

Row-column designs balanced for non-directional neighbour effects

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Abstract. In this paper, a class of row-column design incorporating neighbour effects has been studied. A row-column design is said to be neighbour balanced if every treatment has all other treatments appearing as neighbour(s) constant number of times. We have considered here row-column designs with non-directional neighbour effects and method of constructing these designs has been developed. The characterization properties of the designs have also been studied and the designs are found to be variance balanced for the estimation of direct and neighbour effects of treatments.

Keywords: Neighbour effects, neighbour balanced row-column design, circular design, variance balanced, partially variance balanced

1. Introduction

The experimental units in a field trial are generally isolated from one another so that a treatment applied to one plot does not affect the other plots. This is achieved by introducing border rows or guard plots. But there are situations, where the blocks or groups have units which cannot be sufficiently isolated from each other. For example, the branches of a tree may form plots while the tree serves as a block. In varietal trials, the yield of a variety may be depressed by more aggressive neighbouring varieties. In fungicidal trials, an unsprayed plot provides a source of spores which can infect neighbouring treated plots. Such experiments exhibit neighbour effects, because the response from a given plot is affected by the treatments on neighbouring plots besides the treatment applied to that plot. In order to compare the effects of treatments in this situation, it is important to ensure that no treatment is unduly disadvantaged by its neighbour. Neighbour balance is considered a desirable property for an experiment to possess in situations where experimental units are influenced by their neighbours. Thus, neighbour balanced designs, wherein the allocation of treatments is such that every treatment occurs equally often with every other treatment as neighbour(s), are used for these situations. These designs permit the estimation of direct and neighbour effect(s) of treatments.

Azais et al. [1] obtained series of designs that are balanced in $v - 1$ blocks of size v and v blocks of size $v - 1$ and developed program that generates these designs. Bailey [2] studied designs for one-sided neighbour effects and have presented table of designs with different block sizes. Druilhet [4] studied optimality of circular neighbour-balanced block designs when neighbour effects are present in the model with special attention to designs with $v - 1$ blocks of length v and designs with v blocks of length $v - 1$ of Azais et al. [1]. Tomar et al. [11] obtained some methods of constructing totally balanced incomplete block designs for competition effects. Jaggi et al. [7] obtained some methods of constructing partially balanced block designs for neighbouring competition effects. Pateria et al. [9] proposed a series of incomplete non-circular block designs for competition effects. Jaggi et al. [8] studied optimal complete block designs for neighbouring competition effects. Pateria et al. [10] obtained some series of self – neighboured block designs that are strongly balanced for neighbour effects.

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It is seen that most of the work on designs with neighbour effects is concentrated under block design setup. There may be situations when the heterogeneity present in the experimental material is in two directions and the treatments applied to a unit may be affected by treatments in neighbouring units. The practical application of these designs is in polycross designs [12] for seed production wherein each genotype has every other genotype as its immediate neighbour once in the North, South, East and West positions. Freeman [6] has given some row-column designs balanced for neighbours with and without border plots. Federer and Basford [5] have given three methods of constructing balanced nearest neighbour row-column or competition effect designs. Chan and Eccleston [3] have given an algorithm which generates neighbour balanced row–column Designs. However, the designs obtained are found to be only combinatorially balanced.

We have considered here the row-column model with non-directional neighbour effects from adjacent sides. The experimental setup and some definitions related to row-column designs are described in Section 2. We consider here row-column design with border plots on all sides. The treatments applied on the border plots are taken in such a way so as to achieve balance with respect to neighbours. It may be noted that the border plots are considered for neighbour effects and the observations are not taken from these plots. For example, in a polycross design the plants in the interior plots are used as both seed parents and pollinators but plants in the border plots are used as just pollinators. The information matrices under this setup for estimating the direct and neighbour effects of treatments have also been derived. Method of constructing row-column designs balanced for neighbours has been given in Section 3 and its characterization properties have been investigated, followed by conclusions in Section 4.

2. Experimental setup

Let v be the number of treatments arranged in p rows and q columns and $y_{ij(m)}$ be response from the experimental unit occurring in the i^{th} row and the j^{th} column to which m^{th} treatment is applied. Broadly two types of model have been considered.

2.1. Row-column model with equal neighbour effects from all four sides

Assuming that units are affected by the adjacent neighbouring units from four sides (left, right, top and bottom) and the neighbour effects from all the directions are same, the following is the row-column model with non-directional neighbours:

$$y_{ij(m)} = \mu + \tau_{[i,j](m)} + \delta_{[i-1,j](m)} + \delta_{[i+1,j](m)} + \delta_{[i,j-1](m)} + \delta_{[i,j+1](m)} + \alpha_i + \beta_j + e_{ij(m)} \quad (1)$$

where, $\tau_{[i,j](m)}$ is the direct effect of m^{th} treatment, $\delta_{[i-1,j](m)}$, $\delta_{[i+1,j](m)}$, $\delta_{[i,j-1](m)}$ and $\delta_{[i,j+1](m)}$ are the neighbour effects (assumed to be same) due to the treatment applied in the adjacent top plot of i^{th} row, adjacent bottom plot of i^{th} row, adjacent left plot of j^{th} column and adjacent right plot of j^{th} column respectively, α_i is the i^{th} row effect, β_j is the j^{th} column effect and $e_{ij(m)}$ is the random errors assumed to be independent with $E(e_{ij(m)}) = 0$ and constant variance σ^2 .

