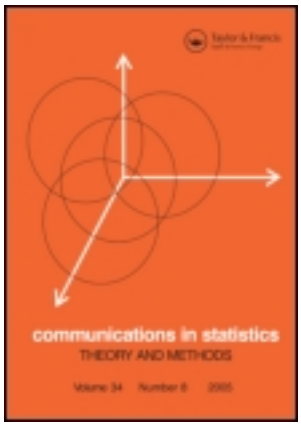

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Neighbor-Balanced Bipartite Block Designs

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This article deals with the neighbor-balanced block design setting when there are two disjoint sets of treatments, one set consisting of test treatments and the other of control treatments. The interest here is to estimate the contrasts pertaining to test treatments vs. control treatments (with respect to direct and neighbors) with as high precision as possible. Some series of neighbor-balanced block designs for comparing a set of test treatments to a set of control treatments have been developed. The designs obtained are totally balanced in the sense that all the contrasts among test treatments for direct and neighbor effects are estimated with same variance and all the contrasts pertaining to test vs. control for direct and neighbor effects are estimated with the same variance.

Keywords Circular design; Control treatments; Direct effects; Neighbor effects; Test treatments; Totally balanced design.

Mathematics Subject Classification 62K10.

1. Introduction

While conducting large scale experiments with crop varieties for identification of varieties for release, the experimenters sometimes would like to keep more than one variety as control with which they would like to make all comparisons. Sometimes the experiment is conducted for a region, but there are different sub regions in it with some locally adopted varieties and comparison of every new variety is bound to be made before a decision can be made on it for release. In such cases the experimenter prefers to keep some common control varieties or checks and some local checks in series of trials. Sometimes one would like to keep different controls which are known to be good for different purposes and conditions. In such situations the experimenters, while planning for large scale trials, prefer to ensure that a number of control varieties are included in the trial, than to take the risk with one control so that after the experiment is conducted one has enough

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freedom and scope in interpreting the performance of the varieties and identifying suitable varieties to be released for cultivation. Under this experimental situation, it is desired to compare a set of treatments called the test treatments to another set of treatments called standard or control treatments. The main interest here lies in making test vs. control treatment comparisons with as much precision as possible and comparisons within the test treatments are of less consequence (Bechhofer and Tamhane, 1993). Jaggi et al. (1996) studied A-efficient block designs for comparing two disjoint sets of treatments.

In varietal trials, adjacent plots are planted to different varieties. The yields of the plots with shorter varieties are low as compared to their normal yield because of the shading effect of the tall varieties in the adjoining neighboring plots. This interference or competition from neighboring units can contribute to variability in experimental results and lead to substantial loss in efficiency. To avoid bias when comparing the effects of treatments in this situation, it is important to ensure that no treatment is unduly disadvantaged by its neighbor. This is done by using the neighbor balanced designs wherein the allocation of treatments is such that each treatment occurs equally often (say λ times) with every other treatment as neighbor (Azais et al., 1993; Tomar et al., 2005; Jaggi et al., 2006, 2007).

In the conventional neighbor-balanced designs, the interest is to make all possible pair-wise comparisons among the direct effects as well as the neighbor effects of treatments. This article deals with neighbor balanced block (NBB) designs when there are two disjoint sets of treatments, one set consisting of test treatments and the other of control treatments. The interest here is to estimate the contrasts pertaining to test treatments (direct effects, left and right neighbor effects) vs. control treatments with as high precision as possible. Series of NBB designs for comparing a set of test treatments to a set of control treatments have been developed.

2. Model and Definitions

We consider a set of v_t test treatments and a set of v_c control treatments. Under the block design set-up with b blocks of size k each, following is the model for analyzing a design for test treatments-control treatments comparison with neighbor effects:

$$\mathbf{Y} = \mu\mathbf{1} + \boldsymbol{\Delta}'\boldsymbol{\tau} + \boldsymbol{\Delta}_1'\boldsymbol{\delta} + \boldsymbol{\Delta}_2'\boldsymbol{\rho} + \mathbf{D}'\boldsymbol{\beta} + \mathbf{e}, \tag{2.1}$$

where $\mathbf{Y}$ is a $n \times 1$ vector of observations, μ is grand mean, and $\mathbf{1}$ is $n \times 1$ vector of unities. $\boldsymbol{\Delta}'$ is $n \times (v_t + v_c)$ incidence matrix of observations vs. direct treatments pertaining to v_t tests and v_c controls, $\boldsymbol{\tau}$ is $(v_t + v_c) \times 1$ vector of direct treatment effects of tests and controls; $\boldsymbol{\tau} = (\boldsymbol{\tau}_t\ \boldsymbol{\tau}_c)'$, $\boldsymbol{\Delta}_1'$ is $n \times (v_t + v_c)$ incidence matrix of observations vs. left neighbor of test treatments and controls, $\boldsymbol{\delta}$ is $(v_t + v_c) \times 1$ vector of left neighbor effects of tests and controls; $\boldsymbol{\delta} = (\boldsymbol{\delta}_t\ \boldsymbol{\delta}_c)'$, $\boldsymbol{\Delta}_2'$ is $n \times (v_t + v_c)$ incidence matrix of observations vs. right neighbor of test treatments and controls, $\boldsymbol{\rho}$ is $(v_t + v_c) \times 1$ vector of right neighbor effects of tests and controls; $\boldsymbol{\rho} = (\boldsymbol{\rho}_t\ \boldsymbol{\rho}_c)'$, $\mathbf{D}'$ is $n \times b$ incidence matrix of observations vs. blocks, $\boldsymbol{\beta}$ is $b \times 1$ vector of block effects and $\mathbf{e}$ is $n \times 1$ vector of errors with $E(\mathbf{e}) = 0$ and $V(\mathbf{e}) = \sigma^2\mathbf{I}_n$. Further,

$$\boldsymbol{\Delta}\boldsymbol{\Delta}_1' = \mathbf{M}_1 = \begin{bmatrix} \mathbf{M}_{1t} & \mathbf{m}_{10} \\ \mathbf{m}_{10}' & \mathbf{M}_{1c} \end{bmatrix},$$

