

Climatic variability and prediction of annual rainfall using stochastic time series model at Jhansi in central India

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सार – इस शोध पत्र में झाँसी (25°27' उ. अक्षांश, 78°35' पू. देशांतर मा.स. तल 271 मी. से अधिक) के 72 वर्षों (1939 से 2010 तक) की अवधि के वर्षा आंकड़ों का उपयोग करते हुए वार्षिक वर्षा पूर्वानुमान देने हेतु एक स्टाकिस्टिक टाइम सिरीज मॉडल विकसित करने के लिए वर्षा के बदलाव का अध्ययन किया गया है। 77 वर्षों अर्थात् 1939 से 2015 तक की इस लंबी अवधि के वर्षा आंकड़ों का विश्लेषण करने से पता चला है कि इन 77 वर्षों में वार्षिक वर्षा 375 से 1510 मि.मी. के बीच रही है। इन 77 वर्षों में वर्षा में 4.2 मि.मी प्रतिवर्ष की दर से कमी ही प्रवृत्ति रही है। दीर्घअवधि वार्षिक वर्षा का औसत 908.3 ± 248.2 मि.मी. रहा है जिसमें भिन्नता गुणांक 27.3% रहा है। इस क्षेत्र में 77 वर्षों की अवधि में वर्षा में 319.5 मि.मी. की गिरावट आई है और यह 1068.4 मि.मी. से घटकर 748.4 मि.मी. हो गई है। इसके लिए 0, 1 और 2 श्रेणी के ऑटोरिग्रेसिव (AR) मॉडलों का प्रयोग एवं विकास किया गया है श्रेणी 2 का ऑटोरिग्रेसिव मॉडल झाँसी में 74% वर्षों में 20% के घट-बढ़ के साथ वार्षिक वर्षा का पूर्वानुमान देने में सफल रहा है। वार्षिक वर्षा के पूर्वानुमान और प्रेक्षित की गई वर्षा के बीच सहसंबंध (r) में जलवायिक औसत विसंगति 0.76 पाई गई है। इन मॉडलों की अनुकूलता और उपयुक्तता का परीक्षण बॉक्स-पियर्स पोर्ट मनटाओ टेस्ट के अकाई क्रिटेरियन इन्फारमेशन तथा ऐतिहासिक एवं प्राप्त किए गए आंकड़ों की तुलना के आधार पर किया गया है। ऐतिहासिक और प्राप्त किए गए वर्षा आंकड़ों की आरेखी प्रस्तुति एक दूसरे के काफी निकट है। ऑटो रिग्रेशन (2) मॉडल द्वारा मापित और पूर्वानुमानित वर्षा आंकड़ों के बीच तुलना करने पर स्पष्ट रूप से पता चलता है कि झाँसी में वार्षिक वर्षा का पूर्वानुमान देने के लिए विकसित किए गए मॉडल का प्रयोग दक्षतापूर्वक किया जा सकता है।

ABSTRACT. A study was conducted on rainfall variability/change and to develop a stochastic time series model for annual rainfall prediction using rainfall data for the period of 72 years (1939-2010) at Jhansi (25°27' N latitude, 78°35' E longitude, 271 m above mean sea level). The analysis of long term rainfall data for the period of 77 years *i.e.*, 1939-2015 revealed that annual rainfall varied between 375 to 1510 mm over 77 years with a decreasing trend of 4.2 mm/year. The long term mean annual rainfall is 908.3 ± 248.2 mm with a coefficient of variation of 27.3%. The rainfall of the region had been decreased by 319.5 mm over the period of 77 years from 1068.4 mm to 748.4 mm. Autoregressive (AR) models of order 0, 1 and 2 were tried and developed. The autoregressive model of the order 2 was able to predict the annual rainfall of Jhansi within ±20% in 74% of the years. Correlation (r) between the anomaly of observed and predicted annual rainfall from the climatological mean was 0.76. The goodness of fit and adequacy of models were tested by Box- Pierce Portmanteau test, Akaike information Criterion and by comparison of historical and generated data. The graphical representation between historical and generated rainfall was a very close agreement between them. The comparison between the measured and predicted rainfall by AR (2) model clearly shows that the developed model can be used efficiently for the annual prediction of rainfall at Jhansi.

Key words – Akaike information criterion, Autoregressive (AR) models, Box-Pierce Portmanteau test, Long term trend, Seasonal rainfall variation, Stochastic time series model.