Model given in Eq. (1) can be written in matrix form as

$$\mathbf{Y} = \mu \mathbf{1} + \Delta' \boldsymbol{\tau} + \Delta'_1 \boldsymbol{\delta} + \mathbf{D}'_1 \boldsymbol{\alpha} + \mathbf{D}'_2 \boldsymbol{\beta} + \mathbf{e}, \quad (2)$$

where \mathbf{Y} is a $n \times 1$ vector of responses from n units, μ is the grand mean, $\mathbf{1}$ is a $n \times 1$ vector of ones, $\boldsymbol{\tau}$ and $\boldsymbol{\delta}$ are $v \times 1$ vector of direct and neighbour effects respectively, $\boldsymbol{\alpha}$ and $\boldsymbol{\beta}$ are $p \times 1$ and $q \times 1$ vectors of row and column effects respectively. Δ' and Δ'_1 are $n \times v$ matrix of observations versus direct treatments and observations versus neighbour treatments respectively, \mathbf{D}'_1 is a $n \times p$ incidence matrix of observations versus rows and \mathbf{D}'_2 is a $n \times q$ incidence matrix of observations versus columns. \mathbf{e} is a $n \times 1$ vector of errors.

Further, let $\Delta\Delta'_1 = \mathbf{M}$, a $v \times v$ incidence matrix of direct treatments versus neighbour treatments; $\Delta\mathbf{D}'_1 = \mathbf{N}_1$, a $v \times p$ incidence matrix of direct treatments versus rows; $\Delta\mathbf{D}'_2 = \mathbf{N}_2$, a $v \times q$ incidence matrix of direct treatments versus columns; $\Delta_1\mathbf{D}'_1 = \mathbf{N}_3$, $v \times p$ incidence matrix of neighbour treatments versus rows; $\Delta_1\mathbf{D}'_2 = \mathbf{N}_4$, a $v \times q$ incidence matrix of neighbour treatments versus columns; $\mathbf{D}_1\mathbf{D}'_2 = \mathbf{W}$, a $v \times q$ incidence matrix of rows versus columns and $\Delta_1\Delta'_1 = \mathbf{G}$, a $v \times v$ neighbour matrix.

Let $\mathbf{r} = (r_1, r_2, \dots, r_v)'$ be the $v \times 1$ replication vector of direct treatments with r_s ($s = 1, 2, \dots, v$) being the number of times the s^{th} treatment appears in the design and $\mathbf{r}_1 = (r_{11}, r_{12}, \dots, r_{1v})'$ be the $v \times 1$ replication vector of the neighbour treatments with r_{1s} being the number of times the treatments in the design has s^{th} treatment as neighbour. $\Delta\Delta' = \mathbf{R}_\tau = \text{diag}(r_1, r_2, \dots, r_v)$ and $\mathbf{R}_\delta = \text{diag}(r_{11}, r_{12}, \dots, r_{1v})$. $\mathbf{k} = (k_1, k_2, \dots, k_p)'$ be the $p \times 1$ vector of row sizes, $\mathbf{D}_1\mathbf{D}'_1 = \mathbf{K} = \text{diag}(k_1, k_2, \dots, k_p)$ = diagonal matrix of row sizes, $\mathbf{h} = (h_1, h_2, \dots, h_q)'$ be the $q \times 1$ vector of column sizes, $\mathbf{D}_2\mathbf{D}'_2 = \mathbf{H} = \text{diag}(h_1, h_2, \dots, h_q)$ = diagonal matrix of column sizes.

The $2v \times 2v$ symmetric, non-negative definite information matrix for estimating the direct and neighbour effects with zero row and column sums is obtained as below:

$$\mathbf{C} = \begin{bmatrix} \mathbf{C}_{11} & \mathbf{C}_{12} \\ \mathbf{C}_{21} & \mathbf{C}_{22} \end{bmatrix}$$

where

$$\mathbf{C}_{11} = \mathbf{R}_\tau - (\mathbf{N}_1\mathbf{K}^{-1}\mathbf{N}'_1 + \mathbf{N}_1\mathbf{K}^{-1}\mathbf{W}\mathbf{E}^{-}\mathbf{W}'\mathbf{K}^{-1}\mathbf{N}'_1 - \mathbf{N}_2\mathbf{E}^{-}\mathbf{W}'\mathbf{K}^{-1}\mathbf{N}'_1 - \mathbf{N}_1\mathbf{K}^{-1}\mathbf{W}\mathbf{E}^{-}\mathbf{N}'_3 + \mathbf{N}_2\mathbf{E}^{-}\mathbf{N}'_3)$$

$$\mathbf{C}_{12} = \mathbf{M} - (\mathbf{N}_1\mathbf{K}^{-1}\mathbf{N}'_2 + \mathbf{N}_1\mathbf{K}^{-1}\mathbf{W}\mathbf{E}^{-}\mathbf{W}'\mathbf{K}^{-1}\mathbf{N}'_2 - \mathbf{N}_2\mathbf{E}^{-}\mathbf{W}'\mathbf{K}^{-1}\mathbf{N}'_2 - \mathbf{N}_1\mathbf{K}^{-1}\mathbf{W}\mathbf{E}^{-}\mathbf{N}'_4 + \mathbf{N}_2\mathbf{E}^{-}\mathbf{N}'_4)$$

$$\mathbf{C}_{22} = \mathbf{G} - (\mathbf{N}_3\mathbf{K}^{-1}\mathbf{N}'_2 + \mathbf{N}_3\mathbf{K}^{-1}\mathbf{W}\mathbf{E}^{-}\mathbf{W}'\mathbf{K}^{-1}\mathbf{N}'_2 - \mathbf{N}_4\mathbf{E}^{-}\mathbf{W}'\mathbf{K}^{-1}\mathbf{N}'_2 - \mathbf{N}_3\mathbf{K}^{-1}\mathbf{W}\mathbf{E}^{-}\mathbf{N}'_4 + \mathbf{N}_4\mathbf{E}^{-}\mathbf{N}'_4)$$

Here, $E = H - W'K^{-1}W$ and $\mathbf{C}_{21} = \mathbf{C}_{12}$. The information matrix for estimating the direct effects can be obtained as follows:

$$\mathbf{C}_\tau = \mathbf{C}_{11} - \mathbf{C}_{12}\mathbf{C}_{22}^{-1}\mathbf{C}_{21}$$

where \mathbf{C}_{22}^{-1} is a generalized inverse of \mathbf{C}_{22} .