$(v_t + v_c) \times (v_t + v_c)$ matrix with $\mathbf{M}_{1t}$ as the incidence of direct test vs. left neighbor test treatments, $\mathbf{m}_{10}$ is the incidence of direct test vs. left neighbor control treatments, and $\mathbf{M}_{1c}$ is the incidence of direct control vs. left neighbor control treatments,

$$\mathbf{\Delta}\mathbf{\Delta}_2' = \mathbf{M}_2 = \begin{bmatrix} \mathbf{M}_{2t} & \mathbf{m}_{20} \\ \mathbf{m}_{20}' & \mathbf{M}_{2c} \end{bmatrix},$$

$(v_t + v_c) \times (v_t + v_c)$ matrix with $\mathbf{M}_{2t}$ as the incidence of direct test vs. right neighbor test treatments, $\mathbf{m}_{20}$ is the incidence of direct test vs. right neighbor control treatments, and $\mathbf{M}_{2c}$ is the incidence of direct control vs. right neighbor control treatments,

$$\mathbf{\Delta}_1\mathbf{\Delta}_2' = \mathbf{M}_3 = \begin{bmatrix} \mathbf{M}_{3t} & \mathbf{m}_{30} \\ \mathbf{m}_{30}' & \mathbf{M}_{3c} \end{bmatrix},$$

$(v_t + v_c) \times (v_t + v_c)$ matrix with $\mathbf{M}_{3t}$ as the incidence of left neighbor vs. right neighbor test treatments, $\mathbf{m}_{30}$ is the incidence of left neighbor test vs. right neighbor control treatments, and $\mathbf{M}_{3c}$ is the incidence of left neighbor vs. right neighbor control treatments,

$$\mathbf{\Delta}\mathbf{D}' = \mathbf{N}_1 = \begin{bmatrix} \mathbf{N}_{1t} \\ \mathbf{N}_{1c} \end{bmatrix},$$

$(v_t + v_c) \times b$ matrix with $\mathbf{N}_{1t}$ as the incidence of direct test treatments vs. block and $\mathbf{N}_{1c}$ as the incidence of direct control treatments vs. block.

$$\mathbf{\Delta}_1\mathbf{D}' = \mathbf{N}_2 = \begin{bmatrix} \mathbf{N}_{2t} \\ \mathbf{N}_{2c} \end{bmatrix},$$

$(v_t + v_c) \times b$ matrix with $\mathbf{N}_{2t}$ as the incidence of left neighbor test treatments vs. block and $\mathbf{N}_{2c}$ as the incidence of left neighbor control treatments vs. block.

$$\mathbf{\Delta}_2\mathbf{D}' = \mathbf{N}_3 = \begin{bmatrix} \mathbf{N}_{3t} \\ \mathbf{N}_{3c} \end{bmatrix},$$

$(v_t + v_c) \times b$ matrix with $\mathbf{N}_{3t}$ as the incidence of right neighbor test treatment vs. block and $\mathbf{N}_{3c}$ as the incidence of right neighbor control treatment vs. block.

$\mathbf{r} = [\mathbf{r}_{\tau t}' \; \mathbf{r}_{\tau c}']'$ is the $(v_t + v_c) \times 1$ replication vector of direct treatments with $\mathbf{r}_{\tau t}$ as the replication vector of test treatments and $\mathbf{r}_{\tau c}$ as the replication of controls.

$\mathbf{r_1} = [\mathbf{r}_{\delta t}' \; \mathbf{r}_{\delta c}']'$ is the $(v_t + v_c) \times 1$ replication vector of the treatments appearing as left neighbor with $\mathbf{r}_{\delta t}$ as the replication vector of test treatments appearing as left neighbor and $\mathbf{r}_{\delta c}$ as the replication of controls appearing as left neighbor.

$\mathbf{r}_2 = [\mathbf{r}_{\rho t}' \; \mathbf{r}_{\rho c}']'$ is the $(v_t + v_c) \times 1$ replication vector of the treatments appearing as right neighbor with $\mathbf{r}_{\rho t}$ as the replication vector of test treatments appearing as right neighbor and $\mathbf{r}_{\rho c}$ as the replication of controls appearing as right neighbor.

$$\mathbf{R}_\tau = \begin{bmatrix} \mathbf{R}_{\tau t} & \mathbf{0} \\ \mathbf{0} & \mathbf{R}_{\tau c} \end{bmatrix}; \quad \mathbf{R}_\delta = \begin{bmatrix} \mathbf{R}_{\delta t} & \mathbf{0} \\ \mathbf{0} & \mathbf{R}_{\delta c} \end{bmatrix}; \quad \mathbf{R}_\rho = \begin{bmatrix} \mathbf{R}_{\rho t} & \mathbf{0} \\ \mathbf{0} & \mathbf{R}_{\rho c} \end{bmatrix},$$

$\mathbf{R}_{\tau t}(\mathbf{R}_{\tau c})$ is the diagonal matrix of replications of direct test (control) treatments, $\mathbf{R}_{\delta t}(\mathbf{R}_{\delta c})$ is the diagonal matrix of replications of the test (control) treatments appearing as left neighbors and $\mathbf{R}_{\rho t}(\mathbf{R}_{\rho c})$ is the diagonal matrix of replications of the test (control) treatments appearing as right neighbors.

The joint information matrix for estimating direct effects, left and right neighbor effects of treatments (tests and controls) is obtained as follows:

