



Mating plans for breeding trials using generalized row-column designs

ANINDITA DATTA¹, SEEMA JAGGI², CINI VARGHESE¹, ELDHO VARGHESE³,
MOHD HARUN¹ and ARPAN BHOWMIK^{1*}

ICAR-Indian Agricultural Statistics Research Institute, New Delhi 110 012, India

Received: 10 February 2021; Accepted: 30 March 2021

ABSTRACT

Generalized Row-Column (GRC) designs are used where there is more than one unit in each row-column intersection. Mating plan is a systematic procedure of producing the progenies. Diallel and triallel crosses are some examples of mating plans. In literature there are many mating plans which are developed using block/row-column designs. The objective of the present study is to obtain mating plans using GRC designs. The research work has been done at ICAR-IASRI during 2019–20. Here, methods of obtaining partial diallel crosses (PDC) and partial triallel crosses (PTC) plans using different classes of GRC designs have been described. The plans obtained using these designs will yield smaller degree of fractionation thereby reducing resources.

Keywords: Degree of fractionation, Partial diallel cross, Partial triallel cross, Row-column design

There are two types of designs, viz. mating designs and environmental designs in breeding trials. Mating design is a procedure of producing the progenies. Subjecting these progenies to the environmental conditions in a planned manner is known as environmental design. Diallel, partial diallel, triallel, partial triallel and four-way crosses are examples of mating designs. Proper selection of mating design is essential to attain the breeder’s goal. In literature, there are many mating plans which are developed using block/row-column designs. Here mating plans are obtained using Generalized Row-Column (GRC) designs. When the number of treatments is large with limited experimental resources, GRC designs are used where there is more than one unit in each row-column intersection. GRC design is an arrangement of v treatments in p rows and q columns such that the intersection of each row and column (cell) consists of more than one unit. Consider an experiment conducted to compare the colour intensities of apple sauce (Edmondson 1998). The treatments were 12 blends of apple sauce with 4 concentration of cinnamon. Treatments could be stored for 4 different lengths of time. A GRC design as shown below was used in which rows represent cinnamon concentrations, columns as storage times and symbols as blends.

Rows (Cinnamon Concentrations)	Columns (Storage Time)			
	I	II	III	IV
I	1 5 9	2 6 10	3 7 11	4 8 12
II	2 7 10	1 8 9	4 5 12	3 6 11
III	3 8 12	4 7 11	1 6 10	2 5 9
IV	4 6 11	3 5 12	2 8 9	1 7 10

These designs are studied in the literature in different names such as Semi-Latin square (Bedford and Whitaker 2001, Bailey and Monod 2001), Trojan square (Bailey 1988, 1992 and Edmondson 1998), Generalized incomplete Trojan-type designs (Jaggi *et al.* 2010) and row-column designs with multiple units per cell (Datta *et al.* 2014, 2015). Dharmalingam (2002) obtained PTC plans using Trojan square designs. Varghese and Jaggi (2011) constructed mating designs like PDC plans and PTC plans using cell contents of appropriate generalized incomplete Trojan-type designs.

MATERIALS AND METHODS

We consider a GRC design with v treatments arranged in p rows, q columns and in each row-column intersection (i.e. cells) there are k units or plots resulting in total $n = pqk$ experimental units or observations. The observation from a GRC design can be modeled based on a three-way classified model with treatments, rows and columns as the known sources of variation (Datta *et al.* 2014, 2015).

Partial diallel and partial triallel crosses: Diallel cross is a set of all possible mating between several genotypes which may be clones, homozygous lines, etc. These type of crosses are usually used in plant breeding trials for estimating

Present address: ¹ICAR-Indian Agricultural Statistics Research Institute; ²Krishi Anusandhan Bhavan II, New Delhi; ³ICAR-Central Marine Fisheries Research Institute, Kochi. *Corresponding author e-mail: arpan.stat@gmail.com.