2.2. Row-column model with equal row-neighbour and equal column-neighbour effects

Here it is assumed that the effects from left and right neighbours in a row is same and from the top and bottom neighbours in a column is also same. Hence the model considered is of the form

$$y_{ij(m)} = \mu + \tau_{[i,j](m)} + \delta_{[i,j-1](m)} + \delta_{[i,j+1](m)} + \gamma_{[i-1,j](m)} + \gamma_{[i+1,j](m)} + \alpha_i + \beta_j + e_{ij(m)} \quad (3)$$

δ is the neighbour effect due to the treatment applied in the adjacent left or right plot in a row, called row-neighbour effect and γ is the neighbour effect due to the treatment applied in the adjacent top or bottom plot, called column-neighbour effect. This model can be represented as

$$\mathbf{Y} = \mu \mathbf{1} + \Delta' \boldsymbol{\tau} + \Delta'_1 \boldsymbol{\delta} + \Delta'_2 \boldsymbol{\gamma} + \mathbf{D}'_1 \boldsymbol{\alpha} + \mathbf{D}'_2 \boldsymbol{\beta} + \mathbf{e}, \quad (4)$$

where Δ'_1 is a $n \times v$ matrix of observations versus row neighbour treatment, δ is $v \times 1$ vector of row neighbour effects, Δ'_2 is a $n \times v$ matrix of observations versus column neighbour treatment, γ is $v \times 1$ vector of column neighbour effects. Now, the $n \times (3v + p + q + 1)$ design matrix \mathbf{X} can be partitioned into that of parameters of interest (\mathbf{X}_1) and nuisance parameters (\mathbf{X}_2).

$$\mathbf{X}_1 = [\Delta' \ \Delta'_1 \ \Delta'_2] \text{ and } \mathbf{X}_2 = [\mathbf{1} \ \mathbf{D}'_1 \ \mathbf{D}'_2]$$

$$\mathbf{X}'_1 \mathbf{X}_1 = \begin{bmatrix} \Delta\Delta' & \Delta\Delta'_1 & \Delta\Delta'_2 \\ \Delta_1\Delta' & \Delta_1\Delta'_1 & \Delta_1\Delta'_2 \\ \Delta_2\Delta' & \Delta_2\Delta'_1 & \Delta_2\Delta'_2 \end{bmatrix} = \begin{bmatrix} \mathbf{R}_\tau & \mathbf{M} & \mathbf{M}_1 \\ \mathbf{M}' & \mathbf{G} & \mathbf{M}_2 \\ \mathbf{M}'_1 & \mathbf{M}'_2 & \mathbf{G}_1 \end{bmatrix}$$

$$\mathbf{X}'_1 \mathbf{X}_2 = \begin{bmatrix} \Delta\mathbf{1} & \Delta\mathbf{D}'_1 & \Delta\mathbf{D}'_2 \\ \Delta_1\mathbf{1} & \Delta_1\mathbf{D}'_1 & \Delta_1\mathbf{D}'_2 \\ \Delta_2\mathbf{1} & \Delta_2\mathbf{D}'_1 & \Delta_2\mathbf{D}'_2 \end{bmatrix} = \begin{bmatrix} \mathbf{r} & \mathbf{N}_1 & \mathbf{N}_2 \\ \mathbf{r}_1 & \mathbf{N}_3 & \mathbf{N}_4 \\ \mathbf{r}_2 & \mathbf{N}_5 & \mathbf{N}_6 \end{bmatrix}$$

$$\mathbf{X}'_2 \mathbf{X}_2 = \begin{bmatrix} \mathbf{1}'\mathbf{1} & \mathbf{1}'\mathbf{D}'_1 & \mathbf{1}'\mathbf{D}'_2 \\ \mathbf{D}_1\mathbf{1} & \mathbf{D}_1\mathbf{D}'_1 & \mathbf{D}_1\mathbf{D}'_2 \\ \mathbf{D}_2\mathbf{1} & \mathbf{D}_2\mathbf{D}'_1 & \mathbf{D}_2\mathbf{D}'_2 \end{bmatrix} = \begin{bmatrix} \mathbf{n} & \mathbf{k}' & \mathbf{h}' \\ \mathbf{k} & \mathbf{K} & \mathbf{W} \\ \mathbf{h} & \mathbf{W}' & \mathbf{H} \end{bmatrix}$$

Let $\mathbf{r} = (r_1, r_2, \dots, r_v)'$ is the $v \times 1$ replication vector of direct treatments with r_s ($s = 1, 2, \dots, v$) being the number of times the s^{th} treatment appears in the design, $\mathbf{r}_1 = (r_{11}, r_{12}, \dots, r_{1v})'$ be the $v \times 1$ replication vector of the neighbour treatments with r_{1s} being the number of times the treatments in the design has s^{th} treatment as row-neighbour and $\mathbf{r}_2 = (r_{21}, r_{22}, \dots, r_{2v})'$ be the $v \times 1$ replication vector of the neighbour treatments with r_{2s} being the number of times the treatments in the design has s^{th} treatment as column-neighbour.