1. Introduction

The Bundelkhand region is spread over 71618 square kilometers of the central plains and many of the districts are included in the list of most backward districts of India by Planning Commission, GOI. The region supports 18.31 million (79.1% in rural areas) human populations as per the 2011 census with 10.7 million animal population and more than one third of the households in these areas are considered to be Below the Poverty Line (BPL).

Agriculture in Bundelkhand is rainfed, diverse, complex, under-invested, risky and vulnerable. In addition, extreme weather conditions, like droughts, short-term rain and flooding in fields add to the uncertainties in agricultural production and seasonal human migrations for the search of employment. The scarcity of water in the semi-arid region, with poor soil and low productivity further aggravates the problem of food security. Climate change in world is always one of the most important aspects in water resources management (Rai *et al.*, 2014;

Palsaniya *et al.*, 2016). Weather parameter such as precipitation could be practically useful in making decisions, risk management and optimum usage of water resources (Baigorria and Jones, 2010; Chattopadhyay and Chattopadhyay, 2010) in country like India. India has been traditionally dependent on agriculture as 70% of its population is engaged in farming. Rainfall in India is dependent on south-west and north east monsoons, on shallow cyclonic depression and disturbances and on local storms. India receives annual precipitation of about 4000 km³ including snowfall. Out of this, monsoon rainfall is of the order 3000 km³. Climate variability and change affects individuals and societies. Thus for a given region it is important before developing a prediction model. Since, an understanding of the variations of rainfall is indispensable for the design of water harvesting structure, development of soil moisture conservation measures, drainage systems, storm water management plans etc. (Brissette *et al.*, 2007). Within agricultural systems, climate forecasting can increase preparedness and lead to better social, economic and environmental outcomes. Information on rainfall is also important in various types of hydrological studies concerned with the determination of peak runoff and its volume. Time series analysis and forecasting has become a major tool in numerous hydro-meteorological applications, to study trends and variations of variables like rainfall and many other environmental parameters (Alexendar *et al.*, 2006, Kwon *et al.*, 2007). Before designing suitable adaptation and mitigation strategies for agricultural production system against changing climate, it becomes inevitable to analyse the long term variability, rate of rainfall and its trend. Therefore, forecasting of annual rainfall for efficient and sustainable utilization of water resources need to be explored in view of the changing climatic conditions in Bundelkhand region. Rainfall series are the hydrological time series composed of deterministic and stochastic components. In order to consider the deterministic part, the nuances of the series, which is noise of signal, have to be eliminated (Tantaneel *et al.*, 2005; Chakraborty *et al.*, 2014). Thus, the deterministic part can describe the mathematical characteristics of the series. However, the dependency of stochastic components of the series can be analyzed using the autoregressive (AR) models. Moving average model (MA) or auto regressive integrated moving average model (ARIMA) & are widely used to predict annual rainfall. Autoregressive (AR) model with pth (0, 1, 2, ...n) order is a representation of a type of random process describe certain time-varying processes in nature and it specifies that the output variable depends linearly on its own previous values and on a stochastic (an imperfectly predictable term), term thus the model is in the form of a stochastic difference equation. The random component in time series, which represents the characteristics that are purely probabilistic, needs

special attention. The data generated through these models are used for various water resources management. Iyenger (1982) used stochastic modeling to predict the monthly rainfall and reported that the developed model is suitable for a certain range and applicable to particular zone of climate. Stochastic time series modeling was used to predict the annual rainfall and runoff in lidar catchment of South Kashmir (Sherring *et al.*, 2009). Dhar *et al.* (1982) analyzed the average rainfall for the north east monsoon using standard methods. Sundaram and Lakshmi (2014) tried to predict the monthly rainfall using Box-Jenkins Seasonal Auto Regressive Integrated Moving Average model, with 136 years of rainfall data of Tamilnadu. They analyzed trend, periodicities and variability for prediction of annual rainfall in Tamilnadu. Keeping this into mind two aspects were studied (1) to quantify monthly and seasonal rainfall variability and trend (2) Prediction of the time series changes by means of autoregressive models.