$$\mathbf{C} = \begin{bmatrix} \mathbf{R}_{\tau t} - \frac{1}{k}\mathbf{N}_{1t}\mathbf{N}'_{1t} & -\frac{1}{k}\mathbf{N}_{1t}\mathbf{N}'_{1c} & \mathbf{M}_{1t} - \frac{1}{k}\mathbf{N}_{1t}\mathbf{N}'_{2t} & \mathbf{m}_{10} - \frac{1}{k}\mathbf{N}_{1t}\mathbf{N}'_{2c} & \mathbf{M}_{2t} - \frac{1}{k}\mathbf{N}_{1t}\mathbf{N}'_{3t} & \mathbf{m}_{20} - \frac{1}{k}\mathbf{N}_{1t}\mathbf{N}'_{3c} \\ -\frac{1}{k}\mathbf{N}_{1c}\mathbf{N}'_{1t} & \mathbf{R}_{\tau c} - \frac{1}{k}\mathbf{N}_{1c}\mathbf{N}'_{1c} & \mathbf{m}'_{10} - \frac{1}{k}\mathbf{N}_{1c}\mathbf{N}'_{2t} & \mathbf{M}_{1c} - \frac{1}{k}\mathbf{N}_{1c}\mathbf{N}'_{2c} & \mathbf{m}'_{20} - \frac{1}{k}\mathbf{N}_{1c}\mathbf{N}'_{3t} & \mathbf{M}_{2c} - \frac{1}{k}\mathbf{N}_{1c}\mathbf{N}'_{3c} \\ \mathbf{M}'_{1t} - \frac{1}{k}\mathbf{N}_{2t}\mathbf{N}'_{1t} & \mathbf{m}_{10} - \frac{1}{k}\mathbf{N}_{2t}\mathbf{N}'_{1c} & \mathbf{R}_{\delta t} - \frac{1}{k}\mathbf{N}_{2t}\mathbf{N}'_{2t} & -\frac{1}{k}\mathbf{N}_{2t}\mathbf{N}'_{2c} & \mathbf{M}_{3t} - \frac{1}{k}\mathbf{N}_{2t}\mathbf{N}'_{3t} & \mathbf{m}_{30} - \frac{1}{k}\mathbf{N}_{2t}\mathbf{N}'_{3c} \\ \mathbf{m}'_{10} - \frac{1}{k}\mathbf{N}_{2c}\mathbf{N}'_{1t} & \mathbf{M}'_{1c} - \frac{1}{k}\mathbf{N}_{2c}\mathbf{N}'_{1c} & -\frac{1}{k}\mathbf{N}_{2c}\mathbf{N}'_{2t} & \mathbf{R}_{\delta c} - \frac{1}{k}\mathbf{N}_{2c}\mathbf{N}'_{2c} & \mathbf{m}'_{30} - \frac{1}{k}\mathbf{N}_{2c}\mathbf{N}'_{3t} & \mathbf{M}'_{3c} - \frac{1}{k}\mathbf{N}_{2c}\mathbf{N}'_{3c} \\ \mathbf{M}'_{2t} - \frac{1}{k}\mathbf{N}_{3t}\mathbf{N}'_{1t} & \mathbf{m}_{20} - \frac{1}{k}\mathbf{N}_{3t}\mathbf{N}'_{1c} & \mathbf{M}'_{3t} - \frac{1}{k}\mathbf{N}_{3t}\mathbf{N}'_{2t} & \mathbf{m}_{30} - \frac{1}{k}\mathbf{N}_{3t}\mathbf{N}'_{2c} & \mathbf{R}_{\rho t} - \frac{1}{k}\mathbf{N}_{3t}\mathbf{N}'_{3t} & -\frac{1}{k}\mathbf{N}_{3t}\mathbf{N}'_{3c} \\ \mathbf{m}'_{20} - \frac{1}{k}\mathbf{N}_{3c}\mathbf{N}'_{1t} & \mathbf{M}'_{2c} - \frac{1}{k}\mathbf{N}_{3c}\mathbf{N}'_{1c} & \mathbf{m}'_{30} - \frac{1}{k}\mathbf{N}_{3c}\mathbf{N}'_{2t} & \mathbf{M}'_{3c} - \frac{1}{k}\mathbf{N}_{3c}\mathbf{N}'_{2c} & -\frac{1}{k}\mathbf{N}_{3c}\mathbf{N}'_{3t} & \mathbf{R}_{\rho c} - \frac{1}{k}\mathbf{N}_{3c}\mathbf{N}'_{3c} \end{bmatrix}.$$

The information matrix for estimating the direct effect of treatments is obtained as given below:

$$\mathbf{C}_{\tau} = \mathbf{C}_{11} - \mathbf{C}_{12}\mathbf{C}_{22}^{-}\mathbf{C}_{21}, \tag{2.2}$$

where

$$\mathbf{C}_{11} = \begin{bmatrix} \mathbf{R}_{\tau t} - \frac{1}{k}\mathbf{N}_{1t}\mathbf{N}'_{1t} & -\frac{1}{k}\mathbf{N}_{1t}\mathbf{N}'_{1c} \\ -\frac{1}{k}\mathbf{N}_{1c}\mathbf{N}'_{1t} & \mathbf{R}_{\tau c} - \frac{1}{k}\mathbf{N}_{1c}\mathbf{N}'_{1c} \end{bmatrix},$$

$$\mathbf{C}_{12} = \begin{bmatrix} \mathbf{M}_{1t} - \frac{1}{k}\mathbf{N}_{1t}\mathbf{N}'_{2t} & \mathbf{m}_{10} - \frac{1}{k}\mathbf{N}_{1t}\mathbf{N}'_{2c} & \mathbf{M}_{2t} - \frac{1}{k}\mathbf{N}_{1t}\mathbf{N}'_{3t} & \mathbf{m}_{20} - \frac{1}{k}\mathbf{N}_{1t}\mathbf{N}'_{3c} \\ \mathbf{m}'_{10} - \frac{1}{k}\mathbf{N}_{1c}\mathbf{N}'_{2t} & \mathbf{M}_{1c} - \frac{1}{k}\mathbf{N}_{1c}\mathbf{N}'_{2c} & \mathbf{m}'_{20} - \frac{1}{k}\mathbf{N}_{1c}\mathbf{N}'_{3t} & \mathbf{M}_{2c} - \frac{1}{k}\mathbf{N}_{1c}\mathbf{N}'_{3c} \end{bmatrix},$$

$$\mathbf{C}_{22} = \begin{bmatrix} \mathbf{R}_{\delta t} - \frac{1}{k}\mathbf{N}_{2t}\mathbf{N}'_{2t} & -\frac{1}{k}\mathbf{N}_{2t}\mathbf{N}'_{2c} & \mathbf{M}_{3t} - \frac{1}{k}\mathbf{N}_{2t}\mathbf{N}'_{3t} & \mathbf{m}_{30} - \frac{1}{k}\mathbf{N}_{2t}\mathbf{N}'_{3c} \\ -\frac{1}{k}\mathbf{N}_{2c}\mathbf{N}'_{2t} & \mathbf{R}_{\delta c} - \frac{1}{k}\mathbf{N}_{2c}\mathbf{N}'_{2c} & \mathbf{m}'_{30} - \frac{1}{k}\mathbf{N}_{2c}\mathbf{N}'_{3t} & \mathbf{M}_{3c} - \frac{1}{k}\mathbf{N}_{2c}\mathbf{N}'_{3c} \\ \mathbf{M}'_{3t} - \frac{1}{k}\mathbf{N}_{3t}\mathbf{N}'_{2t} & \mathbf{m}_{30} - \frac{1}{k}\mathbf{N}_{3t}\mathbf{N}'_{2c} & \mathbf{R}_{\rho t} - \frac{1}{k}\mathbf{N}_{3t}\mathbf{N}'_{3t} & -\frac{1}{k}\mathbf{N}_{3t}\mathbf{N}'_{3c} \\ \mathbf{m}'_{30} - \frac{1}{k}\mathbf{N}_{3c}\mathbf{N}'_{2t} & \mathbf{M}'_{3c} - \frac{1}{k}\mathbf{N}_{3c}\mathbf{N}'_{2c} & -\frac{1}{k}\mathbf{N}_{3t}\mathbf{N}'_{3t} & \mathbf{R}_{\rho c} - \frac{1}{k}\mathbf{N}_{3c}\mathbf{N}'_{3c} \end{bmatrix}.$$

