000	10010 1011

660	661	662	663	664	665	666	667	668	669
10010 0011	10010 0100	10010 0101	10010 0110	10010 0111	10011 0000	10011 0001	10011 0010	10011 0011	10011 0100
10010 1010	10010 1101	10010 1100	10010 1111	10010 1110	10011 1001	10011 1000	10011 1011	10011 1010	10011 1101

670	671	672	673	674	675	676	677	678	679
10011 0101	10011 0110	10011 0111	10100 0000	10100 0001	10100 0010	10100 0011	10100 0100	10100 0101	10100 0110
10011 1100	10011 1111	10011 1110	10100 1001	10100 1000	10100 1011	10100 1010	10100 1101	10100 1100	10100 1111

680	681	682	683	684	685	686	687	688	689
10100 0111	10101 0000	10101 0001	10101 0010	10101 0011	10101 0100	10101 0101	10101 0110	10101 0111	10110 0000
10100 1110	10101 1001	10101 1000	10101 1011	10101 1010	10101 1101	10101 1100	10101 1111	10101 1110	10110 1001

690	691	692	693	694	695	696	697	698	699
10110 0001	10110 0010	10110 0011	10110 0100	10110 0101	10110 0110	10110 0111	10111 0000	10111 0001	10111 0010
10110 1000	10110 1011	10110 1010	10110 1101	10110 1100	10110 1111	10110 1110	10111 1001	10111 1000	10111 1011

700	701	702	703	704	705	706	707	708	709
10111 0011	10111 0100	10111 0101	10111 0110	10111 0111	11000 0000	11000 0001	11000 0010	11000 0011	11000 0100
10111 1010	10111 1101	10111 1100	10111 1111	10111 1110	11000 1001	11000 1000	11000 1011	11000 1010	11000 1101

710	711	712	713	714	715	716	717	718	719
11000 0101	11000 0110	11000 0111	11001 0000	11001 0001	11001 0010	11001 0011	11001 0100	11001 0101	11001 0110
11000 1100	11000 1111	11000 1110	11001 1001	11001 1000	11001 1011	11001 1010	11001 1101	11001 1100	11001 1111

720	721	722	723	724	725	726	727	728	729
11001 0111	11010 0000	11010 0001	11010 0010	11010 0011	11010 0100	11010 0101	11010 0110	11010 0111	11011 0000
11001 1110	11010 1001	11010 1000	11010 1011	11010 1010	11010 1101	11010 1100	11010 1111	11010 1110	11011 1001

730	731	732	733	734	735	736	737	738	739
11011 0001	11011 0010	11011 0011	11011 0100	11011 0101	11011 0110	11011 0111	11100 0000	11100 0001	11100 0010
11011 1000	11011 1011	11011 1010	11011 1101	11011 1100	11011 1111	11011 1110	11100 1001	11100 1000	11100 1011

740	741	742	743	744	745	746	747	748	749
11100 0011	11100 0100	11100 0101	11100 0110	11100 0111	11101 0000	11101 0001	11101 0010	11101 0011	11101 0100
11100 1010	11100 1101	11100 1100	11100 1111	11100 1110	11101 1001	11101 1000	11101 1011	11101 1010	11101 1101

750	751	752	753	754	755	756	757	758	759
11101 0101	11101 0110	11101 0111	11110 0000	11110 0001	11110 0010	11110 0011	11110 0100	11110 0101	11110 0110
11101 1100	11101 1111	11101 1110	11110 1001	11110 1000	11110 1011	11110 1010	11110 1101	11110 1100	11110 1111

760	761	762	763	764	765	766	767	768	769
11110 0111	11111 0000	11111 0001	11111 0010	11111 0011	11111 0100	11111 0101	11111 0110	11111 0111	00000 0000
11110 1110	11111 1001	11111 1000	11111 1011	11111 1010	11111 1101	11111 1100	11111 1111	11111 1110	11111 1111

770	771	772	773	774	775	776	777	778	779
11111 1111	01111 1111	10000 0000	00000 0000	11111 1111	10111 1111	01000 0000	00000 0000	11111 1111	11011 1111
00000 0000	10000 0000	01111 1111	11111 1111	00000 0000	01000 0000	10111 1111	11111 1111	00000 0000	00100 0000

780	781	782	783	784	785	786	787	788	789
00100 0000	00000 0000	11111 1111	11101 1111	00010 0000	00000 0000	11111 1111	11110 1111	00001 0000	00000 0000
11011	11111	00000	00010	11101	11111	00000	00001	11110	11111

1111	1111	0000	0000	1111	1111	0000	0000	1111	1111
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790	791	792	793	794	795	796	797	798	799
11111	00111	11000	00000	11111	01011	10100	00000	11111	01101
1111	1111	0000	0000	1111	1111	0000	0000	1111	1111
00000	11000	00111	11111	00000	10100	01011	11111	00000	10010
0000	0000	1111	1111	0000	0000	1111	1111	0000	0000

800	801	802	803	804	805	806	807	808	809
10010	00000	11111	01110	10001	00000	11111	10011	01100	00000
0000	0000	1111	1111	0000	0000	1111	1111	0000	0000
01101	11111	00000	10001	01110	11111	00000	01100	10011	11111
1111	1111	0000	0000	1111	1111	0000	0000	1111	1111

810	811	812	813	814	815	816	817	818	819
11111	10101	01010	00000	11111	10110	01001	00000	11111	11001
1111	1111	0000	0000	1111	1111	0000	0000	1111	1111
00000	01010	10101	11111	00000	01001	10110	11111	00000	00110
0000	0000	1111	1111	0000	0000	1111	1111	0000	0000

820	821	822	823	824	825	826	827	828
00110	00000	11111	11010	00101	00000	11111	11100	00011
0000	0000	1111	1111	0000	0000	1111	1111	0000
11001	11111	00000	00101	11010	11111	00000	00011	11100
1111	1111	0000	0000	1111	1111	0000	0000	1111

Number of factors	Replication number	Independent factorial effects confounded	Generalized factorial effects cofounded	Some Factorial Effects(higher order interactions) which are not confounded*	Some main effects and two factor interactions which are confounded in all the replication
9	R ₁	1, 2, 3, 4, 5,6,7, 8,9	123456789	12345678, 12345679	1, 2, 12, 3, 13, 23, 4, 14, 24, 34, 5, 15, 25, 35, 45
	R ₂	1, 2, 3, 4, 5,6, 8, 7,9	123456789	12345678, 12345689	
	R ₃	1, 2, 3,4,5, 7, 8, 6,9	123456789	12345678, 12345789	

Appendix II: Catalogue of efficient block designs for mixed level (number of levels: 2, 3 and 4; number of factors: 2 and 3) factorial experiments based on baseline parameterization