2. Materials and method

The annual rainfall data for the period of 77 years (1939-2015) have been used to analyze monthly, seasonal rainfall and 71 years (1939-2009) of data have used to develop stochastic time series model to predict annual rainfall at Jhansi (25° 27' N latitude, 78° 35' E longitude and 271 m above mean sea level) and rest five years (2011-2015) data were used to validate the model for its evaluation. About 90% of the annual rainfall is received during June to September and rest 10% in the remaining period. Autoregressive model was developed using the method given below:

2.1. Autoregressive model

Let us consider a stationary time series Y_t normally distributed with mean ' μ ' and variance ' σ^2 ' which has an auto regressive correlation (or time dependent structure) with constant parameters (Salas and Smith, 1981). The auto regressive model of order 'p', denoted by AR(p) representing the variable Y_t may written as,

$$Y_t = \mu + z_t \quad (1)$$

$$z_t = \sum_{j=1}^p \phi_j z_{t-j} + \varepsilon_t, \text{ or}$$

$$z_t = \sum_{j=1}^p \phi_j z_{t-j} + \sigma_\varepsilon \xi_t \quad (2)$$

where, Y_t is the dependent time series (variable), ' ε_t ' is independent of Y_t and is normally distributed with mean zero and variance one, \bar{Y} is the mean of annual rainfall data and $\phi_1, \phi_2, \dots, \phi_p$ are the Autoregressive parameters.

TABLE 1

Mean month, seasonal and annual rainfall (1939-2015) of Jhansi along with standard deviation (SD) and CV(%)

Month/season	Rainfall (mm)	SD (mm)	CV (%)
January	15.2	20.7	135.9
February	13.5	23.8	176.4
March	5.3	12.5	233.5
April	3.8	6.4	169.7
May	12.7	21.5	169.6
June	98.7	105.3	106.7
July	311.6	142.8	45.8
August	252.5	138.7	54.9
September	147.8	140.1	94.8
October	31.4	56.3	179.5
November	8.0	25.2	313.7
December	7.8	21.8	279.1
Monsoon (Jun-Sep)	810.6	236.2	29.1
Post-Monsoon (Oct-Nov)	39.4	59.5	150.9
Winter (Dec-Feb)	35.8	39.5	110.1
Summer (Marc-May)	21.8	27.4	126.0
Annual	908.3	248.2	27.3

The modeling procedure described herein contains various parts. The first part outlines some preliminary analysis and criteria for selecting the order of the model to be fitted to the historical series. The second part estimates the parameters of the selected model. The third part describes some tests of goodness of fit based essentially on tests for independence and normality of the residual, graphical comparison of the historical and model correlograms and a model parsimony test. The fourth part ‘‘Optional Tests of the Model’’ deals with further testing the model by comparing the statistical characteristics of the historical series with the corresponding characteristics of the synthetic time series generated with the model. The last part ‘‘Reliability of Estimated Parameters’’ deals with determine the confidence limits of the estimated parameters of the model.

2.1.1. Preliminary analysis and order of the model

The main purpose of this part are to check the normality of the original series by applying either the chi-square goodness of fit test or the test of the skewness coefficient as described by Cochran and Cox, 1957. If the data is not normal then transform the non-normal time series into normal. After assuming the

normality of the data, determine autocorrelation and partial functions described below to find out the order of the model.

2.1.1.1. Autocorrelation function

Autocorrelation function (r_k) of lag ‘k’ was estimated as proposed by Kottogoda and Horder (1980) :

$$r_k = \frac{\sum_{t=1}^{N-k} (Y_t - \bar{Y})(Y_{t+k} - \bar{Y})}{\sum_{t=1}^{\infty} (Y_t - \bar{Y})^2} \tag{3}$$

Where r_k is the Auto correlation function of time series Y_t at lag k , Y_t is Stream flow series (observed data), is Mean of time series (Y_t), N are the total number of discrete values of time series (Y_t). The auto correlogram was used for identifying the order of the model for given time series as well as for comparing the sample correlogram with model correlogram. The 95 per cent probability level for the auto correlation function was estimated (Anderson, 1942).

2.1.1.2. Partial auto correlation function

The partial autocorrelation was calculated to identify both the type and order of the model (Durbin, 1960).

$$PC_{k,k} = \frac{r_{k-\sum_{j=1}^{k-1} PC_{k-1,j}} r_{k-j}}{1 - \sum_{j=1}^{k-1} PC_{k-1,j} r_{k-j}} \tag{4}$$

where, $PC_{k,k}$ is the Partial autocorrelation function at lag ‘k’ and ‘ r_k ’ is the autocorrelation function at lag ‘k’. The 95% probability limit for partial autocorrelation function was also estimated (Anderson, 1942).

