DESIGNS FOR VARIETAL CUM MANURIAL TRIALS

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(R.B.D. SHARMA)

CONTENTS

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CHAPTER I

INTRODUCTION AND REVIEW OF LITERATURE

For agricultural experimentation, there are mainly two types of designs. These are (i) factorial designs, which are adopted when the treatments are the combinations of the levels of several factors; and (ii) complete and incomplete block designs suitable for varietal trials, though the complete block designs are used for factorial treatments also.

Fisher first introduced the concept of factorial experiments and confounding for such experimentation. The theory of confounding in factorial experiments was systematically discussed first by Yates (1933). Nair's (1938) paper was the first attempt to develop a general method of construction of confounded symmetrical factorial designs with prime power numbers of levels of each of the factors. The theory of confounding in symmetrical factorial experiments with prime power number of levels of the factors was developed further by Bose and Kishen (1940) and by Bose (1947).

Confounded asymmetrical factorial designs were first introduced by Yates (1937). The problem of confounding in the designs of the type $3^m \times 2^n$ was solved by him also. Afterwards Li (1944) obtained some more designs following Yates. Later, Nair and Rao (1948) developed a set of conditions which led to the construction of some balanced confounded designs.

More recently much research has been directed to construct what are known as:

(1) Fractionally replicated designs introduced by Finney (1945)

(2) Orthogonal arrays introduced by Rao (1950).

(3) Optimum multifactorial designs by Plackett and Burman (1946), and

(4) Multifactorial designs for exploring the response surfaces introduced by Box and his coworkers (1951, 1952, 1957).

Das (1961) has given in a series of papers a systematic method of construction of all different types of asymmetrical factorial designs. All these designs actually come out as a fractional replicate of some symmetrical confounded factorial design. He has also introduced a method of construction of confounded symmetrial factorial designs where the number of levels of each of the factors is the product of different primes.

Coming to designs suitable for varietal trials, Yates (1936_a , 1936_b , 1937_a) first introduced incomplete block designs suitable for experiments involving a large number of treatments. Lattice designs and balanced incomplete block (b.i.b) designs are the two designs evolved by him. A systematic study of the various methods of construction of b.i.b. designs was first made by Bose (1939). Next Bose and

Nair (1939) introduced another class of incomplete designs called partially balanced incomplete block designs. These designs included as special cases the b.i.b. and square lattice designs. Later Nair and Rao (1942_a) generalized these designs so as to include the cubic and other higher dimensional lattices as special cases of such designs.

One more type of designs involving blocks of two plots only was introduced by Kempthorne (1953). He called these designs circulant designs as the solution of the normal equations of such designs can be obtained by inverting a circulant matrix. Das(1960) introduced circular designs, which are available for any number of treatments and replications. In these designs the block size is either equal to or a multiple of the replication number. There are several other series of incomplete block designs like rectangular lattice, chain block designs, linked block designs etc.

The incomplete block designs considered so far have blocks of constant size and a constant number of replicates for all the treatments. Some amount of research has also been done to get designs with unequal block sizes as also unequal numbers of replications. The quasi-factorial designs given by Yates (1936_a) when $v = pq$ have blocks of sizes p and q. Again the designs developed by Kishen (1941) have, in general, m different block sizes, k_1, k_2 ... k_m

More recently Graybill and Pruitt (1958) has introduced a series of designs called staircase designs which accommodates block of all sizes less than and equal to the number of treatments. Recently Bose and Srikhande (1959) have used such incomplete block designs with unequal block sizes and $\lambda = 1$ to get orthogonal latin squares of sides $4t + 2$, $(t \neq 1)$.

Federer (1956) developed a new class of designs. These designs have one set of treatments replicated b times and a second set only once. In these designs the blocks also need not be of the same size.

One other direction of interest in which the investigations have been directed concerns evolving designs with two-way elimination of heterogenity. These designs started with Youden's square introduced by Yates (1937). Much work has also been done to get designs by dualising the existing designs and a good number of designs of interest could be obtained through this technique.

All these references indicate that so long factorial experimentation and varietal trials have been dealt with separately. Of late some necessity has been felt to combine these two types of experimentation. For example in varietal trials for selection of promising varieties some uniform manurial dose is taken and all the varieties are tested on this dose.

It is often found after selection from such experiments that a variety selected from experimentation at a lower dose of manure lodges when it is grown at some higher manurial dose. This suggests that at the time of varietal trial the varieties should be selected through experimentation involving more than one manurial dose, so that the experiment can bring out if there is any interaction between the varieties and the manurial levels. The necessity for obtaining such design was first pointed out by Dr. Panse.

The usual designs which are available in literature are not very suitable for this type of experiment, excepting that when the number of manurial treatments is small, incomplete block designs augmented by associating each variety with a number of treatments can be adopted. Such designs already exist in literature (Bose and Connor, 1952), though their application to meet the present contingency has not been attempted.

If we have n manures each at s levels to be tried with v varieties the total number of combinations will be v x s^n. So the designs suitable for this type of experimentation will essentially be some asymmetrical factorial design. As v is expected to be very large, the usual asymmetrical design where no main effect is affected will not be of much help, because the block size in these designs must be a multiple of v and hence will be very large. This points to the necessity of evolving other types of asymmetrical designs where some amount of information of the main effect of one of the factors may have to be lost. In these designs not only the block size should be small but the number of replications should also be small, otherwise, the experimental material required may be too large, and the experiment will be too big.

Several series of designs which are suitable for this type of experimentation have been evolved and presented in the present thesis. The analysis of asymmetrical design offers some difficulty which is more pronounced in the case of these designs. A method of analysis of such and other assymmetrical designs through which besides getting a complete analysis of the data it can be easily known whether for any design two affected interactions are estimable mutually independently, has also been presented in the thesis.

CHAPTER II

A SYSTEMATIC METHOD OF ANALYSIS OF DATA FROM ASYMMETRICAL FACTORIAL DESIGNS

No easy and systematic method of analysis of asymmetrical factorial designs seems available in literature. Kempthorne (1952) has given a method of analysing such designs through fitting of constants. But he has not generelised it and has given it only for a particular design. Here it has been intended to put forward a general method of analysis for the design of the type $v \times 2^n$.

By adopting the factorial model for observation obtained from experiments of the type $v \times 2^n$ it is always possible to obtain the sum of squares of the main effect or any interaction with (v-1) degrees of freedom from a comparison of v treatments defined suitably as detailed below.

We shall denote the factor with v levels by X and the other factors will be denoted by A,B,C, etc, as usual. Any interaction involving X, say, XI where, I stands for a group of letters from A,B,C, etc., the S.S. due to X I can be obtained from a comparison of the v treatments given by $t_i = v_i$ (a-b), $i = 1,2, \ldots, v$ where, v_i stands for the ith level of X, 'a' stands for the sum of all those combinations of the levels of the factors A,B,C,.... each of which has an even number of letters common with I when such combinations are written using small letters a,b,c, etc., as usual in factorial designs. While b stands for the sum of the rest of the combinations of the factors A,B,C, etc. For example, if we have three factors X,A and B with number of levels 9,2 and 2 respectively, the S.S. due to the two factor interaction XA can be obtained by estimating the 9 treatments $t_i = v_i\ (a-b)\ i = 1,2,...9$; where, $a = (00) + (01)$ and $b = (10) + (11)$. (00), (01), (10) and (11) being the four combinations of the factors A and B each at levels 0 and 1.

After defining the treatments in this way a two-way table giving the frequencies of occurrence of such treatments in the different blocks of a design can be obtained as described below.

If any combination which is present in the treatment t_i occurs in the jth block, a frequency of +1 or - 1 will be its contribution in the cell of the ith treatment and jth block according as it occurs in the treatment t_i with + or - sign. The algebric sum of all the frequencies of the different combinations constituting t_i and occurring in the jth block of the design will be the frequency in the cell of the treatment t_i in the jth block. The justification for obtaining the frequencies as above follows from the fact that by taking the factorial model, if the normal equations for each combination be written through least squares and the equations corresponding to the combinations in t_1 be combined so as to give t_1 together with block effects, the coefficients of the different block effects will provide exactly the same frequencies as provided by the above table of the treatment t_1 in the different blocks.

Given any asymmetrical design together with the method of its construction, it can be ascertained what are the interactions and main effects which are evidently unaffected by block differences. A frequency table has to be prepared for each of the interactions and main effects which are affected by block differences and the frequency tables of these interactions are to be placed side by side so that the block numbering for each of the tables is common. If any interaction not involving X be affected its S.S. can be obtained from a comparison of two treatments, which in general, shall be denoted by a and b, such that a is the sum of all those combinations each of which has an even number of letters common with the interaction under consideration, while b denotes the sum of the rest. The frequencies of these two treatments in the different blocks of a design can be obtained similarly though no negative sign is involved in such cases. The frequency tables of such affected interactions having 1 d.f. should also be placed beside the other tables so as to have the same block numbering.