Factorial experiments: 2x2

Block	1	2	3	4
Unit1	01	10	11	11
Unit2	00	00	01	10

Factorial experiments: 2x3

Block	1	2	3	4	5	6	7
Unit1	01	02	10	11	12	11	12

Unit2	00	00	00	01	02	10	10
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Factorial experiments: 3x2

Block	1	2	3	4	5	6	7
Unit1	01	10	11	20	21	11	21
Unit2	00	00	01	00	01	10	20

Factorial experiments: 3x3

Block	1	2	3	4	5	6	7	8	9	10	11	12
Unit1	01	02	10	11	12	20	21	22	11	12	21	22
Unit2	00	00	00	01	02	00	01	02	10	10	20	20

Factorial experiments: 2x2x2

Block	1	2	3	4	5	6	7	8	9	10	11	12
Unit1	001	010	011	100	101	110	111	011	101	110	111	111
Unit2	000	000	001	000	001	010	011	010	100	100	101	110

Factorial experiments: 2x3x2

Block	1	2	3	4	5	6	7	8	9	10
Unit1	001	010	011	020	021	100	101	110	111	120
Unit2	000	000	001	000	001	000	001	010	011	020
Block	11	12	13	14	15	16	17	18	19	20
Unit1	121	011	021	101	110	120	111	111	121	121
Unit2	021	010	020	100	100	100	101	110	101	120

Factorial experiments: 2x2x3

Block	1	2	3	4	5	6	7	8	9	10
Unit1	001	002	010	011	012	100	101	102	110	111
Unit2	000	000	000	001	002	000	001	002	010	011
Block	11	12	13	14	15	16	17	18	19	20
Unit1	112	011	012	101	102	110	111	112	111	112
Unit2	012	010	010	100	100	100	101	102	110	110

Factorial experiments: 2x3x3

Block	1	2	3	4	5	6	7	8	9
Unit1	001	002	010	011	012	020	021	022	100
Unit2	000	000	000	001	002	000	001	002	000
Block	10	11	12	13	14	15	16	17	18
Unit1	101	102	110	111	112	120	121	122	011
Unit2	001	002	010	011	012	020	021	022	010
Block	19	20	21	22	23	24	25	26	27
Unit1	012	021	022	101	102	110	120	111	112
Unit2	010	020	020	100	100	100	100	101	102
Block	28	29	30	31	32	33			

Unit1	121	122	111	112	121	122
Unit2	101	102	110	110	120	120

Factorial experiments: 3x2x2

Block	1	2	3	4	5	6	7	8	9	10
Unit1	001	010	011	100	101	110	111	200	201	210
Unit2	000	000	001	000	001	010	011	000	001	010
Block	11	12	13	14	15	16	17	18	19	20
Unit1	211	011	101	110	201	210	111	211	111	211
Unit2	011	010	100	100	200	200	101	201	110	210

Factorial experiments: 3x3x2

Block	1	2	3	4	5	6	7	8	9
Unit1	001	010	011	020	021	100	101	110	111
Unit2	000	000	001	000	001	000	001	010	011
Block	10	11	12	13	14	15	16	17	18
Unit1	120	121	200	201	210	211	220	221	011
Unit2	020	021	000	001	010	011	020	021	010
Block	19	20	21	22	23	24	25	26	27
Unit1	021	101	110	120	201	210	220	111	121
Unit2	020	100	100	100	200	200	200	101	101
Block	28	29	30	31	32	33			
Unit1	211	221	111	121	211	221			
Unit2	201	201	110	120	210	220			

Factorial experiments: 3x2x3

Block	1	2	3	4	5	6	7	8	9
Unit1	001	002	010	011	012	100	101	102	110
Unit2	000	000	000	001	002	000	001	002	010
Block	10	11	12	13	14	15	16	17	18
Unit1	111	112	200	201	202	210	211	212	011
Unit2	011	012	000	001	002	010	011	012	010
Block	19	20	21	22	23	24	25	26	27
Unit1	012	101	102	110	201	202	210	111	112
Unit2	010	100	100	100	200	200	200	101	102
Block	28	29	30	31	32	33			
Unit1	211	212	111	112	211	212			
Unit2	201	202	110	110	210	210			

Factorial experiments: 3x3x3

Block	1	2	3	4	5	6	7	8	9
Unit1	001	002	010	011	012	020	021	022	100

Unit2	000	000	000	001	002	000	001	002	000
Block	10	11	12	13	14	15	16	17	18
Unit1	101	102	110	111	112	120	121	122	200
Unit2	001	002	010	011	012	020	021	022	000
Block	19	20	21	22	23	24	25	26	27
Unit1	012	101	102	110	201	202	210	111	112
Unit2	010	100	100	100	200	200	200	101	102
Block	28	29	30	31	32	33	34	35	36
Unit1	012	021	022	101	102	110	120	201	202
Unit2	010	020	020	100	100	100	100	200	200
Block	37	38	39	40	41	42	43	44	45
Unit1	210	220	111	112	121	122	211	212	221
Unit2	200	200	101	102	101	102	201	202	201
Block	46	47	48	49	50	51	52	53	54
Unit1	222	111	112	121	122	211	212	221	222
Unit2	202	110	110	120	120	210	210	220	220

Factorial experiments: 2x2x2x2

Block	1	2	3	4	5	6	7	8	9
Unit1	0001	0010	0011	0100	0101	0110	0111	1000	1001
Unit2	0000	0000	0001	0000	0001	0010	0011	0000	0001
Block	10	11	12	13	14	15	16	17	18
Unit1	1010	1011	1100	1101	1110	1111	0011	0101	0110
Unit2	0010	0011	0100	0101	0110	0111	0010	0100	0100
Block	19	20	21	22	23	24	25	26	27
Unit1	1001	1010	1100	0111	1011	1101	1110	0111	1011
Unit2	1000	1000	1000	0101	1001	1001	1010	0110	1010
Block	28	29	30	31	32				
Unit1	1101	1110	1111	1111	1111				
Unit2	1100	1100	1011	1101	1110				

Factorial experiments: 2x3x2x2

Block	1	2	3	4	5	6	7	8	9	10
Unit1	0001	0010	0011	0100	0101	0110	0111	0200	0201	0210
Unit2	0000	0000	0001	0000	0001	0010	0011	0000	0001	0010
Block	11	12	13	14	15	16	17	18	19	20
Unit1	0211	1000	1001	1010	1011	1100	1101	1110	1111	1200
Unit2	0011	0000	0001	0010	0011	0100	0101	0110	0111	0200
Block	21	22	23	24	25	26	27	28	29	30
Unit1	1201	1210	1211	0011	0101	0110	0201	0210	1001	1010
Unit2	0201	0210	0211	0010	0100	0100	0200	0200	1000	1000
Block	31	32	33	34	35	36	37	38	39	40