Similarly, the information matrices pertaining to right and left neighbor effects can also be obtained.

We now give some definitions of block designs with neighbor effects for two disjoint sets of treatments, with one set of test treatments and other of control treatments.

Definition 2.1. A block design with neighbor effects for two disjoint sets of treatments is said to be neighbor-balanced bipartite block (NBBPB) design if every test treatment from first set has every other test treatment appearing constant (say λ_1) number of times as a right and as a left neighbor and every test treatment from first set has every control treatment from second set appearing constant (say λ_2) number of times as a right and as a left neighbor.

We consider here the neighbor-balanced block designs that are circular, i.e., the treatment in the left border is the same as the treatment in the right-end inner plot and the treatment in the right border is same as the treatment in the left-end inner plot. It may be mentioned here that the observations are not recorded from the border plots; these plots are taken only to have the neighbor effects of treatments at the end plots of the blocks.

Definition 2.2. A NBBPB design with circular blocks, permitting the estimation of direct test and neighbor effects, is called variance balanced if the variance of any estimated elementary contrast among the direct test effects (left neighbor test effects, right neighbor test effects) is constant, say $V_{t1}(V_{t2}, V_{t3})$ and the variance of any estimated elementary contrast among the direct test (left neighbor test, right neighbor test) vs. direct control treatment effects is constant, say $V_{tc1}(V_{tc2}, V_{tc3})$. A block design with circular blocks, permitting the estimation of direct and neighbor effects, is totally balanced if $V_{t1} = V_{t2} = V_{t3}$ and $V_{tc1} = V_{tc2} = V_{tc3}$.

3. Methods of Constructing NBBPB Designs

Method 3.1. Consider the circular complete NBB design with parameters v (v prime ≥ 7), $b = v - 1$, $r = v - 1$, $k = v$, $\lambda = 1$ obtained by taking the jth block ($j = 1, 2, \ldots, v - 1$) of the design as $v, j, 2j, \ldots, (v-1)j$ modulo v (Azais et al., 1993). Out of v treatments, pi treatments ($p > 1, i > 1, pi \leq v - 2$) are divided into p sets of size i each. Replace all the treatments of 1st set of size i with 1st control treatment, 2nd set with 2nd control treatment and so on pth set with pth control treatment. The resulting design is totally balanced complete NBBPB design for comparing a set of $v_t = v - pi$ test treatments, $v_c = p$ control treatments, $b^* = v - 1$ blocks and $r_{\tau t} = r_{\delta t} = r_{\rho t} = r_t(\text{say}) = (v - 1)$ replications of test treatments appearing as direct, left and right, $r_{\tau c} = r_{\delta c} = r_{\rho c} = r_c(\text{say}) = i(v - 1)$ replications of control treatments appearing as direct, left and right, $k^* = v$ block size and $\lambda_1 = \lambda$, $\lambda_2 = i$.

The structure of the incidence matrices of the design obtained is as follows:

$$\mathbf{M}_1 = \mathbf{M}_2 = \mathbf{M}_3 = \begin{bmatrix} \mathbf{J}_{v-pi} - \mathbf{I}_{v-pi} & i\mathbf{J}_{v-pi,p} \\ i\mathbf{J}_{p,v-pi} & i^2\mathbf{J}_p - i\mathbf{I}_p \end{bmatrix},$$

$$\mathbf{R}_\tau = \mathbf{R}_\delta = \mathbf{R}_\rho = \begin{bmatrix} (v-1)\mathbf{I}_v & \mathbf{0}_{v-pi,p} \\ \mathbf{0}'_{v-pi,p} & i(v-1)\mathbf{I}_p \end{bmatrix},$$

$$\mathbf{N}_1\mathbf{N}_1' = \mathbf{N}_1\mathbf{N}_2' = \mathbf{N}_1\mathbf{N}_3' = \mathbf{N}_2\mathbf{N}_1' = \mathbf{N}_2\mathbf{N}_2' = \mathbf{N}_2\mathbf{N}_3' = \mathbf{N}_3\mathbf{N}_1' = \mathbf{N}_3\mathbf{N}_2' = \mathbf{N}_3\mathbf{N}_3'$$
$$= \begin{bmatrix} (v-1)\mathbf{J}_{v-pi} & i(v-1)\mathbf{J}_{v-pi,p} \\ i(v-1)\mathbf{J}_{p,v-pi} & i^2(v-1)\mathbf{J}_p \end{bmatrix},$$

Further using 2.2,

$$\mathbf{C}_{11} = \begin{bmatrix} (v-1)\left(\mathbf{I}_{v-pi} - \frac{1}{v}\mathbf{J}_{v-pi}\right) & -\frac{i(v-1)}{v}\mathbf{J}_{v-pi,p} \\ -\frac{i(v-1)}{v}\mathbf{J}_{p,v-pi} & i(v-1)\left(\mathbf{I}_p - \frac{i}{v}\mathbf{J}_p\right) \end{bmatrix},$$

$$\mathbf{C}_{12} = \begin{bmatrix} \frac{1}{v}\mathbf{J}_{v-pi} - \mathbf{I}_{v-pi} & \frac{i}{v}\mathbf{J}_{p,v-pi} & \frac{1}{v}\mathbf{J}_{v-pi} - \mathbf{I}_{v-pi} & \frac{i}{v}\mathbf{J}_{p,v-pi} \\ \frac{i}{v}\mathbf{J}_{v-pi,p} & \frac{i^2}{v}\mathbf{J}_p - i\mathbf{I}_p & \frac{i}{v}\mathbf{J}_{v-pi,p} & \frac{i^2}{v}\mathbf{J}_p - i\mathbf{I}_p \end{bmatrix},$$