2.2. Estimation of parameter of the selected model

For estimation of the model parameter method of maximum likelihood will be used (Box and Jenkins, 1970). Consider the sum of cross-products,

$$Z_i Z_j + Z_{i+1} Z_{j+1} + \dots + Z_{N+1-j} Z_{N+1-i}$$

and define,

$$D_{i,j} = \frac{N}{N+2-i-j} \sum_{L=0}^{N+i-(i+j)} Z_{i+L} Z_{j+L} \tag{5}$$

where,

D = Difference operator

N = Sample size

$i, j =$ Maximum possible order

$I =$ Autocorrelation function

Estimation of Autoregressive parameters (ϕ)

$$\text{For AR (1): } \phi_1 = \frac{D_{1,2}}{D_{2,2}} \quad (6)$$

$$\text{For AR (2): } \phi_1 = \frac{D_{1,2}D_{3,3} - D_{1,3}D_{2,3}}{D_{2,2}D_{3,3} - D_{2,3}^2} \quad (7)$$

$$\phi_2 = \frac{D_{1,3}D_{2,2} - D_{1,2}D_{2,3}}{D_{2,2}D_{3,3} - D_{2,3}^2} \quad (8)$$

The variance of white noise ' σ_ε^2 ' may be estimated by :

$$\hat{\sigma}_\varepsilon^2 = \frac{1}{(N-p)} (D_{11} - \sum_{j=1}^p \phi_j D_{1,j+1}) \quad (9)$$

For AR(0)

$$\hat{\sigma}_\varepsilon^2 = \frac{1}{N} (D_{11}) \quad (10)$$

For AR (1)

$$\hat{\sigma}_\varepsilon^2 = \frac{1}{(N-1)} (D_{11} - \phi_1 D_{12}) \quad (11)$$

For AR (2)

$$\hat{\sigma}_\varepsilon^2 = \frac{1}{(N-2)} (D_{11} - \phi_1 D_{12} - \phi_2 D_{13}) \quad (12)$$

2.2.1. Stationarity conditions of estimated parameters

Test the stationarity conditions of the estimated autoregressive parameters ϕ_1, \dots, ϕ_p by obtaining the p roots of equation (13-16) and check whether they lie within the unit circle. In particular, for $p = 1$ expression (13) must be met while for $p = 2$ expression (14-16) must be met.

$$-1 < \phi_1 < 1 \quad (13)$$

$$\phi_1 + \phi_2 < 1 \quad (14)$$

$$\phi_2 - \phi_1 < 1 \quad (15)$$

$$-1 < \phi_2 < 1 \quad (16)$$

2.3. Tests of goodness of fit of selected model

Port Manteau lack of fit test in which autocorrelations of the ε_t are taken as a whole.

In this case equation is applied to determine the statistics Q :

$$Q = N \sum_{k=1}^L r_k^2(\hat{\varepsilon}) \quad (17)$$

L may be the order of 30% of the sample size N . The statistics Q is approximately $X^2 (L-p)$. If $Q < X^2 (L-p)$ then ε_t of the expression (17) is independent which in turn implies that the selected model [AR(P) is adequate or otherwise, the model is inadequate and another model say of order $p+1$] should be selected for analysis.

2.3.1. Test for the parsimony of the parameters

The Akaike Information Criterion (AIC) is also used for checking whether the order of the fitted model is adequate compared with other orders of the dependence model. From equation the AIC for an AR(p) model is

$$\text{AIC}(p) = N \ln(\hat{\sigma}_\varepsilon^2) + 2p \quad (18)$$

where, $\hat{\sigma}_\varepsilon^2$ the maximum likelihood estimates of the variance. Therefore, a comparison can be made between the AIC (p) and the AIC ($p-1$) and AIC ($p+1$). If the AIC (p) is less than both the AIC ($p-1$) and AIC ($p+1$), then the AR (p) model is best. Otherwise, the model with less AIC become the new candidate model. In a way the AIC is a criterion for the selection of the order of the model, thus the plot of the AIC (p) against p as well as the plot of the sample and population partial correlograms could be used for the final modal selection.

