Journal of Community Mobilization and Sustainable Development
Vol. 16(3), September-December 2021, 668-672

Cost Effective Two Level Factorial Run Orders for Agricultural Experimentation

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ABSTRACT

Sustainable agriculture systems over the long term satisfies human food, enhances environmental quality, sustain the economic viability of farm operations and thus enhance the quality of life for farmers and society as a whole. Agricultural experimentations over the years are the backbone of sustainable developments and the importance of agricultural experiments are still increasing day by day as the observable phenomena is affected by a combination of several factors. Here factorial experiments plays a crucial role. In the present article, the concept of factorial run order where randomization of the experiment is difficult due to the presence of costly factors in the experiment has been discussed. These factorial run orders can be effective in reducing the cost of the experiments when the experiments involved costly factors.

Keywords: Hard-to-change factors, Minimal cost, Minimum level changes, Multifactor experiment, Run orders

INTRODUCTION

Sustainability in agricultural systems is a global problem today. Sustainable agriculture systems over the long term satisfies human food, enhances environmental quality, sustain the economic viability of farm operations and thus enhance the quality of life for farmers and society as a whole. One of the most important challenges facing humanity today is to conserve or sustain natural resources, including soil and water for increasing food production while protecting the environment. As the world population grows, stress on natural resources increases, making it difficult to maintain food security. Long term food security requires a balance between increasing crop production, maintaining soil health and environmental sustainability. It thus need continuous science based interventions in agricultural technology based on observation of natural phenomena. Science based experimentation in the area of agriculture can thus pave the way of achieving the goal of sustainable developments. Day by day the importance of agricultural experiments is increasing rapidly towards achieving the goal of sustainable development as the observable phenomena not only affected by a single factor or source of variation but also a combination of several factors. Here lies the role of factorial experiments. In statistical terminology, factorial experiment is a particular type of multifactor experiments which are having profound applications in many field of agricultural research. Research findings based on results obtained from the multifactor agricultural experiments are getting rapid momentum for achieving the goal of sustainable developments as factorial experiments not only allow us to study the effect of individual factors but also it helps us in getting a clear cut idea about how two or more factors interact with each other. It is always desirable to randomly execute the factorial run orders of any multifactor experiment to make the observation independent in order to get a valid estimate of the error by minimizing the bias which will ultimately increase the precision of the experiment. However, if proper randomization is carried out during the planning stage of multifactor experiment, the experimenter may witness a large number of factor level changes which ultimately make the experimentation expensive particularly if the experiment involves hard-to-change factors (where changing the levels of factors are difficult due to the cost structure of factors involved in such experiments or due to operational procedure). Consider the following is an experimental situation:

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Experimental situation [Bhowmik *et al.,* 2020]: "In an experiment of soil microbial diversity (community level physiological profiling) through BIOLOG ecoplates with the purpose of identifying the best treatment, three factors were tried *viz.* (i) CO_2 [Two levels: Elevated CO_2 and Ambient CO_2], (ii) Fertilizer (Two levels: Organic and Inorganic) and (iii) Variety [two wheat varieties]. It was known in advance that changing the levels of CO_2 was very expensive. Therefore, if randomization was done on the level combinations, it would have increase the cost of the experiment to a great extent since the experiment involved hard-to-change factors. In this situation, use of a factorial design where the number of factor level changes remains small, is advisable".

The number of factor level changes is a matter of great concern to experimenters in different agricultural, post-harvest and processing, engineering and industrial research as in such experiments it may be physically very difficult to change levels of some factors. In such situations one should try to have factorial run orders where total number of factor level changes vis-a-vis the cost of the experiment is minimum. A lot of work is available in literature in that aspects [see for reference Cox (1951), Draper and Stoneman (1968), Dickinson (1974), De León *et al.* (2005), Correa *et al.* (2009), Correa *et al.* (2012), Hilow (2013), Bhowmik *et al.* (2015) Varghese *et al.* (2017), Bhowmik et al. (2017), Oprime *et al.* (2017), Varghese *et al.* (2019), Bhowmik *et al.* (2020), Pureza *et al.* (2020) etc.]. Here, we have discussed the concept of factorial run order with minimum number of level changes viz cost of the experiment in the context of two level factorial experiments. These run orders will serve the purpose of minimizing the cost of the experiment and thus will be very effective for those experiments which involved costly factors.