$$\mathbf{C}_{22} = \begin{bmatrix} (v-1)\mathbf{I}_{v-pi} - \frac{(v-1)}{v}\mathbf{J}_{v-pi} & -\frac{i(v-1)}{v}\mathbf{J}_{v-pi,p} & \frac{1}{v}\mathbf{J}_{v-pi} - \mathbf{I}_{v-pi} & \frac{i}{v}\mathbf{J}_{v-pi,p} \\ -\frac{i(v-1)}{v}\mathbf{J}_{p,v-pi} & i(v-1)\mathbf{I}_p - \frac{i^2(v-1)}{v}\mathbf{J}_p & \frac{i}{v}\mathbf{J}_{p,v-pi} & \frac{i^2}{v}\mathbf{J}_p - i\mathbf{I}_p \\ \frac{1}{v}\mathbf{J}_{v-pi} - \mathbf{I}_{v-pi} & \frac{i}{v}\mathbf{J}_{v-pi,p} & (v-1)\mathbf{I}_{v-pi} - \frac{(v-1)}{v}\mathbf{J}_{v-pi} & -\frac{i(v-1)}{v}\mathbf{J}_{v-pi,p} \\ \frac{i}{v}\mathbf{J}_{p,v-pi} & \frac{i^2}{v}\mathbf{J}_p - i\mathbf{I}_p & -\frac{i(v-1)}{v}\mathbf{J}_{p,v-pi} & i(v-1)\mathbf{I}_p - \frac{i^2(v-1)}{v}\mathbf{J}_p \end{bmatrix}$$

To obtain the $\mathbf{C}_{22}^-$, the following result is used.
If

$$\mathbf{X} = \begin{bmatrix} \mathbf{A} & \mathbf{B} \\ \mathbf{B}' & \mathbf{D} \end{bmatrix} \quad \text{then } \mathbf{X}^- = \begin{bmatrix} \mathbf{A}^- + \mathbf{F}\mathbf{E}^-\mathbf{F}' & -\mathbf{F}\mathbf{E}^- \\ -\mathbf{E}^-\mathbf{F}' & \mathbf{E}^- \end{bmatrix}, \tag{3.1}$$

where $\mathbf{F} = \mathbf{A}^-\mathbf{B}$ and $\mathbf{E} = \mathbf{D} - \mathbf{B}'\mathbf{A}^-\mathbf{B}$

Here,

$$\mathbf{A} = \begin{bmatrix} (v-1)\mathbf{I}_{v-pi} - \frac{(v-1)}{v}\mathbf{J}_{v-pi} & -\frac{i(v-1)}{v}\mathbf{J}_{v-pi,p} \\ -\frac{i(v-1)}{v}\mathbf{J}_{p,v-pi} & i(v-1)\mathbf{I}_p - \frac{i^2(v-1)}{v}\mathbf{J}_p \end{bmatrix},$$

$$\mathbf{B} = \begin{bmatrix} \frac{1}{v}\mathbf{J}_{v-pi} - \mathbf{I}_{v-pi} & \frac{i}{v}\mathbf{J}_{v-pi,p} \\ \frac{i}{v}\mathbf{J}_{p,v-pi} & \frac{i^2}{v}\mathbf{J}_p - i\mathbf{I}_p \end{bmatrix},$$

$$\mathbf{D} = \begin{bmatrix} (v-1)\mathbf{I}_{v-pi} - \frac{(v-1)}{v}\mathbf{J}_{v-pi} & -\frac{i(v-1)}{v}\mathbf{J}_{v-pi,p} \\ -\frac{i(v-1)}{v}\mathbf{J}_{p,v-pi} & i(v-1)\mathbf{I}_p - \frac{i^2(v-1)}{v}\mathbf{J}_p \end{bmatrix}.$$

Thus, the information matrices for estimating the direct effects, left and right neighbor effects pertaining to test and control treatments for above NBBPB designs are obtained as:

$$\mathbf{C}_\tau = \mathbf{C}_\delta = \mathbf{C}_\rho = \begin{bmatrix} \frac{(v-3)}{(v-2)}(v\mathbf{I}_{v-pi} - \mathbf{J}_{v-pi}) & -\frac{i(v-3)}{(v-2)}\mathbf{J}_{v-pi,p} \\ -\frac{i(v-3)}{(v-2)}\mathbf{J}_{v-pi,p} & \frac{i(v-3)}{(v-2)}(v\mathbf{I}_p - i\mathbf{J}_p) \end{bmatrix}$$

with

$$\mathrm{V}_{t1} = \mathrm{V}_{t2} = \mathrm{V}_{t3} = \frac{2(v-2)}{v(v-3)}\sigma^2$$

and variance between treatments from two sets as

$$\mathrm{V}_{tc1} = \mathrm{V}_{tc2} = \mathrm{V}_{tc3} = \frac{(v-2)(i+1)}{iv(v-3)}\sigma^2.$$

Example 3.1. Consider the following circular NBB design with parameters $v = 7$, $b = 6$, $r = 6$, $k = 7$, and $\lambda = 1$:

$$\begin{array}{c|ccccccc|c} 6 & 7 & 1 & 2 & 3 & 4 & 5 & 6 & 7 \\ 5 & 7 & 2 & 4 & 6 & 1 & 3 & 5 & 7 \\ 4 & 7 & 3 & 6 & 2 & 5 & 1 & 4 & 7 \\ 3 & 7 & 4 & 1 & 5 & 2 & 6 & 3 & 7 \\ 2 & 7 & 5 & 3 & 1 & 6 & 4 & 2 & 7 \\ 1 & 7 & 6 & 5 & 4 & 3 & 2 & 1 & 7 \end{array}.$$