genetic variances of total variance of a quantitative character and in estimating general combining abilities (gca) and specific combining abilities (sca) of inbred lines involved in the crosses. There are $n(n-1)/2$ possible crosses among a set of n lines with exclusion of reciprocal crosses and parental inbred, that increases rapidly with increase in n . A diallel cross may only be feasible for a comparably small number of inbred lines. It may be desirable to have a large number of inbred lines but raise only a sample of all possible crosses among them. Such a diallel cross is known as partial diallel cross (PDC). The set of all possible three-way hybrids based on n lines will constitute $N_T = n(n-1)(n-2)/2$ distinct triallel crosses in distinct three-way hybrids. As the number of lines (n) involved increase, the number of crosses increase manifold and becomes very large for the investigator to handle. Partial Triallel Crosses (PTC) is the alternative for that situation. Let there be n lines denoted by $1, 2, \dots, n$. A three-way cross is represented by $(i \times j) \times k$, where the offspring of the cross $i \times j$ is crossed with k and hence i and j are half-parents whereas k is a full-parent. Also, $i \neq j \neq k = 1, 2, \dots, n$.

On the lines of Hinkelmann (1965), a set of matings is said to be a PTC if it satisfies the following conditions:

Each line occurs exactly r_H times as half-parent and r_F times as full-parent; Each cross $(i \times j) \times k$ occurs either once or not at all.

Condition (ii) also include the simultaneous occurrence of $(i \times j) \times k$, $(i \times k) \times j$ and $(j \times k) \times i$ indicating structural symmetric property of the triallel crosses. Since each line is equally often represented as half-parent, it follows that $r_H = 2r_F$. Further a PTC plan has to be connected.

The statistical model underlying the analysis of variance of diallel crosses is given by

$$y_{ij} = \mu + g_i + g_j + e_{ij} \quad i < j = 1, 2, \dots, n, \quad (1)$$

with restriction $\sum g_i = 0$ for $i = 1, 2, \dots, (n-1)$. y_{ij} is the response of crosses, μ is the overall mean, g_i, g_j is the gca effect of i^{th} and j^{th} line and e_{ij} is the error term with mean zero and variance σ^2 .

The statistical model underlying the analysis of variance of triallel crosses is given by

$$y_{(ij)k} = \mu + h_i + h_j + g_k + e_{(ij)k} \quad (2)$$

($i, j, k = 1, 2, \dots, n, i \neq j \neq k$), where $y_{(ij)k}$ stands for the response of triple cross $(i \times j) \times k$, μ is the overall mean, h is the g.c.a effect of half parents and g is the gca effect of full parents and $e_{(ij)k}$ are considered to be independent random variables with mean zero and variance σ^2 .

RESULTS AND DISCUSSION

Construction of PDC plans using Generalized Row-Column Designs: Consider a Latin square of order s and another orthogonal Latin square of the same order. Renumber the s treatments of the second Latin square by $s+1, s+2, \dots, 2s$. Superimpose the second Latin square on the first Latin square. This results in a GRC design (Bailey 1988) with parameter $v=2s$ ($s>2$), $p=s$, $q=s$ and $k=2$. A PDC plan can

be obtained by making all possible distinct 2-way crosses within each cell of the GRC design. The parameters of the developed PDC plan will be n (no. of lines/genotypes) = v , N (no. of crosses) = s^2 and f (degree of fractionation) = $s/(2s-1)$. The developed PDC plan is partially balanced following a group divisible association scheme. The $v=2s$ treatments are arranged in two rows of size s each as shown below.

1	2	...	s
s+1	s+2	...	2s

The treatments in the other rows are first associates to each other and the treatments in the first row are second associates. The information matrix for PDC plan is;

$$C = a_0 I_v + a_1 A_v + a_2 B_v$$

where,

$$a_0 = \frac{(v-k)}{2}, a_1 = -1, a_2 = 0$$

$B_v = \{b_{ij}\} = 1$, if i and j are second associate = 0, otherwise

Example 1 Following (Table 1) is a GRC design with parameters $v = 10, p = 5, q = 5$ and $k = 2$:

Now, considering each treatment as a line in the breeding programme, the following crosses PDC plans are obtained by making crosses within each cell.