The $3v \times 3v$ joint information matrix for estimating the direct effects, left-neighbour effects and right-neighbour effects of treatments is as follows:

$$\mathbf{C} = \begin{bmatrix} \mathbf{C}_{11} & \mathbf{C}_{12} & \mathbf{C}_{13} \\ \mathbf{C}_{21} & \mathbf{C}_{22} & \mathbf{C}_{23} \\ \mathbf{C}_{31} & \mathbf{C}_{32} & \mathbf{C}_{33} \end{bmatrix}$$

where

$$\mathbf{C}_{11} = \mathbf{R}_\tau - (\mathbf{N}_1 \mathbf{K}^{-1} \mathbf{N}'_1 + \mathbf{N}_1 \mathbf{K}^{-1} \mathbf{W} \mathbf{E}^{-1} \mathbf{W}' \mathbf{K}^{-1} \mathbf{N}'_1 - \mathbf{N}_2 \mathbf{E}^{-1} \mathbf{W}' \mathbf{K}^{-1} \mathbf{N}'_1 - \mathbf{N}_1 \mathbf{K}^{-1} \mathbf{W} \mathbf{E}^{-1} \mathbf{N}'_2 + \mathbf{N}_2 \mathbf{E}^{-1} \mathbf{N}'_2)$$

$$\mathbf{C}_{12} = \mathbf{M} - (\mathbf{N}_1 \mathbf{K}^{-1} \mathbf{N}'_3 + \mathbf{N}_1 \mathbf{K}^{-1} \mathbf{W} \mathbf{E}^{-1} \mathbf{W}' \mathbf{K}^{-1} \mathbf{N}'_3 - \mathbf{N}_2 \mathbf{E}^{-1} \mathbf{W}' \mathbf{K}^{-1} \mathbf{N}'_3 - \mathbf{N}_1 \mathbf{K}^{-1} \mathbf{W} \mathbf{E}^{-1} \mathbf{N}'_4 + \mathbf{N}_2 \mathbf{E}^{-1} \mathbf{N}'_4)$$

$$\mathbf{C}_{13} = \mathbf{M}_1 - (\mathbf{N}_1 \mathbf{K}^{-1} \mathbf{N}'_5 + \mathbf{N}_1 \mathbf{K}^{-1} \mathbf{W} \mathbf{E}^{-1} \mathbf{W}' \mathbf{K}^{-1} \mathbf{N}'_5 - \mathbf{N}_2 \mathbf{E}^{-1} \mathbf{W}' \mathbf{K}^{-1} \mathbf{N}'_5 - \mathbf{N}_1 \mathbf{K}^{-1} \mathbf{W} \mathbf{E}^{-1} \mathbf{N}'_6 + \mathbf{N}_2 \mathbf{E}^{-1} \mathbf{N}'_6)$$

$$\mathbf{C}_{22} = \mathbf{G} - (\mathbf{N}_3 \mathbf{K}^{-1} \mathbf{N}'_3 + \mathbf{N}_3 \mathbf{K}^{-1} \mathbf{W} \mathbf{E}^{-1} \mathbf{W}' \mathbf{K}^{-1} \mathbf{N}'_3 - \mathbf{N}_4 \mathbf{E}^{-1} \mathbf{W}' \mathbf{K}^{-1} \mathbf{N}'_3 - \mathbf{N}_3 \mathbf{K}^{-1} \mathbf{W} \mathbf{E}^{-1} \mathbf{N}'_4 + \mathbf{N}_4 \mathbf{E}^{-1} \mathbf{N}'_4)$$

$$\mathbf{C}_{23} = \mathbf{M}_2 - (\mathbf{N}_3 \mathbf{K}^{-1} \mathbf{N}'_5 + \mathbf{N}_3 \mathbf{K}^{-1} \mathbf{W} \mathbf{E}^{-1} \mathbf{W}' \mathbf{K}^{-1} \mathbf{N}'_5 - \mathbf{N}_4 \mathbf{E}^{-1} \mathbf{W}' \mathbf{K}^{-1} \mathbf{N}'_5 - \mathbf{N}_3 \mathbf{K}^{-1} \mathbf{W} \mathbf{E}^{-1} \mathbf{N}'_6 + \mathbf{N}_4 \mathbf{E}^{-1} \mathbf{N}'_6)$$

$$\mathbf{C}_{33} = \mathbf{G}_1 - (\mathbf{N}_5 \mathbf{K}^{-1} \mathbf{N}'_5 + \mathbf{N}_5 \mathbf{K}^{-1} \mathbf{W} \mathbf{E}^{-1} \mathbf{W}' \mathbf{K}^{-1} \mathbf{N}'_5 - \mathbf{N}_6 \mathbf{E}^{-1} \mathbf{W}' \mathbf{K}^{-1} \mathbf{N}'_5 - \mathbf{N}_5 \mathbf{K}^{-1} \mathbf{W} \mathbf{E}^{-1} \mathbf{N}'_6 + \mathbf{N}_6 \mathbf{E}^{-1} \mathbf{N}'_6)$$

Also $\mathbf{C}_{21} = \mathbf{C}_{12}$, $\mathbf{C}_{31} = \mathbf{C}_{13}$ and $\mathbf{C}_{32} = \mathbf{C}_{23}$. Therefore, the information matrix for estimating the direct effect of treatments can be obtained as

$$\mathbf{C}_\tau = \mathbf{C}_{11} - [\mathbf{C}_{12} \quad \mathbf{C}_{13}] \begin{bmatrix} \mathbf{C}_{22} & \mathbf{C}_{23} \\ \mathbf{C}_{32} & \mathbf{C}_{33} \end{bmatrix}^- \begin{bmatrix} \mathbf{C}_{21} \\ \mathbf{C}_{31} \end{bmatrix}$$

We now give some definitions associated with row-column designs with neighbour effects.