Let $p = 2, i = 2$ and the $pi = 4$ treatments considered in 2 sets of size 2 each are (4, 5) and (6, 7). Replacing the two treatments 4 and 5 by 1st control (0_1) and 6 and 7 with second control (0_2) in each block results in a NBBPB design with parameters $v_t = 3$, $v_c = 2$, $b^* = 6$, $r_t = 6$, $r_c = 12$, $k^* = 7$, $\lambda_1 = 1$ and $\lambda_2 = 2$ obtained as follows:

$$\begin{array}{c|ccccccc|c} 0_2 & 0_2 & 1 & 2 & 3 & 0_1 & 0_1 & 0_2 & 0_2 \\ 0_1 & 0_2 & 2 & 0_1 & 0_2 & 1 & 3 & 0_1 & 0_2 \\ 0_1 & 0_2 & 3 & 0_2 & 2 & 0_1 & 1 & 0_1 & 0_2 \\ 3 & 0_2 & 0_1 & 1 & 0_1 & 2 & 0_2 & 3 & 0_2 \\ 2 & 0_2 & 0_1 & 3 & 1 & 0_2 & 0_1 & 2 & 0_2 \\ 1 & 0_2 & 0_2 & 0_1 & 0_1 & 3 & 2 & 1 & 0_2 \end{array}$$

Here, treatments from the first set are 1, 2, and 3 whereas treatments from second set are 0_1 and 0_2. It can be seen that for this design,

$$\mathbf{C}_\tau = \mathbf{C}_\delta = \mathbf{C}_\rho = \begin{bmatrix} 5.6\mathbf{I}_3 - 0.8\mathbf{J}_3 & -1.6\mathbf{J}_{3,2} \\ -1.6\mathbf{J}_{2,3} & 11.2\mathbf{I}_2 - 3.2\mathbf{J}_2 \end{bmatrix}$$

with

$$V_{t1} = V_{t2} = V_{t3} = 0.3571\sigma^2 \text{ and}$$
$$V_{tc1} = V_{tc2} = V_{tc3} = 0.2678\sigma^2.$$

Method 3.2. Let v (≥ 7) be a prime or prime power. Develop $v-1$ mutually orthogonal Latin squares (MOLS) of order v as given in Fisher and Yates (1963) tables. Transpose these Latin squares and juxtapose them one below the other so that we obtain an arrangement of v symbols in $v(v-1)$ rows and v columns. Deleting the last q columns ($q = 0, 1, 2, \ldots, v-4$) and taking rows as blocks along with border plots, would result in an incomplete NBB design with parameters $v, b = v(v-1), r = (v-1)(v-q), k = v-q$ and $\lambda = v-q$ (Tomar et al., 2005). Following the same procedure as given in Method 3.1, i.e., out of v treatments, pi treatments ($p > 1, i > 1$) are taken such that $pi \leq (v-2)$ and these pi treatments are divided into p sets of size i each. Replace all the treatments of 1st set of size i with 1st control treatment, 2nd set with 2nd control treatment and so on pth set with pth control treatment. The resulting design is totally balanced incomplete NBBPB design with $v_t = (v-pi), v_c = p, b^* = v(v-1), r_t = (v-q)(v-1), r_c = i(v-1)(v-q), k^* = v-q, \lambda_1 = (v-q+1)(v-q)$, and $\lambda_2 = i(v-q)$.

The structure of the incidence matrices of the design are as follows:

$$\mathbf{M}_1 = \mathbf{M}_2 = \mathbf{M}_3 = \begin{bmatrix} (v-q)(\mathbf{J}_{v-ip} - \mathbf{I}_{v-ip}) & i(v-q)\mathbf{J}_{v-ip,p} \\ i(v-q)\mathbf{J}_{p,v-ip} & i(v-q)(i\mathbf{J}_p - \mathbf{I}_p) \end{bmatrix}.$$

Consider a pair of test treatments. The number of blocks in which this pair appear together in the entire design as direct test treatments is $(v-q)(v-q-1)$. Consider a pair of treatments involving one test and one control treatment. The number of blocks in which this pair appear together in the entire design as direct treatments is $i(v-q)(v-q-1)$. Consider a pair of control treatments. The number of blocks in which this pair appear together in the entire design as direct control treatments is $i^2(v-q)(v-q-1)$. Therefore,

$$\mathbf{N}_1\mathbf{N}_1' = \begin{bmatrix} (v-q)\{q\mathbf{I}_{v-ip} + (v-q-1)\mathbf{J}_{v-ip}\} & i(v-q)(v-q-1)\mathbf{J}_{v-ip,p} \\ i(v-q)(v-q-1)\mathbf{J}_{p,v-ip} & (v-q)\{iq\mathbf{I}_p + i^2(v-q-1)\mathbf{J}_p\} \end{bmatrix}.$$

Similarly, the concurrence of all other pairs can be obtained and it is found that

$$\mathbf{N}_1\mathbf{N}_1' = \mathbf{N}_1\mathbf{N}_2' = \mathbf{N}_1\mathbf{N}_3' = \mathbf{N}_2\mathbf{N}_1' = \mathbf{N}_2\mathbf{N}_2' = \mathbf{N}_2\mathbf{N}_3' = \mathbf{N}_3\mathbf{N}_1' = \mathbf{N}_3\mathbf{N}_2' = \mathbf{N}_3\mathbf{N}_3'$$

$$\mathbf{R}_\tau = \mathbf{R}_\delta = \mathbf{R}_\rho = \begin{bmatrix} (v-1)(v-q)\mathbf{I}_{v-ip} & \mathbf{0}_{v-ip,p} \\ \mathbf{0}'_{p,v-ip} & i(v-1)(v-q)\mathbf{I}_p \end{bmatrix}$$

$$\mathbf{C}_{11} = \begin{bmatrix} (v-q-1)(v\mathbf{I}_{v-ip} - \mathbf{J}_{v-ip}) & -i(v-q-1)\mathbf{J}_{v-ip} \\ -i(v-q-1)\mathbf{J}_{v-ip} & i(v-q-1)(v\mathbf{I}_p - i\mathbf{J}_p) \end{bmatrix}$$

$$\mathbf{C}_{12} = \begin{bmatrix} \mathbf{J}_{v-ip} - v\mathbf{I}_{v-ip} & i\mathbf{J}_{v-ip,p} & \mathbf{J}_{v-pi} - v\mathbf{I}_{v-pi} & i\mathbf{J}_{v-ip,p} \\ i\mathbf{J}_{p,v-ip} & -i(v\mathbf{I}_p - i\mathbf{J}_p) & i\mathbf{J}_{p,v-ip} & -i(v\mathbf{I}_p - i\mathbf{J}) \end{bmatrix}$$