2.4. Optional test of the model

The modeler is usually interested in finding a model which can replace the historical statistical characteristics may be the historical mean, standard deviation, skewness coefficient, correlogram, mean ranges. Therefore the main purpose of this part is to compare the statistical characteristics of the generated data with those of the historical data. Some of statistics such mean forecast error (MFE), mean absolute error (MAE) and root mean square error (RMSE) given below were computed to test the adequacy of the model

$$\text{MFE} = \frac{\sum_{t=1}^n \chi_c(t) - \sum_{t=1}^n \chi_0(t)}{\eta}$$

where,

$\chi_c(t) =$ Predicated rainfall

$\chi_0(t) =$ Observed rainfall

$\eta =$ Number of observation

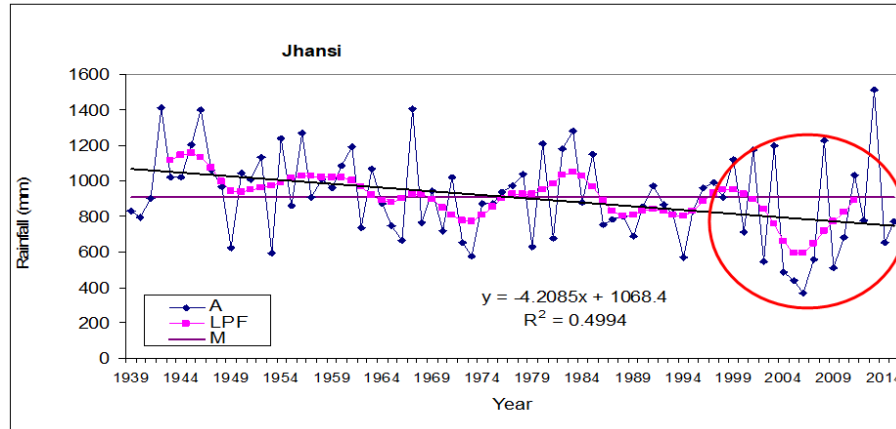


Fig. 1. Annual rainfall variability along with Gaussian low pass filter curve along with long term trend at Jhansi (Source: Palsaniya *et al.*, 2016)

TABLE 2
Statistical parameters of autoregressive (AR) model for annual rainfall

Model	Autoregressive Parameters	White noise	White noise variance	AIC (p)	Value of Q test	DF	Chi-square at 5% level
AR(0)	-	57262.7	239.3	796	21.7	24	36.4
AR(1)	$\phi_1 = -0.0042$	58068.5	240.9	794.6	23.5	23	35.1
AR(2)	$\phi_1 = -0.0025$ $\phi_2 = 0.27$	54484.7	233.4	794.1	20.5	22	33.9

$$MAE = \frac{\sum_{t=1}^n |\chi_c(t) - \hat{\chi}_c(t)|}{\eta}$$

$$RMSE = \left[\frac{\sum_{t=1}^n [\chi_c(t) - \hat{\chi}_c(t)]^2}{\eta} \right]^{1/2}$$

3. Results and discussion

3.1. Mean monthly and seasonal variation and trends in rainfall

The long term mean annual rainfall of Jhansi is 908.3 ± 248.2 mm with coefficient of variation of 27.3% and it varied in the range of 375.2 in the year 2006 and 1510 mm in the year 2013. High degree of variability was observed in annual rainfall series since standard deviation and coefficient of variation (CV) was higher (Table 1). Coefficient of variation of monthly rainfall varied in the range of 45-54% in July and August to more than 300% in November to December. The season wise - rainfall distribution over Jhansi indicated that 89.3% (810.6 mm) of annual rainfall is received during the monsoon. However, contribution of annual rainfall during post-monsoon and winter seasons are very small to the order 4.3 and 3.9% with high CV, thus indicated that these

rainfall are not dependable for *rabi* crops (Table 1). The seasonal rainfall during monsoon is dependable as the CV is 29.1%. A significant long term decreasing trend ($R^2 = 50\%$, $N = 77$, $P < 0.05$) was observed in annual rainfall series at the rate of 4.21 mm/year (Fig. 1). The rainfall of the region had been decreased by 319.5 mm from 1068.4 mm (Mean = 908.3) to 748.6 mm (588.8) over the period of 76 years. The recent section of the graph (2001-2015), it can be seen that the 9 years experienced moderate to disastrous drought in region, wherein rainfall deficiency ranged between 24 to 60% from normal value. Of special mention was the year, 2006, which experienced a worst drought ever recorded in this region and the rainfall deficiency was 60% from the normal value. The year 2015 experienced worst terminal drought (Rainfall ranged between 300-450 mm) affecting crop production badly during both seasons, *kharif* season contributed only 400 mm rainfall and the deficiency was more than 45% from its normal (765 mm). The deficient rainfall and the ensuing drought caused extensive damage to major crops grown in this region.