2.3. Definitions

Definition 1. A row-column design under Model 1 with four-sided neighbour effects is said to be balanced if every treatment has every other treatment appearing as neighbours constant number of times (say μ_1 times) and strongly neighbour balanced if same treatment also appear as neighbours constant number of times (say μ_2 times). μ_1 may be equal to μ_2 .

Definition 2. A row-column design under Model 3 with four-sided neighbour effects is said to be balanced if every treatment has every other treatment appearing as neighbours in rows constant number of times (say μ_{1r} times), in columns constant number of times (say μ_{1c} times) and strongly neighbour balanced if same treatment also appears as neighbours constant number of times in rows (say μ_{2r} times) and constant number of times in columns (say μ_{2c} times).

Definition 3. A neighbour balanced row-column (NBRC) design with four-sided neighbour effects is said to be circular if the treatments in the left border of the rows are the same as the treatments in the right-end inner plots, the treatments in the right border of the rows are the same as the treatments in the left-end inner plots, the treatments in the top border of the columns are the same as the treatments in the bottom-end inner plots and the treatments in the bottom border of the columns are the same as the treatment in the top-end inner plot.

Definition 4. A row-column design with four sided neighbour effects is said to be variance balanced for direct effects, if all the contrasts pertaining to the direct effects of treatments are estimated with the same variance.

3. Method of construction

Let v be a prime number. Obtain a basic array of $v - 1$ columns each of size v from the following initial sequence for values of $\omega = 0, 1, \dots, v - 2$ by taking modulo v :

$$\begin{aligned} &v \\ &\omega + 1 \\ &2(\omega + 1) \\ &\cdot \\ &\cdot \\ &\cdot \\ &(v - 1)(\omega + 1) \end{aligned}$$

Develop the columns of this array cyclically mod v to get v sets of $v - 1$ columns each. Making design circular by adding border plots results in a NBRC design for v treatments in v rows and $v(v - 1)$ columns.

Based on Model 1, for the above class of designs, $\mu_1 = 2(2v - 1)$ and $\mu_2 = 2(v - 1)$. Further, the various incidence matrices obtained are as follows:

$$\begin{aligned} \mathbf{N}_1 &= (v - 1)\mathbf{J}_v; \mathbf{N}_2 = \mathbf{J}_{v \times v(v-1)}; \mathbf{N}_3 = 4(v - 1)\mathbf{J}_v; \\ \mathbf{N}_4 &= 4\mathbf{J}_{v \times v(v-1)}; \mathbf{M} = 2[(2v - 1)\mathbf{J}_v - v\mathbf{I}_v]; \mathbf{W} = \mathbf{J}_{v \times v(v-1)} \text{ and} \\ \mathbf{G} &= 2[v(2v - 3)\mathbf{I}_v + (6v - 5)\mathbf{J}_v]. \end{aligned}$$

Also, $\mathbf{R}_\tau = \mathbf{K} = v(v - 1)\mathbf{I}_v$; $\mathbf{H} = v\mathbf{I}_{v(v-1)}$. \mathbf{I} is the identity matrix and \mathbf{J} is the matrix of unities.

The joint information matrix for estimating the direct and neighbour effects is worked out as follows:

$$\mathbf{C}_{\tau\delta} = \begin{pmatrix} v(v - 1)(\mathbf{I} - \frac{1}{v}\mathbf{J}) & -2v(\mathbf{I} - \frac{1}{v}\mathbf{J}) \\ -2v(\mathbf{I} - \frac{1}{v}\mathbf{J}) & 2v(2v - 3)(\mathbf{I} - \frac{1}{v}\mathbf{J}) \end{pmatrix}$$

Further, the information matrix for estimating the direct effects is

$$\mathbf{C}_\tau = \frac{v(2v^2 - 5v + 1)}{2v - 3} \left(\mathbf{I} - \frac{1}{v}\mathbf{J} \right)$$

It can be easily seen that variance of estimated elementary contrast pertaining to direct effects of treatments is

$$V(\hat{\tau}_i - \hat{\tau}_{i'}) = \frac{2(2v - 3)}{v(2v^2 - 5v + 1)} \sigma^2, \quad i \neq i' = 1, 2, \dots, v$$

Thus the designs obtained are variance balanced for estimating the contrasts pertaining to direct effects of treatments.

The information matrix for estimating the neighbour effects is obtained as

$$\mathbf{C}_\delta = \frac{2v(2v^2 - 5v + 1)}{v - 1} \left(\mathbf{I} - \frac{1}{v}\mathbf{J} \right)$$

Hence, the design is also variance balanced for estimating the contrasts pertaining to neighbour effects of treatments.

Example 1. Let $v = 5$. The basic array of order 5×4 is obtained as given by taking $\omega = 0, 1, 2, 3$.

$\omega = 0$	$\omega = 1$	$\omega = 2$	$\omega = 3$
5	5	5	5
1	2	3	4
2	4	1	3
3	1	4	2
4	3	2	1

Developing the columns of this array cyclically mod 5 and making it circular as given in Definition 3, the following NBRC design in 5 rows and 20 columns is obtained:

4	3	2	1	5	4	3	2	1	5	4	3	2	1	5	4	3	2	1	5
5	5	5	5	1	1	1	1	2	2	2	2	3	3	3	3	4	4	4	4
3	1	2	3	4	2	3	4	5	3	4	5	1	4	5	1	2	5	1	2
2	2	4	1	3	3	5	2	4	4	1	3	5	5	2	4	1	1	3	5
1	3	1	4	2	4	2	5	3	5	3	1	4	1	4	2	5	2	5	3
5	4	3	2	1	5	4	3	2	1	5	4	3	2	1	5	4	3	2	1
5	5	5	5	1	1	1	1	2	2	2	2	3	3	3	3	4	4	4	4