$$\mathbf{C}_{22} = \begin{bmatrix} (v-q-1)(v\mathbf{I}_{v-ip} - \mathbf{J}_{v-ip}) & -i(v-q-1)\mathbf{J}_{v-ip,p} & \mathbf{J}_{v-ip} - v\mathbf{I}_{v-ip} & i\mathbf{J}_{v-ip,p} \\ -i(v-q-1)\mathbf{J}_{p,v-ip} & i(v-q-1)(v\mathbf{I}_p - i\mathbf{J}_p) & i\mathbf{J}_{p,v-ip} & -i(v\mathbf{I}_p - i\mathbf{J}_p) \\ \mathbf{J}_{v-ip} - v\mathbf{I}_{v-ip} & i\mathbf{J}_{v-ip,p} & (v-q-1)(v\mathbf{I}_{v-ip} - \mathbf{J}_{v-ip}) & -i(v-q-1)\mathbf{J}_{v-ip,p} \\ i\mathbf{J}_{p,v-ip} & -i(v\mathbf{I}_p - i\mathbf{J}_p) & -i(v-q-1)\mathbf{J}_{p,v-ip} & i(v-q-1)(v\mathbf{I}_p - i\mathbf{J}_p) \end{bmatrix}.$$

$\mathbf{C}_{22}^{-}$ has been obtained using (3.1). Here,

$$\mathbf{A} = \begin{bmatrix} (v-q-1)(v\mathbf{I}_{v-ip} - \mathbf{J}_{v-ip}) & -i(v-q-1)\mathbf{J}_{v-ip,p} \\ -i(v-q-1)\mathbf{J}_{p,v-ip} & i(v-q-1)(v\mathbf{I}_p - i\mathbf{J}_p) \end{bmatrix},$$

$$\mathbf{B} = \begin{bmatrix} \mathbf{J}_{v-ip} - v\mathbf{I}_{v-ip} & i\mathbf{J}_{v-ip,p} \\ i\mathbf{J}_{p,v-ip} & -i(v\mathbf{I}_p - i\mathbf{J}_p) \end{bmatrix},$$

$$\mathbf{D} = \begin{bmatrix} (v-q-1)(v\mathbf{I}_{v-ip} - \mathbf{J}_{v-ip}) & -i(v-q-1)\mathbf{J}_{v-ip,p} \\ -i(v-q-1)\mathbf{J}_{p,v-ip} & i(v-q-1)(v\mathbf{I}_p - i\mathbf{J}_p) \end{bmatrix},$$

$$\mathbf{A}^{-} = \begin{bmatrix} \frac{1}{iv(v-q-1)}(\mathbf{I}_{v-ip} - \frac{v^2}{v^2-p^2i^2}\mathbf{J}_{v-ip}) & -\frac{1}{(v-q-1)}\frac{(-v^2+p^2i^2+v^2p)}{v^2(v^2-p^2i^2)}\mathbf{J}_{v-ip,p} \\ -\frac{1}{(v-q-1)}\frac{(-v^2+p^2i^2+v^2p)}{v^2(v^2-p^2i^2)}\mathbf{J}_{p,v-ip} & \frac{1}{iv(v-q-1)}(\mathbf{I}_p - \frac{v^2}{v^2-p^2i^2}\mathbf{J}_p) \end{bmatrix}$$

Thus,

$$\mathbf{C}_\tau = \mathbf{C}_\delta = \mathbf{C}_\rho = \begin{bmatrix} \frac{(v-q)(v-q-3)}{(v-q-2)}(v\mathbf{I}_{v-ip} - \mathbf{J}_{v-ip}) & -\frac{i(v-q)(v-q-3)}{(v-q-2)}\mathbf{J}_{v-ip,p} \\ -\frac{i(v-q)(v-q-3)}{(v-q-2)}\mathbf{J}_{p,v-ip} & \frac{i(v-q)(v-q-3)}{(v-q-2)}(v\mathbf{I}_p - i\mathbf{J}_p) \end{bmatrix}$$

with

$$\mathrm{V}_{t1} = \mathrm{V}_{t2} = \mathrm{V}_{t3} = \frac{2(v-q-2)}{v(v-q)(v-q-3)}\sigma^2 \quad \text{and}$$
$$\mathrm{V}_{tc1} = \mathrm{V}_{tc2} = \mathrm{V}_{tc3} = \frac{(v-q-2)(i+1)}{iv(v-q)(v-q-3)}\sigma^2.$$

Example 3.2. Consider the NBB design with parameters $v = 7$, $b = 42$, $r = 24$, $q = 3$ so that $k = (v-q) = 4$ and $\lambda = 4$. Let $p = 2$ and $i = 2$, replacing the two treatments 4 and 5 by 1st control (0_1) and 6 and 7 with second control (0_2) in each block results in a incomplete NBBPB design with parameters $v_t = 3$, $v_c = 2$, $b^* = 42$,

$r_t = 24$, $r_c = 48$, $k^* = 4$, $\lambda_1 = 4$ and $\lambda_2 = 8$ obtained as follows:

0_2	1	3	0_1	0_2	1
1	2	0_1	0_2	1	2
2	3	0_1	0_2	2	3
3	0_1	0_2	1	3	0_1
0_1	0_1	0_2	2	0_1	0_1
0_1	0_2	1	3	0_1	0_2
0_2	0_2	2	0_1	0_2	0_2
0_1	1	2	3	0_1	1
0_1	2	3	0_1	0_1	2
0_2	3	0_1	0_1	0_2	3
0_2	0_1	0_1	0_2	0_2	0_1
1	0_1	0_2	0_2	1	0_1
2	0_2	0_2	1	2	0_2
3	0_2	1	2	3	0_2
3	1	0_1	0_2	3	1
0_1	2	0_1	1	0_1	2
0_1	3	0_2	2	0_1	3
0_2	0_1	0_2	3	0_2	0_1
0_2	0_1	1	0_1	0_2	0_1
1	0_2	2	0_1	1	0_2
2	0_2	3	0_2	2	0_2
0_2	1	0_1	2	0_2	1
0_2	2	0_2	3	0_2	2
1	3	0_2	0_1	1	3
2	0_1	1	0_1	2	0_1
3	0_1	2	0_2	3	0_1
0_1	0_2	3	0_2	0_1	0_2
0_1	0_2	0_1	1	0_1	0_2
2	1	0_2	0_1	2	1
3	2	0_2	0_1	3	2
0_1	3	1	0_2	0_1	3
0_1	0_1	2	0_2	0_1	0_1
0_2	0_1	3	1	0_2	0_1
0_2	0_2	0_1	2	0_2	0_2
1	0_2	0_1	3	1	0_2
0_1	1	0_2	0_2	0_1	1
0_2	2	1	0_2	0_2	2
0_2	3	2	1	0_2	3
1	0_1	3	2	1	0_1
2	0_1	0_1	3	2	0_1
3	0_2	0_1	0_1	3	0_2
0_1	0_2	0_2	0_1	0_1	0_2