The normal equations for the different treatments for any interactions can be obtained from these frequency tables as in the case of non-orthogonal data excepting for some modification necessary because of (1) the possible negative frequncies and (2) adjoining the frequency tables for other affected interactions together with that of its own. Let k_1 interactions each with v - 1 d.f. and k_2 interactions each with 1 d.f. be affected. We shall denote the ith treatment of the kth affected interaction with v-1 d.f. by t_i^k ($k=1,2,\ldots k_1$ and $i = 1,2,\ldots v$). Similarly the treatment a of the kth affected interaction with 1 d.f. will be denoted by a_1^k and the b treatment by a_2^k. In general by a_i^k ($i = 1, 2$; and $k = 1,2,\ldots k_2$).

The frequency of the t_i^k treatment in the jth block will be denoted by n_{ij}^k and that of a_i^k by m_{ij}^k.

The normal equations for any treatment t_i^k after eliminating the block effects may contain all the other treatments present in all the frequency tables. The normal equations for t_i^k will be of the form

$$r_i^k \, t_i^k - \sum_{m,p} C_{im}^{kp} \, t_m^p = Q_i^k ,$$

where, r_i^k is equal to the total number of observations coming from all the combinations present in t_i^k, and

$$C_{im}^{kp} = \sum_j n_{ij}^k \times n_{mj}^p / n_{.j} , \quad n_{.j} \text{ being}$$

the total number of observations in the jth block, and

$$Q_i^k = T_i^k - \sum_j n_{ij}^k B_j / n_{.j} ,$$

where, T_i^k is the algebric total of all observations corresponding to the combinations in t_i^k and B_j is the total of jth block. It will be noticed that $\sum_i Q_i^k$ need not be equal to zero always. If for the kth and pth interactions the coefficients C_{im}^{kp} for different m's given any i be zero or a constant (though the constant may have different signs for different i's) in a design, these two interactions are estimable independently of each other through this design. If again given the kth interaction the coefficients C_{im}^{kp} be similarly zero or constant for each of the p's not equal to k, then this interaction is estimable independently of all other interactions.

In a design through which each affected interaction is estimable independently of all other affected interactions, if C_{im}^{kk} is constant for all $m \neq i$, this interaction will be balanced. If for a design through which the affected interactions are estimable mutually independently, $\hat{t}_i^k$ is the solution for the treatment effect t_i^k, the S.S. due to the kth interaction is given by

$$\sum_i \hat{t}_i^k \; Q_i^k - \left(\sum_i \hat{t}_i^k \right) \left(\sum_i Q_i^k \right) / v.$$

With such indication of the method of analysis we shall now give the method of obtaining several series of asymmetrical designs of the form $v \times 2^n$ which are particularly suitable for varietal cum factorial trials, through each of which the different affected interactions are estimable mutually independently. The different steps involved in the analysis have been exemplified in Chapter IV, page no. 30, while discussing a particular design.

CHAPTER III

VARIETAL CUM MANURIAL DESIGNS BASED ON BALANCED INCOMPLETE BLOCK DESIGNS

A) Without confounding factorial effects:

If there be v varieties and n manurial treatments each at two levels, there will be $v \times 2^n$ treatment combinations. One type of varietal cum manurial designs with such factors can be obtained by including each of the $v \times 2^n$ combinations in a block and adopting a randomised block design. The analysis of such designs will not involve any complication.

As such designs will often require very large block size, they are not so suitable from the point of view of precision of comparison. As v is usually large a suitable design with small block size can be obtained first by taking an incomplete block design for the v varieties with block size k (say) and then associate each of the 2^n manurial combinations with each of the v varieties whenever they occur in the blocks. The block size for such a design will be $k \times 2^n$. If in a block the k varieties denoted by 1,2,....k are present and 2^n combinations are associated with each of them (Bose and Connor, 1952), the $k \times 2^n$ treatment combinations can be alloted to the $k \times 2^n$ plots in a block in two different ways. One method is to allot them at random to the plots. The other method is to form k groups of 2^n contiguous plots each and allot first the k varieties at random to the bigger plots of size 2^n and then allot the manurial treatments at random to the 2^n plots in each group. This allotment will make the design a split plot with the main plot treatments in an incomplete block design. It will involve two plot sizes and hence two error variances. Judging from the nature of the treatments for main plots, namely varieties no bigger plot may be necessary for the varieties. So it will not be of any advantage to adopt a split plot design, rather through this design the main problem namely the varietal comparison will have less precision. In the alternative form of designs where all the $k \times 2^n$ treatments are allotted at random in a block, no main effect or interaction excepting the varietal main effect will be affected and hence the analysis of this design will be simpler. All the main effects and interactions have to be obtained as usual. While for the S.S. due to the varietal effects (to be called main effect X hereafter) the normal equations are the same as those for the original incomplete block design excepting that the block size in this case is 2^n times the block size of the original design and the adjusted totals Q_i's are to be divided by 2^n and the replication as also the λ's are to be taken the same as in the original design. No fresh problem arises for the solution of these equations and hence the S.S. due to the main effect X which alone is affected by block differences, can be obtained without any fresh difficulty.

B) With some of the factorial effects confounded:

If k be the block size of an incomplete block design and there are n manurial factors each at two levels, the block size of a varietal cum manurial design described in the last section will be $k \times 2^n$. This size also will often be large. One way of reducing the block size is not to include all the 2^n manurial trials associated with each of the varieties in a block but to include only a fraction say $1/2^a$ of the 2^n combinations associated with a variety. The problem in this case reduces to combining an incomplete block design and a confounded factorial design. One way of getting such designs is (1) to obtain a balanced incomplete block design in v varieties with block size k; (2) to obtain a balanced confounded factorial design $k \times 2^n$ with suitable block size, such that the main effect of the first factor Y with k levels is not confounded. It will be seen that the factor Y has no real existence but is required for association. The ith level of this factor will be denoted by y_i.

Now the design suitable for varietal cum manurial trials can be obtained by suitably associating these two designs. For such association let us first take a block of the factorial design and any block of the b.i.b. design with k plots, the plots in the different blocks being numbered from 1 to k in order of the b.i.b design, the ith plot in this order being denoted by p_i. Let X_i denote the group of all combinations of the levels of n factors excluding the factor Y which occur with the level y_i of Y in the block under consideration. Now the rule of association of these two blocks of the two designs is to write the combinations X_i in that plot of the b.i.b. design which is denoted by p_i, i varying from 1 to k. This type of association has to be effected for all possible pairs of blocks of the two designs. Thus in all there will be $b_1 \times b_2$ blocks in the design where b_1 is the number of blocks in the b.i.b. design, and b_2 that of the other design. Similarly if r_1 and r_2 be the number of replications of the two designs, the number of replications of the combined design will be $r_1 \times r_2$. These designs follow from Shah's (1960) method also if one of the balanced designs in his case has only one factor.

Though such a type of design exists, they are not very suitable for varietal cum manurial trials for any value of k, as very large number of replications will often be necessary for such designs. One way of reducing the number of replications is to take k = 2 and in that case one replication of the factorial design which will now be of the form 2×2^n will serve the purpose, and the number of replications of the design will be the same as in b.i.b. design. These designs, namely, 2×2^n need not be balanced always but some of the manurial interaction may be completely confounded. Let the design $v \times 2^n$ be constructed by obtaining (1) the b.i.b. design with v treatments and k = 2 as block size and (2) 2×2^n factorial design with 2×2^{n_1} plots per block. We shall denote the factors of the second design as Y, a pseudo factor with 2 levels and A,B,C etc. each at 2 levels. This design can always be obtained in one replication without confounding the main effect Y. While obtaining the design 2×2^n in blocks of 2×2^{n_1} plots, some of the $2^{n-n_1}-1$ interactions each with 1 d.f. which are confounded, will contain Y, while there may be others without containing Y. All the interactions confounded and containing Y will be said to form the set A of confounded interactions, while the set of the confounded interactions without Y will be said to form the set B of confounded interactions. All the interactions of the set B considered as interactions of the design v x 2^n will be confounded in the design v x 2^n constructed as above. All the generalised interactions of the main effect X and the interactions in the set B will be affected but will remain balanced. Again all the interactions of the set A with Y replaced by X will also be affected but will be balanced. It will thus be seen that when only one interaction is confounded while obtaining the design 2 x 2^n in 2 x 2^{n-1} plot blocks and that toocontains Y, the design will have only one affected interaction together with the main effect X both of which are balanced and mutually independent. One more fact emerges from this investigation that the interactions in set B should be so chosen that they are always of higher order.

