National Fellow Scheme

Efficient Design of Experiments for Quality Agricultural Research

Annual Progress Report
(27.01.2005-26.01.2006)

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1. **Title of the Scheme** : **Efficient Design of Experiments for Quality Agricultural Research**
2. **Location** : Indian Agricultural Statistics Research Institute, Library Avenue, New Delhi - 110 012
3. **Date of Start** : January 27, 2005
4. **Objectives** :

(i) To obtain economic, efficient and robust designs for single response experiments and to prepare catalogues of such designs for use of experimenters.

(ii) To develop optimal/efficient designs for multi-response experiments and to prepare the catalogue of such designs for use of experimenters.

(iii) To develop the analytical procedures for the analysis of experimental data related to multi-response experiments with special emphasis on contrast analysis for identifying the best treatment.

(iv) To develop analytical techniques for simultaneous optimization of several responses with special emphasis on food processing experiments.

(v) To develop analytical techniques of groups of experiments when the designs are different at different locations/environments with some or all treatments in common.

(vi) To develop the statistical software package for designing of the experiments catalogued in objectives 1 and 2 and for analysis of experimental data using the techniques developed under objectives 3, 4 and 5.

5. Research Achievements

During the period under report, emphasis was made on obtaining economic, efficient and robust designs for single response experiments and to prepare catalogues of such designs for use of experimenters. Efficient α-designs, nested balanced incomplete block designs, nested partially balanced incomplete block designs, nested block designs for making test treatments-control treatment comparisons, minimally connected designs with some extra observations, semi-Latin squares have been obtained and catalogued. To disseminate the knowledge available on the combinatorial aspects of designs and analytical procedures acquired to the scientists engaged in research in the National Agricultural Research System. The advisory services are pursued rigorously. For the benefit of the experimenters and practicing statisticians, a beginning has been made to initiate a "WEB DESIGN". Designs for various experimental situations would be available for the experimenters. Discussion forum is also being created so as to introduce E-advisory services. A brief description of the achievements made is given in the sequel.

5.1 α-Designs: Designs useful for varietal trials

In National Agricultural Research System (NARS), a large number of experiments are conducted for improvement of crop varieties. A large number of varieties in these trials are fixed, *i.e.*, not at the choice of statistician, and are large enough to require use of incomplete block designs or some other method of controlling the variability in the experimental units. Number of replications is also fixed based on the availability of land and other experimental resources. Further, it has been seen that in many crop improvement programmes, initial varietal trials and advanced varietal trials are generally conducted using a Randomized Complete Block (RCB) design. The analysis of the data reveals that the percent coefficient of variation (CV) in many of these experiments is high and as a consequence, the precision of treatment comparisons is low. This amount to saying that the error sum of squares is large compared to the sum of squares attributable to the model and hence, small treatment differences may not be detected as significant. A large number of trials are rejected due to high CV. In these experiments, the block mean squares are small as compared to error mean square. This may happen due to any of the following:

1. Bad management of the controllable factors during experimentation (managerial aspects)
2. Faulty formation of blocks (designing)
3. Lack of identification of ancillary information that could have been used as covariate (analysis of covariance).

The first point may be taken care of if the experimenter is very cautious and experienced. Analysis of covariance is an analytical procedure and is very effective in controlling the experimental error although it has nothing to do with the designing of the experiment. The most important point that has to be taken care of during the allocation of treatments to different experimental units is to adopt proper blocking techniques. In field trials, the formation of blocks *i.e.* shape and size should be preceded by uniformity trials to determine the fertility pattern through a contour map. In most situations particularly in the plains, the contour maps show the variations in the fertility patches rather than in a single direction. Therefore, there is a need of effective control of variation through blocking.

This necessitates the use of incomplete block designs like balanced incomplete block (BIB) designs, partially balanced incomplete block (PBIB) designs, cyclic designs, etc. One may, however, argue that in these designs the purpose of demonstration of a variety effect in the field cannot be done as all the treatments are not appearing in adjacent piece of land. To overcome this problem it is recommended that resolvable block designs with smaller block sizes may be used.

In a resolvable block design, the blocks can be grouped such that in each group, every treatment appears exactly once. Resolvable block designs allow performing an experiment one replication at a time. For example, field trials with large number of crop varieties cannot always be laid out in a single location or a single season. Therefore, it is desired that variation due to location or time periods may also be controlled along with controlling within location or time period variation. This can be handled by using resolvable block designs. Here, locations or time periods may be taken as replications and the variation within a location or a time period can be taken care of by blocking. In an agricultural field experiment, the land may be divided into a number of large areas corresponding to the replications and then each area is subdivided into blocks. These designs are also quite useful for varietal trials conducted in NARS and will help in improving the precision of treatment comparisons.

α-designs are also essentially resolvable block designs. α-designs were introduced by Patterson and Williams (1976). A critical look at the experimentation in the NARS reveals that α-designs have not found much favour from the experimenters. It may possibly be due to the fact that the experimenters may find it difficult to lay their hands on the α-designs. The construction of these designs is not easy. An experimenter has to get associated with a statistician to get a randomized layout of this design. For the benefit of the experimenters, a comprehensive catalogue of α-designs for $6 \le v(= sk) \le 150$, $2 \le r \le 5$, $3 \le k \le 10$ and $2 \le s \le 15$ has been prepared along with lower bounds to A- and D- efficiencies and generating arrays. The distribution of the designs in different efficiency ranges is:

Frequency Class	<0.8000	0.8000-0.8500	0.8500-0.9000	0.9000-0.9500	0.9500-0.9750	0.9750-0.9900	>0.9900	Total
A-Efficiency	18	17	43	116	151	102	1	448
D-Efficiency	0	5	10	49	123	203	58	448

The layout of these designs along with block contents has also been prepared. The designs obtained have been compared with corresponding square lattice, rectangular lattice, resolvable two-associate class partially balanced incomplete block {PBIB (2)} designs and the α-designs obtainable from basic arrays given by Patterson, Williams and Hunter (1978). Eleven designs are more efficient than the corresponding resolvable PBIB (2) designs (S11, S38, S69, S114, LS8, LS30, LS54, LS76, LS89, LS126 and LS140). It is interesting to note here that for the PBIB (2) designs based on Latin square association scheme, the concurrences of the treatments were 0 or 2 and for singular group divisible designs the concurrences are either 1 or 5. Further all the designs LS8, LS30, LS54, LS76, LS89, LS126 and LS140 were obtained by taking two copies of a design with 2-replications. 10 designs were found to be more efficient than the designs obtainable from basic arrays. 48 designs (29 with k = 4 and 19 with k = 3) are more efficient than the designs obtainable by dualization of basic arrays. 25 designs have been obtained for which no corresponding resolvable solution of PBIB (2) designs is available in literature. The list of corresponding resolvable PBIB(2) designs is S28, S86, SR18, SR41, SR52, SR58, SR66, SR75, SR80, R42, R70, R97, R109, R139, T14, T16, T20, T44, T48, T49, T72, T73, T86, T87 and M16. Here X# denotes the design of type X at serial number # in Clatworthy (1973). The catalogue of the designs is given in Table 5.1.1 in the Appendix. Table 5.1.2 in the Appendix contains the α-arrays for generating α-designs. The procedure of generation of α-designs for $v = ks$ treatments in r replication from reduced α-array and terminology involved is described in the sequel.

1. **α-array:** It is an $k \times r$ array of elements consisting of residue class of s, *i.e.*, the elements can be 0, 1, 2, …, s − 1. If the first row and first column of that array consists of zeros as elements, then that array is called reduced array. We shall describe the

construction of the α-designs by the help of the reduced arrays. We can determine the concurrences of the different treatment pairs from the α-array.

2. **Intermediate Array:** Construction of the intermediate array is the intermediate step to develop α-design. One can have ($s-1$) new columns from a column of α-array by developing the column cyclically modulo s. By this way we can construct $r(s-1)$ new columns by cyclically developing each column modulo s of the α-array. The arrangements of $k \times sr$ array is called intermediate array.
3. **α-Design:** Now, take the intermediate array and add $(1 + (j-1)s)$ to the elements of j^{th} row of that array (j = 1, 2, …, k). This gives us an array of size ($k \times sr$). Now taking one to one correspondence between the treatments and the elements of the final array, we get an α-design with the parameters $v = ks,\ b = rs, r, k$.

The steps of construction are described with the help of following example:

Example 5.1.1: Consider the construction of an α-design for $v = 12, b = 12, r = 3,$ $k = 3,\ s = 4$. The generating array α for this design is

$$\begin{matrix} 0 & 0 & 0 \\ 0 & 2 & 3 \\ 0 & 3 & 1 \end{matrix}$$

Given this generating array, develop the intermediate array α^* by cyclically developing each column modulo s. The intermediate array α^* is

$$\begin{matrix} 0 & 1 & 2 & 3 & \quad & 0 & 1 & 2 & 3 & \quad & 0 & 1 & 2 & 3 \\ 0 & 1 & 2 & 3 & & 2 & 3 & 0 & 1 & & 3 & 0 & 1 & 2 \\ 0 & 1 & 2 & 3 & & 3 & 0 & 1 & 2 & & 1 & 2 & 3 & 0 \end{matrix}$$

Now add $(1 + (j-1)s)$ to the elements of j^{th} row of that array (j = 1, 2,…, k). The α-design with columns as blocks is

Replication-I				Replication-II				Replication-III			
B1	**B2**	**B3**	**B4**	**B5**	**B6**	**B7**	**B8**	**B9**	**B10**	**B11**	**B12**
1	2	3	4	1	2	3	4	1	2	3	4
5	6	7	8	7	8	5	6	8	5	6	7
9	10	11	12	12	9	10	11	10	11	12	9

B#: denotes the block number

Following the above procedure, one can generate the α-designs from the α-arrays given in Table 5.1.2 in the Appendix. A monograph of the α-designs has also been prepared that contains the block layouts of the α-designs catalogued in Appendix.

In some experimental situations, however, the user may be interested in getting designs outside the parametric range given in the monograph. To circumvent such situations, we have developed a computer algorithm based on exchange and interchange of treatments in an α-array. This algorithm is converted into a user-friendly module for generation of α-designs and is described in the sequel.

Software for Generation of α-designs

β- Version of user friendly software module for the generation of α-designs has been developed. This module generates the α-array along with lower bounds to A and D-efficiency. The α-array and the design is generated once the user enters the number of treatments (v), number of replications (r) and the block size (k). The module generates the design for any v, k, r provided v is a multiple of k. It also gives the block contents of the design generated.