Unit1	1100	1200	0111	0211	1011	1101	1110	1201	1210	0111
Unit2	1000	1000	0101	0201	1001	1001	1010	1001	1010	0110
Block	41	42	43	44	45	46	47	48	49	50
Unit1	0211	1011	1101	1110	1201	1210	1111	1211	1111	1211
Unit2	0210	1010	1100	1100	1200	1200	1011	1011	1101	1201
Block	51	52								
Unit1	1111	1211								
Unit2	1110	1210								

Factorial experiments: 2x2x3x2

Block	1	2	3	4	5	6	7	8	9	10	11	12
Unit1	0001	0010	0011	0020	0021	0100	0101	0110	0111	0120	0121	1000
Unit2	0000	0000	0001	0000	0001	0000	0001	0010	0011	0020	0021	0000
Block	13	14	15	16	17	18	19	20	21	22	23	24
Unit1	1001	1010	1011	1020	1021	1100	1101	1110	1111	1120	1121	0011
Unit2	0001	0010	0011	0020	0021	0100	0101	0110	0111	0120	0121	0010
Block	25	26	27	28	29	30	31	32	33	34	35	36
Unit1	0021	0101	0110	0120	1001	1010	1020	1100	0111	0121	1011	1021
Unit2	0020	0100	0100	0100	1000	1000	1000	1000	0101	0101	1001	1001
Block	37	38	39	40	41	42	43	44	45	46	47	48
Unit1	1101	1110	1120	0111	0121	1011	1021	1101	1110	1120	1111	1121
Unit2	1001	1010	1020	0110	0120	1010	1020	1100	1100	1100	1011	1021
Block	49	50	51	52								
Unit1	1111	1121	1111	1121								
Unit2	1101	1101	1110	1120								

Factorial experiments: 2x2x2x3

Block	1	2	3	4	5	6	7	8	9	10	11	12
Unit1	0001	0002	0010	0011	0012	0100	0101	0102	0110	0111	0112	1000
Unit2	0000	0000	0000	0001	0002	0000	0001	0002	0010	0011	0012	0000
Block	13	14	15	16	17	18	19	20	21	22	23	24
Unit1	1001	1002	1010	1011	1012	1100	1101	1102	1110	1111	1112	0011
Unit2	0001	0002	0010	0011	0012	0100	0101	0102	0110	0111	0112	0010
Block	25	26	27	28	29	30	31	32	33	34	35	36
Unit1	0012	0101	0102	0110	1001	1002	1010	1100	0111	0112	1011	1012
Unit2	0010	0100	0100	0100	1000	1000	1000	1000	0101	0102	1001	1002
Block	37	38	39	40	41	42	43	44	45	46	47	48

Unit1	1101	1102	1110	0111	0112	1011	1012	1101	1102	1110	1111	1112
Unit2	1001	1002	1010	0110	0110	1010	1010	1100	1100	1100	1011	1012
Block	49	50	51	52								
Unit1	1111	1112	1111	1112								
Unit2	1101	1102	1110	1110								

Factorial experiments: 2x3x2x3

Block	1	2	3	4	5	6	7	8	9	10	
Unit1	0001	0002	0010	0011	0012	0100	0101	0102	0110	0111	
Unit2	0000	0000	0000	0001	0002	0000	0001	0002	0010	0011	
Block	11	12	13	14	15	16	17	18	19	20	
Unit1	0112	0200	0201	0202	0210	0211	0212	1000	1001	1002	
Unit2	0012	0000	0001	0002	0010	0011	0012	0000	0001	0002	
Block	21	22	23	24	25	26	27	28	29	30	
Unit1	1010	1011	1012	1100	1101	1102	1110	1111	1112	1200	
Unit2	0010	0011	0012	0100	0101	0102	0110	0111	0112	0200	
Block	31	32	33	34	35	36	37	38	39	40	
Unit1	1201	1202	1210	1211	1212	0011	0012	0101	0102	0110	
Unit2	0201	0202	0210	0211	0212	0010	0010	0100	0100	0100	
Block	41	42	43	44	45	46	47	48	49	50	
Unit1	0201	0202	0210	1001	1002	1010	1100	1200	0111	0112	
Unit2	0200	0200	0200	1000	1000	1000	1000	1000	0101	0102	
Block	51	52	53	54	55	56	57	58	59	60	
Unit1	0211	0212	1011	1012	1101	1102	1110	1201	1202	1210	
Unit2	0201	0202	1001	1002	1001	1002	1010	1001	1002	1010	
Block	61	62	63	64	65	66	67	68	69	70	
Unit1	0111	0112	0211	0212	1011	1012	1101	1102	1110	1201	
Unit2	0110	0110	0210	0210	1010	1010	1100	1100	1100	1200	
Block	71	72	73	74	75	76	77	78	79	80	
Unit1	1202	1210	1111	1112	1211	1212	1111	1112	1211	1212	
Unit2	1200	1200	1011	1012	1011	1012	1101	1102	1201	1202	
Block	81	82	83	84							
Unit1	1111	1112	1211	1212							
Unit2	1110	1110	1210	1210							

Factorial experiments: 2x3x3x2

Block	1	2	3	4	5	6	7	8	9	10	11	12
Unit1	0001	0010	0011	0020	0021	0100	0101	0110	0111	0120	0121	0200
Unit2	0000	0000	0001	0000	0001	0000	0001	0010	0011	0020	0021	0000
Block	13	14	15	16	17	18	19	20	21	22	23	24
Unit1	0201	0210	0211	0220	0221	1000	1001	1010	1011	1020	1021	1100
Unit2	0001	0010	0011	0020	0021	0000	0001	0010	0011	0020	0021	0100
Block	25	26	27	28	29	30	31	32	33	34	35	36
Unit1	1101	1110	1111	1120	1121	1200	1201	1210	1211	1220	1221	0011
Unit2	0101	0110	0111	0120	0121	0200	0201	0210	0211	0220	0221	0010
Block	37	38	39	40	41	42	43	44	45	46	47	48

Unit1	0021	0101	0120	0201	0210	0220	1001	1010	1020	1200	1100	0021
Unit2	0020	0100	0100	0200	0200	0200	1000	1000	1000	1000	1000	0020
Block	49	50	51	52	53	54	55	56	57	58	59	60
Unit1	0111	0121	0211	0221	1011	1021	1101	1110	1120	1201	1210	1220
Unit2	0101	0101	0201	0201	1001	1001	1001	1010	1020	1001	1010	1020
Block	61	62	63	64	65	66	67	68	69	70	71	72
Unit1	0111	0121	0211	0221	1011	1021	1101	1110	1120	1201	1210	1220
Unit2	0110	0120	0210	0220	1010	1020	1100	1100	1100	1200	1200	1200
Block	73	74	75	76	77	78	79	80	81	82	83	84
Unit1	1111	1121	1211	1221	1111	1121	1211	1221	1111	1121	1211	1221
Unit2	1011	1021	1011	1021	1101	1101	1201	1201	1110	1120	1210	1220