CHAPTER IV

VARIETAL CUM MANURIAL DESIGN BASED ON CIRCULAR DESIGNS

So far we have discussed methods of obtaining varietal cum manurial designs obtained by combining a b.i.b. design and a factorial design. As b.i.b. designs suffer from certain limitations that they are not available for any number of treatments with a reasonable number of replications particularly when the block size is 2, we have to search for other types of incomplete block designs which are available for any number of treatments and block size 2. The Circular designs introduced by Das (1960) satisfy both these conditions, and as such can profitably be used as the varietal design for obtaining varietal cum manurial designs. Construction of a circular design with any number of treatments, say, v having blocks of 2 plots can be easily be done by developing the initial block 1,2, mod (v). There will be v blocks and 2 replications.

For obtaining the varietal cum manurial design we shall use in this section (1) a circular design in v treatments with two plot blocks, and (2) the factorial design 2×2^n in 2^a plot blocks. By combining these two designs exactly in the same way as described earlier, we shall get the design $v \times 2^n$ in $v \times 2^{n-a+1}$ blocks of 2^a plots each and two replications. While obtaining the design 2×2^n in 2^a plot blocks $2^{n-a+1} -1$ interactions will be confounded. As in the case of $v \times 2^n$ designs obtained through b.i.b. designs all of these confounded interactions which do not contain the pseudo factor Y will be confounded in the design $v \times 2^n$, the generalized interactions of these interactions with the main effect X will be affected but can be recovered. All those interactions of the design $v \times 2^n$ which are obtained by replacing the factor Y of the remaining confounded interactions of the design 2×2^n by the factor X will be affected but recoverable. Each of these last mentioned interactions of the design $v \times 2^n$ will further lose 1 d.f. when v is even, since in such case a particular contrast namely, sum of the treatments with odd suffix minus sum of the remaining treatments is not estimable.

<u>ANALYSIS:</u> Each of the affected interactions together with the main effect X is estimable mutually independently thragh the data collected from such designs. This becomes evident from a scrutiny of the frequency tables of such design.

The frequency tables for the affected interactions in the design are given in table No.1 for the 2^{n-a+1} blocks obtainable by combining the single block containing 1,2 of the circular design with all the blocks of the factorial design. Thus the table is not complete.

No other tables have been presented for the reason that the frequency table for any other affected interactions will give normal equations obtainable from one or the other of the first two tables. For such designs the sum of products of the frequencies of any two treatments belonging to two effects or interactions is zero. Let X_{k_0} and X_{k_1} be the two groups of treatment combinations of the real factors A, B, C, etc., each at two levels, which occur with 0 and 1 levels respectively of the factor Y in the kth block of the design 2 x 2^n in 2^a plot blocks. Further let G_{k_0} and G_{k_1} denote the sums of the treatment combinations present respectively in the groups X_{k_0} X_{k_1}. It can be seen from the design that each of the possible groups X_{k_0} and X_{k_1} for different values of k will occur twice in the blocks of the design 2 x 2^n. If there is a block containing X_{k_0} and X_{k_1} in that order, then there is another block in which the same two groups occur but in the reverse order i.e. the first group of treatments occur with the

Table No. 1. Frequency table for $v \times 2^n$ design in 2^a plot blocks

Blocks	Main effect X Treatments					Generalised interaction of set A with X					Generalised interaction of set B with X				
	t_1^0	t_2^0	t_3^0	....	t_v^0	t_1^1	t_2^1	t_3^1	....	t_v^1	t_1^2	t_2^2	t_3^2	....	t_v^2
1	2^{a-1}	2^{a-1}	0	...	0	2^{a-1}	-2^{a-1}	0	....	0	2^{a-1}	2^{a-1}	0	....	0
2	2^{a-1}	2^{a-1}	0	...	0	-2^{a-1}	2^{a-1}	0	....	0	2^{a-1}	2^{a-1}	0	....	0
3	2^{a-1}	2^{a-1}	0	...	0	2^{a-1}	-2^{a-1}	0	....	0	-2^{a-1}	-2^{a-1}	0	....	0
4	2^{a-1}	2^{a-1}	0	...	0	-2^{a-1}	2^{a-1}	0	....	0	-2^{a-1}	$+2^{a-1}$	0	....	0
⋮															
2^{n-a+1}	2^{a-1}	2^{a-1}	0	...	0	-2^{a-1}	2^{a-1}	0	....	0	2^{a-1}	2^{a-1}	0	....	0

level 1 of Y while in the second block it occurs with 0 level of Y. Any affected interaction is expressible as a contrast of the totals G_{k_0} and G_{k_1} when k varies over the blocks, till each possible group occurs only once.

It can be seen from the design that each such group will occur with any treatment twice. Now given any interaction, if the contrast among G_{k_0} etc. representing this interaction be such that any group G_{k_1} is having -ve or +ve sign, the frequency of the treatments in all those blocks where this group occurs with the treatments will have the same sign. Thus the frequencies under any treatment in the different blocks will be in form of a contrast which is the same as that of the interaction to which the treatment belong. Thus if there be two treatments belonging to two different affected interactions whose contrasts in terms of G_{k_0} etc. are mutually orthogonal, the sum of products of the frequencies will be zero.

From such frequency tables the normal equations for estimating the treatments come out as

$$(2x2^{n-a+1} x\ 2^{a-1} - 2x2^{n-a+1} x\ 2^{2a-2} / 2^{a})\ t_i^m$$

$$\pm (2^{n-a+1} x2^{2a-2}/2^{a})\ (t_{i-1}^m + t_{i+1}^m) = P_{i,m} \ldots.(1)$$

$$i = 1,2,\ldots\ldots\ v.$$

For some interactions as also the main effect X, the normal equations will be with - sign after t_i^m, while the sign will be + for the rest of the interactions. We shall call in future these two types of equations as type B and type A respectively.

The set of equations (1) when simplified becomes

$$2^n t_i^m \pm 2^{n-1} (t_{i-1}^m + t_{i+1}^m) = P_{i,m} \qquad i = 1,2,..v.$$

Here t_i^m denotes the ith treatment of the mth affected interaction, t_{i+1}^m denote the treatment which is just after the ith treatment when the treatments are written according to ascending order, the treatment next to the vth one being the first treatment. Similarly t_{i-1}^m is that treatment which is just before the ith treatment. $P_{i,m}$ is the adjusted treatment total of t_i^m. These equations are exactly like those obtainable from the circular designs.

The solution of the normal equations differ according as v is odd or even and hence have been obtained for odd and even number of varieties separately.

For odd values of v, by solving the equations as in the case of circular designs we get the estimates of treatments of different affected interactions as below.

(i) The solution for t_i^m from the normal equations of type B has been obtained as

$$2^{n-1} v \hat{t}_i^m = \sum_{r=1}^{p} \frac{r(r+1)}{2} P_{i,m}^{(p-r)}$$

where, $p = \overline{v-1}/2$, $P_{i,m}^{(j)} = P_{i-j,m} + P_{i+j,m}$

where $i-j$ and $i+j$ are reducible mod (v);

and $P_{i,m}^{(0)} = P_{i,m}$

(ii) The solution for t_i^m from the normal equations of type A again depends on whether v is of the form 4q + 1 or 4q - 1. The solution when v is of the form 4q + 1 comes out as

$$(a)\ 2^{n-1} v \hat{t}_i^m = (p/2) P_{i,m} - (p/2) P_{i,m}^{(1)}$$

$$+ (p/2 - 1) P_{i,m}^{(2)} - (p/2 - 1) P_{i,m}^{(3)}$$

$$+ \ldots\ldots - P_{i,m}^{(p-1)} + 2^{n-1} C_m$$

where, p is as above $(= \overline{v-1}/2)$, $C_m = P_{i,m} / 2^{n+1}$.

when v is of the form 4q-1 the solution is

$$(b)\ -2^{n-1} v \hat{t}_i^m = -(\overline{p+1}/2) P_{i,m} + (\overline{p+1}/2 - 1) P_{i,m}^{(1)}$$

$$-(\overline{p+1}/2 - 1) P_{i,m}^{(2)} + \ldots\ldots\ldots$$

$$\ldots\ldots P_{i,m}^{(p+1)} + 2^{n-1} C_m$$

where, p and C_m are as above.