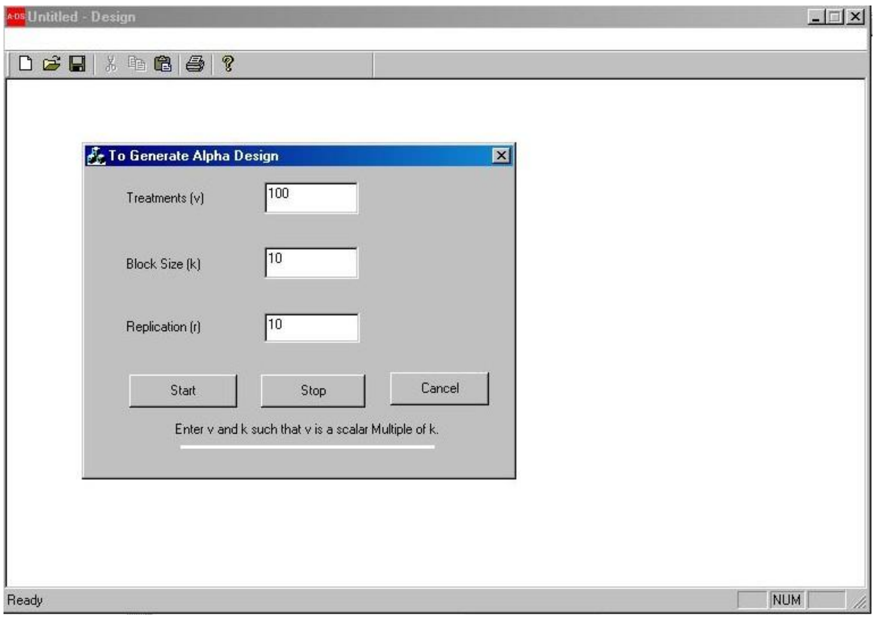

Figure 5.1.1: The Screen for entering the parameters for generation of α-designs

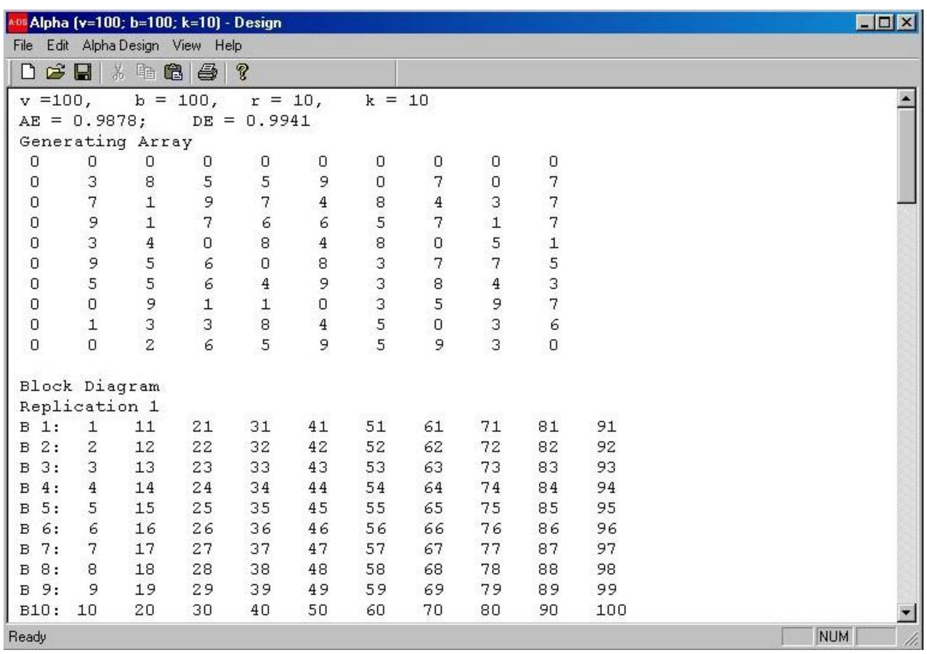

Figure 5.1.2: Generated α-array and α-design for the parameters in Figure 1.

Some α-designs have already been recommended to the experimenters in NARS. Some details of the recommended designs are the following:

- Scientists from the Division of Crop Improvement, Central Institute of Cotton Research, Nagpur, were advised on the designing of four experiments to be conducted with cotton varieties. First two of these experiments are to be conducted to morphologically characterize varieties and hybrids with respect to about 45 morphological characters both qualitative and quantitative and document the database as a part of Distinctness, Uniformity and Stability (DUS) testing. The parameters and lower bound to the A-efficiency of the designs recommended are:

S.No.	Parameters of the Design	A-efficiency (Lower Bound)	Plot Size
1a.	v = 70, b = 20, r = 4 , k = 14	0.9826	2.7m x 4.8m
1b	v = 70, b = 28, r = 4 , k = 10	0.9785	2.7m x 4.8m
2.	v = 84, b = 24, r = 4 , k = 14	0.9830	2.7m x 4.8m
3.	v = 28, b = 12, r = 3, k = 7	0.9603	2.7 m x 6.0 m
4.	v = 14, b = 6, r = 3, k = 7	0.9684	2.7 m x 6.0 m

The block contents of these designs are:

Design 1a: α-design for v = 70, b = 20, k = 14; A-efficiency: 0.9826

Replication I					Replication II					Replication III					Replication IV				
B1	**B2**	**B3**	**B4**	**B5**	**B6**	**B7**	**B8**	**B9**	**B10**	**B11**	**B12**	**B13**	**B14**	**B15**	**B16**	**B17**	**B18**	**B19**	**B20**
1	2	3	4	5	1	2	3	4	5	1	2	3	4	5	1	2	3	4	5
6	7	8	9	10	9	10	6	7	8	10	6	7	8	9	7	8	9	10	6
11	12	13	14	15	12	13	14	15	11	14	15	11	12	13	13	14	15	11	12
16	17	18	19	20	19	20	16	17	18	20	16	17	18	19	17	18	19	20	16
21	22	23	24	25	25	21	22	23	24	23	24	25	21	22	24	25	21	22	23
26	27	28	29	30	27	28	29	30	26	29	30	26	27	28	30	26	27	28	29
31	32	33	34	35	33	34	35	31	32	32	33	34	35	31	35	31	32	33	34
36	37	38	39	40	40	36	37	38	39	37	38	39	40	36	38	39	40	36	37
41	42	43	44	45	44	45	41	42	43	45	41	42	43	44	43	44	45	41	42
46	47	48	49	50	48	49	50	46	47	49	50	46	47	48	47	48	49	50	46
51	52	53	54	55	52	53	54	55	51	55	51	52	53	54	54	55	51	52	53
56	57	58	59	60	59	60	56	57	58	58	59	60	56	57	60	56	57	58	59
61	62	63	64	65	63	64	65	61	62	62	63	64	65	61	64	65	61	62	63
66	67	68	69	70	70	66	67	68	69	68	69	70	66	67	67	68	69	70	66

Design 1b: Layout of the α-design for v = 70, b = 28, k = 10; A-efficiency: 0.9785

Replication I							Replication II							Replication III							Replication IV						
B1	**B2**	**B3**	**B4**	**B5**	**B6**	**B7**	**B8**	**B9**	**B10**	**B11**	**B12**	**B13**	**B14**	**B15**	**B16**	**B17**	**B18**	**B19**	**B20**	**B21**	**B22**	**B23**	**B24**	**B25**	**B26**	**B27**	**B28**
1	2	3	4	5	6	7	1	2	3	4	5	6	7	1	2	3	4	5	6	7	1	2	3	4	5	6	7
8	9	10	11	12	13	14	9	10	11	12	13	14	8	11	12	13	14	8	9	10	12	13	14	8	9	10	11
15	16	17	18	19	20	21	17	18	19	20	21	15	16	16	17	18	19	20	21	15	18	19	20	21	15	16	17
22	23	24	25	26	27	28	27	28	22	23	24	25	26	24	25	26	27	28	22	23	23	24	25	26	27	28	22
29	30	31	32	33	34	35	35	29	30	31	32	33	34	31	32	33	34	35	29	30	33	34	35	29	30	31	32
36	37	38	39	40	41	42	37	38	39	40	41	42	36	41	42	36	37	38	39	40	39	40	41	42	36	37	38
43	44	45	46	47	48	49	49	43	44	45	46	47	48	47	48	49	43	44	45	46	48	49	43	44	45	46	47
50	51	52	53	54	55	56	53	54	55	56	50	51	52	56	50	51	52	53	54	55	55	56	50	51	52	53	54
57	58	59	60	61	62	63	61	62	63	57	58	59	60	62	63	57	58	59	60	61	59	60	61	62	63	57	58
64	65	66	67	68	69	70	66	67	68	69	70	64	65	67	68	69	70	64	65	66	70	64	65	66	67	68	69

Design 2: α-Design for v = 84, b = 24, r = 4, k = 14; A-efficiency: 0.9830

Replication I						Replication II						Replication III						Replication IV					
B1	**B2**	**B3**	**B4**	**B5**	**B6**	**B7**	**B8**	**B9**	**B10**	**B11**	**B12**	**B13**	**B14**	**B15**	**B16**	**B17**	**B18**	**B19**	**B20**	**B21**	**B22**	**B23**	**B24**
1	2	3	4	5	6	1	2	3	4	5	6	1	2	3	4	5	6	1	2	3	4	5	6
10	11	12	7	8	9	9	10	11	12	7	8	7	8	9	10	11	12	12	7	8	9	10	11
16	17	18	13	14	15	17	18	13	14	15	16	13	14	15	16	17	18	15	16	17	18	13	14
21	22	23	24	19	20	24	19	20	21	22	23	19	20	21	22	23	24	20	21	22	23	24	19
26	27	28	29	30	25	30	25	26	27	28	29	25	26	27	28	29	30	29	30	25	26	27	28
32	33	34	35	36	31	34	35	36	31	32	33	31	32	33	34	35	36	35	36	31	32	33	34
41	42	37	38	39	40	42	37	38	39	40	41	37	38	39	40	41	42	38	39	40	41	42	37
45	46	47	48	43	44	46	47	48	43	44	45	43	44	45	46	47	48	44	45	46	47	48	43
54	49	50	51	52	53	53	54	49	50	51	52	49	50	51	52	53	54	52	53	54	49	50	51
60	55	56	57	58	59	56	57	58	59	60	55	55	56	57	58	59	60	58	59	60	55	56	57
62	63	64	65	66	61	64	65	66	61	62	63	61	62	63	64	65	66	63	64	65	66	61	62
70	71	72	67	68	69	68	69	70	71	72	67	67	68	69	70	71	72	72	67	68	69	70	71
77	78	73	74	75	76	75	76	77	78	73	74	73	74	75	76	77	78	78	73	74	75	76	77
83	84	79	80	81	82	80	81	82	83	84	79	79	80	81	82	83	84	81	82	83	84	79	80

Design 3: α Design for v = 28, b = 12, r = 3, k = 7; A-efficiency: 0.9603

Replication I				Replication II				Replication III			
B1	**B2**	**B3**	**B4**	**B5**	**B6**	**B7**	**B8**	**B9**	**B10**	**B11**	**B12**
1	2	3	4	1	2	3	4	1	2	3	4
5	6	7	8	6	7	8	5	7	8	5	6
9	10	11	12	10	11	12	9	12	9	10	11
13	14	15	16	15	16	13	14	14	15	16	13
17	18	19	20	19	20	17	18	20	17	18	19
21	22	23	24	24	21	22	23	22	23	24	21
25	26	27	28	28	25	26	27	27	28	25	26

Design 4: α Design for v =14, b=6, r=3, k=7; A-efficiency: 0.9684

Replication I		Replication II		Replication III	
B1	**B2**	**B3**	**B4**	**B5**	**B6**
1	2	1	2	1	2
3	4	4	3	4	3
5	6	5	6	6	5
7	8	8	7	7	8
9	10	10	9	9	10
11	12	11	12	12	11
13	14	14	13	14	13

- A reinforced α-design in 55 (50 tests and 5 check varieties of tomato) arranged in 10 blocks each of size 15 was recommended to the scientists of Division of Germplasm Evaluation, National Bureau of Plant Genetic Resources, New Delhi. The layout of the design recommended is:

Replication 1						Replication 2				
B1	**B2**	**B3**	**B4**	**B5**		**B1**	**B2**	**B3**	**B4**	**B5**
21	12	18	9	25		38	27	9	36	24
31	47	48	14	45		34	35	49	50	44
16	22	28	19	35		7	20	31	4	37
26	37	23	34	20		25	8	15	29	6
36	17	3	39	40		19	41	3	23	46
41	32	8	24	15		1	2	22	32	30
11	42	33	4	30		45	48	42	10	5
6	27	38	49	5		13	14	40	17	18
46	7	13	44	50		26	21	28	43	33
1	2	43	29	10		47	39	16	11	12
51	51	51	51	51		51	51	51	51	51
52	52	52	52	52		52	52	52	52	52
53	53	53	53	53		53	53	53	53	53
54	54	54	54	54		54	54	54	54	54
55	55	55	55	55		55	55	55	55	55

- Through the presentations and discussions held during the 12th Annual Workshop of All India Co-ordinated Research Project on Rapeseed and Mustrad; National Research Centre on Rapeseed and Mustard, Bharatpur and a **Brain Storming session** on **Statistical Issues in Rapeseed-Mustard Trials** held at Indian Agricultural Statistics Research Institute, New Delhi, it was realized that α-designs would be very useful for Rapeseed and Mustard varietal trials.