Comparison of the climate during 1956-85 and 1986 to 2015 was made to see the monthly shift of rainfall. The mean annual rainfall during the period 1956-85 and 1986-2015 were 933.6 and 819.6 mm which indicate that the

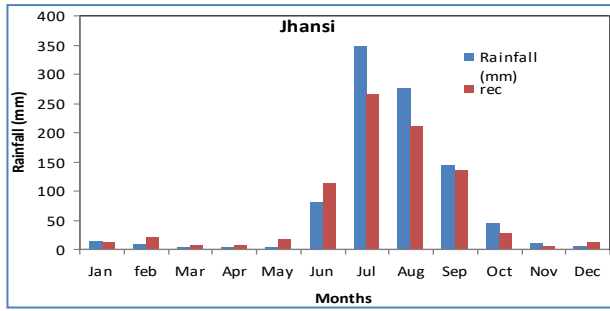


Fig. 2. Shift in monthly rainfall pattern over Jhansi

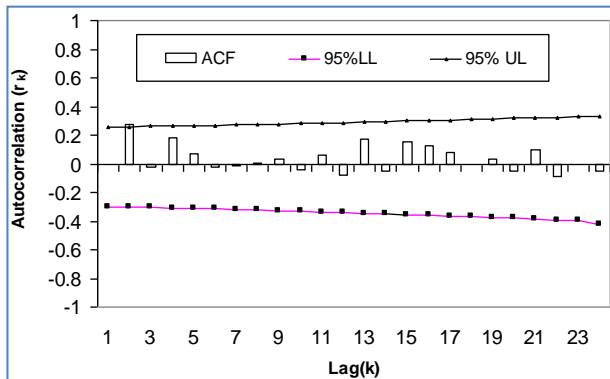


Fig. 3. Autocorrelation function of annual rainfall at Jhansi

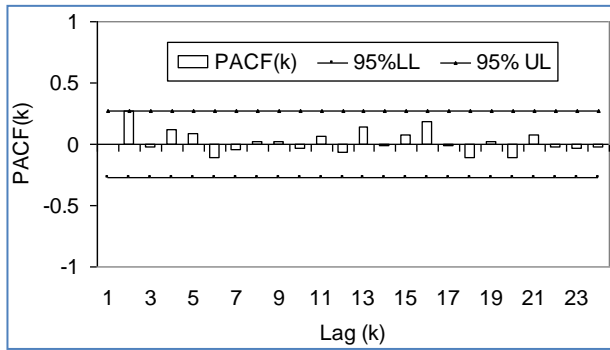


Fig. 4. PACF of annual rainfall at Jhansi

rainfall of Jhansi has changed (144 mm) significantly (Crammer's t_k test = -2.12*, $P < 0.05$). It can be seen from the Fig. 2 that there is a shift in monthly rainfall pattern moving toward early part of the south-west monsoon season. A decrease of 83.4 mm in July, 67.3 mm in August and 8.0 mm in September was noticed except in June (an increase in 29.9 mm from a period of 1956-85 over a period of 1986-2015). It is interesting to note that during winter season particularly, December and February showed positive shift in rainfall by 147.5 and 166.5% during the climate of 1986-2015 as compared to 1956-1985. This has a direct bearing of agricultural crops, as it influences cropping pattern. Upadhaya *et al.* (2015) also

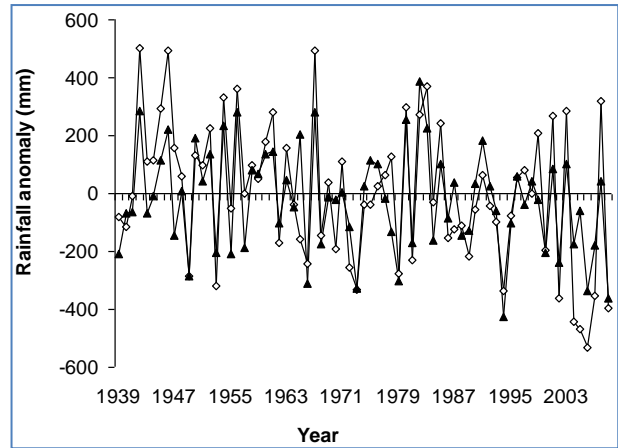


Fig. 5. Comparison between measured (M) and predicted (P) annual rainfall anomaly at Jhansi