The parameters of the above PDC plan (as given in Table 2) are $n = 10, N = 25$ and $f = 5/9$. Here, f is the degree of fractionation. It is to be noted that the degree of fractionation is sufficiently smaller in this case which is well desirable property from breeder point of view. The developed PDC plan is partially balanced following a group divisible association scheme which is given as follows

1	2	3	4	5
6	7	8	9	10

Here, treatments in the other row are first associate of each other and treatments in the same row are second

Table 1 Layout of GRC design with parameters $v = 10, p = 5, q = 5$ and $k = 2$

Column	Rows				
	I	II	III	IV	V
I	1 6	2 7	3 8	4 9	5 10
II	2 8	3 9	4 10	5 6	1 7
III	3 10	4 6	5 7	1 8	2 9
IV	4 7	5 8	1 9	2 10	3 6
V	5 9	1 10	2 6	3 7	4 8

Table 2 PDC plan developed based on a GRC design as mentioned in Table 1

1 × 6	2 × 7	3 × 8	4 × 9	5 × 10
2 × 8	3 × 9	4 × 10	5 × 6	1 × 7
3 × 10	4 × 6	5 × 7	1 × 8	2 × 9
4 × 7	5 × 8	1 × 9	2 × 10	3 × 6
5 × 9	1 × 10	2 × 6	3 × 7	4 × 8

associates of each other. For example, the first associates of treatment 1 is 6,7,8,9, and 10, whereas the second associates are 2,3,4 and 5.

Construction of PTC Plans using Generalized Row-Column Designs: PTC plans can be obtained from the cell contents of appropriate GRC designs with cells of size 3. The treatments in the design are to be considered as the lines and then possible distinct 3-way crosses in a systematic order are to be made. If the condition of structural symmetry of PTC is not met, repeat the crosses by changing the role of full-parents and half-parents in circular manner. For v (prime) treatments, consider a set of $2v$ mutually orthogonal Latin square (MOLS) juxtaposing one after other horizontally giving rise to an array (A) of dimension $v \times 2v$. The cell contents of the first row of the GRC design is formed by taking the first k ($2 \leq k \leq v-1$) rows of the above array (A). Similarly, cell contents of the second row are obtained by taking the k consecutive rows starting from the second row of the array (A). The resulting design is a GRC design (Datta *et al.* 2015) with v treatments in $p = 2$, $q = 2v$ and each cell of size k . A PTC plan can be obtained by making all possible distinct 3-way crosses within each cell of first or second row in a systematic order. In order to meet the condition of a structural symmetry of PTC, distinct crosses of the form $(i \times j) \times k$, $(i \times k) \times j$ and $(j \times k) \times i$ ($i \neq j \neq k=1,2,\dots,v$) are taken in a cell. Degree of fractionation for the developed plans is $12/(v-1)(v-2)$.

Example 2 Consider a GRC design with parameters $v = 7$, $p = 2$, $q = 14$ and $k = 3$. Consider the treatments as lines. Form all distinct three-way crosses using each cell contents in a particular order, i.e. by considering two lines as half-parents and third one as full parent. There are 42 three-way crosses, each of the form $(i \times j) \times k$, $(i \times k) \times j$ and $(j \times k) \times i$. The developed PTC plans (Table 3) based on a GRC design with parameters $v = 7$, $p = 2$, $q = 14$ and $k = 3$ is given as follows:

Thus there are in total 42 crosses in the above final PTC plan and this PTC plan satisfies the structural symmetric

Table 3 PTC plan developed based on a GRC design with parameters $v = 7$, $p = 2$, $q = 14$ and $k = 3$

$(1 \times 2) \times 3$	$(6 \times 7) \times 1$	$(4 \times 6) \times 1$
$(1 \times 3) \times 2$	$(6 \times 1) \times 7$	$(4 \times 1) \times 6$
$(2 \times 3) \times 1$	$(1 \times 7) \times 6$	$(1 \times 6) \times 4$
$(2 \times 3) \times 4$	$(7 \times 1) \times 2$	$(5 \times 7) \times 2$
$(2 \times 4) \times 3$	$(7 \times 2) \times 1$	$(5 \times 2) \times 7$
$(3 \times 4) \times 2$	$(1 \times 2) \times 7$	$(2 \times 7) \times 5$
$(3 \times 4) \times 5$	$(1 \times 3) \times 5$	$(6 \times 1) \times 3$
$(3 \times 5) \times 4$	$(1 \times 5) \times 3$	$(6 \times 3) \times 1$
$(4 \times 5) \times 3$	$(3 \times 5) \times 1$	$(1 \times 3) \times 6$
$(4 \times 5) \times 6$	$(2 \times 4) \times 6$	$(7 \times 2) \times 4$
$(4 \times 6) \times 5$	$(2 \times 6) \times 4$	$(7 \times 4) \times 2$
$(5 \times 6) \times 4$	$(4 \times 6) \times 2$	$(2 \times 4) \times 7$
$(5 \times 6) \times 7$		$(3 \times 5) \times 7$
$(5 \times 7) \times 6$		$(3 \times 7) \times 5$
$(6 \times 7) \times 5$		$(5 \times 7) \times 3$

property. It is to be noted here that a complete diallel crosses (CTC) plan for 7 lines requires 105 three-way crosses. However, in the above PTC plans only 52 crosses are involved which saves the experimental resources to a great extent. The degree of fractionation for the above plan is $f = 42/105 = 2/5$ which is also small and well desirable from breeder's point of view.