5.2 Nested block designs

A nested block design is defined as two systems of blocks such that the second system of blocks is nested within the first system of blocks. These designs are quite useful in many experimental situations. For example, consider a field experiment conducted using a block design and harvesting is done block wise. Harvested samples are to be analyzed for their contents either by different technicians at same time or by a technician over different periods of time. The variation due to technicians or time periods may be controlled by another blocking system. Technicians or time periods form a system of blocks that are nested within blocks. Such experimental situations are also common in post harvest storage and value addition of horticultural and vegetable crops. Block designs with nested factors are quite useful for these experimental situations.

Based on the analysis of several uniformity trials data, Nigam, Parsad and Gupta (2005) have shown that the nested block designs with smaller block sizes are beneficial in comparison to usual procedure of blocking, in the sense that it reduces the coefficient of variation and hence the precision of the treatment comparisons. They have also shown that the resolvable block designs and nested block designs are quite useful for on-farm research experiments.

Preece (1967) was the first to introduce nested block designs and termed them as nested balanced incomplete block (NBIB) designs.

➢ NBIB designs

A NBIB design is a design where block classification ignoring sub-blocks is a balanced incomplete block (BIB) design and sub-block classification ignoring blocks is also a BIB design. A NBIB design is defined as follows:

Definition 5.2.1: An arrangement of *v* treatments each replicated *r* times in two systems of blocks is said to be a NBIB design with parameters $v, b_1, b_2, r, k_1, k_2, \lambda_1, \lambda_2, q$ if

(i) the second system is nested within the first, with each block from the first system (block) containing exactly *q* blocks from the second system (sub-blocks);

(ii) ignoring the second system leaves a balanced incomplete block (BIB) design with b_1 blocks each of k_1 units with λ_1 concurrences; and

(iii) ignoring the first system leaves a BIB design with $b_2 = b_1 q$ blocks each of size $k_2 = k_1 / q$, with λ_2 concurrences.

The work on combinatorial aspects of NBIB designs was initiated by Preece (1967). Jimbo and Kuriki (1983), Dey, Das and Banerjee (1986), Parsad, Gupta and Srivatava (1999) and Morgan, Preece and Rees (2001) gave methods of construction of NBIB designs. Morgan, Preece and Rees (2001) gave a catalogue of NBIB designs with number of treatments (v) ≤ 16 and number of replications (r) ≤30. This catalogue gave designs only upto 16 treatments and method of obtaining the block contents of the NBIB designs were quite involved algebraically.

Therefore, for the benefit of the experimenters, we have prepared a complete catalogue of NBIB designs with number of replications $r \le 30$. The catalogue contains a total of 299 designs. Out of 299 designs, 8 designs are non-existent. A new method of construction of NBIB designs has been obtained.

Method 5.2.1: A NBIB design with parameters $v = mt + 1$ (a prime or prime power), $b_1 = mt + 1, b_2 = t(mt + 1), r = mt, k_1 = mt, k_2 = m, \lambda_1 = mt - 1, \lambda_2 = m - 1$, can be obtained by developing the following initial block cyclically by modulo *v*:

$$\left[\left(x^{i}, x^{i+t}, x^{i+2t}, \ldots, x^{i+(m-1)t}\right); \forall\, i=0,1,\ldots,t-1\right]$$

This method gives NBIB designs when v is prime or prime power and (v – 1) is a multiple of k_1 and k_2 both.

Method 5.2.2: (Trial and Error Solutions). Let there exists a BIB design with parameters $v, b_1, r, k_1, \lambda_1$. The solution of BIB design is obtained by developing s initial blocks. Rearrange each of s initial blocks of size $k_1 = qk_2$ into q-sub-blocks of size k_2 consists of the elements of residue class of v. Such that all the elements of the residue class v appears in equal frequencies in the difference set of the initial blocks as well as in the sub-blocks. The number s, the number of initial blocks tried, will be minimal for a particular choice of v, q and k_2.

Using these methods, block layouts of 22 new NBIB designs have been obtained. A list of the designs obtained is given in Table 5.2.1 in Appendix.

The layout of 199 designs with block contents has been completed. The solution for the block layout for remaining 92 designs is unknown and the statisticians need to develop methods of construction of these NBIB designs. The designs catalogued have also been identified for 1-resolvable and 2-resolvable sets.

➢ Nested Partially Balanced Incomplete Block Designs

A NBIB design may not exist for all parametric combinations or even if it exists may require a large number of replications, which the experimenter may not be able to afford. To deal with such situations, Homel and Robinson (1975) introduced nested partially balanced incomplete block (NPBIB) designs.

Definition 5.2.2: An NPBIB design based on m (≥ 2)-class association scheme defined in v symbols, is an arrangement of v symbols into b_2 sub-blocks of size k_2 nested within b_1 (= b_2 / t , t is an integer) blocks of size k_1 (= $tk_2 < v$) such that

(i) every symbol occurs at most once in a block;
(ii) every symbol appears at most r times in the design;
(iii) if two symbols, say α and β, are i^{th} associates, then they occur together in λ_{1i} blocks and λ_{2i} sub-blocks, the numbers λ_{1i}, λ_{2i} being independent of the particular pair of i^{th} associates α and β, i = 1, 2, ..., m.

The numbers $v, b_1, b_2, r, k_1, k_2, \lambda_{1i}$, λ_{2i} (i = 1, 2, ..., m) are called parameters of the design. If $\lambda_{1i} = \lambda_1$ and $\lambda_{2i} = \lambda_2$; $\forall$ i = 1, 2, ..., m, then an NPBIB design reduces to NBIB design. Several methods of construction of NPBIB designs are available in the literature. Satpati and Parsad (2005) gave comprehensive catalogues of two and three associate class NPBIB designs for $v \le 30$ and $r \le 15$. Using this catalogue, obtaining the block layouts of a NPBIB design is quite difficult and cannot be obtained without the help of a statistician. Therefore, to prepare a ready reckoner of NPBIB designs, the block layouts of the NPBIB designs catalogued by Satpati and Parsad (2005) have been obtained. Further, this catalogue does not contain all the NPBIB designs based on group divisible association scheme. Hence, a catalogue of NPBIB designs based on group divisible association scheme for $r \leq 15$ has been prepared. Some new methods of construction of NPBIB designs have also been given. These are described in the sequel.

Method 5.2.3: Consider a group divisible (GD) association scheme with $v = 2n$ treatment symbols arranged in a $2 \times n$ array as

1	3	…	$2n$-3	$2n$-1
2	4	…	$2n$-2	$2n$

From this array, generate n isomorphic arrays using the following procedure:

(i) Write a Latin square of order n with elements as symbols from second row of the array.
(ii) Write the first row of the array as first row of the new n arrays each of order $2 \times n$.
(iii)Take the rows of the Latin square one by one as the second row of the 'n' arrays of the order $2 \times n$.

All the n arrays are isomorphic in the sense that 1st and 2nd associates of the treatments do not change. (Consider $v = mn$ treatments are defined in a GD association scheme. In GD association scheme symbols in a row are 1st associate to each other and two symbols present in different rows are second associates to each other. Therefore, the association scheme will not change if we permute the rows or the columns of the association scheme)

If we take all possible combinations of $c < n$ columns for each of the arrays separately. This gives $n\binom{n}{c}$ subsets of $2 \times c$ arrays. Consider each subset as bigger block with column contents as sub-blocks of size 2 each. This yields an NPBIB design with parameters

$v = 2n$, $b_1 = n\binom{n}{c}$, $b_2 = nc\binom{n}{c}$, $r = n\binom{n-1}{c-1} = c\binom{n}{c}$, $k_1 = 2c$, $k_2 = 2$, $\lambda_{11} = n\binom{n-2}{c-2}$, $\lambda_{12} = c\binom{n-1}{c-1}$ {from the array in which two second associates say 1, 2 appear in the same column, the concurrences will be $\binom{n-1}{c-1}$}, whereas from other n-1 arrays, the concurrences will be $\binom{n-2}{c-2}$ and $\binom{n-1}{c-1}$+(n-1) $\binom{n-2}{c-2}$ gives $c\binom{n-1}{c-1}$, $\lambda_{21} = 0$, $\lambda_{22} = \binom{n-1}{c-1}$.

Example 5.2.3.1: For $v = 2 \times 4$, GD association scheme as $\begin{matrix} 1 & 3 & 5 & 7 \\ 2 & 4 & 6 & 8 \end{matrix}$. Now write the Latin Square of order 4 with symbols from the second row of the association scheme as

2	4	6	8
4	6	8	2
6	8	2	4
8	2	4	6

New 4 arrays of isomorphic GD association scheme are obtained as

1	3	5	7	1	3	5	7	1	3	5	7	1	3	5	7
2	4	6	8	4	6	8	2	6	8	2	4	8	2	4	6

Now taking all possible combinations of three columns from each of the arrays, we get 16 subsets as

1	3	5		1	3	7		1	5	7		3	5	7
2	4	6		2	4	8		2	6	8		4	6	8

1	3	5		1	3	7		1	5	7		3	5	7
4	6	8		4	6	2		4	8	2		6	8	2

1	3	5		1	3	7		1	5	7		3	5	7
6	8	2		6	8	4		6	2	4		8	2	4

1	3	5		1	3	7		1	5	7		3	5	7
8	2	4		8	2	6		8	4	6		2	4	6

Now considering subsets as bigger blocks and columns within subsets as blocks, we get an NPBIB design based on group divisible association scheme with parameters as v = 8, b_1 = 16, b_2 = 48, r = 12, k_1 = 6, k_2 = 2, λ_{11} = 8, λ_{12} = 9, λ_{21} = 0, λ_{22} = 3. The block contents are

[(1, 2); (3, 4); (5, 6)]; [(1, 2); (3, 4); (7, 8)]; [(1, 2); (5, 6); (7, 8)]; [(3, 4); (5, 6); (7, 8)];
[(1, 4); (3, 6); (5, 8)]; [(1, 4); (3, 6); (7, 2)]; [(1, 4); (5, 8); (7, 2)]; [(3, 6); (5, 8); (7, 2)];
[(1, 6); (3, 8); (5, 2)]; [(1, 6); (3, 8); (7, 4)]; [(1, 6); (5, 2); (7, 4)]; [(3, 8); (5, 2); (7, 4)];
[(1, 8); (3, 2); (5, 4)]; [(1, 8); (3, 2); (7, 6)]; [(1, 8); (5, 4); (7, 6)]; [(3, 2); (5, 4); (7, 6)].

Corollary 5.2.3.1: For c = 2, the parameters of the NPBIB design obtained from method 5.2.3 are $v = 2n$, $b_1 = n^2(n-1)/2$, $b_2 = n^2(n-1)$, $r = n(n-1)$, $k_1 = 4$, $k_2 = 2$, $\lambda_{11} = n$, $\lambda_{12} = 2(n-1)$, $\lambda_{21} = 0$, $\lambda_{22} = n-1$.