Factorial experiments: 2x2x3x3

Block	1	2	3	4	5	6	7	8	9	10	11	12
Unit1	0001	0002	0010	0011	0012	0020	0021	0022	0100	0101	0102	0110
Unit2	0000	0000	0000	0001	0002	0000	0001	0002	0000	0001	0002	0010
Block	13	14	15	16	17	18	19	20	21	22	23	24
Unit1	0111	0112	0120	0121	0122	1000	1001	1002	1010	1011	1012	1020
Unit2	0011	0012	0020	0021	0022	0000	0001	0002	0010	0011	0012	0020
Block	25	26	27	28	29	30	31	32	33	34	35	36
Unit1	1021	1022	1100	1101	1102	1110	1111	1112	1120	1121	1122	0011
Unit2	0021	0022	0100	0101	0102	0110	0111	0112	0120	0121	0122	0010
Block	37	38	39	40	41	42	43	44	45	46	47	48
Unit1	0012	0021	0022	0101	0102	0110	0120	1001	1002	1010	1020	1100
Unit2	0010	0020	0020	0100	0100	0100	0100	1000	1000	1000	1000	1000
Block	49	50	51	52	53	54	55	56	57	58	59	60
Unit1	0111	0112	0121	0122	1011	1012	1021	1022	1101	1102	1110	1120
Unit2	0101	0102	0101	0102	1001	1002	1001	1002	1001	1002	1010	1020
Block	61	62	63	64	65	66	67	68	69	70	71	72
Unit1	0111	0112	0121	0122	1011	1012	1021	1022	1101	1102	1110	1120
Unit2	0110	0110	0120	0120	1010	1010	1020	1020	1100	1100	1100	1100
Block	73	74	75	76	77	78	79	80	81	82	83	84
Unit1	1111	1112	1121	1122	1111	1112	1121	1122	1111	1112	1121	1122
Unit2	1011	1012	1021	1022	1101	1102	1101	1102	1110	1110	1120	1120

Factorial experiments: 2x3x3x3

Block	1	2	3	4	5	6	7	8	9
Unit1	0001	0002	0010	0011	0012	0020	0021	0022	0100
Unit2	0000	0000	0000	0001	0002	0000	0001	0002	0000
Block	10	11	12	13	14	15	16	17	18
Unit1	0101	0102	0110	0111	0112	0120	0121	0122	0200
Unit2	0001	0002	0010	0011	0012	0020	0021	0022	0000
Block	19	20	21	22	23	24	25	26	27

Unit1	0201	0202	0210	0211	0212	0220	0221	0222	1000
Unit2	0001	0002	0010	0011	0012	0020	0021	0022	0000
Block	28	29	30	31	32	33	34	35	36
Unit1	1001	1002	1010	1011	1012	1020	1021	1022	1100
Unit2	0001	0002	0010	0011	0012	0020	0021	0022	0100
Block	37	38	39	40	41	42	43	44	45
Unit1	1101	1102	1110	1111	1112	1120	1121	1122	1200
Unit2	0101	0102	0110	0111	0112	0120	0121	0122	0200
Block	46	47	48	49	50	51	52	53	54
Unit1	1201	1202	1210	1211	1212	1220	1221	1222	0011
Unit2	0201	0202	0210	0211	0212	0220	0221	0222	0010
Block	55	56	57	58	59	60	61	62	63
Unit1	0012	0021	0022	0101	0102	0110	0120	0201	0202
Unit2	0010	0020	0020	0100	0100	0100	0100	0200	0200
Block	64	65	66	67	68	69	70	71	72
Unit1	0210	0220	1001	1002	1010	1020	1100	1200	0111
Unit2	0200	0200	1000	1000	1000	1000	1000	1000	0101
Block	73	74	75	76	77	78	79	80	81
Unit1	0112	0121	0122	0211	0212	0221	0222	1011	1012
Unit2	0102	0101	0102	0201	0202	0201	0202	1001	1002
Block	82	83	84	85	86	87	88	89	90
Unit1	1021	1022	1101	1102	1110	1120	1201	1202	1210
Unit2	1001	1002	1001	1002	1100	1100	1200	1200	1200
Block	91	92	93	94	95	96	97	98	99
Unit1	1220	0111	0112	0121	0122	0211	0212	0221	0222
Unit2	1200	0110	0110	0120	0120	0210	0210	0220	0220
Block	100	101	102	103	104	105	106	107	108
Unit1	1011	1012	1021	1022	1101	1102	1110	1120	1201
Unit2	1010	1010	1020	1020	1100	1100	1100	1100	1200
Block	109	110	111	112	113	114	115	116	117
Unit1	1202	1210	1220	1111	1112	1121	1122	1211	1212
Unit2	1200	1200	1200	1011	1012	1021	1022	1011	1012
Block	118	119	120	121	122	123	124	125	126
Unit1	1221	1222	1111	1112	1121	1122	1211	1212	1221
Unit2	1021	1022	1101	1102	1101	1102	1201	1202	1201
Block	127	128	129	130	131	132	133	134	135
Unit1	1222	1111	1112	1121	1122	1211	1212	1221	1222
Unit2	1202	1110	1110	1120	1120	1210	1210	1220	1220

Factorial experiments: 3x2x2x2

Block	1	2	3	4	5	6	7	8	9
Unit1	0001	0010	0011	0100	0101	0110	0111	1000	1001
Unit2	0000	0000	0001	0000	0001	0010	0011	0000	0001
Block	10	11	12	13	14	15	16	17	18
Unit1	1010	1011	1100	1101	1110	1111	2000	2001	2010
Unit2	0010	0011	0100	0101	0110	0111	0000	0001	0010

Block	19	20	21	22	23	24	25	26	27
Unit1	2011	2100	2101	2110	2111	0011	0101	0110	1001
Unit2	0011	0100	0101	0110	0111	0010	0100	0100	1000
Block	28	29	30	31	32	33	34	35	36
Unit1	1010	1100	2001	2010	2100	0111	1011	1101	1110
Unit2	1000	1000	2000	2000	2000	0101	1001	1001	1010
Block	37	38	39	40	41	42	43	44	45
Unit1	2011	2101	2110	0111	1011	1101	1110	2011	2101
Unit2	2001	2001	2010	0110	1010	1100	1100	2010	2100
Block	46	47	48	49	50	51	52		
Unit1	2110	1111	2111	1111	2111	1111	2111		
Unit2	2100	1011	2011	1101	2101	1110	2110		