(iii) For the main effect X, the solution is given by

$$2^{n-1} v \hat{t}_i^{o} = \sum_{r=1}^{p} \frac{r(r+1)}{2} Q_i^{(p-r)}$$

for $i = 1, 2, \ldots\ldots v$.

where, Q_i is the adjusted total of the ith variety t_i^{o}, and $Q_i^{(j)} = Q_{i-j} + Q_{i+j}$, and $Q_i^{(o)} = Q_i$.

The average variance for any two varietal comparison is given by

$$(v + 1) \times \sigma^2 / (3 \times 2^n)$$

The efficiency factor (E.F.) can be found from the following relation

$$\text{Efficiency} = \frac{\text{Mean variance of treatment differences in Randomised block design}}{\text{Mean variance of intrablock estimate of treatment differences in incomplete block design}}$$

$$\times \frac{\text{Within block variance in incomplete block design}}{\text{Within block variance in Randomised block design}}$$

In this case E.F. = $3/(v + 1)$.

For even values of v, proceeding as in the case of odd values of v, we get the estimates of t_i^m

(i) for the normal equations of type B as

$$2^n v \hat{t}_i^m = \sum_{r=1}^{p} r^2 P_{i,m}^{(p-r)}$$

where, $p = v/2$.

and (ii) for the normal equations of type $\not{B}A$ as

$$\text{(a)} \quad -2^n v \hat{t}_i^m = + p^2 P_{i,m} + (p-1)^2 P_{i,m}^{(1)} + (p-2)^2 P_{i,m}^{(2)} + \ldots\ldots + P_{i,m}^{(p-1)}$$

where, p is as above ($= v/2$) and v is of the form 4q.

The solution for even v becomes when v is of the form 4q-2,

$$\text{(b)} \quad 2^n v \hat{t}_i^m = p^2 P_{i,m} - (p-1)^2 P_{i,m}^{(1)} + (p-2)^2 P_{i,m}^{(2)} - \ldots + P_{i,m}^{(p-1)} .$$

Here in the case of even values of v as already stated we loose 1 d.f. for the interactions with X obtained by substituting X for the pseudo factor Y in the interactions containing Y confounded in 2×2^n design. These are the interactions whose normal equations are of type $\not{B}A$ discussed above. We loose one d.f. since the contrast

$$C_1 \equiv t_i^m - (t_{i-1}^m + t_{i+1}^m) + (t_{i-2}^m + t_{i+2}^m) \pm \ldots + t_{p+1}^m$$

where, p is as usual $v/2$ and $v = 4q$,

or $C_2 = t_i^m - (t_{i-1}^m + t_{i+1}^m) + \ldots\ldots - t_{p+1}^m$

when $v = 4q - 2$, is not estimable.

It may be noted that the contrasts C_1 and C_2 remain same for any $i = 1, 2, \ldots, v$.

(iii) Particularly for the main effect X the solution is given by

$$2^n v \hat{t}_i^o = \sum_{r=1}^{p} r^2 Q_i^{(p+r)}$$

The average variance for any two varietal comparison is $(v+1) \times \sigma^2 / (3 \times 2^n)$ and $E.F. = 3/(v+1)$.

The splitting up of d.f. for different components in the analysis of variance table for $v \times 2^n$ design in 2^a plot blocks is given below.

Let us first suppose that out of $2^{n-a+1} - 1$ interactions confounded in the design 2×2^n in 2^a plot blocks z interactions do not involve the pseudo factor Y.

Then for odd values of v, we have the following analysis of variance partitioning

Sources of variation	d.f.
Due to blocks	$v\ 2^{n-a+1} - 1$
Due to main effect X	$v - 1$
Due to M	$2^n - 1 - z$
Due to X M	$(v-1)(2^n - 1)$

Error	$2^n v - 2^{n-a+1} v + z + 1$
Total	$2^{n+1} v - 1$

where, M denotes all the main effects and interactions of the real factors at 2 levels each.

While for even values of v we have

Sources of variation	d.f.
Blocks	$v\, 2^{n-a+1} - 1$
X	$v - 1$
M	$2^n - 1 - z$
XM	$(v-1)(2^n-1) - (2^{n-a+1} - 1 - z)$
Error	$2^n v - 2^{n-a+1}(v - 1)$
Total	$2^{n+1} v - 1$

ILLUSTRATION:

The method of construction and analysis of varietal cum manurial trials has been illustrated by constructing and analysing the 19×2^2 design obtained by combining the circular design in 2 plot blocks with 19 treatments and one replication of the factorial design 2×2^2 in 4 plot blocks.

CONSTRUCTION: Denoting the three factors of the second design by Y, A, and B, let us obtain one replication of the design by confounding the interaction YAB. The two blocks of this design are given below:

Levels of Y	Blocks 1	2
0	X_{10}	X_{20}
1	X_{11}	X_{21}

where $X_{10} \equiv X_{21}$ stands for 00,11 and $X_{20} \equiv X_{11}$ stands for 01,11. Let the circular design be obtained by developing the initial block 1,2. The two blocks of the 1∅9x 2^2 design obtained by combining the block containing the treatments(1,2) of the circular design and the two blocks of the factorial design are

Block	Block content	
1	1_a	2_b
2	1_b	2_a

where $a = X_{10}$ and $b = X_{20}$. Similarly 36 more blocks can be obtained out of the remaining 18 blocks of the circular design.

<u>ANALYSIS:</u> As such designs have not yet been applied, no actual data could be found for illustration of the method of analysis. However, the observations in the different plots of the design have been taken from the data of an uniformity trial on Malvi Cotton reported by Panse and Sukhatme (1957)

The observations in the blocks have been presented in table No.2, together with the treatment combinations given in brackets, in which the first number denotes the varietal level and the rest two the levels of A and B respectively.

Table No.2. The table showing yield of seed cotton in gm. per plot of size 1/2000 acre.

Blocks				
1	94	121	122	108
	(100)	(111)	(210)	(201)
2	143	138	165	135
	(110)	(101)	(200)	(211)
3	103	82	97	38
	(200)	(211)	(310)	(301)
4	62	102	64	68
	(210)	(201)	(300)	(311)
5	73	87	74	65
	(300)	(311)	(410)	(401)
6	49	81	36	28
	(310)	(301)	(400)	(411)
7	56	97	35	66
	(400)	(411)	(510)	(501)
8	67	76	89	99
	(410)	(401)	(500)	(511)
9	82	52	60	72
	(500)	(511)	(610)	(601)

10	92 (510)	57 (501)	78 (600)	63 (611)
11	61 (600)	62 (611)	36 (710)	42 (701)
12	87 (610)	56 (601)	76 (700)	75 (711)
13	51 (700)	58 (711)	56 (810)	93 (801)
14	80 (710)	12 (701)	50 (800)	91 (811)
15	56 (800)	87 (811)	78 (910)	79 (901)
16	109 (810)	54 (801)	87 (900)	54 (911)
17	81 (900)	78 (911)	61 (1010)	59 (1001)
18	107 (910)	62 (901)	80 (1000)	87 (1011)
19	107 (1000)	101 (1011)	82 (1110)	116 (1101)
20	92 (1010)	126 (1001)	55 (1100)	67 (1111)
21	73 (1100)	104 (1111)	108 (1210)	116 (1201)
22	79 (1110)	82 (1101)	120 (1200)	111 (1211)
23	78 (1200)	99 (1211)	69 (1310)	62 (1301)
24	110 (1210)	127 (1201)	91 (1300)	88 (1311)

25	53	62	99	75
	(1300)	(1311)	(1410)	(1401)
26	88	75	89	72
	(1310)	(1301)	(1400)	(1411)
27	90	98	104	95
	(1400)	(1411)	(1510)	(1501)
28	86	96	64	85
	(1410)	(1401)	(1500)	(1511)
29	67	70	61	83
	(1500)	(1511)	(1610)	(1601)
30	99	85	80	75
	(1510)	(1501)	(1600)	(1611)
31	79	89	59	48
	(1600)	(1611)	(1710)	(1701)
32	45	78	60	134
	(1610)	(1601)	(1700)	(1711)
33	56	89	70	66
	(1700)	(1711)	(1810)	(1801)
34	62	106	49	67
	(1710)	(1701)	(1800)	(1811)
35	85	78	76	77
	(1800)	(1811)	(1910)	(1901)
36	128	80	157	121
	(1810)	(1801)	(1900)	(1911)
37	133	101	125	133
	(1900)	(1911)	(110)	(101)
38	119	79	111	108
	(1910)	(1901)	(100)	(111)

The total S.S., block S.S. and unaffected main effects and interactions S.S.'s can be found as usual, and have been given below.