Example 5.2.3.2: In example 5.2.3.1, if we take all possible combinations of two columns from the 2×4 array of the GD association scheme, then we get a NPBIB design based on GD association scheme with parameters v = 8, b_1 = 24, b_2 = 48, r = 12, k_1 = 4, k_2 = 2, λ_{11} = 4, λ_{12} = 6, λ_{21} = 0, λ_{22} = 3. The block contents of NPBIB design are

[(1, 2); (3, 4)]; [(1, 2); (5, 6)]; [(1, 2); (7, 8)]; [(3, 4); (5, 6)]; [(3, 4); (7, 8)]; [(5, 6); (7, 8)];
[(1, 4); (3, 6)]; [(1, 4); (5, 8)]; [(1, 4); (7, 2)]; [(3, 6); (5, 8)]; [(3, 6); (7, 2)]; [(5, 8); (7, 2)];
[(1, 6); (3, 8)]; [(1, 6); (5, 2)]; [(1, 6); (7, 4)]; [(3, 8); (5, 2)]; [(3, 8); (7, 4)]; [(5, 2); (7, 4)];
[(1, 8); (3, 2)]; [(1, 8); (5, 4)]; [(1, 8); (7, 6)]; [(3, 2); (5, 4)]; [(3, 2); (7, 6)]; [(5, 4); (7, 6)].

Example 5.2.3.3: For v = 6 (2×3) and c = 2, using the Method 5.2.3 and Corollary 5.2.3.1, we get an NPBIB design with parameters as $v = 6$, $b_1 = 9$, $b_2 = 18$, $r = 6$, $k_1 = 4$, $k_2 = 2$, $\lambda_{11} = 3$, $\lambda_{12} = 4$, $\lambda_{21} = 0$, $\lambda_{22} = 2$. The block contents are:

[(1, 2); (3, 4)]; [(1, 2); (5, 6)]; [(3, 4); (5, 6)]; [(1, 4); (3, 6)]; [(1, 4); (5, 2)]; [(3, 6); (5, 2)]; [(1, 6); (3, 2)]; [(1, 6); (5, 4)]; [(3, 2); (5, 4)].

Remark 5.2.3.1: If m = 2 and n = 3, then take one GD association scheme. Take a column of the association scheme as sub-block and form another sub-block by taking elements of a row except the element which is already in the 1st sub-block. Now take these two sub-blocks to form a bigger block. Selection of one column yields two blocks, one by selecting the elements from the second row and one by selecting the elements from the first row. Repeat this for all the columns and for all isomorphic GD association scheme arrays. We get an NPBIB design with $v = 6$, $b_1 = 18$, $b_2 = 36$, $r = 12$, $k_1 = 4$, $k_2 = 2$, $\lambda_{11} = 9$, $\lambda_{12} = 6$, $\lambda_{21} = 3$, $\lambda_{22} = 2$. The block contents are:

[(1, 2); (4, 6)]; [(2, 1); (3, 5)]; [(3, 4); (6, 2)]; [(4, 3); (5, 1)]; [(5, 6); (2, 4)]; [(6, 5); (1, 3)]; [(1, 4); (2, 6)]; [(4, 1); (5, 3)]; [(3, 6); (4, 2)]; [(6, 3); (1, 5)]; [(2, 5); (3, 1)]; [(5, 2); (6, 4)]; [(1, 6); (2, 4)]; [(6, 1); (3, 5)]; [(2, 3); (5, 1)]; [(3, 2); (4, 6)]; [(4, 5); (1, 3)]; [(5, 4); (6, 2)].

This is same as design at serial number 4 by Satpati and Parsad (2005).

Method 5.2.4: Let there exists two NPBIB designs with parameters $v^*, b_1^*, b_2^*, r^*, k_1^*, k_2^*, \lambda_{11}^*, \lambda_{12}^*, \lambda_{21}^*, \lambda_{22}^*$; $v^*, b_1', b_2', r', k_1^*, k_2^*, \lambda_{11}', \lambda_{12}', \lambda_{21}', \lambda_{22}'$ based on the group divisible association scheme. Taking union of the blocks of these two NPBIB designs, we get another NPBIB design with same association scheme and parameters as

$$v = v^*,\ b_1 = b_1^* + b_1',\ b_2 = b_2^* + b_2',\ r = r^* + r',\ k_1 = k_1^*,\ k_2 = k_2^*,\ \lambda_{11} = \lambda_{11}^* + \lambda_{11}',$$

$$\lambda_{12} = \lambda_{12}^* + \lambda_{12}',\ \lambda_{21} = \lambda_{21}^* + \lambda_{21}',\ \lambda_{22} = \lambda_{22}^* + \lambda_{22}'$$

Remark 5.2.4.1: Method 5.2.4 is quite useful in obtaining NPBIB designs when one of the NPBIB designs is a disconnected design. A disconnected NPBIB design can be obtained using the following procedure: take one GD association scheme with $v = 2n$. Take one row of the association scheme and select two elements of that row to form one sub-block. Now from the second row select two elements from the same columns as used to form 1st sub-block from the first row to form 2nd sub-block. Now, form one bigger-block by combining two sub-blocks. Repeat this procedure for all possible pairs of elements in a row. Repeat this for each of the isomorphic arrays of the GD association scheme. The parmeters of the design so obtained are $v = 2n,\ b_1 = n(n_2),\ b_2 = 2n(n_2),\ r = n(n\!-),\ k_1 = 4,\ k_2 = 1,\ \lambda_{11} = n,$ $\lambda_{12} = 2(n-1),\ \lambda_1 = n,\ \lambda_{22} = 0.$ The design is disconnected with respect to sub-blocks

Example 5.2.4.1: For n = 3, following the above procedure, we get a NPBIB design with parameters $v = 6, b_1 = 9, b_2 = 18, r = 6, k_1 = 4, k_2 = 2,\ \lambda_{11} = 3, \lambda_{12} = 4, \lambda_{21} = 3, \lambda_{22} = 0.$ The block contents are

[(1, 3); (2, 4)]; [(1, 5); (2, 6)]; [(3, 5); (4, 6)]; [(1, 3); (4, 6)]; [(1, 5); (4, 2)]; [(3, 5); (6, 2)]; [(1, 3); (6, 2)]; [(1, 5); (6, 4)]; [(3, 5); (2, 4)].

The design is disconnected in sub-blocks.

Remark 5.2.4.2: Consider a GD association scheme with $v = mn$ symbols, where n is even. Write all possible $\binom{n}{2}$ duplets of elements from each row of the association scheme. Take these duplets as sub-blocks. Arrange the (n/2) sub-blocks containing distinct treatments in a bigger block such that bigger blocks contains all the treatments pertaining to the same row. Repeat this procedure for all m rows of the association scheme. This yields a NPBIB design based on GD association scheme with parameters
$v = mn,\ b_1 = 2m(n-1)\ b_2 = mn(n-1)/2,\ r = n-1,\ k_1 = n,\ k_2 = 2, \lambda_{11} = n-1, \lambda_{12} = 0,$
$\lambda_{21} = 1, \lambda_{22} = 0.$

This design is disconnected with respect to bigger as well as sub-blocks.

Example 5.2.4.2: For $m = 2$ and $n = 4$, following the procedure of Remark 5.2.4.2, we get a NPBIB design based on GD association scheme with parameters $v = 8, b_1 = 6,$ $b_2 = 12, r = 3, k_1 = 4, k_2 = 2, \lambda_{11} = 3, \lambda_{12} = 0, \lambda_{21} = 1, \lambda_{22} = 0.$ The block contents of the design are
[(1, 3); (5, 7)]; [(1, 5); (3, 7)]; [(1, 7); (3, 5)]; [(2, 4); (6, 8)]; [(2, 6); (4, 8)]; [(2, 8); (4, 6)].

Remark 5.2.4.3: A disconnected NPBIB design based on GD association scheme with $v = 2 \times 4$ can be obtained as: Select any two elements from row 1 to form one sub-block. Now from row 2, select two elements from the same columns as used to form 1st sub-block to form the 2nd sub-block. Putting these rwo sub-blocks together, form a bigger block. Repeat this procedure for all possible pairs of elements in row. Now, further 6 blocks are formed by taking any two elements from row 1 to form a sub-block and the elements from row 2 and columns other than those selected for row 1 to form another sub-block. Repeat this for all possible pairs of elements in rows. We get a NPBIB design with parameters v = 8, b_1 = 12, b_2 = 24, r = 3, k_1 = 4, k_2 = 2, λ_{11} = 2, λ_{12} = 3, λ_{21} = 2, λ_{22} = 0. The block contents of the design are
[(1, 3); (2, 4)]; [(5, 7); (6, 8)]; [(1, 5); (2, 6)]; [(3, 7); (4, 8)]; [(1, 7); (2, 8)]; [(3, 5); (4, 6)]; [(1, 3); (6, 8)]; [(5, 7); (2, 4)]; [(1, 5); (4, 8)]; [(3, 7); (2, 6)]; [(1, 7); (4, 6)]; [(3, 5); (2, 8)].

Combining these sets with NPBIB designs of same association scheme or with NBIB designs for same v, we get NPBIB designs based on same association scheme.

Using these methods of construction and trial and error solutions, 31 new NPBIB designs based on GD association scheme have been obtained. These are given in the Table 5.2.2 in Appendix.

➢ Nested block designs for making test treatments-control treatment comparisons

The nested block (NBIB and NPBIB) designs are useful for the experimental situations where the experimenter is interested in making all possible pairwise treatment comparisons with as high a precision as possible. However, there do occur experimental situations where the experimenter is interested in comparing several new treatments (called test treatments) with existing practice (a control treatment) with high precision and the comparisons among the test treatments are not of much importance. In the general block design setting, a lot of literature is available for obtaining efficient designs for such experimental situations. For more details, one may refer to Gupta and Parsad (2001) and references cited therein. No work seems to have been done for obtaining nested block designs for making comparisons between test treatments and a control treatment. Therefore, in this investigation we deal with the combinatorial aspects of nested block designs for making comparisons between test treatments and a control treatment.

It is well known that for a nested block design set up, the coefficient matrix of reduced normal equations for estimating the linear functions of treatment effects is the same as that obtained if the blocks are ignored in the analysis. The properties of the coefficient matrix of reduced normal equations are completely determined by the treatments versus sub-blocks incidence matrix. From this, it follows that the arrangement of treatments in blocks is of no consequence. Therefore, a nested balanced treatment sub-bock (NBTSB) design that estimates all test treatments versus control treatment contrasts with the same variance can always be constructed if there exists a balanced treatment block (BTB) design of Jacroux and Majumdar (1989). To be clearer, consider a BTB design in v tests and a control arranged in $b_2 = qb_1$ blocks of size k_2 each. Let each of the test treatments be replicated r times and the control treatment be replicated r_0 times. Regroup b_2 blocks in b_1 sets such that there are q blocks of the BTB design in each set. Take the sets as blocks and the blocks of the original design as sub-blocks. The above procedure yields a NBTSB design in which v test treatments and a control treatment are arranged in b_1 blocks of size k_1 each, there being q sub-blocks of size $k_2 = k_1/q$ within each block. The other parameters of the NBTSB design are r, r_0, $\lambda_2 = \sum_{j'=1}^{b_2} n_{2ij'} n_{2i'j'}$; $\forall\, i \neq i' = 1, 2, \ldots, v$ and $\lambda_{20} = \sum_{j'=1}^{b_2} n_{20j'} n_{2ij'}$ $\forall\, i = 1, 2, \ldots, v$, where $n_{tj'}$ is the number of times treatment t occurs in sub-block j'; $t = 0, 1, \ldots, v$; $j' = 1, 2, \ldots, b_2$. However, in this arrangement, the characterization of the coefficient matrix of the reduced normal equations for estimating treatment effects using the block classification ignoring sub-blocks is of no consequence. The property of variance balance may also be desirable on the block classification ignoring sub-blocks, particularly when inference is required on the characters that are observed on the blocks. Therefore, here we concentrate on combinatorial aspects of nested block designs in which the block classification ignoring sub-blocks leaves a BTB design and the sub-block classification also forms a BTB design. Such designs have been termed nested balanced treatment block (NBTB) designs. An NBTB design will be called a nested balanced treatment incomplete block (NBTIB) design if block classifications as well as sub-block classifications ignoring the other classification give a balanced treatment incomplete block (BTIB) design. The parameters of the NBTB designs will be denoted by v, b_1, b_2, r, r_0, k_1, k_2, λ_1, λ_{10}, λ_2, λ_{20}, where r and r_0 are respectively the replications of the test treatments and control treatment and $\lambda_1 = \sum_{j=1}^{b_1} n_{2ij} n_{2i'j}$; $\forall\, i \neq i' = 1, 2, \ldots, v$ and $\lambda_{10} = \sum_{j=1}^{b_1} n_{20j} n_{2ij}$ $\forall\, i = 1, 2, \ldots, v$, where n_{tj} is the number of times treatment t occurs in block j ; $t = 0, 1, \ldots, v$; $j = 1, 2, \ldots, b_1$.