Factorial experiments: 3x3x2x2

Block	1	2	3	4	5	6	7	8	9
Unit1	0001	0010	0011	0100	0101	0110	0111	0200	0201
Unit2	0000	0000	0001	0000	0001	0010	0011	0000	0001
Block	10	11	12	13	14	15	16	17	18
Unit1	0210	0211	1000	1001	1010	1011	1100	1101	1110
Unit2	0010	0011	0000	0001	0010	0011	0100	0101	0110
Block	19	20	21	22	23	24	25	26	27
Unit1	1111	1200	1201	1210	1211	2000	2001	2010	2011
Unit2	0111	0200	0201	0210	0211	0000	0001	0010	0011
Block	28	29	30	31	32	33	34	35	36
Unit1	2100	2101	2110	2111	2200	2201	2210	2211	0011
Unit2	0100	0101	0110	0111	0200	0201	0210	0211	0010
Block	37	38	39	40	41	42	43	44	45
Unit1	0101	0110	0201	0210	1001	1010	1100	1200	2001
Unit2	0100	0100	0200	0200	1000	1000	1000	1000	2000
Block	46	47	48	49	50	51	52	53	54
Unit1	2010	2100	2200	0111	0211	1011	1101	1110	1201
Unit2	2000	2000	2000	0101	0201	1001	1001	1010	1001
Block	55	56	57	58	59	60	61	62	63
Unit1	1210	2011	2101	2110	2201	2210	0111	0211	1011
Unit2	1010	2001	2001	2010	2001	2010	0110	0210	1010
Block	64	65	66	67	68	69	70	71	72
Unit1	1101	1110	1201	1210	2011	2101	2110	2201	2210
Unit2	1100	1100	1200	1200	2010	2100	2100	2200	2200
Block	73	74	75	76	77	78	79	80	81
Unit1	1111	1211	2111	2211	1111	1211	2111	2211	1111
Unit2	1011	1011	2011	2011	1101	1201	2101	2201	1110
Block	82	83	84						
Unit1	1211	2111	2211						
Unit2	1210	2110	2210						

Factorial experiments: 3x2x3x2

Block	1	2	3	4	5	6	7	8	9
Unit1	0001	0010	0011	0020	0021	0100	0101	0110	0111
Unit2	0000	0000	0001	0000	0001	0000	0001	0010	0011
Block	10	11	12	13	14	15	16	17	18
Unit1	0120	0121	1000	1001	1010	1011	1020	1021	1100
Unit2	0020	0021	0000	0001	0010	0011	0020	0021	0100
Block	19	20	21	22	23	24	25	26	27
Unit1	1101	1110	1111	1120	1121	2000	2001	2010	2011
Unit2	0101	0110	0111	0120	0121	0000	0001	0010	0011
Block	28	29	30	31	32	33	34	35	36
Unit1	2020	2021	2100	2101	2110	2111	2120	2121	0011
Unit2	0020	0021	0100	0101	0110	0111	0120	0121	0010
Block	37	38	39	40	41	42	43	44	45
Unit1	0021	0101	0110	0120	1001	1010	1020	1100	2001
Unit2	0020	0100	0100	0100	1000	1000	1000	1000	2000
Block	46	47	48	49	50	51	52	53	54
Unit1	2010	2020	2100	0111	0121	1011	1021	1101	1110
Unit2	2000	2000	2000	0101	0101	1001	1001	1001	1010
Block	55	56	57	58	59	60	61	62	63
Unit1	1120	2011	2021	2101	2110	2120	0111	0121	1011
Unit2	1020	2001	2001	2001	2010	2020	0110	0120	1010
Block	64	65	66	67	68	69	70	71	72
Unit1	1021	1101	1110	1120	2011	2021	2101	2110	2120
Unit2	1020	1100	1100	1100	2010	2020	2100	2100	2100
Block	73	74	75	76	77	78	79	80	81
Unit1	1111	1121	2111	2121	1111	1121	2111	2121	1111
Unit2	1011	1021	2011	2021	1101	1101	2101	2101	1110
Block	82	83	84						
Unit1	1121	2111	2121						
Unit2	1120	2110	2120						

Factorial experiments: 3x2x2x3

Block	1	2	3	4	5	6	7	8	9
Unit1	0001	0002	0010	0011	0012	0100	0101	0102	0110
Unit2	0000	0000	0000	0001	0002	0000	0001	0002	0010
Block	10	11	12	13	14	15	16	17	18
Unit1	0111	0112	1000	1001	1002	1010	1011	1012	1100
Unit2	0011	0012	0000	0001	0002	0010	0011	0012	0100
Block	19	20	21	22	23	24	25	26	27
Unit1	1101	1102	1110	1111	1112	2000	2001	2002	2010
Unit2	0101	0102	0110	0111	0112	0000	0001	0002	0010
Block	28	29	30	31	32	33	34	35	36
Unit1	2011	2012	2100	2101	2102	2110	2111	2112	0011

Unit2	0011	0012	0100	0101	0102	0110	0111	0112	0010
Block	37	38	39	40	41	42	43	44	45
Unit1	0012	0101	0102	0110	1001	1002	1010	1100	2001
Unit2	0010	0100	0100	0100	1000	1000	1000	1000	2000
Block	46	47	48	49	50	51	52	53	54
Unit1	2002	2010	2100	0111	0112	1011	1012	1101	1102
Unit2	2000	2000	2000	0101	0102	1001	1002	1001	1002
Block	55	56	57	58	59	60	61	62	63
Unit1	1110	2011	2012	2101	2102	2110	0111	0112	1011
Unit2	1010	2001	2002	2001	2002	2010	0110	0110	1010
Block	64	65	66	67	68	69	70	71	72
Unit1	1012	1101	1102	1110	2011	2012	2101	2102	2110
Unit2	1010	1100	1100	1100	2010	2010	2100	2100	2100
Block	73	74	75	76	77	78	79	80	81
Unit1	1111	1112	2111	2112	1111	1112	2111	2112	1111
Unit2	1011	1012	2011	2012	1101	1102	2101	2102	1110
Block	82	83	84						
Unit1	1112	2111	2112						
Unit2	1110	2110	2110						