Total S.S.	=	103766.520
Block S.S.	=	60321.770
S.S. due to A	=	497.533
S.S. due to B	=	18.480
S.S. due to AB	=	43.164
S.S. due to XA	=	4429.342
S.S. due to XB	=	7800.895

We have now to find the S.S.'s due to the affected main effect X and interaction XAB. Let the 19 varietal effects be denoted by x_i's (i=1,2 .....19), and the 19 treatments for the interaction XAB by t_i's (i = 1,2,.......19), where, $t_i = x_i$ (00 + 11 - 10 - 01). The frequency tables for both these sets of treatments have been presented in table No.3. These show that, X and XAB are independently estimable, as sum of products of the frequencies of any column of one table with any column of the other table is zero. The normal equations for X come out as

$$4 x_i - 2 (x_{i-1} + x_{i+1}) = Q_i , \quad i = 1,2,\ldots 19 \qquad \ldots(I)$$

and that for XAB are

$$4 t_i + 2(t_{i-1} + t_{i+1}) = P_i , \quad i = 1,2,\ldots 19 \qquad \ldots(II)$$

Here, $i - 1$ and $i + 1$ are to be reduced mod (19), if necessary, and Q_i is the adjusted total of the ith variety, while P_i, is that of ith treatment of XAB defined by t_i.

The solution of the set of 19 equations given by (I) provide the estimates of x_i's by writing down the reduced normal equations, which are obtained by adding the normal equations corresponding to the treatments which are equidistant from the ith treatment, when the treatments $x_1, x_2, \ldots, x_{19}$ are written in a circle in descending order. As for example for treatment x_i, (x_{i-1}, x_{i+1}), (x_{i-2}, x_{i+2}), etc., are the pairs of equidistant treatments from x_i, and will be called different associates of x_i. Here there are 9 pairs of equidistant treatments with respect to any particular treatment. In general for odd values of v treatments there are $(v-1)/2$ (= p say) such equidistant treatments or associates, and for even values of v there are p (= v/2) associates. Let us denote $x_{i-j} + x_{i+j}$ by $s_i^{(j)}$ and $Q_{i-j} + Q_{i+j}$ by $Q_i^{(j)}$. Then the reduced normal equations for first p-1 (=8) associates for treatment x_i along with the normal equation for x_i are as follows

$$4\,x_i - 2\,s_i^{(1)} \quad \ldots\ldots\ldots\ldots = Q_i \qquad \ldots.(1)$$

$$-4\,x_i + 4\,s_i^{(1)} - 2\,s_i^{(2)} \quad \ldots\ldots\ldots = Q_i^{(1)} \qquad \ldots.(2)$$

Main effect X																			Interaction XAB																		
x_1	x_2	x_3	x_4	x_5	x_6	x_7	x_8	x_9	x_{10}	x_{11}	x_{12}	x_{13}	x_{14}	x_{15}	x_{16}	x_{17}	x_{18}	x_{19}	t_1	t_2	t_3	t_4	t_5	t_6	t_7	t_8	t_9	t_{10}	t_{11}	t_{12}	t_{13}	t_{14}	t_{15}	t_{16}	t_{17}	t_{18}	t_{19}
2	2	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	2	-2	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
2	2	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	-2	2	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
0	2	2	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	2	-2	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
0	2	2	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	-2	2	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
0	0	2	2	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	2	-2	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
0	0	2	2	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	-2	2	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
0	0	0	2	2	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	2	-2	0	0	0	0	0	0	0	0	0	0	0	0	0	0
0	0	0	2	2	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	-2	2	0	0	0	0	0	0	0	0	0	0	0	0	0	0
0	0	0	0	2	2	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	2	-2	0	0	0	0	0	0	0	0	0	0	0	0	0
0	0	0	0	2	2	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	-2	2	0	0	0	0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	2	2	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	2	-2	0	0	0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	2	2	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	-2	2	0	0	0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	2	2	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	2	-2	0	0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	2	2	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	-2	2	0	0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	2	2	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	2	-2	0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	2	2	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	-2	2	0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	2	2	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	2	-2	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	2	2	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	-2	2	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	2	2	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	2	-2	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	2	2	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	-2	2	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0	2	2	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	2	-2	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0	2	2	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	-2	2	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0	0	2	2	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	2	-2	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0	0	2	2	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	-2	2	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0	0	0	2	2	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	2	-2	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0	0	0	2	2	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	-2	2	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0	0	0	0	2	2	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	2	-2	0	0	0	0
0	0	0	0	0	0	0	0	0	0	0	0	0	2	2	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	-2	2	0	0	0	0
0	0	0	0	0	0	0	0	0	0	0	0	0	0	2	2	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	2	-2	0	0	0
0	0	0	0	0	0	0	0	0	0	0	0	0	0	2	2	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	-2	2	0	0	0
0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	2	2	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	-2	-2	0	0
0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	2	2	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	-2	2	0	0
0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	2	2	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	2	-2	0
			0	0	0	0	0	0	0	0	0	0	0	0	0	2	2	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	-2	2	0
			0	0	0	0	0	0	0	0	0	0	0	0	0	0	2	2	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	2	-2

$$-2s_i^{(1)} +4s_i^{(2)} -2s_i^{(3)} \quad .. \; .. \; .. \; .. \; .. \; .. \quad = Q_i^{(2)} \quad ...(3)$$

$$-2s_i^{(2)} +4s_i^{(3)} -2s_i^{(4)} \quad .... \quad = Q_i^{(3)} \quad ...(4)$$

$$-2s_i^{(3)} +4s_i^{(4)} -2s_i^{(5)} \quad .... \quad = Q_i^{(4)} \quad ...(5)$$

$$-2s_i^{(4)} +4s_i^{(5)} -2s_i^{(6)} \quad .... \quad = Q_i^{(5)} \quad ...(6)$$

$$-2s_i^{(5)} +4s_i^{(6)} -2s_i^{(7)} \quad .... \quad = Q_i^{(6)} \quad ...(7)$$

$$-2s_i^{(6)} + 4s_i^{(7)} -2s_i^{(8)} \quad .... \quad = Q_i^{(7)} \quad ...(8)$$

$$-2s_i^{(7)} +4s_i^{(8)} -2s_i^{(9)} \quad .... \quad = Q_i^{(8)} \quad ...(9)$$

and, as there are 10 unknowns in 9 equations which are not all independent we add to these set of 9 equations the restriction $\sum_{i=1}^{19} x_i = 0$ which is same as the equation,

$$x_i + s_i^{(1)} + s_i^{(2)} + \quad .... \quad + s_i^{(9)} = 0 \qquad ...(10)$$

With the help of these ten equations the solution for x_i is obtained as

$$\hat{x}_i = (45Q_i + 36Q_i^{(1)} + 28Q_i^{(2)} + 21Q_i^{(3)} + 15Q_i^{(4)} + 10Q_i^{(5)} + 6Q_i^{(6)}$$

$$+ 3Q_i^{(7)} + Q_i^{(8)}) / 38$$

for, $i = 1, 2, ...19.$

The estimates of different x_i for ($i=1,2,\ldots 19$) are given below

x_i	1	2	3	4	5
Estimates	18.276	29.079	10.882	-8.566	-8.013
	6	7	8	9	10
	-8.211	-15.158	9.395	9.697	1.750
	11	12	13	14	15
	-22.447	9.105	-14.592	1.961	-1.237
	16	17	18	19	
	-4.434	0.368	-12.579	4.724	

The S.S. due to X is as usual $\sum_{i=1}^{19} \hat{x}_i Q_i$, and is found to be here equal to 9303.145.

The variances of the comparison between varieties for different associates are given below.

Between 1st associates = $9\sigma^2/19$. e.g. for x_i, $V(x_i - x_{i-1})$ or $V(x_i - x_{i+1})$.

" 2nd " $=17\sigma^2/19$

" 3rd " $=24\sigma^2/19$

" 4th " $=30\sigma^2/19$

" 5th " $=35\sigma^2/19$

" 6th " $=39\sigma^2/19$

" 7th " $=42\sigma^2/19$

Between 8th associates $= 44\sigma^2/19$

" 9th " $= 45\sigma^2/19$

The average variance is found to be equal to $5\sigma^2/3$, and E.F. = 0.15.