Some methods of construction of NBTB designs are given. Once the designs are obtained, the next question arises "How efficient are these designs with respect to treatment contrasts of interest?". For studying the efficiencies of these designs, we make use of the results of A-optimality of general block designs for comparing test treatments with a control. The block [sub-block] classification is ignored for studying the efficiency of the design with the sub-block [block] classification. For this purpose, we consider **D** (v, b_1, b_2, k_1, k_2) as the class of all connected nested block designs in which v tests and a control are arranged in b_1 blocks of size k_1 each, there being k_1 / k_2 sub-blocks of size k_2 nested within each block. We make use of the sufficient condition for establishing the A-optimality of BTB designs obtained by Jacroux and Majumdar (1989). The sufficient condition gives the lower bound to the trace of the variance-covariance matrix of all the test treatments versus control treatment contrasts. A design that attains the lower bound is termed A-optimal. The condition is given in result 5.2.3.1.

Result 5.2.3.1: An NBTB design is A-optimal in the class of all designs with the same values of v, b_1, b_2, k_1, k_2 if

$$g(t_h, s_h) = \min\{g(x_h, z_h);\ (x_h, z_h) \in \Delta_h\} \quad \forall\, h = 1, 2 \qquad (5.2.3.1)$$

where $\Delta_h = \{(x_h, z_h);\ x_h = 0, 1, \ldots, \mathrm{int}[k_h/2] - 1;\ z_h = 0, 1, \ldots, b_h$ with $z_h > 0$, when $x_h = 0\}$ $\forall\, h = 1, 2$, and where

$$g(x_h, z_h) = v/A(x_h, z_h) + v(v-1)^2/B(x_h, z_h),$$

with $A(x_h, z_h) = (k_h e_h - g_h)/k_h$; $B(x_h, z_h) = (c_h - p_h e_h + g_h)/k_h$ $\forall\, h = 1, 2$ and
$e_h = b_h x_h + z_h$; $g_h = b_h x_h^2 + 2x_h z_h + z_h$; $c_h = v b_h k_h (k_h - 1) + v b_h V_h (v - 2k_h + vV_h)$;
$p_h = v(k_h - 1) + k_h - 2vV_h$; $V_h = \mathrm{int}[(b_h k_h)/v b_h]$. Here int[.] denotes the greatest integer function.

It is not possible to give a general method of construction which yields an A-optimal design in D (v, b_1, b_2, k_1, k_2) by satisfying the condition given in (5.2.3.1). Hence, we adopt the indirect approach of using the A-efficiency criterion, considered by Stufken (1987), to obtain A-optimal NBTB designs. The A-efficiency is the ratio of the A-value of a hypothetical A-optimal design whose criterion value given in (5.2.3.1) is minimum for making test treatments-control treatment comparisons in a given class of designs, to the A-value of the design whose A-efficiency is to be obtained in the same class of designs. Here, A-value is the trace of the variance-covariance matrix of the estimated treatment contrasts of interest. A-efficiencies for the block designs are obtained by taking $h = 1$ in result (5.2.3.1) and by taking $h = 2$ to get the same for the sub-block designs. Further, a design {either block or sub-block design or both} is A-optimal if the A-efficiency is 1.0000.

Methods of construction of NBTB designs

Some methods of construction of NBTB designs based on BTIB designs, NBIB designs and initial block solutions. In most of these methods, sub-blocks give a BTIB design. The block classification ignoring sub-blocks may be a BTIB design or a BTB design.

Method 5.2.5: Let there exist an NBIB design with parameters v', b_1', b_2', r', k_1', k_2', λ_1', λ_2' such that $k_1'/ k_2' = q$. Adding the control treatment once to each of the sub-blocks of the NBIB design, we get an NBTIB design with parameters $v = v'$, $b_1 = b_1'$, $b_2 = b_2'$, $r = r'$, $r_0 = b_2'$, $k_1 = k_1' + q$, $k_2 = k_2' + 1$, $\lambda_1 = \lambda_1'$, $\lambda_{10} = qr$, $\lambda_2 = \lambda_2'$, $\lambda_{20} = r$.

Method 5.2.6: Suppose there exists a BTIB design with parameters v', b', r', r_0', k', λ', λ_0', where the symbols have their usual meaning. For more details on BTIB designs one may refer to Gupta and Parsad (2001) and references cited therein. Let there also exist an NB(I)B design with parameters k', b_1^*, b_2^*, r^*, k_1^*, k_2^*, λ_1^*, λ_2^*. Then writing each of the block contents of the BTIB design as an NB(I)B design, we get an NBTIB design with parameters $v = v'$, $b_1 = b'b_1^*$, $b_2 = b'b_2^*$, $r = r'r^*$, $r_0 = r_0'r^*$, $k_1 = k_1^*$, $k_2 = k_2^*$, $\lambda_1 = \lambda'\lambda_1^*$, $\lambda_{10} = \lambda_0'\lambda_1^*$, $\lambda_2 = \lambda'\lambda_2^*$, $\lambda_{20} = \lambda_0'\lambda_2^*$.

Remark 5.2.6.1: A BTIB design with parameters v', b', r', $r_0' = b'$, k', λ', $\lambda_0' = r'$, can be obtained by adding a control treatment once to each of the blocks of a BIB design v', b', r', $k' - 1$, λ'. Now, let there exist a BIB design with parameters v', b', r', $k' = 3$, λ' and j^{th} block contents as (x_{1j}, x_{2j}, x_{3j}), $j = 1, 2, ..., b'$. On adding the control treatment 0 once to each of the b' blocks, we get a BTIB design with parameters v', b', r', $r_0' = b'$, $k' = 4$, λ', $\lambda_0' = r'$. Let $(x_{1j}, x_{2j}, x_{3j}, 0)$ denote the j^{th} block contents of the BTIB design, $j = 1, 2, ..., b'$. Now arrange $(x_{1j}, x_{2j}, x_{3j}, 0)$ in three blocks in the following manner:

$$[(x_{1j}, 0); (x_{3j}, x_{2j})]; [(x_{2j}, 0); (x_{3j}, x_{1j})]; [(x_{3j}, 0); (x_{1j}, x_{2j})]$$

Repeating this process for each of the b' blocks of the BTIB design, we get an NBTIB design with parameters $v = v'$, $b_1 = 3b'$, $b_2 = 6b'$, $r = 3r'$, $r_0 = 3b'$, $k_1 = 4$, $k_2 = 2$, $\lambda_1 = 3\lambda'$, $\lambda_{10} = 3r'$, $\lambda_2 = \lambda'$, $\lambda_{20} = r'$.

Method 5.2.7: Let there exist an NBIB design with parameters as $v + \alpha$, b_1, b_2, r, k_1, k_2, λ_1, λ_2. Let the treatments be denoted by 1, 2, ..., v, $v + 1$, $v + 2$, ..., $v + \alpha$. On merging the treatments $v + 1$, $v + 2$, ..., and $v + \alpha$ to the $(v + 1)^{th}$ treatment and calling this $(v + 1)^{th}$ treatment a control treatment, we get an NBTIB design with parameters as v, b_1, b_2, r, $r_0 = \alpha r$, k_1, k_2, $\lambda_1 = \lambda_1$, $\lambda_{10} = \alpha\lambda_1$, $\lambda_2 = \lambda_2$, $\lambda_{20} = \alpha\lambda_2$.

Note: This method can produce designs with useless sub-blocks, that is, sub-blocks containing only the control and which, therefore, provide no information for the experiment. This may lead to sacrificing the efficiency for balance. However, a small sacrifice in efficiency can be a worthwhile trade for the case of interpretation offered by balance. However, this method should not be used when it produces more than a very few useless blocks, nor when it produces useless blocks of large size.

A catalogue of designs obtainable from these methods of construction along with lower bounds to A-efficiencies has been prepared. Sometimes, NBTIB designs require large number of replications, which the experimenter may not be able to afford. To deal with such experimental situations, some methods of nested block designs with smaller number of experimental units have been obtained. These are described in the sequel.

Method 5.2.8: Consider a group divisible association scheme with v = mn treatments arranged in $m \times n$ array as

$$\begin{array}{ccccc} 1 & m+1 & 2m+1 & \cdots & m(n-1)+1 \\ 2 & m+2 & 2m+2 & \cdots & m(n-1)+2 \\ 3 & m+3 & 2m+3 & \cdots & m(n-1)+3 \\ \vdots & \vdots & \vdots & \vdots & \vdots \\ m & 2m & 3m & \cdots & mn \end{array}$$

Let '0' denotes the control treatment. Then the block layout

$$\begin{array}{l} [(0,1), (0, m+1), (0, 2m+1), ..., (0, m(n-1)+1)]; \\ [(0,2), (0, m+2), (0, 2m+2), ..., (0, m(n-1)+2)]; \\ \qquad\quad \vdots \qquad \vdots \qquad \vdots \\ [(0,m), \ (0, \ 2m), \ (0, \ 3m), \ ..., \ (0, \ mn)]. \end{array}$$

is a NPBTIB design with the following parameters, $v = 2m$, $b_1 = m$, $b_2 = 2m$, $r = 1$, $k_1 = 4$, $k_2 = 2$, $r_0 = 2m$, $\lambda_{10} = 2$, $\lambda_{11} = 1$, $\lambda_{12} = 0$, $\lambda_{20} = 1$, $\lambda_{21} = 0$, $\lambda_{22} = 0$.

A-efficiency only for bigger blocks are computed. The design is A-optimal with respect to sub-blocks as it is a minimally connected design in sub blocks. A –efficiency of the designs in sub-blocks decreases as the number of test treatments increase.

5.3 Block designs for making test treatments-control treatment comparisons

Two new methods of construction of block designs for making test treatment control comparisons which require smaller number of experimental units have been obtained and are described in the sequel.

Method 5.3.1: This method relates to construction of block designs for making test treatments (v test treatments) versus a control treatment comparisons that are combinatorial balance in respect to concurrence of test treatments and control treatment and partially balanced in concurrences of the test versus test treatments where test treatments are defined in GD association scheme.

Consider a $m \times 2$ Group divisible association scheme. Form first v blocks of size 2 each by taking control treatment and one of the v test treatments in a block so that no treatment is chosen more than once. Now form next m blocks of size 2 each by taking rows of the association scheme as blocks. The block layout of the design is

$(0,1), (0,2), (0,3), ..., (0, v)$;
$(1, m+1), (2, m+2), (3, m+3), ..., (m, 2m)$.

The parameters of the group divisible treatmnent design so obtained are $v = 2m, b = 3m, r = 2, k = 2, \lambda_0 = 1, \lambda_1 = 1, \lambda_2 = 0$.