Here,

$$\mathbf{C}_\tau = \mathbf{C}_\delta = \mathbf{C}_\rho = \begin{bmatrix} 14\mathbf{I}_3 - 2\mathbf{J}_3 & -4\mathbf{J}_{3,2} \\ -4\mathbf{J}_{2,3} & 28\mathbf{I}_2 - 8\mathbf{J}_2 \end{bmatrix}$$

with

$$\mathrm{V}_{t1} = \mathrm{V}_{t2} = \mathrm{V}_{t3} = 0.1428\sigma^2$$

$$V_{tc1} = V_{tc2} = V_{tc3} = 0.1071\sigma^2.$$

Method 3.3. Consider a Balanced Incomplete Block (BIB) design in v' treatments replicated r' times arranged in b' blocks each of size k' such that each pair of treatments appear together in λ' blocks. Augment p controls once in each block of the BIB design such that $(k' + p)$ is a prime number. From each block, develop a NBB design as given in Method 3.1 (Azais et al., 1993), with parameters $v = k' + p$, $b = k' + p - 1$, $r = k' + p - 1$, $k = k' + p$, $\lambda = 1$. Juxtapose the blocks of all NBB designs obtained from each block contents one below other and make it circular by adding border plots. Resulting design is a totally balanced NBBPB with parameters $v_t = v'$, $v_c = p$, $b^* = b'(k' + p - 1)$, $r_t = r'(k' + p - 1)$, $r_c = b'(k' + p - 1)$, $k^* = (k + p)$, $\lambda_1 = \lambda'$, and $\lambda_2 = r'$.

Example 3.3. Consider a BIB design with parameter $v' = 4$, $b' = 6$, $r' = 3$, $k' = 2$ and $\lambda' = 1$ and augment $p = 3$ control treatments (0_1, 0_2 and 0_3) as follows:

$$\begin{array}{ccccc} 1 & 2 & 0_1 & 0_2 & 0_3 \\ 1 & 3 & 0_1 & 0_2 & 0_3 \\ 1 & 4 & 0_1 & 0_2 & 0_3 \\ 2 & 3 & 0_1 & 0_2 & 0_3 \\ 2 & 4 & 0_1 & 0_2 & 0_3 \\ 3 & 4 & 0_1 & 0_2 & 0_3 \end{array}.$$

From each block, a NBB design of parameters $v = 5$, $b = 4$, $r = 4$, $k = 5$, $\lambda = 1$ is obtained. Juxtaposing these NBB designs one below other and making it circular, the resulting design is NBBPB with parameters $v_t = 4$, $v_c = 3$, $b^* = 24$, $r_t = 12$, $r_c = 24$, $k^* = 5$, $\lambda_1 = 1$, and $\lambda_2 = 3$.

$$\begin{array}{c|ccccc|c} 0_3 & 1 & 2 & 0_1 & 0_2 & 0_3 & 1 \\ 0_2 & 1 & 0_1 & 0_3 & 2 & 0_2 & 1 \\ 0_1 & 1 & 0_2 & 2 & 0_3 & 0_1 & 1 \\ 2 & 1 & 0_3 & 0_2 & 0_1 & 2 & 1 \\ 0_3 & 1 & 3 & 0_1 & 0_2 & 0_3 & 1 \\ 0_2 & 1 & 0_1 & 0_3 & 3 & 0_2 & 1 \\ 0_1 & 1 & 0_2 & 3 & 0_3 & 0_1 & 1 \\ 3 & 1 & 0_3 & 0_2 & 0_1 & 3 & 1 \\ 0_3 & 1 & 4 & 0_1 & 0_2 & 0_3 & 1 \\ 0_2 & 1 & 0_1 & 0_3 & 4 & 0_2 & 1 \\ 5 & 1 & 0_2 & 4 & 0_3 & 0_1 & 1 \\ 4 & 1 & 0_3 & 0_2 & 0_1 & 4 & 1 \\ 0_3 & 2 & 3 & 0_1 & 0_2 & 0_3 & 2 \\ 0_2 & 2 & 0_1 & 0_3 & 3 & 0_2 & 2 \\ 0_1 & 2 & 0_2 & 3 & 0_3 & 0_1 & 2 \\ 3 & 2 & 0_3 & 0_2 & 0_1 & 3 & 2 \\ 0_3 & 2 & 4 & 0_1 & 0_2 & 0_3 & 2 \\ 0_2 & 2 & 0_1 & 0_3 & 4 & 0_2 & 2 \\ 0_1 & 2 & 0_2 & 4 & 0_3 & 0_1 & 2 \\ 4 & 2 & 0_3 & 0_2 & 0_1 & 4 & 2 \\ 0_3 & 3 & 4 & 0_1 & 0_2 & 0_3 & 3 \\ 0_2 & 3 & 0_1 & 0_3 & 4 & 0_2 & 3 \\ 0_1 & 3 & 0_2 & 4 & 0_3 & 0_1 & 3 \\ 4 & 3 & 0_3 & 0_2 & 0_1 & 4 & 3 \end{array}$$

For this design,

$$\mathbf{C}_\tau = \mathbf{C}_\delta = \mathbf{C}_\rho = \begin{bmatrix} 8.68\mathbf{I}_4 - 0.67\mathbf{J}_4 & -2\mathbf{J}_{4,3} \\ -2\mathbf{J}_{3,4} & 20\mathbf{I}_3 - 4\mathbf{J}_3 \end{bmatrix}$$

with

$$\mathrm{V}_{t1} = \mathrm{V}_{t2} = \mathrm{V}_{t3} = 0.2307\sigma^2 \quad \text{and}$$
$$\mathrm{V}_{tc1} = \mathrm{V}_{tc2} = \mathrm{V}_{tc3} = 0.1615\sigma^2.$$

It is thus concluded that all the NBBPB designs for comparing two disjoint sets of treatments are totally balanced and contrasts pertaining to test treatment vs. control treatment are estimated with more precision.

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