Factorial experiments: 3x3x3x2

Block	1	2	3	4	5	6	7	8	9
Unit1	0001	0010	0011	0020	0021	0100	0101	0110	0111
Unit2	0000	0000	0001	0000	0001	0000	0001	0010	0011
Block	10	11	12	13	14	15	16	17	18
Unit1	0120	0121	0200	0201	0210	0211	0220	0221	1000
Unit2	0020	0021	0000	0001	0010	0011	0020	0021	0000
Block	19	20	21	22	23	24	25	26	27
Unit1	1001	1010	1011	1020	1021	1100	1101	1110	1111
Unit2	0001	0010	0011	0020	0021	0100	0101	0110	0111
Block	28	29	30	31	32	33	34	35	36
Unit1	1120	1121	1200	1201	1210	1211	1220	1221	2000
Unit2	0120	0121	0200	0201	0210	0211	0220	0221	0000
Block	37	38	39	40	41	42	43	44	45
Unit1	2001	2010	2011	2020	2021	2100	2101	2110	2111
Unit2	0001	0010	0011	0020	0021	0100	0101	0110	0111
Block	46	47	48	49	50	51	52	53	54
Unit1	2120	2121	2200	2201	2210	2211	2220	2221	0011
Unit2	0120	0121	0200	0201	0210	0211	0220	0221	0010
Block	55	56	57	58	59	60	61	62	63
Unit1	0021	0101	0110	0120	0201	0210	0220	1001	1010
Unit2	0020	0100	0100	0100	0200	0200	0200	1000	1000
Block	64	65	66	67	68	69	70	71	72
Unit1	1020	1100	1200	2001	2010	2020	2100	2200	0111
Unit2	1000	1000	1000	2000	2000	2000	2000	2000	0101
Block	73	74	75	76	77	78	79	80	81

Unit1	0121	0211	0221	1011	1021	1101	1110	1120	1201
Unit2	0101	0201	0201	1001	1001	1001	1010	1020	1001
Block	82	83	84	85	86	87	88	89	90
Unit1	1210	1220	2011	2021	2101	2110	2120	2201	2210
Unit2	1010	1020	2010	2001	2001	2010	2020	2001	2010
Block	91	92	93	94	95	96	97	98	99
Unit1	2220	0111	0121	0211	0221	1011	1021	1101	1110
Unit2	2020	0110	0120	0210	0220	1010	1020	1100	1100
Block	100	101	102	103	104	105	106	107	108
Unit1	1120	1201	1210	1220	2011	2021	2101	2110	2120
Unit2	1100	1200	1200	1200	2010	2020	2100	2100	2100
Block	109	110	111	112	113	114	115	116	117
Unit1	2201	2210	2220	1111	1121	1211	1221	2111	2121
Unit2	2200	2200	2200	1011	1021	1011	1021	2011	2021
Block	118	119	120	121	122	123	124	125	126
Unit1	2211	2221	1111	1121	1211	1221	2111	2121	2211
Unit2	2011	2021	1101	1101	1201	1201	2101	2101	2201
Block	127	128	129	130	131	132	133	134	135
Unit1	2221	1111	1121	1211	1221	2111	2121	2211	2221
Unit2	2201	1110	1120	1210	1220	2110	2120	2210	2220

Factorial experiments: 3x3x2x3

Block	1	2	3	4	5	6	7	8	9
Unit1	0001	0002	0010	0011	0012	0100	0101	0102	0110
Unit2	0000	0000	0000	0001	0002	0000	0001	0002	0010
Block	10	11	12	13	14	15	16	17	18
Unit1	0111	0112	0200	0201	0202	0210	0211	0212	1000
Unit2	0011	0012	0000	0001	0002	0010	0011	0012	0000
Block	19	20	21	22	23	24	25	26	27
Unit1	1001	1002	1010	1011	1012	1100	1101	1102	1110
Unit2	0001	0002	0010	0011	0012	0100	0101	0102	0110
Block	28	29	30	31	32	33	34	35	36
Unit1	1111	1112	1200	1201	1202	1210	1211	1212	2000
Unit2	0111	0112	0200	0201	0202	0210	0211	0212	0000
Block	37	38	39	40	41	42	43	44	45
Unit1	2001	2002	2010	2011	2012	2100	2101	2102	2110
Unit2	0001	0002	0010	0011	0012	0100	0101	0102	0110
Block	46	47	48	49	50	51	52	53	54
Unit1	2111	2112	2200	2201	2202	2210	2211	2212	0011
Unit2	0111	0112	0200	0201	0202	0210	0211	0212	0010
Block	55	56	57	58	59	60	61	62	63
Unit1	0012	0101	0102	0110	0201	0202	0210	1001	1002
Unit2	0010	0100	0100	0100	0200	0200	0200	1000	1000
Block	64	65	66	67	68	69	70	71	72
Unit1	1010	1100	1200	2001	2002	2010	2100	2200	0111
Unit2	1000	1000	1000	2000	2000	2000	2000	2000	0101

Block	73	74	75	76	77	78	79	80	81
Unit1	0112	0211	0212	1011	1012	1101	1102	1110	1201
Unit2	0102	0201	0202	1001	1002	1001	1002	1010	1001
Block	82	83	84	85	86	87	88	89	90
Unit1	1202	1210	2011	2012	2101	2102	2110	2201	2202
Unit2	1002	1010	2001	2002	2001	2002	2010	2001	2002
Block	91	92	93	94	95	96	97	98	99
Unit1	2210	0111	0112	0211	0212	1011	1012	1101	1102
Unit2	2010	0110	0110	0210	0210	1010	1010	1100	1100
Block	100	101	102	103	104	105	106	107	108
Unit1	1110	1201	1202	1210	2011	2012	2101	2102	2110
Unit2	1100	1200	1200	1200	2010	2010	2100	2100	2100
Block	109	110	111	112	113	114	115	116	117
Unit1	2201	2202	2210	1111	1112	1211	1212	2111	2112
Unit2	2200	2200	2200	1011	1012	1011	1012	2011	2012
Block	118	119	120	121	122	123	124	125	126
Unit1	2211	2212	1111	1112	1211	1212	2111	2112	2211
Unit2	2011	2012	1101	1102	1201	1202	2101	2102	2201
Block	127	128	129	130	131	132	133	134	135
Unit1	2212	1111	1112	1211	1212	2111	2112	2211	2212
Unit2	2202	1110	1110	1210	1210	2110	2110	2210	2210