Here, $\mu_1 = 18$ and $\mu_2 = 8$. The various incidence matrices for this design are as follows:

$$\mathbf{N}_1 = 4\mathbf{J}_4; \mathbf{N}_2 = \mathbf{J}_{4 \times 20}; \mathbf{N}_3 = 16\mathbf{J}_5; \mathbf{N}_4 = 4\mathbf{J}_{5 \times 20}; \mathbf{M} = 18\mathbf{J}_5 - 10\mathbf{I}_5;$$

$$\mathbf{W} = \mathbf{J}_{5 \times 20} \text{ and } \mathbf{G} = 70\mathbf{I}_5 + 50\mathbf{J}_5.$$

The corresponding \mathbf{C} -matrices are obtained as

$$\mathbf{C}_\tau = 14.84\mathbf{I} - 3.71\mathbf{J} \text{ and}$$

$$\mathbf{C}_\delta = 52\mathbf{I} - 13\mathbf{J}.$$

Here, $V(\hat{\tau}_i - \hat{\tau}_{i'}) = 0.1076\sigma^2$ and $V(\hat{\delta}_i - \hat{\delta}_{i'}) = 0.0307\sigma^2$, $i \neq i' = 1, 2, \dots, 5$. Thus, the design is variance balanced for estimating direct effects as well as the neighbour effects.

Remark: Deleting any $t (= 1, 2, \dots, p-2)$ rows from the design obtained above results in an incomplete NBRC design for v treatments in $v-t$ rows and $v(v-1)$ columns which is a partially variance balanced design following circular association scheme. Any treatment i ($i = 1, 2, \dots, v = 2t+1$) is said to be u^{th} $\{u = 1, 2, \dots, t\}$ associate of i' if $i' = i \pm u, mod(v)$. As a result the number of u^{th} associates of any treatment is 2.

Example 2. The following design obtained after deleting third row is a two-associate class incomplete NBRC for 5 treatments in 4 rows and 20 columns:

4	3	2	1	5	4	3	2	1	5	4	3	2	1	5	4	3	2	1	5
5	5	5	5	1	1	1	1	2	2	2	2	3	3	3	3	4	4	4	4
3	1	2	3	4	2	3	4	5	3	4	5	1	4	5	1	2	5	1	2
1	3	1	4	2	4	2	5	3	5	3	1	4	1	4	2	5	2	5	3
5	4	3	2	1	5	4	3	2	1	5	4	3	2	1	5	4	3	2	1
5	5	5	5	1	1	1	1	2	2	2	2	3	3	3	3	4	4	4	4

The information matrices for estimating the direct and neighbour effects are obtained as follows:

$$\mathbf{C}_\tau = \begin{pmatrix} 10.46 & -2.16 & -3.07 & -3.07 & -2.16 \\ -2.16 & 10.46 & -2.16 & -3.07 & -3.07 \\ -3.07 & -2.16 & 10.46 & -2.16 & -3.07 \\ -3.07 & -3.07 & -2.16 & 10.46 & -2.16 \\ -2.16 & -3.07 & -3.07 & -2.16 & 10.46 \end{pmatrix} \mathbf{C}_\delta = \begin{pmatrix} 41.04 & -14.10 & -6.42 & -6.42 & -14.10 \\ -14.10 & 41.04 & -14.10 & -6.42 & -6.42 \\ -6.42 & -14.10 & 41.04 & -14.10 & -6.42 \\ -6.42 & -6.42 & -14.10 & 41.04 & -14.10 \\ -14.10 & -6.42 & -6.42 & -14.10 & 41.04 \end{pmatrix}$$

Here it can be observed that treatment 1 has treatment number 2 and 5 as first associates and 3 and 4 as second associates.

A SAS code (using PROC IML) has been developed for generation of NBRC designs for any given v (prime) and to obtain the information matrices for estimating direct and neighbour effects of treatments. The code has been given in the Annexure.

Based on Model 3, for the above class of designs, $\mu_{1r} = \mu_{2r} = 2(v-1)$ and $\mu_{1c} = 2v$ and $\mu_{2c} = 0$. Further, the various incidence matrices are as follows:

$$\mathbf{N}_1 = (v-1)\mathbf{J}_v; \mathbf{N}_2 = \mathbf{J}_{v \times v(v-1)}; \mathbf{N}_3 = \mathbf{N}_5 = 2(v-1)\mathbf{J}_v; \mathbf{N}_4 = \mathbf{N}_6 = 2\mathbf{J}_{v \times v(v-1)}$$

$$\mathbf{M} = 2(v-1)\mathbf{J}_v; \mathbf{M}_1 = 2v(\mathbf{J}_v - \mathbf{I}_v); \mathbf{M}_2 = 4(v-1)\mathbf{J}_v; \mathbf{W} = \mathbf{J}_{v \times v(v-1)};$$

$$\mathbf{G} = 2v(v-1) \left[\mathbf{I}_v + \frac{1}{v} \mathbf{J}_v \right]; \mathbf{G}_1 = 2v [(v-2)\mathbf{I}_v + \mathbf{J}_v].$$

Also, $\mathbf{R}_\tau = \mathbf{K} = v(v-1)\mathbf{I}_v$ and $\mathbf{H} = v\mathbf{I}_{v(v-1)}$;