Proceeding as in the case of main effect X, and denoting $t_{i-j}+t_{i+j}$ by $u_i^{(j)}$ and $P_{i-j}+P_{i+j}$ by $P_i^{(j)}$, the reduced normal equations over first 8 associates for t_i along with the normal equation for t_i obtained from set of equations (II) are given below.

$$4t_i + 2u_i^{(1)} \quad \ldots\ldots\ldots\ldots\ldots\ldots = P_i \qquad \ldots(1)$$

$$4t_i + 4u_i^{(1)} + 2u_i^{(2)} \quad \ldots\ldots\ldots\ldots\ldots = P_i^{(1)} \qquad \ldots(2)$$

$$2u_i^{(1)} + 4u_i^{(2)} + 2u_i^{(3)} \quad \ldots\ldots\ldots\ldots = P_i^{(2)} \qquad \ldots(3)$$

$$2u_i^{(2)} + 4u_i^{(3)} + 2u_i^{(4)} \quad \ldots\ldots\ldots\ldots = P_i^{(3)} \qquad \ldots(4)$$

$$2u_i^{(3)} + 4u_i^{(4)} + 2u_i^{(5)} \quad \ldots\ldots\ldots = P_i^{(4)} \qquad \ldots(5)$$

$$2u_i^{(4)} + 4u_i^{(5)} + 2u_i^{(6)} \quad \ldots\ldots\ldots = P_i^{(5)} \qquad \ldots(6)$$

$$2u_i^{(5)} + 4u_i^{(6)} + 2u_i^{(7)} \quad \ldots\ldots\ldots = P_i^{(6)} \qquad \ldots(7)$$

$$2u_i^{(6)} + 4u_i^{(7)} + 2u_i^{(8)} \quad \ldots\ldots = P_i^{(7)} \qquad \ldots(8)$$

$$2u_i^{(7)} + 4u_i^{(8)} + 2u_i^{(9)} \quad \ldots\ldots = P_i^{(8)} \qquad \ldots(9)$$

The solution of t_i can be obtained with the help of these 9 equations and the restriction $\sum_{i=1}^{19} t_i$ = C (constant), i.e. the equation

$$t_i + u_i^{(1)} + u_i^{(2)} + \ldots\ldots + u_i^{(9)} = C$$

It is found to be

$$\hat{t}_i = \tfrac{1}{8}(5P_i - 4P_i^{(1)} + 4P_i^{(2)} - 3P_i^{(3)} + 3P_i^{(4)} - 2P_i^{(5)} + 2P_i^{(6)} - P_i^{(7)} + P_i^{(8)} - 2C)$$

for $i = 1, 2, \ldots\ldots 19$

Substituting for t_i in equation (10), and solving for C we get, $C = \frac{1}{8}\sum_{i=1}^{19} P_i$.

The algebric expression for P_i's for i =1, 2, ...19 are given as follows.

$$2P_1 = 2T_1 - (B_1 - B_2 - B_{37} + B_{38})$$

$$2P_2 = 2T_2 - (-B_1 + B_2 + B_3 - B_4)$$

$$2P_3 = 2T_3 - (-B_3 + B_4 + B_5 - B_6)$$

$$2P_4 = 2T_4 - (-B_5 + B_6 + B_7 - B_8)$$

$$2P_5 = 2T_5 - (-B_7 + B_8 + B_9 - B_{10})$$

$$2P_6 = 2T_6 - (-B_9 + B_{10} + B_{11} - B_{12})$$

$$2P_7 = 2T_7 - (-B_{11} + B_{12} + B_{13} - B_{14})$$

$$2P_8 = 2T_8 - (-B_{13} + B_{14} + B_{15} - B_{16})$$

$$2P_9 = 2T_9 - (-B_{15} + B_{16} + B_{17} - B_{18})$$

$$2P_{10} = 2T_{10} - (-B_{17} + B_{18} + B_{19} - B_{20})$$

$$2P_{11} = 2T_{11} - (-B_{19} + B_{20} + B_{21} - B_{22})$$

$$2P_{12} = 2T_{12} - (-B_{21} + B_{22} + B_{23} - B_{24})$$

$$2P_{13} = 2T_{13} - (-B_{23} + B_{24} + B_{25} - B_{26})$$

$$2P_{14} = 2T_{14} - (-B_{25} + B_{26} + B_{27} - B_{28})$$

$$2P_{15} = 2T_{15} - (-B_{27} + B_{28} + B_{29} - B_{30})$$

$$2P_{16} = 2T_{16} - (-B_{29} + B_{30} + B_{31} - B_{32})$$

$$2P_{17} = 2T_{17} - (-B_{31} + B_{32} + B_{33} - B_{34})$$

$$2P_{18} = 2T_{18} - (-B_{33} + B_{34} + B_{35} - B_{36})$$

$$2P_{19} = 2T_{19} - (-B_{35} + B_{36} + B_{37} - B_{38})$$

where, T_i is the difference of the sum of the plot yields having treatments (ioo), (iii) and (i10), (i01) and B_i is the total of the ith block. S.S. due to XAB is given by

$$\sum_{i=1}^{19} \hat{t}_i P_i - \left(\sum_{i=1}^{19} \hat{t}_i \sum_{i=1}^{19} P_i \right)/19 = 7215.461$$

The analysis of variance table is as follows.

Table No. 4 . Analysis of Variance Table

....

Sources of Variation	d.f.	S.S.	m.s.
Between Blocks	37	60321.770	1630.318
X	18	9303.145	516.841
A	1	497.533	
B	1	18.480	
AB	1	43.164	
XA	18	4429.342	346.075
XB	18	7800.895	433.383
XAB	18	7215.461	400.859
Error	39	14136.730	362.480
Total	151	103766.520	

VARIETAL CUM MANURIAL DESIGN BASED ON OTHER INCOMPLETE BLOCK DESIGNS WITH TWO PLOT BLOCKS

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When the varietal design is any other incomplete block design with two plot blocks a varietal cum manurial design $v \times 2^n$ can be constructed exactly similarly as described earlier. In these designs also there will be two types of normal equations, and the analysis will be similar as in the case where the varietal design is a circular design in two plot blocks with v odd. The method of solving the normal equations will follow the same line as in the case of the incomplete block design used for obtaining varietal cum manurial design.

VARIETAL CUM MANURIAL DESIGNS BASED ON CIRCULAR DESIGN WITH FOUR PLOT BLOCKS

* * * * * *

We have discussed so far varietal cum manurial designs based on varietal designs with only two plots per block. Other varietal designs with four plots or in general 2^p plot blocks can also be used to obtain varietal cum manurial design with as many replications as that in the varietal design. In the present investigation we shall consider only varietal designs with four plot blocks, and one replication of the factorial design 4×2^n in 2^n plot blocks so as to give the combined design $v \times 2^n$ in 2^n plot blocks. Further circular designs in 4 plot blocks will be taken as the varietal design.

CONSTRUCTION: For construction of the design $v \times 2^n$, we shall first obtain the circular design in 4 plot blocks by developing the initial block (1,2,3,4) mod(v). Next one replication of the factorial design 4×2^n in 2^n plot blocks will be obtained (following Das 1961) from the symmetrical design 2^{n+2} in 2^n plot blocks. The four combinations of the two extra factors each at two levels in the design 2^{n+2} give the four levels of the first factor in the design 4×2^n. These two factors will be denoted by X_1 and X_2. By confounding three interactions of the type $X_1 I_1$, $X_2 I_2$ and $X_1 X_2 (I_1 I_2)$, where, I_1 stands for a group of factors containing only real factors i.e. other than X_1 and X_2, and I_2 similarly stands for another group and not identical with I_1 group of such factors, while $(I_1 I_2)$ stands for all the factors in I_1 and I_2 which are not common to both the groups. In each of the four blocks of this design each of the four combinations of the factors X_1 and X_2 will occur an equal number of times as none of X_1, X_2 and $X_1 X_2$ is confounded in the design. The two designs can now be combined as indicated earlier. In the combined design the main effect X will be affected together with the interactions XI_1, XI_2 and $X(I_1 I_2)$; I_1, I_2, having the same meaning indicated earlier.

ANALYSIS: In the factorial design 4×2^n, we shall denote by G_0 the group of 2^{n-2} combinations of the n real factors which occur in the first block with (00) combination of X_1 and X_2. Similarly

G_1, G_2, G_3 will denote three other groups of combinations of n factors each of size 2^{n-2} which occur in first block with the other three combinations of the factors X_1 and X_2, namely, (01), (10) and (11). This block will be written in order $00G_0$, $01G_1$, $10G_2$ and $11G_3$. In the other three blocks these four groups only occur such that they form a latin square in all the four blocks each occurring once with each of the four combinations of the factor X_1 and X_2. Each of interactions I_1, I_2 and (I_1I_2) can always be expressed as a contrast between (G_0), (G_1), (G_2) and (G_3), where (G_i) denotes the sum of all the combinations forming G_i.