The lower bound to the A-efficiency of the designs obtainable from this method of construction were obtained using the lower bound to the A-efficiency of group divisible treatment designs given in Hedayat, Jacroux and Majumdar (1988). The lower bound to the A-efficiency decreases as the number of test treatments increase.

Method 5.3.2: Let $v = m(m - 1)/2$ test treatments are defined in a Triangular Association scheme. Take each row and the control to form a block. By this procedure we can get m blocks of size m each. The design obtained is a partially balanced treatment incomplete block design with parameters: $v = m(m - 1)/2, b = m, r = 2, k = m, r_0 = m, \lambda_0 = 2, \lambda_1 = 1, \lambda_2 = 0$.

A catalogue of designs obtainable from this method of construction along with lower bounds to A-efficiency (computed using the lower bound to A-efficiency of group divisible treatment designs given in Hedayat, Jacroux and Majumdar (1988)) is given in Table. 5.3.1 in Appendix. The designs obtained from this method have very good efficiency to compare test treatments with the control treatment.

5.4 Semi-Latin squares

In many agricultural experiments, there are two sources of heterogeneity in experimental units that may influence the response variable. To deal with such experimental situations, row-column designs such as Latin square designs (LSD), Youden square designs (YSD), generalized Youden designs (GYD), Youden type designs, etc. have been developed in the literature. For details on these designs, a reference may be made to Shah and Sinha (1996).

Most of the row-column designs developed have only one unit in each row column intersection. There, however, do occur situations, where there are more than one unit in each row-column intersection. For such experimental situations, Semi-Latin square designs were introduced.

A semi-Latin square is an arrangement of $v = nk$ symbols in n^2k units arranged in n rows and n columns, each row-column intersection contains $k > 1$ units and each symbol occurs exactly once in each row and column.

A semi-Latin square with above parameters is represented by $(n \times n)/k$. A design based on Semi-Latin square is called a Semi-Latin square design. In fact semi-Latin squares are also a type of designs with nested structures. Here, the row-column intersection is called the blocks and the treatments are orthogonal to the rows and columns strata in semi-Latin squares. Therefore, it is important that the block design at the bottom stratum (ignoring row-column) classifications has some optimal properties.

Semi-Latin squares are useful for situations when the number of levels of both the nuisance factors (factors of heterogeneity) is same. A large number of such experimental situations in consumer testing, glass house crops, residual effect experiments, sugar beet trials, food industry, etc. have been given by Bailey (1992). Some of these experimental situations are described in the sequel.

Experimental Situation 5.4.1: (Organoleptic Evaluation of Food Products). Consider a food sensory experiment where 12 food items are to be compared. The experiment will be conducted in 4 sessions. There are 4 panelists and each of them will taste 3 food items at each session. In this case a semi-Latin square with 4 rows, 4 columns with each row-column intersection having size 3 can be used. Here the rows, columns and symbols represent sessions, panelists and food items respectively. This ensures that each of the 12 food items are organoleptically evaluated by each panelists, each food item was evaluated in each of the sessions and food items were orthogonal with respect to panelists and session.

Experimental Situation 5.4.2: (Residual Effects: Bailey, 1992). The effects of some treatments may persist during the next experiment. This is particularly true if the experimental units are long lived such as trees, animals, but it may also occur on arable crops if the treatments affect the soil directly, for example, by inhibiting or encouraging nematode growth. Suppose that in last year five varieties of potato were compared in 5 replicates. This year, a single standard variety is grown and ten chemicals are tested for their ability to control nematodes. Last year's varieties will affect the number of nematodes in the soil, but it is assumed that there is no interaction between those varieties and this year's chemicals.

Chemicals can be applied to smaller areas of land than varieties, so each plot from last year is split into two for the chemicals. Use of a semi-Latin square, with rows representing old replicates, columns varieties and symbols chemicals, ensures that each chemical occurs once on soil that had each variety last year.

The semi-Latin squares can be useful for crop sequence experiments, wherein treatments applied on the first season crop has a residual effect on the soil and in turn affect the effects of the treatments applied in the subsequent season.

A semi-Latin square may also be used for obtaining fractional factorial plans with three factors say R (rows), C (columns) and S (symbols) with *n*, *n* and *nk* levels respectively. Then a one to one correspondence between a semi-Latin square and fractional replication plan gives a fractional factorial plant for a $n \times n \times nk$ factorial in n^2k runs.

Preece and Freeman (1983), Bailey (1988, 1992), Bedford and Whitaker (2001) have given several methods of construction of semi-Latin squares. Preece and Freeman (1983) have given a relationship between double resolvability of semi-regular group divisible designs and semi-Latin squares. They have also given a catalogue of semi regular group divisible designs given in Clatworthy (1973) that can be converted into semi-Latin squares. They indicated that by rearrangement of treatments in a semi regular group divisible designs, no semi-Latin square can be obtained for $k=2, n=6, 8, 10$ and $k=3, n=6, 9, 10$. In the present investigation, we have given a method of construction of semi-Latin square designs whose bottom stratum is a semi regular group divisible design and the semi-Latin squares for $k=2, n=6, 8, 10$ can also be obtained through this method of construction.

Method 5.4.2: A $(n \times n)/2$ semi-Latin square with $v=2n,\quad r=n,\ c=n,\ k=2$ can be constructed by developing the initial column

$$\left\{\begin{bmatrix} (1, 2n \\ (2, 2n-1) \\ \vdots \\ (n, n+1) \end{bmatrix} \qquad \begin{matrix} \text{mod } (2n) \\ \text{with steps as } 2 \end{matrix}\right\}$$

The bottom stratum design is a doubly resolvable semi-regular group divisible design with parameters $v=2n, b=n^2, r=n,\ k=2, \lambda_1=0, \lambda_2=1, m=2,\ n=n$.

Example 5.4.1: For $v=12, i.e.,\ n=6,$ we get a $(6 \times 6)/2$ semi-Latin square is

$$\begin{bmatrix} (1,12) & (3,2) & (5,4) & (7,6) & (9,8) & (11,10) \\ (2,11) & (4,1) & (6,3) & (8,5) & (10,7) & (12,9) \\ (3,10) & (5,12) & (7,2) & (9,4) & (11,6) & (1,8) \\ (4,9) & (6,11) & (8,1) & (10,3) & (12,5) & (2,7) \\ (5,8) & (7,10) & (9,12) & (11,2) & (1,4) & (3,6) \\ (6,7) & (8,9) & (10,11) & (12,1) & (2,3) & (4,5) \end{bmatrix}.$$

The method of construction is very general in nature and gives doubly resolvable semi-regular designs and semi-Latin squares. Three new semi-Latin squares with $k=2, n=6$; $k=2, n=8$ and $k=2, n=10$ which cannot be constructed by rearrangement of the designs given in Clatworthy (1973) can also be obtained. Further, the semi-Latin squares with $k=2$ and $n>10$ can also be obtained.

5.5 Minimally connected designs with some extra observations

In NARS some experiments are conducted to study the effect of soil erosion on crop yield. In such experiments, the soil erosion is done artificially at different levels in different experimental plots and their effect is seen on the yield. Artificial creation of soil erosion is quite difficult to be made. Moreover, this also amounts to destroying some of the upper layers of the soil from a part of the land. Therefore, it is always better to plan such experiments in the minimum possible number of experimental units. To ensure that all pairwise treatment comparisons are possible in a block design, the minimum number of experimental units required is equal to one less than the sum of the number of blocks and treatments. A design in minimal number of experimental units that provides all possible pairwise treatment comparisons is called a minimally connected design. For such experimental situations, the minimally connected designs, minimally connected designs with some extra observations may be useful. The basic objection to this kind of designs with minimum number of observations in agricultural experimentation is that they do not provide an estimate of error. Therefore, to get an estimate of error, some modifications in these designs are required to be made, possibly by adding some more experimental units.

Keeping these problems in mind, a catalogue of block designs with $n = v+b-1+i$, $i=1, 2, 3$ observations has been prepared, where v is the number of treatments; b the block size; k is the block size and n is the total number of experimental units. Block contents along with lower bounds to A- and D-efficiencies are also given. The lower bounds to A-efficiencies for making test treatments-control treatment comparisons are being obtained. Further, the work on obtaining efficient designs with $n = v+b-1+i$, $i=4, 5, 6, 7$ and 8 is also in progress.

5.6 Web Design and E-advisory services

The design resources server has been initiated and launched on the web site of the Institute. The main objective of this design resources server is to develop WEB DESIGN and E-advisory services in NARS. At present the material on binary balanced block designs and designs for making test treatments-control treatment(s) comparisons and the Electronic Book on Design and Analysis of Agricultural Experiments is available on this site. A discussion board has also been created. The suggestions for improving and inclusion of new material in this web site are invited. Some screens are given in the Sequel:

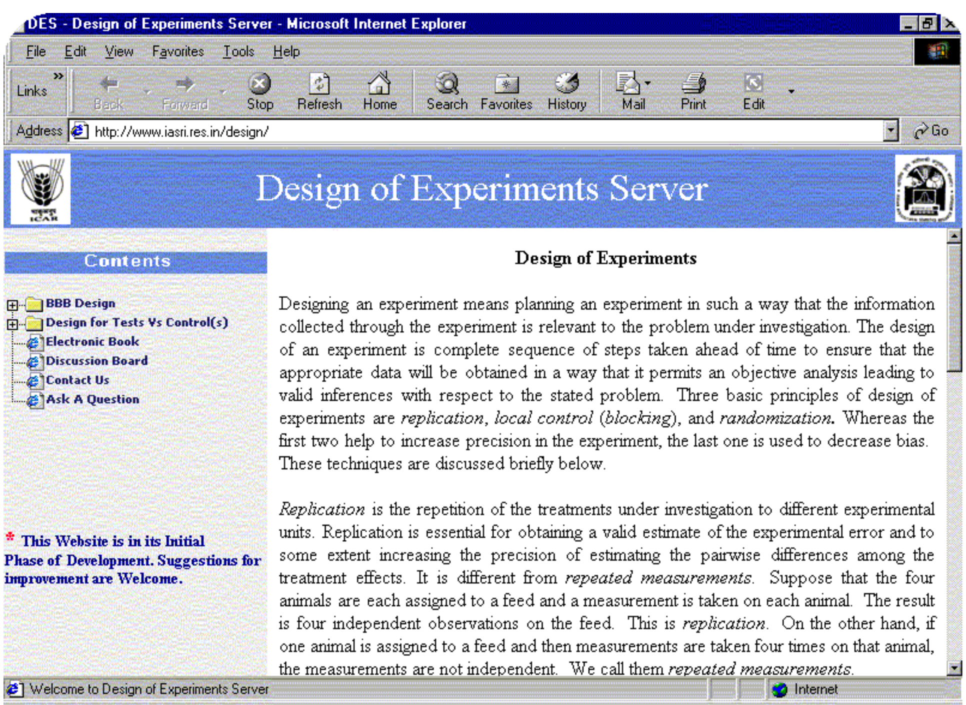

Figure 5.6.1: First Screen of Design of Experiments Server

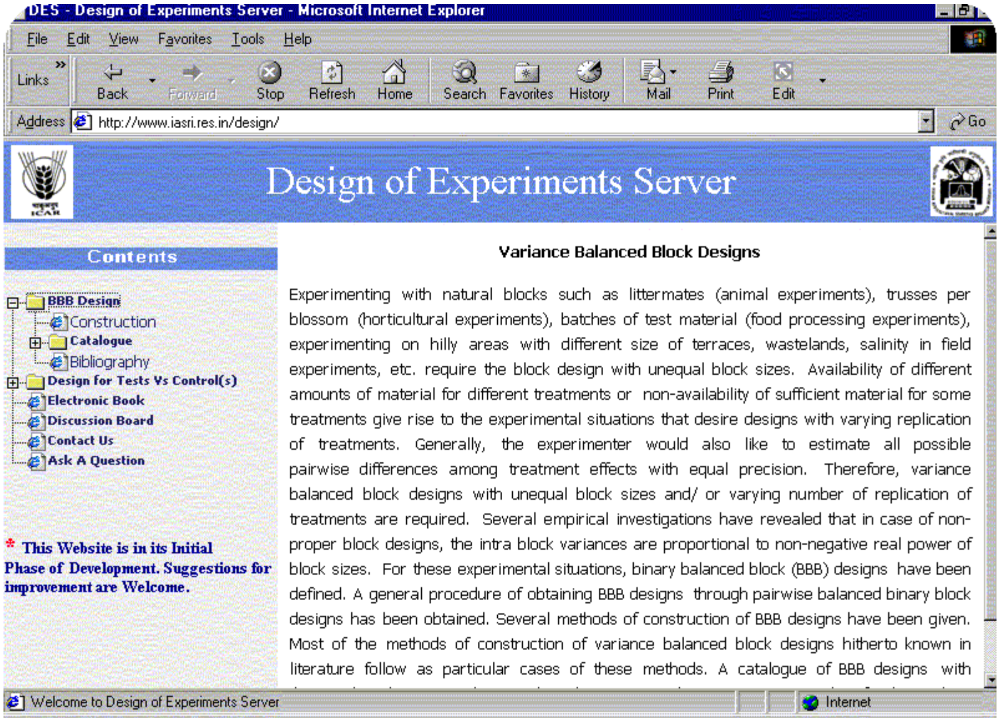

Figure 5.6.2: Binary Balanced Block Designs

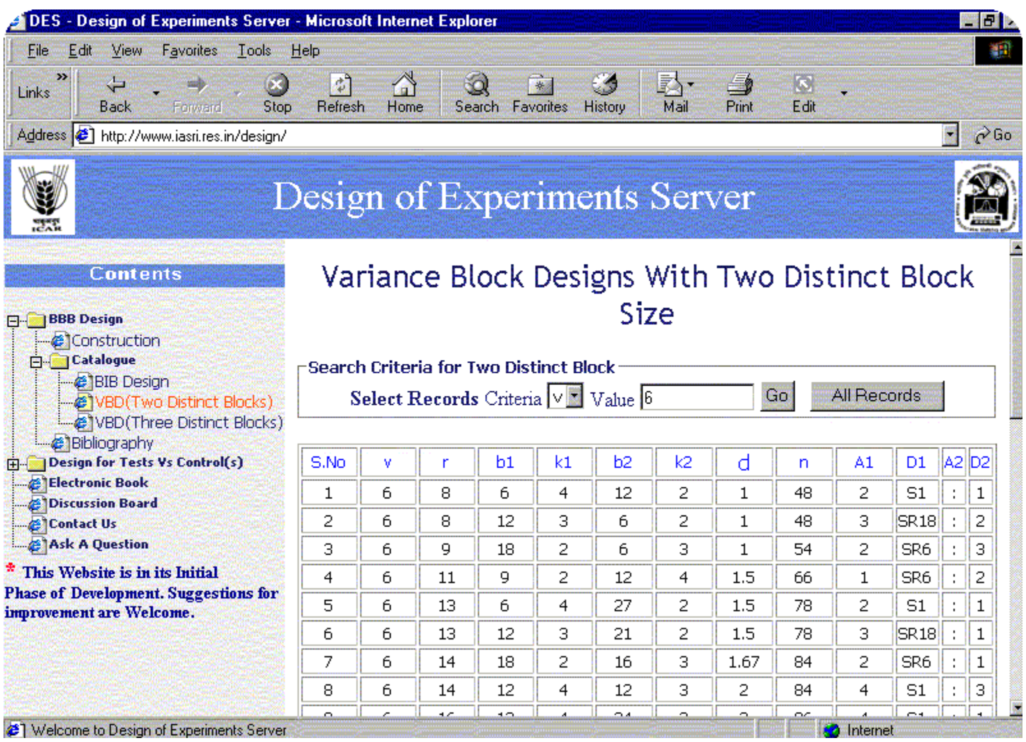

Figure 5.6.3: Catalogue of Binary Balanced Block Designs with two Distinct Block Sizes

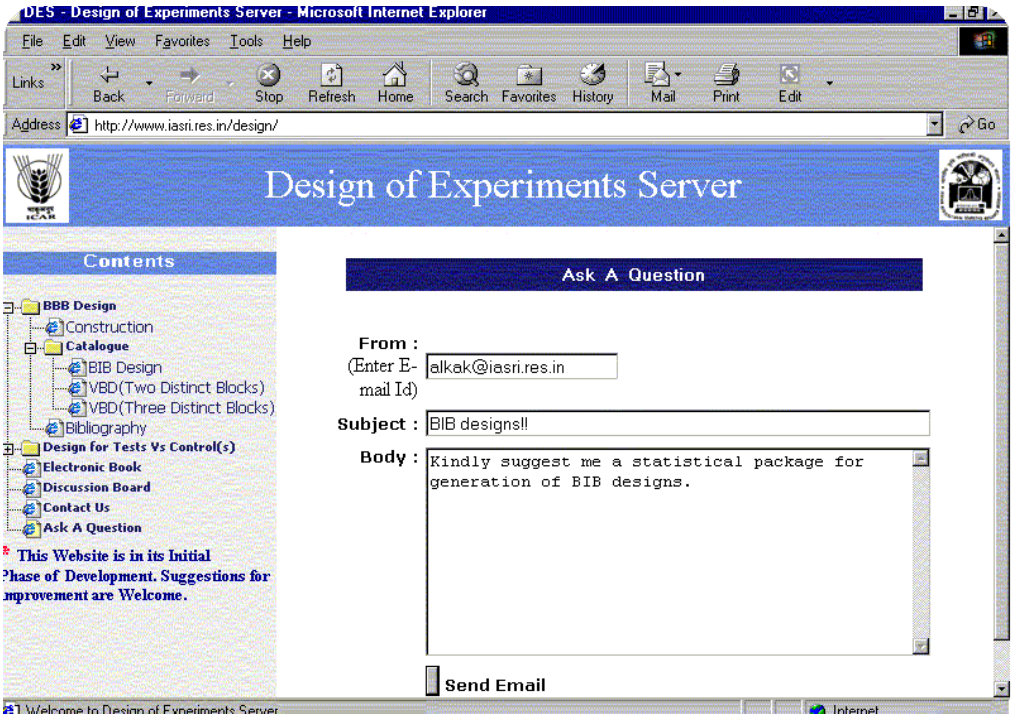

Figure 5.6.4: Sending a mail for Asking a Question

5.7 Software packages developed and commercialized

Following software packages were finalized. These packages were released by Professor Mangala Rai, Secretary, DARE and Director General, ICAR on the Institute's Annual Day July 02, 2005.

I. **Statistical Package for Augmented Designs (SPAD)** is useful for designing agricultural experiments conducted for comparing existing practices / check varieties, called controls, with new practices / varieties / germplasm collections, called tests, where the experimental material for the tests is limited and it is not possible to replicate them in the design. The package generates a randomized layout of an augmented randomized complete block design and augmented complete block design with equal or unequal block sizes. The optimal replication number of the control treatments in every block is obtained by maximizing the efficiency per observation for making tests versus controls comparisons. User has a flexibility to choose the replication number of the control(s) in each of the blocks. The package generates randomized layout of the design as per the procedure of Federer (1956), which is generally overlooked while conducting such experiments. The package also performs the analysis of data generated from augmented block designs (complete or incomplete). The treatment sum of squares is partitioned into different components of interest *viz.* (i) among test treatments, (ii) among control treatments and (iii) among test treatments and control treatments. Multiple comparison procedures for making all possible pairwise treatment comparisons can also be employed through this package. A null hypothesis on any other contrast of interest can also be tested. A window depicting the different features is given in Figure 5.7.1.

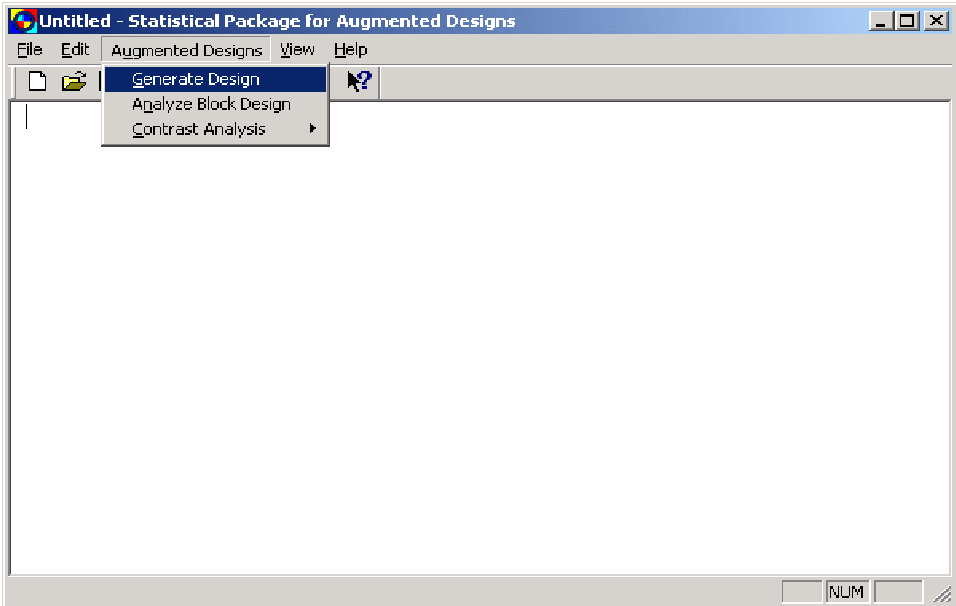

Figure 5.7.1: Window Depicting Features of SPAD

II. Statistical Package for Agricultural Research (SPAR 2.0) is useful for the analysis of experimental research data in Plant Breeding and Genetics. The package consists of eight modules (i) Data Management (ii) Descriptive Statistics (iii) Estimation of Breeding values (iv) Correlation and Regression Analysis (v) Variance and Covariance Components Estimation (vi) Stability Analysis (vii) Multivariate Analysis (viii) Mating Design Analysis. Flash Screen of SPAR 2.0 is A window depicting the different features is given in Figure 5.7.2. These features are discussed in the sequel.

Figure 5.7.2: Flash Screen of SPAR 2.0

The cost of each of these packages is Rs. 1000.00 + Rs. 50.00 for Postage for National Agricultural Research System (NARS) and Rs. 2500.00 + Rs. 50.00 for Postage for the organizations outside NARS. For each additional license, the organization from NARS has to pay Rs. 500.00 and the organization from outside the NARS has to pay Rs. 1000.00.

5.8 Advisory services

Advisory services were pursued rigorously. Details of some of them are given below:

➢ Fertilizer Response Ratios

For the task force on Balanced use of fertilizers, fertilizer response ratios were obtained using the data pertaining to an experiment conducted to find out the response of nitrogen (N), phosphorus (P) and potassium (K) under different sub agro-ecological zones/NARP zones under the aegis of Project Directorate of Cropping Systems Research, Modipuram 1999-2000. The experiment is being conducted with following five treatments:

Treatment Number	**Treatment Details**
T_1	Control
T_2	Recommended Dose of N
T_3	Recommended Dose of NP
T_4	Recommended Dose of NK
T_5	Recommended Dose of NPK

Data generated from this experiment for the years 1999-2000, 2000-2001, 2001-2002 and 2002-2003 has been used for the computation of 8 different Fertilizer Response Ratios *viz*. N over control; NP over control; NK over control; NPK over control; P over N; P over NK; K over N; K over NP. Response ratios for cereals, pulses, oilseeds and foodgrains at state and country level have been evaluated for different nutrients and their combinations and response ratio at country level is as given in Table 5.8.1 in Appendix. The percentage increase in yield were also obtained and are given in Table 5.8.2 in Appendix.