Factorial experiments: 3x2x3x3

Block	1	2	3	4	5	6	7	8	9
Unit1	0001	0002	0010	0011	0012	0020	0021	0022	0100
Unit2	0000	0000	0000	0001	0002	0000	0001	0002	0000
Block	10	11	12	13	14	15	16	17	18
Unit1	0101	0102	0110	0111	0112	0120	0121	0122	1000
Unit2	0001	0002	0010	0011	0012	0020	0021	0022	0000
Block	19	20	21	22	23	24	25	26	27
Unit1	1001	1002	1010	1011	1012	1020	1021	1022	1100
Unit2	0001	0002	0010	0011	0012	0020	0021	0022	0100
Block	28	29	30	31	32	33	34	35	36
Unit1	1101	1102	1110	1111	1112	1120	1121	1122	2000
Unit2	0101	0102	0110	0111	0112	0120	0121	0122	0000
Block	37	38	39	40	41	42	43	44	45
Unit1	2001	2002	2010	2011	2012	2020	2021	2022	2100
Unit2	0001	0002	0010	0011	0012	0020	0021	0022	0100
Block	46	47	48	49	50	51	52	53	54
Unit1	2101	2102	2110	2111	2112	2120	2121	2122	0011
Unit2	0101	0102	0110	0111	0112	0120	0121	0122	0010
Block	55	56	57	58	59	60	61	62	63
Unit1	0012	0021	0022	0101	0102	0110	0120	1001	1002
Unit2	0010	0020	0020	0100	0100	0100	0100	1000	1000
Block	64	65	66	67	68	69	70	71	72

Unit1	1010	1020	1100	2001	2002	2010	2020	2100	0111
Unit2	1000	1000	1000	2000	2000	2000	2000	2000	0101
Block	73	74	75	76	77	78	79	80	81
Unit1	0112	0121	0122	1011	1012	1021	1022	1101	1102
Unit2	0102	0101	0102	1001	1002	1001	1002	1001	1002
Block	82	83	84	85	86	87	88	89	90
Unit1	1110	1120	2011	2012	2021	2022	2101	2102	2110
Unit2	1010	1020	2001	2002	2001	2002	2001	2002	2010
Block	91	92	93	94	95	96	97	98	99
Unit1	2120	0111	0112	0121	0122	1011	1012	1021	1022
Unit2	2020	0110	0110	0120	0120	1010	1010	1020	1020
Block	100	101	102	103	104	105	106	107	108
Unit1	1101	1102	1110	1120	2011	2012	2021	2022	2101
Unit2	1100	1100	1100	1100	2010	2010	2020	2020	2100
Block	109	110	111	112	113	114	115	116	117
Unit1	2102	2110	2120	1111	1112	1121	1122	2111	2112
Unit2	2100	2100	2100	1011	1012	1021	1022	2011	2012
Block	118	119	120	121	122	123	124	125	126
Unit1	2121	2122	1111	1112	1121	1122	2111	2112	2121
Unit2	2021	2022	1101	1102	1101	1102	2101	2102	2101
Block	127	128	129	130	131	132	133	134	135
Unit1	2122	1111	1112	1121	1122	2111	2112	2121	2122
Unit2	2102	1110	1110	1120	1120	2110	2110	2120	2120

Factorial experiments: 3x3x3x3

Block	1	2	3	4	5	6	7	8	9
Unit1	0001	0002	0010	0011	0012	0020	0021	0022	0100
Unit2	0000	0000	0000	0001	0002	0000	0001	0002	0000
10	11	12	13	14	15	16	17	18	19
0101	0102	0110	0111	0112	0120	0121	0122	0200	0201
0001	0002	0010	0011	0012	0020	0021	0022	0000	0001
20	21	22	23	24	25	26	27	28	29
0202	0210	0211	0212	0220	0221	0222	1000	1001	1002
0002	0010	0011	0012	0020	0021	0022	0000	0001	0002
30	31	32	33	34	35	36	37	38	39
1010	1011	1012	1020	1021	1022	1100	1101	1102	1110
0010	0011	0012	0020	0021	0022	0100	0101	0102	0110
40	41	42	43	44	45	46	47	48	49
1111	1112	1120	1121	1122	1200	1201	1202	1210	1211
0111	0112	0120	0121	0122	0200	0201	0202	0210	0211
50	51	52	53	54	55	56	57	58	59
1212	1220	1221	1222	2000	2001	2002	2010	2011	2012
0212	0220	0221	0222	0000	0001	0002	0010	0011	0012
60	61	62	63	64	65	66	67	68	69
2020	2021	2022	2100	2101	2102	2110	2111	2112	2120
0020	0021	0022	100	101	102	110	111	112	120

70	71	72	73	74	75	76	77	78	79
2121	2122	2200	2201	2202	2210	2211	2212	2220	2221
121	122	200	201	202	210	211	212	220	221
80	81	82	83	84	85	86		87	
2222	0011	0012	0021	0022	0101	0102		0110	
222	0010	0010	0020	0020	0100	0100		0100	
88	89	90	91	92	93	94	95	96	97
0120	0201	0202	0210	0220	1001	1002	1010	1020	1100
0100	0200	0200	0200	0200	1000	1000	1000	1000	1000
98	99	100	101	102	103	104	105	106	107
1200	2001	2002	2010	2020	2100	2200	0111	0112	0121
1000	2000	2000	2000	2000	2000	2000	0101	0102	0101
108	109	110	111	112	113	114	115	116	117
0122	0211	0212	0221	0222	1011	1012	1021	1022	1101
0102	0201	0202	0201	0202	1001	1002	1001	1002	1001
118	119	120	121	122	123	124	125	126	127
1102	1110	1120	1201	1202	1210	1220	2011	2012	2021
1002	1010	1020	1001	1002	1010	1020	2001	2002	2001
128	129	130	131	132	133	134	135	136	137
2022	2101	2102	2110	2120	2201	2202	2210	2220	0111
2002	2001	2002	2010	2020	2001	2002	2010	2020	0110
138	139	140	141	142	143	144	145	146	147
0112	0121	0122	0211	0212	0221	0222	1011	1012	1021
0110	0120	0120	0210	0210	0220	0220	1010	1010	1020
148	149	150	151	152	153	154	155	156	157
1022	1101	1102	1110	1120	1201	1202	1210	1220	2011
1020	1100	1100	1100	1100	1200	1200	1200	1200	2010
158	159	160	161	162	163	164	165	166	167
2012	2021	2022	2101	2102	2110	2120	2201	2202	2210
2010	2020	2020	2100	2100	2100	2100	2200	2200	2200
168	169	170	171	172	173	174	175	176	177
2220	1111	1112	1121	1122	1211	1212	1221	1222	2111
2200	1011	1012	1021	1022	1011	1012	1021	1022	2011
178	179	180	181	182	183	184	185	186	187
2112	2121	2122	2211	2212	2221	2222	1111	1112	1121
2012	2021	2022	2011	2012	2021	2022	1101	1102	1101
188	189	190	191	192	193	194	195	196	197
1122	1211	1212	1221	1222	2111	2112	2121	2122	2211
1102	1201	1202	1201	1202	2101	2102	2101	2102	2201
198	199	200	201	202	203	204	205	206	207
2212	2221	2222	1111	1112	1121	1122	1211	1212	1221
2202	2201	2202	1110	1110	1120	1120	1210	1210	1220
208	209	210	211	212	213	214	215	216	
1222	2111	2112	2121	2122	2211	2212	2221	2222	
1220	2110	2110	2120	2120	2210	2210	2220	2220	