The normal equations for the main effect X will be the same as obtained in the circular design excepting that each coefficient on the left hand side will be multiplied by 2^{n-2}. Actually these equations are as shown below.

$$2^{n-2} \times 12x_i - 2^{n-2} \times 3\,(x_{i-1}+x_{i+1}) - 2^{n-2} \times 2(x_{i-2}+x_{i+2})$$

$$-2^{n-2} \times 1\,(x_{i-3}+x_{i+3}) = Q_i \quad \ldots\ldots (I)$$

$$i = 1, 2, \ldots\ldots v.$$

where, x_{i+1} etc. are to be reduced mod(v), and Q_i is the adjusted varietal total of x_i.

Defining, $R_i = Q_i / 2^{n-2}$, we can write the set of equations (I) as

$$12x_i-3(x_{i-1}+x_{i+1})-2(x_{i-2}+x_{i+2})-(x_{i-3}+x_{i+3})=R_i \quad ..(II)$$

To solve for x_i we write down the normal equation for x_i and p-1 (where, $p=(v-1)/2$ or $v/2$ according as v is odd or even) other equations each of which is obtained by adding two normal equations corresponding to two treatments which form, an associate group of the treatment x_i. The associate groups of treatments are the same as in the case of circular designs in two plot blocks. The associate groups do not change even when the treatments are far interactions. The restriction $\sum_{i=1}^{v} x_i = 0$ is to be taken to get an unique solution. Denoting as before $t_{i-j}+t_{i+j}$ by $s_i^{(j)}$ and $R_{i-j}+R_{i+j}$ by $R_i^{(j)}$, for even values of v we have the p+1 equations as below.

$$12x_i-3s_i^{(1)}-2s_i^{(2)}-s_i^{(3)} \quad .... = R_i \quad ...(1)$$

$$-6x_i+10s_i^{(1)}-4s_i^{(2)}-2s_i^{(3)}-s_i^{(4)} \quad .... = R_i^{(1)} \quad ...(2)$$

$$-4x_i-4s_i^{(1)}+12s_i^{(2)}-3s_i^{(3)}-2s_i^{(4)}-s_i^{(5)} \quad .... = R_i^{(2)} \quad ...(3)$$

$$-2x_i-2s_i^{(1)}-3s_i^{(2)}+12s_i^{(3)}-3s_i^{(4)}-2s_i^{(5)}-s_i^{(6)} \quad .. = R_i^{(3)} \quad ...(4)$$

$$-s_i^{(1)} - 2s_i^{(2)} - 3s_i^{(3)} + 12s_i^{(4)} - 3s_i^{(5)} - 2s_i^{(6)} - s_i^{(7)} \quad \ldots\ldots\ldots\ldots = R_i^{(4)} \quad \ldots..(5)$$

$$-s_i^{(2)} - 2s_i^{(3)} - 3s_i^{(4)} + 12s_i^{(5)} - 3s_i^{(6)} - 2s_i^{(7)} - s_i^{(8)} \quad \ldots\ldots\ldots\ldots = R_i^{(5)} \quad \ldots..(6)$$

$$\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots$$

$$\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots$$

$$-s_i^{(p-6)} - 2s_i^{(p-5)} - 3s_i^{(p-4)} + 12s_i^{(p-3)} - 3s_i^{(p-2)} - 2s_i^{(p-1)} - 2s_i^{(p)} \quad \ldots = R_i^{(p-3)} \quad \ldots..(p-2)$$

$$-s_i^{(p-5)} - 2s_i^{(p-4)} - 3s_i^{(p-3)} + 12s_i^{(p-2)} - 4s_i^{(p-1)} - 4s_i^{(p)} \quad \ldots\ldots\ldots\ldots = R_i^{(p-2)} \quad \ldots.(p-1)$$

$$-s_i^{(p-4)} - 2s_i^{(p-3)} - 4s_i^{(p-2)} + 10s_i^{(p-1)} - 6s_i^{(p)} \quad \ldots\ldots\ldots\ldots = R_i^{(p-1)} \quad \ldots..(p)$$

$$\text{and, } x_i + s_i^{(1)} + s_i^{(2)} + \ldots\ldots + s_i^{(p-1)} + s_i^{(p)} \ldots\ldots\ldots\ldots = 0 \quad \ldots.(p+1)$$

Since the restriction $\sum_{i=1}^{v} x_i = 0$ is same equation as equation (p+1) above.

The solution of x_i can be obtained by solving for x_i from three independent equations in x_i, $s_i^{(1)}$ and $s_i^{(2)}$, which are obtained by eliminating other unknowns from the above set of (p+1) equations. This can be done by finding three independent sets of multipliers $U_p, U_{p-1} \ldots U_o$; $V_p, V_{p-1}, \ldots V_o$; and $W_p, W_{p-1}, \ldots W_o$, such that when the above p+1 equations are multiplied by these in the order stated, that is equation (1) by U_p equation (2) by U_{p-1} and lastly equation (p+1) by U_o and are added all other unknowns except x_i, $s_i^{(1)}$ and $s_i^{(2)}$ are eliminated. The above multipliers are easily obtainable from the following recurrence relations.

$$U_r - U_o = -2U_{r-1} - 3U_{r-2} + 12U_{r-3} - 3U_{r-4} - 2U_{r-5} - U_{r-6}$$

$$V_r - V_o = -2V_{r-1} - 3V_{r-2} + 12V_{r-3} - 3V_{r-4} - 2V_{r-5} - V_{r-6}$$

$$W_r - W_o = -2W_{r-1} - 3W_{r-2} + 12W_{r-3} - 3W_{r-4} - 2W_{r-5} - W_{r-6}$$

for, r = 8,9,.....etc.

The values of U_r, V_r and W_r for r = 0,1,2.....7 are given in table No.6.

So finally we get the equations as

$$(12U_p - 6U_{p-1} - 4U_{p-2} - 2U_{p-3})x_i + (-3U_p + 10U_{p-1} - 4U_{p-2} - 2U_{p-3} - U_{p-4})s_i^{(1)} + (-2U_p - 4U_{p-1} + 12U_{p-2} - 3U_{p-3} - 2U_{p-4} - U_{p-5})s_i^{(2)}$$

$$= \sum_{j=1}^{p} U_j R_i^{(p-j)}$$

where, $R_i^{(0)} = R_i$.

The other two equations can be written down by putting V's and W's instead of U's in the above equation.

For odd values of v as there are changes in the coefficients of $s_i^{(p)}$, $s_i^{(p-1)}$ and $s_i^{(p-2)}$ in the last three reduced normal equations, the multipliers U, V and W's take different values. Whereas the recurrence relations of the multipliers and form of the three independent equations in x_i, $s_i^{(1)}$ and $s_i^{(2)}$ remains unchanged. The initial values of U_r, V_r and W_r for $r = 0,1,2,\ldots,7$ are given in table No.7 when v is odd.

The normal equations for XI_1, where I_1 corresponds to the contrast $(G_0) + (G_1) - (G_2) - (G_3)$, come out as

$$2^{n-2} \times 12t_i^{(1)} - 2^{n-2} \times (t_{i-1}^{(1)} + t_{i+1}^{(1)}) + 2^{n-2} \times 2(t_{i-2}^{(1)} + t_{i+2}^{(1)})$$

$$+ 2^{n-2} x (t_{i-3}^{(1)} + t_{i+3}^{(1)}) = P_{i,1} \quad \text{for } i=1,2,\ldots,v$$

The normal equations for XI_2, where I_2 corresponds to the contrast $(G_0)-(G_1)+(G_2)$, $-(G_3)$, come out as

$$2^{n-2} x 12 t_i^{(2)} + 2^{n-2} x\ 3(t_{i-1}^{(2)} + t_{i+1}^{(2)}) - 2^{n-2}\ x\ 2(t_{i-2}^{(2)}$$

$$+ t_{i+2}^{(2)}) + 2^{n-2} x (t_{i-3}^{(2)} + t_{i+3}^{(2)}) = P_{i,2}$$

$$\text{for } i=1,2,\ldots,v$$

While the normal equations for $X(I_1 I_2)$, where, $(I_1 I_2)$ corresponds to the contrast (G_0) $-(G_1)-(G_2)+(G_3)$, come out as

$$2^{n-2} x\ 12 t_i^{(3)} + 2^{n-2} (t_{i-1}^{(3)} + t_{i+1}^{(3)}) + 2^{n-2} x\ 2(t_{i-2}^{(3)}$$

$$+ t_{i+2}^{(3)}) - 2^{n-2} (t_{i-3}^{(3)} + t_{i+3}^{(3)}) = P_{i,3}$$

$$\text{for } i=1,2,\ldots,v$$

Each of these sets of equations can be solved just like those in case of main effect X. The only difference is that here the restriction taken is $\sum_{i=1}^{v} t_i^m = L_m$ (Constant), and not zero as in main effect X, where $L_m = (\sum_{i=1}^{v} R_{i,m})/16 =$

$$(\sum_{i=1}^{v} P_{i,m})/(2^{n-2} x\ 16).$$

for m =1,2 and 3. Where m =1,2, and 3 refer to three interactions XI_1, XI_2 and $X(I_1 I_2)$ respectively.