MATERIALS AND METHODS

The number of changes in factor levels has significant impact on costs and effort for carrying out an experiment especially when a great amount of effort is required in changing the levels of a factor or when it is necessary to wait a certain amount of time or for some other reasons. The minimum number of factor wise level changes is attained when only one factor level is changed on two successive experimental trials. In other words, one can have a minimal cost factorial run order with minimum number of level changes when only one sign is changed by passing from one row to the next in the design matrix. If there are k factors with i^{th} factor is having s_i levels for $i = 1, 2, ..., k$, then the total number of level changes for the $\prod_{i=1}^{k} s_i$ factorial design will be $\left(\prod_{i=1}^{k} s_i\right) - 1$ i.e. one less than the total number of runs. Hence, for a 2^k factorial run order i.e. for 2-level factorial with k number of factors, minimum number of changes will be 2^k-1.

RESULTS AND DISCUSSION

Base on the above principle, following are some two level factorial run orders with minimum number of level changes. All these run order will minimize the cost of the experimentations when the experiments involved costly or difficult to-change factors.

Table 1[(a) -(d)]: Four different minimal cost 2^2 factorial run order with minimal level changes [presence of letters indicate the higher levels of the factors with *(1)* indicate the lower levels of all the factors]

(a)	***A***	***B***	**Run Order**
	-1	*-1*	*(1)*
	-1	*1*	*b*
	1	*1*	*ab*
	1	*-1*	*a*
	Factorwise level changes		
	1	***2***	
(b)	***A***	***B***	**Run Order**
	-1	*-1*	*(1)*
	1	*-1*	*a*
	1	*1*	*ab*
	-1	*1*	*b*
	Factorwise level changes		
	2	***1***	
(c)	***A***	***B***	**Run Order**
	1	*-1*	*a*
	1	*1*	*ab*
	-1	*1*	*b*
	-1	*-1*	*(1)*
	Factorwise level changes		
	1	***2***	
(d)	***A***	***B***	**Run Order**
	1	*1*	*ab*
	-1	*1*	*b*
	-1	*-1*	*(1)*
	1	*-1*	*a*
	Factorwise level changes		
	2	***1***	

2^2 Factorial run order with minimum number of level changes: For a 2^2 factorial one can have a total of 4! = 24 number of run orders out of which there will be 8 run orders where total number of changes are minimum. All these 8 run orders will minimize the cost of the experiments particularly when the experiment involved hard-to-change factors. For a minimally changed 2^2 factorial run order i.e. for a minimally changed factorial run order with two factors each are having two number of levels, the total number of level change is *3*. Following are some 2^2 factorial run order with minimum number of level changes. In all the following run orders, the factor wise levels changes are 2 and 1 respectively in different permutation.

2^3 Factorial run order with minimum number of level changes: 2^3 factorial experiments where there are three factors each are having two number of levels are very commonly used in agricultural experiments. For a 2^3 factorial one can have a total of 8! = 40, 320 number of run orders out of which there will be 144 run orders where total

Table 2[(a)-(f)]: Six different minimal cost 2^3 factorial run order with minimal level changes [presence of letters indicate the higher levels of the factors with *(1)* indicate the lower levels of all the factors]

(a)	*A*	*B*	*C*	**Run Order**
	-1	-1	-1	(1)
	-1	-1	1	c
	-1	1	1	bc
	-1	1	-1	b
	1	1	-1	ab
	1	1	1	abc
	1	-1	1	ac
	1	-1	-1	a
	Factorwise level changes			
	1	**2**	**4**	

(b)	*A*	*B*	*C*	**Run Order**
	1	1	-1	ab
	1	1	1	abc
	1	-1	1	ac
	1	-1	-1	a
	-1	-1	-1	(1)
	-1	-1	1	c
	-1	1	1	bc
	-1	1	-1	b
	Factorwise level changes			
	1	**2**	**4**	

(c)	**A**	**B**	**C**	**Run Order**
	1	-1	1	ac
	1	1	1	abc
	1	1	-1	ab
	1	-1	-1	a
	-1	-1	-1	(1)
	-1	-1	1	c
	-1	1	1	bc
	-1	1	-1	b
	Factorwise level changes			
	1	**3**	**3**	

(d)	**A**	**B**	**C**	**Run Order**
	-1	-1	-1	(1)
	-1	-1	1	c
	-1	1	1	bc
	-1	1	-1	b
	1	1	-1	ab
	1	-1	-1	a
	1	-1	1	ac
	1	1	1	abc
	Factorwise level changes			
	1	**3**	**3**	

(e)	**A**	**B**	**C**	**Run Order**
	-1	-1	-1	(1)
	-1	1	-1	b
	-1	1	1	bc
	1	1	1	abc
	1	1	-1	ab
	1	-1	-1	a
	1	-1	1	ac
	-1	-1	-1	c
	Factorwise level changes			
	2	**2**	**3**	

(f)	**A**	**B**	**C**	**Run Order**
	1	-1	-1	a
	-1	-1	-1	(1)
	-1	-1	1	c
	-1	1	1	bc
	-1	1	-1	b
	1	1	-1	ab
	1	1	1	abc
	1	-1	1	ac
	Factorwise level changes			
	2	**2**	**3**	