The joint information matrix for estimating direct effects, row-neighbour and column-neighbour effects is obtained as:

$$\mathbf{C}_{\tau\delta\gamma} = \begin{pmatrix} v(v-1)(\mathbf{I} - \frac{1}{v}\mathbf{J}) & \mathbf{0} & -2v(\mathbf{I} - \frac{1}{v}\mathbf{J}) \\ \mathbf{0} & 2v(v-1)(\mathbf{I} - \frac{1}{v}\mathbf{J}) & \mathbf{0} \\ -2v(\mathbf{I} - \frac{1}{v}\mathbf{J}) & \mathbf{0} & 2v(v-2)(\mathbf{I} - \frac{1}{v}\mathbf{J}) \end{pmatrix}$$

Thus, the information matrix for estimating the direct effect of treatments is obtained as

$$\mathbf{C}_\tau = \frac{v^2(v-3)}{(v-2)} \left(\mathbf{I} - \frac{1}{v}\mathbf{J} \right)$$

Similarly, the information matrices for estimating row-neighbour effects and column-neighbour effects are obtained respectively as

$$\mathbf{C}_\delta = 2v(v-1) \left(\mathbf{I} - \frac{1}{v}\mathbf{J} \right) \text{ and}$$

$$\mathbf{C}_\gamma = \frac{2v^2(v-3)}{(v-1)} \left(\mathbf{I} - \frac{1}{v}\mathbf{J} \right).$$

Example 3. For the design obtained in Example 1 for $v = 5$, under the assumption of equal row-neighbour and equal column-neighbour effects, $\mu_{1r} = \mu_{2r} = 8$ and $\mu_{1c} = 10$ and $\mu_{2c} = 0$. The information matrices for estimating the direct, row-neighbour and column-neighbour effects, are obtained as

$$\mathbf{C}_\tau = 16.66\mathbf{I} - 3.33\mathbf{J},$$

$$\mathbf{C}_\delta = 40\mathbf{I} - 8\mathbf{J} \text{ and}$$

$$\mathbf{C}_\gamma = 25\mathbf{I} - 5\mathbf{J}.$$

Hence, it can be seen that the class of designs obtained here is variance balanced.

Deleting any t ($= 1, 2, \dots, p-4$) rows from the above class of design will result in a partially variance balanced design following circular association scheme as defined earlier.

In Example 3, deleting the second row, the information matrices for estimating the direct effects, row-neighbour and column-neighbour effects are obtained as follows:

$$\mathbf{C}_\tau = \begin{pmatrix} 7.72 & -1.78 & -2.08 & -2.08 & -1.78 \\ -1.78 & 7.72 & -1.78 & -2.08 & -2.08 \\ -2.08 & -1.78 & 7.72 & -1.78 & -2.08 \\ -2.08 & -2.08 & -1.78 & 7.72 & -1.78 \\ -1.78 & -2.08 & -2.08 & -1.78 & 7.72 \end{pmatrix}, \quad \mathbf{C}_\delta = \begin{pmatrix} 25.05 & -5.76 & -6.76 & -6.76 & -5.76 \\ -5.76 & 25.05 & -5.76 & -6.76 & -6.76 \\ -6.76 & -5.76 & 25.05 & -5.76 & -6.76 \\ -6.76 & -6.76 & -5.76 & 25.05 & -5.76 \\ -5.76 & -6.76 & -6.76 & -5.76 & 25.05 \end{pmatrix}$$

$$\mathbf{C}_\gamma = \begin{pmatrix} 10.58 & -2.61 & -2.68 & -2.68 & -2.61 \\ -2.61 & 10.58 & -2.61 & -2.68 & -2.68 \\ -2.68 & -2.61 & 10.58 & -2.61 & -2.68 \\ -2.68 & -2.68 & -2.61 & 10.58 & -2.61 \\ -2.61 & -2.68 & -2.68 & -2.61 & 10.58 \end{pmatrix}$$

The design is seen to be a two-associate class incomplete NBRC design following circular association scheme for estimating all the effects under Model 3.

4. Conclusions

The class of variance balanced row-column design incorporating neighbour effects from adjacent four neighbours can be advantageously used in the situations wherein it is assumed that the effects are same irrespective of the direction of neighbours. These designs are more suited to experimental sites with variable winds during the pollination season in polycross designs. In case of scarcity of resources, these designs can also be used as incomplete row-column designs with neighbour effects by deleting the required number of rows from the designs constructed.