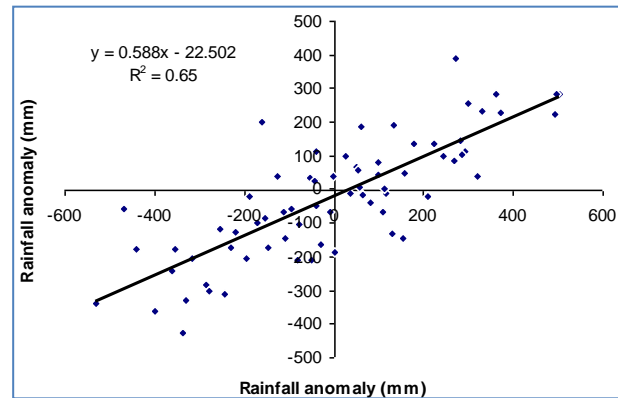


Fig. 6. Relationship between measured (M) and predicted (P) annual rainfall at Jhansi

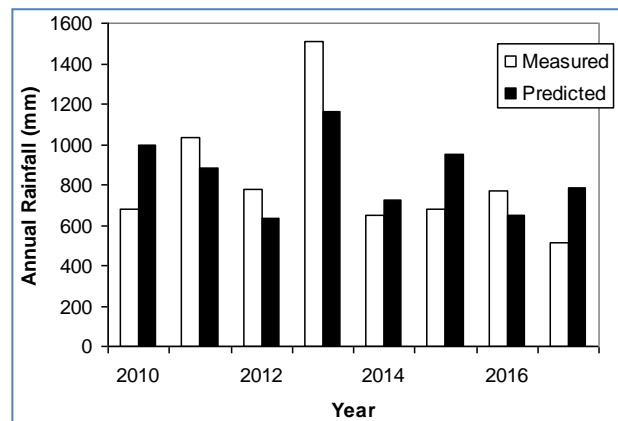


Fig. 7. Validation of the fitted model AR(2)

reported a shift in rainfall pattern at Ranichauri. The climate of Bundelkhand has drastically changed and its impact on agriculture sector is now days become more vulnerable thus a prediction model based on

TABLE 3

Statistical characteristics of observed and predicted rainfall

Statistical characteristics	Rainfall (mm)	
	Observed	Predicted
Mean	904.5	868.9
Standard deviation	247.8	214.7
Coefficient of variation(%)	27.4	24.7
Coefficient of Skewness	0.023	0.27
Coefficient of Kurtosis	-0.58	-0.52
Mean Absolute Error (MAE)	-	54.5
Mean Relative Error(MRE)	-	4.34
Root mean Square Error (RMSE)	-	461.9

autoregressive approach is inevitable to manage risk arising from climate variability.

3.2. Prediction of annual rainfall using autoregressive model (AR)

3.2.1. Preliminary analysis and model identification

In order to test the normality of the time series, the fisher statistics $g_1/(SE(g_1))$ and $g_2/(SE(g_2))$ were computed and it was found to be 0.06 and -0.69 and both statistics were less than 1.96 thus it indicates that the annual rainfall series is normally distributed at 95 % significance level. Also, coefficient of skewness of rainfall series is 0.14, which is less than +/- 0.63, to assure the normality of the annual rainfall series. Auto correlation and partial auto correlation function are used to identify the proper type and order of the model. The sample correlogram $r_k(y)$, $k = 1, \dots, 24$ and partial correlogram $\phi_k(K)$, $k = 1 \dots 24$, are determined from equation 2 and 3, respectively and presented in Figs. 3 and 4. Both correlogram shows that only the second value $r_2(2)$ or $\phi_2(2)$ is outside the 95% probability limits this would suggest that auto regressive model of the order 2 is reliable. Thus AR (2) is selected for further analysis.

3.2.2. Parameter estimation

The maximum likelihood method was used to estimate the parameter for AR(p) along with white noise variance as computed by using equation 9 to 11 and are presented in Table 2. The fitted auto regressive model for order AR (1) and AR(2) are as follows:

For AR(1)

$$Y_t = 904.1 + 0.0042Z_{t-1} + 240.9\xi_t$$

For AR(2)

$$Y_t = 904.1 - 0.0025 Z_{t-1} + 0.27Z_{t-2} + 233.4\xi_t$$

In order to assess the stationarity it becomes imperative to see the roots of the equation must lie inside the unit circle. This condition implies that the parameters Φ_1 and Φ_2 must meet the criteria given in expression (13 to 16).