Table 3[(a),(b)]: Two different minimal cost 2^4 factorial run order with minimal level changes [presence of letters indicate the higher levels of the factors with *(1)* indicate the lower levels of all the factors]

(a)	***A***	***B***	***C***	***D***	**Run Order**
	-1	-1	-1	-1	*(1)*
	-1	-1	-1	1	*d*
	-1	-1	1	1	*cd*
	-1	-1	1	-1	*c*
	-1	1	1	-1	*bc*
	-1	1	1	1	*bcd*
	-1	1	-1	1	*bd*
	-1	1	-1	-1	*b*
	1	1	-1	-1	*ab*
	1	1	-1	1	*abd*
	1	1	1	1	*abcd*
	1	1	1	-1	*abc*
	1	-1	1	-1	*ac*
	1	-1	1	1	*acd*
	1	-1	-1	1	*ad*
	1	-1	-1	-1	*a*
	Factorwise level changes				
	1	**2**	**4**	**8**	

(b)	***A***	***B***	***C***	***D***	**Run Order**
	-1	-1	-1	-1	*(1)*
	1	-1	-1	-1	*a*
	1	-1	-1	1	*ad*
	1	-1	1	1	*acd*
	1	1	1	1	*abcd*
	-1	1	1	1	*bcd*
	-1	1	1	-1	*bc*
	-1	1	-1	-1	*b*
	-1	1	-1	1	*bd*
	1	1	-1	1	*abd*
	1	1	-1	-1	*ab*
	1	1	1	-1	*abc*
	1	-1	1	-1	*ac*
	-1	-1	1	-1	*c*
	-1	-1	1	1	*cd*
	-1	-1	-1	1	*d*
	Factorwise level changes				
	4	**2**	**4**	**5**	

number of changes are minimum. All these 144 run orders will minimize the cost of the experiments particularly when the experiment involved hard-to-change factors. For a minimally changed 2^3 factorial run order the total number of level change is *7* which is one less than the total number of runs i.e. *8*. Out of the 144 run orders for a 2^3 factorial experiments with minimum number of level changes, there are 12 possible combinations where total number of change is 7 i.e. (1, 2, 4), (1, 4, 2), (2, 1, 4), (2, 4, 1), (4, 1, 2), (4, 2, 1), (1, 3, 3), (3, 1, 3), (3, 3, 1), (2, 2, 3), (2, 3, 2) and (3, 2, 2) respectively [Here numbers in brackets indicates factorwise number of level changes]. First 6 combinations are occurring 8 number of times each and remaining 6 combinations are occurring 16 number of times each. Each distinct combinations with factorwise level changes as (1, 2, 4), (2, 2, 3) and (1, 3, 3) in different permutations are coming 48 number of times. Following are some 2^3 factorial run order with minimum number of level changes. In all the following run orders, the factorwise levels changes are 2 and 1 respectively in different permutation.

2^4 Factorial run order with minimum number of level changes: For a 2^4 factorial one can have a total of 16! = 2.092279×10^{13} number of run orders. The number of minimally changed run orders are also large in number. For a minimally changed 2^4 factorial run order the total number of level change is *15* which is one less than the total number of runs i.e. *16*. Following are some minimally changed 2^4 factorial run order:

CONCLUSION

Agricultural experiments now a days are getting visibly importance due to continuous pressure of population growth, continuous depletion of natural resources, changing climatic scenario etc. Findings of agricultural experiments based on different factorial combinations can pave the way for sustainable developments. Multifactor experiments may witness great challenge when experiments involved costly factors for which levels are very difficult to change as randomization in such scenario although minimize the bias but may increase the cost of the experiments. In such situations, the minimally changed factorial designs (where the number of level changes are minimum) as discussed in the present article may be a valid alternative. However, the analysis remains a matter of concern due to lack of proper randomization of run orders. One approach in this direction is to use randomization tests to identify significant factor levels, but the total number of possible randomizations is likely to be very small. Alternatively, an Analysis of Covariance (ANCOVA) type of models by considering the influence of time trend as a covariate may be a feasible solution. Beside, use of split plot type of model for analysis by taking the difficult-to-change factor as main plot factor may be another possible alternative although in such situations only sub plot factor and main plot-sub plot interaction effects will be estimated with more precision.

ACKNOWLEDGEMENTS

First author is thankful to ICAR for providing Junior research fellowship during his M.Sc. tenure. The facilities provided by Director, ICAR-Indian Agricultural Statistics Research Institute, New Delhi is duly acknowledged.

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Received on April 2021; Revised on August 2021