Most of the PDC plans developed in literature using row-column design is either for odd number treatments or prime number of treatments (Varghese and Jaggi 2011). But in the present paper the developed class of PDC plan has been for any even number of treatments. The developed PDC plan is partially balanced following a group divisible association scheme. On the other hand, for PTC plans developed in the present investigation the degree of fractionation is smaller than the PTC plan obtained by Varghese and Jaggi (2011). For example, for 13 numbers of lines, the PTC plans developed by Varghese and Jaggi (2011) is having degree of fractionation as $2/11$ which is larger than the degree of fractionation $1/11$ for the PTC plans obtain based on the method of construction as discussed in the present paper. Due to smaller degree of fractionation for the PTC plans developed under the present investigation as compare to the existing literature, the number of crosses will be less in the present research. As the numbers of crosses are less based on the PTC plans developed under the present investigation as compared to the existing literature, there will be an improvement over saving of experimental resources based on the adaptation of the proposed PTC plans in terms of cost and time. Beside, as the PTC plans developed under the present investigation has been obtained using MOLS, therefore the the breeder has the freedom to choose any MOLS for constructing the PTC of the desired pattern with the required fractionation as the number of MOLS for any prime treatment is one less than the number of treatments. From the above discussion, it can be seen that GRC designs have an important application for obtaining mating plans in breeding trials. The cell contents can be directly used to obtain the crosses. The plans obtained here using these designs yield smaller degree of fractionation thereby reducing the resources and reduce the heterogeneity present in the experimental field, simultaneously which is quite desirable property from breeder's point of view. Beside, the PDC and PTC plans involved fewer numbers of crosses as compared to complete diallel and triallel crosses respectively. Thus the developed PDC and PTC plans can be advantageously used in breeding experiments.

REFERENCES

- Bailey R A. 1988. Semi Latin squares. *Journal of Statistical Planning and Inference* **18**: 299–312.
- Bailey R A. 1992. Efficient semi-Latin squares. *Statistica Sinica* **2**: 413–37.
- Bailey R A and Monod H. 2001. Efficient semi-Latin rectangles: Designs for plant disease experiments. *Scandinavian Journal of Statistics* **28**: 257–70.
- Bedford D and Whitaker R M. 2001. A new construction for efficient semi-Latin squares. *Journal of Statistical Planning*

- and Inference* **98**: 287–92.
- Datta A, Jaggi S, Varghese C and Varghese E. 2014. Structurally incomplete row-column designs with multiple units per cell. *Statistics and Applications* **12**(1&2): 71–79.
- Datta A, Jaggi S, Varghese C and Varghese E. 2015. Some series of row-column designs with multiple units per cell. *Calcutta Statistical Association Bulletin* **67**(265-266): 89–99.
- Datta A, Jaggi S, Varghese C and Varghese E. 2016. Series of incomplete row-column designs with two units per cell. *Advances in Methodology and Statistics* **13**(1): 17–25.
- Datta A, Jaggi S, Varghese E and Varghese C. 2017. Generalized confounded row-column designs. *Communication in Statistics: Theory and Methods* **46**(12): 6213–21.
- Edmondson R N. 1998. Trojan square and incomplete Trojan square design for crop research. *Journal of Agricultural Science* **131**: 135–42.
- Hinkelmann K. 1965. Partial triallel crosses. *Sankhya* **27**: 173–96.
- Jaggi S, Varghese C, Varghese E and Sharma V K. 2010. Generalized incomplete Trojan-type designs. *Statistics and Probability Letters* **80**: 706–10.
- Varghese C and Jaggi S. 2011. Construction of mating plans for breeding experiments using generalized incomplete Trojan type designs. *Journal of Statistics and Application* **6**(3-4): 85–93.