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Appendix

```
*****SAS code to generate design for a given v *****/
%let v = ; /* Enter the number of treatments*/
proc iml;
a0=j(&v,&v*(&v-1),0);
do i=1 to &v-1;
do j=1 to &v-2;
  a0[i+1,j+1]=mod(i*(j+1),&v);
end;
  a0[i+1,1]=i;
  a0[1,i]=&v;
end;
do k=1 to &v-1;
  a000=j(&v,&v-1,k);
do i=1 to &v;
do j=1 to &v-1;
  a0[i,(j+(k*&v-1))]=mod(a0[i,j]+a000[i,j],&v);
  if a0[i,(j+(k*&v-1))]=0 then a0[i,(j+(k*&v-1))]=&v;
end;
end;
end;
a0=a0[&v, ]//a0//a0[1, ];
a=a0[,&v*(&v-1)]||a00||a00[,1];
print a[format=3.0]; /*Design with borders*/
*****SAS code to calculate C-matrix under Model-2.1.1*****
a1=j((nrow(a)-2),ncol(a),0);
do i=2 to nrow(a)-1;
do j=1 to ncol(a);
  a1[i-1,j]=a[i,j];
end;
end;
a2=j(nrow(a),(ncol(a)-2),0);
do i=1 to nrow(a);
do j=2 to ncol(a)-1;
```

```

a2[i,j-1]=a[i,j];
end;
end;
m1=j(nrow(a0)*ncol(a0),1,1);
dir=j(nrow(a0)*ncol(a0),max(a0),0);
k=1;
do i=1 to nrow(a0);
do j=1 to ncol(a0);
  if a0[i,j]>0 then dir[k,a0[i,j]]=1;
  k=k+1;
end;
end;
l_neig = j(nrow(a0)*ncol(a0),max(a0),0);
k=1;
do i = 1 to nrow(a1);
do j = 2 to ncol(a1)-1;
  if a1[i,j]>0 then l_neig[k,a1[i,j-1]]=l_neig[k,a1[i,j-1]]+1;
  k=k+1;
end;
end;
r_neig = j(nrow(a0)*ncol(a0),max(a0),0);
k=1;
do i = 1 to nrow(a1);
do j = 2 to ncol(a1)-1;
  if a1[i,j]>0 then r_neig[k,a1[i,j+1]]=r_neig[k,a1[i,j+1]]+1;
  k=k+1;
end;
end;
t_neig = j(nrow(a0)*ncol(a0),max(a0),0);
k=1;
do i = 2 to nrow(a2)-1;
do j = 1 to ncol(a2);
  if a2[i,j]>0 then t_neig[k,a2[i-1,j]]=t_neig[k,a2[i-1,j]]+1;
  k=k+1;
end;
end;
b_neig = j(nrow(a0)*ncol(a0),max(a0),0);
k=1;
do i = 2 to nrow(a2)-1;
do j = 1 to ncol(a2);
  if a2[i,j]>0 then b_neig[k,a2[i+1,j]]=b_neig[k,a2[i+1,j]]+1;
  k=k+1;
end;
end;
neig=l_neig+r_neig+t_neig+b_neig;
r=j(nrow(a0)*ncol(a0),nrow(a0),0);
k=1;
do i=1 to nrow(a0);
do j=1 to ncol(a0);
  if a0[i,j]>0 then r[k,i]=1;
  k=k+1;
end;

```

```

end;
end;
c=j(nrow(a0)*ncol(a0),ncol(a0),0);
k=1;
do i=1 to nrow(a0);
do j=1 to ncol(a0);
  if a0[i,j]>0 then c[k,j]=1;
  k=k+1;
end;
end;
n1=dir**r;n2=dir*c;n3=neig**r;n4=neig*c;w=r*c;
m=dir*neig;r.dir=dir*dir;g=neig*neig;k=r*r;h=c*c;
e=h-w*inv(k)*w;
c11=r.dir-(n1*inv(k)*n1+n1*inv(k)*w*ginv(e)*w**inv(k)*n1 - n2*inv(e)*w**inv(k)*n1 -
n1*inv(k)*w*ginv(e)*n2+*n2*inv(e)*n2');
c12=m-(n1*inv(k)*n3+n1*inv(k)*w*ginv(e)*w**inv(k)*n3 - n2*inv(e)*w**inv(k)*n3 -
n1*inv(k)*w*ginv(e)*n4+*n2*inv(e)*n4');
c21=m-(n3*inv(k)*n1+n3*inv(k)*w*ginv(e)*w**inv(k)*n1 - n4*inv(e)*w**inv(k)*n1 -
n3*inv(k)*w*ginv(e)*n2+*n4*inv(e)*n2');
c22=g-(n3*inv(k)*n3+n3*inv(k)*w*ginv(e)*w**inv(k)*n3 - n4*inv(e)*w**inv(k)*n3 -
n3*inv(k)*w*ginv(e)*n4+*n4*inv(e)*n4');
c=(c11||c12)/(c21||c22);
c_dir=c11-(c12*inv(c22)*c21); /*C-matrix for estimating direct effect*/
c_neig=c22-(c21*inv(c11)*c12); /*C-matrix for estimating neighbour effect*/
print c_dir[format=3.2],c_neig[format=3.2];
quit;

```

References

- [1] J.M. Azais, R.A. Bailey and H. Monod, A catalogue of efficient neighbour design with border plots, *Biometrics* **49** (1993), 1252–1261.
- [2] R.A. Bailey, Designs for one-sided neighbour effects, *J Ind Soc Ag Statistics* **56**(3) (2003), 302–314.
- [3] Barbara S.P. Chan and J.A. Eccleston, On the Construction of Nearest–Neighbour Balanced Row–Column Designs, *Australian & New Zealand J of Statistics* **45**(1) (2003), 97–106.
- [4] P. Druilhet, Optimality of neighbour balanced designs, *J Statist Plann Inf* **81**(1) (1999), 141–152.
- [5] W.T. Federer and K.E. Basford, Competing effects designs and models for two-dimensional field arrangements, *Biometrics* **47**(1991), 1461–1472.
- [6] G.H. Freeman, Some two dimensional designs balanced for nearest neighbours, *J R Statist Soc, B* **41**(1) (1979), 88–95.
- [7] Seema Jaggi, V.K. Gupta and J. Ashraf, On block designs partially balanced for neighbouring competition effects, *J Ind Statist Assoc* **44**(1) (2006), 27–41.
- [8] Seema Jaggi, Cini Varghese and V.K. Gupta, Optimal block designs for neighbouring competition effects, *J Applied Statistics* **34**(5) (2007), 577–584.
- [9] D.K. Pateria, Seema Jaggi, Cini Varghese and M. N. Das, Incomplete non-circular block designs for competition effects, *Statistics and Applications*, **5**(1&2) (2007), 5–14.
- [10] D.K. Pateria, Seema Jaggi and Cini Varghese, Self – Neighoured Strongly Balanced Block Designs, *J Ind Statist Assoc* **47**(1) (2009), 1–14.
- [11] J.S. Tomar, Seema Jaggi and Cini Varghese, On totally balanced block designs for competition effects, *J Applied Statistics* **32**(1) (2005), 87–97.
- [12] C.E. Wright, A systematic polycross design, *Res Exp Rec Min Agric N.I* **1** (1962), 7–8.