For, $P = 2$ equation must be met for the stationary conditions for AR(2) model. Since $\Phi_1 + \Phi_2 = 0.267 < 1$, $\Phi_2 - \Phi_1 = -0.2725 < 1$ and $-1 < \Phi_2 = 0.25 < 1$ and thus AR (2) model is stationary.

3.2.3. Tests of goodness of fit of selected model

The Box- pierce portmanteau lack of fit test was applied to check the hypothesis that residuals (ϵ_t) is an independent series of autoregressive models for annual rainfall for AR (0), AR (1) and AR (2) models using expression (17). The statistic Q was estimated and compared for L-p degrees of freedom and a given confidence level of 5%. The results revealed that Q values are less than the corresponding chi-square at 5% level significance (Table 2), thus the residuals are independent and the selected model is adequate and AR (2) were given good fit and were acceptable.

3.2.4. Test for the parsimony of parameters

Akaike information criterion test was presented in Table 2. The AIC(0), AIC(1) and AIC(2) was compared and it was found that AIC(2) is slightly less than AIC(1) and the AR(2) is best (Table 2). However, AR(1) model can also be used to predict the rainfall.

3.2.5. Comparison of the observed and predicted annual rainfall

A graphical comparison of observed and predicted anomaly of annual rainfall is given in Fig. 5. The graphical representation of the data shows a closer agreement between observed and predicted annual rainfall by selected AR (2) model. Fig. 6 reports that developed model for rainfall can be utilized for the prediction of future trends with minimum chance of error ($r = 0.81$ and $R^2 = 0.65$). Relationship between observed and predicted anomaly of rainfall revealed that slope did not differ significantly from one. However, intercept significantly differs from zero (Fig. 6). Since, all the errors are comparatively very less (especially MFE with 13.83), the AR (2) model can be used with better confidence for prediction of annual rainfall at Jhansi. Predicted mean annual rainfall and its standard deviation and coefficient of variation are closed to each other (Table 3). Also kurtosis and coefficient of skewness of observed and predicted annual rainfall were closed to each other. The coefficient skewness of predicted data by AR (2) model and observed data is lying between -1 to +1 and therefore

AR(2) model preserved the mean and coefficient of variation better.

3.2.6. Validation and evaluation of the model

Model was validated for the year 2009-2017 using random variables between -1.23 to 1.1. Developed model is able to predict annual rainfall fairly well at Jhansi and the deviation of predicted annual rainfall from observed lies within the range of $\pm 22\%$ (Fig. 7). The greatest error in predicting annual rainfall occurred during the 2013 and 2017 and predicted annual rainfall were under and over estimated by 30 and 35% for the respective years, respectively. Correlation (r) between the anomaly of observed and predicted annual rainfall from the climatological mean was 0.76. Also, independent validation ($n = 8$) showed a relationship ($Y = 0.41x - 54.776$; $R^2 = 0.5815$) between observed and predicted anomaly of rainfall revealed that slope did not differ significantly from one. However, intercept significantly differs from zero

4. Conclusions

Mean rainfall with their standard deviation (247.8) and coefficient of variation (27.4%), indicates that inter-annual variation of rainfall is high. The rainfall of the region had been decreased by 319.5 mm over the period of 76 years from 1068.4 mm. Also a significant long term decreasing trend was observed in annual rainfall series with the rate of 4.2 mm/year. In the recent past 15 years (2001-2015), 9 years experienced moderate to disastrous drought in region, wherein rainfall deficiency ranged between 24 to 60% from normal value. Developed model [AR (2)] can be utilized for the prediction of annual rainfall with minimum chance of error ($r = 0.81$ and $R^2 = 0.65$). On the basis of estimated errors and statistical characteristics between observed and predicted values, it is concluded that the proposed autoregressive AR(2) model has an operational skill and add value to the forecast for predicting the annual rainfall of Jhansi, which can in turn be useful and add value for contingent crop planning, for efficient and sustainable utilization of water resources and soil water conservation measures.

Disclaimer : The contents and views expressed in this research paper/article are the views of the authors and do not necessarily reflect the views of the organizations they belong to.

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