From the above normal equations for any interaction the three independent equations in t_i^m, $s_{i,m}^{(1)}$ and $s_{i,m}^{(2)}$ can be obtained following the same procedure as in the case of the main effect X, where $s_{i,m}^{(j)}$ denotes as before the sum of the treatments of jth associate group to treatment t_i^m, i.e. $s_{i,m}^{(j)} = t_{i-j}^m + t_{i+j}^m$. These three equations come in the form

$$A_1 t_i^m + B_1 s_{i,m}^{(1)} + C_1 s_{i,m}^{(2)} = \sum_{j=1}^{p} U_j R_{i,m}^{(p-j)} \quad \ldots\ldots(1)$$

$$A_2 t_i^m + B_2 s_{i,m}^{(1)} + C_2 s_{i,m}^{(2)} = \sum_{j=1}^{p} V_j R_{i,m}^{(p-j)} \quad \ldots\ldots(2)$$

&, $$A_3 t_i^m + B_3 s_{i,m}^{(1)} + C_3 s_{i,m}^{(2)} = \sum_{j=1}^{p} W_j R_{i,m}^{(p-j)} + W_0 L_m \quad \ldots\ldots(3)$$

The values of A_1, B_1, C_1; A_2, B_2, C_2; and A_3, B_3, C_3 are different for different interactions i.e, for different m's. So is the case for the multipliers U, V, and W's, which apart from being different for different m's are different for odd and even values of v also. The coefficients A_1, B_1, C_1 in terms of the multipliers U_r for different interactions are however, given in the following table No.5; A_2, B_2, C_2 and A_3, B_3, C_3 are obtained by replacing U by V and W respectively.

Table No.5: Table giving A_1, B_1, C_1, for different affected interactions

...

m	Coefficients A_1	B_1	C_1
1	$12U_p - 2U_{p-1} + 4U_{p-2} + 2U_{p-3}$	$-U_p + 14U_{p-1} + 2U_{p-3} + U_{p-4}$	$2U_p + 12U_{p-2} - U_{p-3} + 2U_{p-4} + U_{p-5}$
2	$12U_p + 6U_{p-1} - 4U_{p-2} + 2U_{p-3}$	$3U_p + 10U_{p-1} + 4U_{p-2} - 2U_{p-3}$	$-2U_p + 4U_{p-1} + 12U_{p-2} + 3U_{p-3} - 2U_{p-4} + U_{p-5}$
3	$12U_p + 2U_{p-1} + 4U_{p-2} - 2U_{p-3}$	$U_p + 14U_{p-1} + 2U_{p-3} - U_{p-4}$	$2U_p + 12U_{p-2} + U_{p-3} + 2U_{p-4} - U_{p-5}$

The recurrence relations for the multipliers U_r for different m are given below; they are same for even or odd values of v.

(a) For $m = 1$,

$$U_r + U_o = -2U_{r-1} + U_{r-2} - 12U_{r-3} + U_{r-4} - 2U_{r-5} - U_{r-\bar{6}}$$

for $r = 8, 9, \ldots$ etc.

(b) For $m = 2$,

$$U_r + U_o = 2U_{r-1} - 3U_{r-2} - 12U_{r-3} - 3U_{r-4} + 2U_{r-5} - U_{r-\bar{6}}$$

for $r = 8, 9, \ldots$ etc.

(c) For $m = 3$,

$$U_r - U_o = 2U_{r-1} + U_{r-2} + 12U_{r-3} + U_{r-4} + 2U_{r-5} + U_{r-\bar{6}}$$

for $r = 8, 9, \ldots$ etc.

The recurrence relations for the other two sets of multipliers can be written down by replacing U in the above relations by V and W respectively for each affected interaction. The values of U_r, V_r and W_r for $r = 0, 1, 2, \ldots, 7$ are given in table No. 6 and 7, for even and odd values of v respectively.

The sum of squares for the unaffected interactions are obtained as usual. While S.S. due to interaction m is obtained as

$$\sum_{i=1}^{v} \hat{t}_i^m P_{i,m} - \left(\sum_{i=1}^{v} \hat{t}_i^m \quad \sum_{i=1}^{v} P_{i,m} \right) / v.$$

The analysis of variance table is same as that in the case of odd values of v in two plot blocks.

Table No. 6 Table giving values of U_r, V_r, and W_r for even v.

m	X			XI_1, (m=1)			XI_2, (m=2)			$X(I_1I_2)$, (m=3)		
Multipliers i	U_i	V_i	W_i	U_i	V_i	W_i	U_i	V_i	W_i	U_i	V_i	W_i
0	0	0	2	0	0	-2	0	0	2	0	0	2
1	0	-1	0	0	1	0	0	1	0	0	-1	0
2	1	1	0	1	1	0	1	1	0	1	1	0
3	2	1	1	-2	-1	1	2	-1	1	2	1	1
4	-8	-16	0	4	-12	0	0	-16	4	4	-12	4
5	22	45	-1	-22	11	3	-18	-45	7	22	-11	11
6	1	-31	16	73	-23	-16	-63	-31	-8	73	-23	40
7	-172	-269	-30	-220	197	38	-76	269	-86	220	-197	142

Table No.7 Table giving values of U_r, V_r and W_r for odd v

m	X			XI_1, (m=1)			XI_2, (m=2)			$X(I_1I_2)$, (m=3)		
Multipliers i	U_i	V_i	W_i	U_i	V_i	W_i	U_i	V_i	W_i	U_i	V_i	W_i
0	0	0	1	0	0	-1	0	0	-1	0	0	1
1	0	-1	0	0	1	0	0	1	0	0	1	0
2	1	1	0	1	-1	0	1	2	0	1	-1	0
3	-3	2	1	-3	2	1	1	1	1	1	2	1
4	3	-18	-1	7	-18	-1	-1	-17	3	3	14	3
5	15	45	0	-29	51	4	-17	-64	4	19	19	8
6	-78	-13	16	102	-147	-20	-46	-93	-12	54	77	32
7	154	-332	-46	-322	564	58	-30	210	-74	166	340	110

SUMMARY

The thesis deals with two main problems. These are (1) obtaining designs which are suitable for factorial cum manurial trials and (2) a method of analysis of such and other asymmetrical designs.

So long factorial experimentations and varietal trials have been dealt separately. Since, it has been often found that the varieties selected from a varietal trial conducted at an uniform level of manure lodges at a higher dose of manure, some necessity has been felt to evolve designs through which in addition to selection of varieties some knowledge regarding the interaction between the varieties and manures can also be obtained. The usual designs which are available in literature are not very suitable for this type of experiments excepting that when the number of manurial treatments is small, incomplete block designs augmented by associating each variety with a number of treatments, as introduced by Bose and Connor (1952), can be adopted. The designs suitable for varietal cum manurial trials will essentially be some asymmetrical design. As the number of varieties is usually large the usual asymmetrical design where no main effect is affected is not of much help, as very large block size is required for such experimentation. Therefore attempts have been made to evolve varietal

cum manurial designs by combining a b.i.b. design and a factorial design, and thus a series of such design has been obtained. As b.i.b. designs suffer from certain limitations that they are not available for any number of treatments with a reasonable number of replications particularly when block size is small another series of designs has been obtained by combining circular designs introduced by Das (1960) with suitable block size and a factorial design. In the present thesis we have restricted ourselves to the manurial factors each at two levels. The series obtained are based on a factorial design of the form 2^{n+1} and on (a) circular designs in two plot blocks and (b) circular designs in four plot blocks, the latter design being obtained by developing the initial block 1,2,3,4 mod (v), where, v is the number of varieties and varieties are numbered in natural numbers 1 to v.

Kempthorne (1952) has given a method of analysis for a particular asymmetrical design. An attempt has been made in the present thesis to give a general method of analysis for the design $v \times 2^n$ through which besides getting a complete analysis of data it can be easily known whether for any design two affected interactions are estimable mutually independently. An illustration for the above method has also been given.

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