Appendix III: Catalogue of efficient row-column designs for mixed level (number of levels: 2 and 3; number of factors: 2 and 3) factorial experiments based on baseline parameterization

Factorial experiments: 2x2

Column	1	2	3	4
Row1	01	00	11	10
Row2	00	10	01	11

Factorial experiments: 2x3

Column	1	2	3	4	5	6	7	8
Row1	01	00	10	01	02	11	12	00
Row2	00	02	00	11	12	10	10	01

Factorial experiments: 3x2

Column	1	2	3	4	5	6	7	8
Row1	01	00	11	00	21	10	20	01
Row2	00	10	01	20	01	11	21	00

Factorial experiments: 3x3

Column	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
Row1	01	02	10	11	02	00	21	22	10	12	20	20	00	01	00
Row2	00	00	00	01	12	20	01	02	11	10	21	22	20	00	10

Factorial experiments: 2x2x2

Column	1	2	3	4	5	6	7	8	9	10	11	12
Row1	001	010	011	000	001	010	111	011	100	110	101	111
Row2	000	000	001	100	101	110	011	010	101	100	111	110
Column	13	14	15	16	17	18						
Row1	110	000	000	100	001	101						
Row2	111	010	010	000	011	001						

Factorial experiments: 2x3x2

Column	1	2	3	4	5	6	7	8	9	10
Row1	001	000	011	020	021	000	001	110	111	120
Row2	000	010	001	000	001	100	101	010	011	020
Column	11	12	13	14	15	16	17	18	19	20
Row1	121	010	021	100	110	100	101	111	101	121
Row2	021	011	120	101	100	120	111	110	121	021
Column	21	22	23	24	25	26				
Row1	011	010	100	000	021	001				
Row2	111	011	110	100	001	000				

Factorial experiments: 2x2x3

Column	1	2	3	4	5	6	7	8	9	10
Row1	001	000	010	011	012	000	101	002	110	111
Row2	000	002	000	001	002	100	001	102	010	011
Column	11	12	13	14	15	16	17	18	19	20
Row1	112	011	010	100	100	110	101	102	111	112
Row2	012	010	012	101	102	100	111	112	110	110
Column	21	22	23	24	25	26				
Row1	002	001	102	011	012	010				
Row2	012	101	100	111	010	011				

Factorial experiments: 2x3x3

Column	1	2	3	4	5	6	7	8	9
Row1	001	000	000	011	012	000	021	022	100
Row2	000	002	010	001	002	020	001	002	000
Column	10	11	12	13	14	15	16	17	18
Row1	101	102	010	111	112	120	121	122	011
Row2	001	002	110	011	012	020	021	022	010
Column	19	20	21	22	23	24	25	26	27
Row1	012	021	020	100	102	110	100	101	102
Row2	010	020	022	101	100	100	120	111	112
Column	28	29	30	31	32	33	34	35	36
Row1	101	102	111	112	121	122	010	012	020
Row2	121	122	110	110	120	120	000	010	021
Column	37	38	39						
Row1	022	120	010						
Row2	122	121	011						

Factorial experiments: 3x2x2

Column	1	2	3	4	5	6	7	8	9	10
Row1	001	010	011	100	101	110	111	000	001	210
Row2	000	000	001	000	001	010	011	200	201	010
Column	11	12	13	14	15	16	17	18	19	20
Row1	211	011	100	110	200	210	101	201	111	211
Row2	011	010	101	100	201	200	111	211	110	210
Column	21	22	23	24	25	26				
Row1	201	010	000	001	210	101				
Row2	210	110	001	201	211	100				

Factorial experiments: 3x3x2

Column	1	2	3	4	5	6	7	8	9
Row1	001	000	011	020	021	000	001	110	111
Row2	000	010	001	000	001	100	101	010	011
Column	10	11	12	13	14	15	16	17	18
Row1	120	121	200	201	010	011	220	221	010
Row2	020	021	000	001	210	211	020	021	011
Column	19	20	21	22	23	24	25	26	27

Row1	021	101	100	100	201	210	200	101	121
Row2	020	100	110	120	200	200	220	111	101
Column	28	29	30	31	32	33	34	35	36
Row1	211	221	111	120	211	220	110	210	000
Row2	201	201	110	121	210	221	111	211	100
Column	37	38	39						
Row1	001	021	220						
Row2	021	121	221						

Factorial experiments: 3x2x3

Column	1	2	3	4	5	6	7	8	9
Row1	001	000	010	011	012	000	101	002	010
Row2	000	002	000	001	002	100	001	102	110
Column	10	11	12	13	14	15	16	17	18
Row1	111	112	200	001	002	010	211	212	011
Row2	011	012	000	201	202	210	011	012	010
Column	19	20	21	22	23	24	25	26	27
Row1	012	100	100	110	201	202	200	101	112
Row2	010	101	102	100	200	200	210	111	102
Column	28	29	30	31	32	33	34	35	36
Row1	211	212	111	110	210	210	201	202	101
Row2	201	202	110	112	211	212	211	212	111
Column	37	38	39						
Row1	110	001	000						
Row2	112	101	001						

Factorial experiments: 3x3x3

Column	1	2	3	4	5	6	7	8	9
Row1	001	000	010	011	012	000	021	002	100
Row2	000	002	000	001	002	020	001	022	000
Column	10	11	12	13	14	15	16	17	18
Row1	001	002	110	111	112	020	121	122	000
Row2	101	102	010	011	012	120	021	022	200
Column	19	20	21	22	23	24	25	26	27
Row1	012	101	102	100	200	200	210	111	112
Row2	010	100	100	110	201	202	200	101	102
Column	28	29	30	31	32	33	34	35	36
Row1	010	020	022	101	102	100	100	201	202
Row2	012	021	020	100	100	110	120	200	200
Column	37	38	39	40	41	42	43	44	45
Row1	200	200	121	122	101	102	211	212	201
Row2	210	220	101	102	111	112	201	202	221
Column	46	47	48	49	50	51	52	53	54
Row1	202	110	110	120	120	211	210	221	220
Row2	222	111	112	121	122	210	212	220	222
Column	55	56	57	58	59	60	61	62	63

Row1	021	022	210	201	100	220	222	210	222
Row2	121	122	211	211	110	200	210	200	210
Column	64	65	66	67	68				
Row1	200	200	210	200	000				
Row2	210	201	200	000	100				

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