The response ratios percentage increase in production due to a specified micronutrient over and above the recommended dose of NPK have been obtained using the data from the experiments conducted to study the effect of removal of certain location specific constraints in improving the productivity of major crops and cropping systems under existing farmers' field conditions since1990-2000 (PDCSR, Modipuram).

➢ Contrast Analysis

- Dr. Dinesh Kumar, Senior Scientist, Division of Agronomy, IARI, New Delhi was advised on the analysis of experimental data conducted for standardization of nitrification inhibiting property of Neem oil coated urea for Kharif Rice. 16 treatment were tried in the experiment that were all possible combinations of 5 sources of nitrogen *viz.* prilled urea, 500 ppm oil coated urea, 1000 ppm oil coated urea, 2000 ppm oil coated urea, 5000 ppm oil coated urea and three doses *viz.* 50, 100, 150 kg/ha and one absolute control. The experiment was conducted using a randomized complete block design. The analysis was carried out using the concepts of contrast analysis.

- Sh. Naval Kishor Sepat, Ph.D. (Agronomy) was advised on the analysis of yield data pertaining to an experiment conducted to study the direct and residual effect of organic amendments and levels of NPK on soil health and productivity of rice-wheat cropping system. The experiment was conducted using a split plot design with four main plot treatments as control, green manure/green leaf manure, FYM @10 t/ha and crop residue @5 t/ha. The sub plot treatments for Kharif rice were control (no NPK), 50%, 100% and 150% recommended doses of NPK each replicated twice in a main plot. For rabi wheat, there were 7 distinct treatment combinations viz. control, residual effects of treatments applied as 50%, 100% and 150% NPK to Kharif rice and direct application of 50%, 100% and 150% recommended doses of NPK to Rabi crop. The analysis was carried out using the concepts of contrast analysis.

➢ Response Surface Designs

- A second order rotatable response surface design with orthogonal blocking for 4 factors each at 5 equispaced levels in 30 design points arranged in three blocks each of size 10 has been recommended for an experiment related to osmotic dehydration of the Aonla planned to obtain the optimum combination of levels of solution to sample ratio, concentration of sugar solution, revolutions per minute and temperature of osmosis at Division of Post Harvest Technology, Indian Agricultural Research Institute, New Delhi.
- A second order response surface design with 4 factors each at 4 equispaced levels in 80 design points has been suggested for an experiment related to development of rotating screen grader for selected orchard fruit crops (Ber, Lemon and Aonla) was planned to obtain the optimum combination of levels of rotating speed, diameter of screen, input and length of screen at Division of Agricultural Engineering, Indian Agricultural Research Institute, New Delhi.

➢ Variance Components Estimation

Sh. Rakesh Kumar Ph.D. Student from Department of Animal breeding, CCS HAU, Hissar was advised on the analysis of data pertaining to an experiment conducted with 100 genotypes/hybrids of Field Pea conducted in a simple lattice design at 5 environments. He was also advised on obtaining the genetic parameters *viz.* genotypic variance, phenotypic and heritability coefficient. For this purpose the procedure developed under the project a Diagnostic Study of Design and Analysis of Field Experiments was used. He was also advised on the stability analysis, path analysis and genetic divergence analysis.

6. Executive Summary

❖ α-designs are essentially resolvable block designs. In a resolvable block design, the blocks can be grouped such that in each group, every treatment appears exactly once. Resolvable block designs allow performing an experiment with one replication at a time. For example, field trials with large number of crop varieties cannot always be laid out in a single location or a single season. Therefore, it is desired that variation due to location or time periods may also be controlled along with controlling within location or time period variation. This can be handled by using resolvable block designs. Here, locations or time periods may be taken as replications and the variation within a location or a time period can be taken care of by blocking. In an agricultural field experiment, the land may be divided into a number of large areas corresponding to the replications and then each area is subdivided into blocks. These designs are also quite useful for varietal trials conducted in the National Agricultural Research System (NARS) and will help in improving the precision of treatment comparisons. A critical look at the experimentation in the NARS reveals that α-designs have not found much favour from the experimenters. It may possibly be due to the fact that the experimenters find it difficult to lay their hands on α-designs. The construction of these designs is not easy. An experimenter has to get associated with a statistician to get a randomized layout of this design. For the benefit of the experimenters, a comprehensive catalogue of α-designs for $6 \le v(=sk) \le 150$, $2 \le r \le 5$, $3 \le k \le 10$ and $2 \le s \le 15$ has been prepared along with lower bounds to A- and D- efficiencies and generating arrays. The layout of these designs along with block contents has also been prepared. The designs obtained have been compared with corresponding square lattice, rectangular lattice, resolvable two-associate class partially balanced incomplete block {PBIB (2)} designs and the α-designs obtainable from basic arrays given by Patterson, Williams and Hunter (1978, *J. Agric. Sci.*, **90**, 395-499). Eleven designs are more efficient than the corresponding resolvable PBIB (2) designs (S11, S38, S69, S114, LS8, LS30, LS54, LS76, LS89, LS126 and LS140). It is interesting to note here that for the PBIB (2) designs based on Latin square association scheme, the concurrences of the treatments were 0 or 2 and for singular group divisible designs the concurrences are either 1 or 5. Further all the designs LS8, LS30, LS54, LS76, LS89, LS126 and LS140 were obtained by taking two copies of a design with 2-replications. 10 designs were found to be more efficient than the designs obtainable from basic arrays. 48 designs (29 with k = 4 and 19 with k = 3) are more efficient than the designs obtainable by dualization of basic arrays. 25 designs have been obtained for which no corresponding resolvable solution of PBIB(2) designs is available in the literature. The list of corresponding resolvable PBIB(2) designs is S28, S86, SR18, SR41, SR52, SR58, SR66, SR75, SR80, R42, R70, R97, R109, R139, T14, T16, T20, T44, T48, T49, T72, T73, T86, T87 and M16. Here X# denotes the design of type X at serial number # in Clatworthy, W. H. (1973, *Table of two-associate partially balanced designs*. NBS Applied Maths Series No. **63**. Washington D.C.).

- In some experimental situations, the user may be interested in getting designs outside the above parametric range. To circumvent such situations, a β- Version of user friendly software module for the generation of α-designs has been developed. This module generates the alpha array along with lower bounds to A and D-efficiency. The α-array and the design is generated once the user enter the number of treatments (v), number of replications (r) and the block size (k). The module generates the design for any v, k, r provided v is a multiple of k. It also gives the block contents of the design generated.
- A nested block design is defined as two systems of blocks such that the second system of blocks is nested within the first system of blocks. These designs are quite useful in many experimental situations. For example, consider a field experiment conducted using a block design and harvesting is done block wise. Harvested samples are to be analyzed for their contents either by different technicians at same time or by a

technician over different periods of time. The variation due to technicians or time periods may be controlled by another blocking system. Technicians or time periods form a system of blocks that are nested within blocks. Such experimental situations are also common in post harvest value addition of horticultural and vegetable crops. Nested block designs are also quite useful in agricultural field experiments where the plots with similiar fertility occur in patches rather than in a uniform direction. Preece, D.A. (1967 Biometrika, 54, 479-486) was the first to introduce nested block designs and termed them as nested balanced incomplete block (NBIB) designs. In a NBIB design block classification ignoring sub-blocks is a balanced incomplete block (BIB) design and sub-block classification ignoring blocks is also a BIB design. We have prepared a complete catalogue of NBIB designs with number of replications $r \leq 30$. The catalogue contains a total of 299 designs. Out of 299 designs, 8 designs are non-existent. A new method of construction of NBIB designs has been obtained. Using this method and trial and error solutions, block layouts of 22 new NBIB designs have been obtained. The layout of 199 designs with block contents has been completed. The solution for the block layout for remaining 92 designs is unknown and the statisticians need to develop methods of construction of these NBIB designs. The designs catalogued have also been identified for 1-resolvable and 2-resolvable sets.

- ❖ A NBIB design may not exist for all parametric combinations or even if it exists may require a large number of replications, which the experimenter may not be able to afford. To deal with such situations, nested partially balanced incomplete block (NPBIB) designs have been introduced in the literature. Some new methods of construction of NPBIB designs based on group divisible association scheme have been given using these methods of construction, 31 new NPBIB designs based on group divisible association scheme with $r \leq 15$ have been obtained.
- ❖ Nested block (NBIB and NPBIB) designs are useful for experimental situations where the experimenter is interested in making all possible pairwise treatment comparisons with as high a precision as possible. However, there do occur experimental situations where the experimenter is interested in comparing several new treatments (called test treatments) with existing practice (a control treatment) with high precision and the comparisons among the test treatments are not of much importance. To deal with such situations, nested balanced treatment incomplete block (NBTIB) designs have been introduced. Some new methods of construction of NBTIB designs making use of NBIB designs, initial block solutions, etc. have been developed. A new method of construction of nested block designs for making test treatments-control treatment comparisons has been developed which yields minimally connected designs with respect to sub-blocks. The design with respect to bigger blocks is a group divisible treatment design.
- ❖ A new method of construction of efficient block designs for making test treatments-control treatment comparisons by making use of triangular association scheme has been developed. The number of replications of test treatments developed through this method is always 2.
- ❖ A new method of construction of semi-Latin squares based on initial column solution has been developed. This method yields semi-regular group divisible designs after ignoring the row and column classifications. Preece and Freeman (1983, *J. Royal. Statist. Soc.*, **28**, 154-163) reported that for $k = 2,\ n = 6, 8, 10$ could not be obtained by

rearrangement in Semi-regular group divisible designs. These three semi-Latin squares can be obtained from the proposed method of construction.

- A catalogue of block designs with $n = v+b-1+i$, $i=1, 2, 3$ observations has been prepared, where v is the number of treatments; b the block size; k is the block size and n is the total number of experimental units. Block contents along with lower bounds to A- and D-efficiencies are also given. The lower bounds to A-efficiencies for making test treatments-control treatment comparisons are being obtained.
- The design resources server has been initiated and launched on the web site of the Institute. The main objective of this design resources server is to develop a WEB DESIGN in NARS. At present material on binary balanced block designs and designs for making test treatments- control treatment comparisons along with Electronic Book on Design and Analysis of Agricultural Experiments are available on this site. A discussion board has been created.
- Advisory services have been pursued rigourously in the NARS. Alpha designs, second order response surface designs have been recommened. The sophisticated statistical analytical procedures like contrast analysis and variance component estimation have also been recommended.
- Fertilizer response ratios were obtained using the data pertaining to an experiment conducted to find out the response of N, P and K under different sub agro-ecological zones/NARP zones under the aegis of Project Directorate of Cropping Systems Research, Modipuram 1999-2000.
- Following softwares have been finalized and released commercially.

  1. **Statistical Package on Augmented Designs (SPAD)**
  2. **Statistical Package on Agricultural Research (SPAR) 2.0**

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➢ Publications

Research Papers: 11 (4 published, 3 Accepted for Publication and 4 Communicated/Under Revision); Popular Articles: 2; Book Chapters: 2.

(A) Research Papers
Published

1. Pramila Menon, Madhuban Gopal and **Rajender Prasad** (2005). Effects of chlorpyrifos and quinalphos on dehydrogenase activities and reduction of Fe^{3+} in the soils of two semi-arid fields of tropical India. *Agriculture Ecosystems Environment*, **108**, 73-83.
2. D.K.Panda, V.K.Sharma and **Rajender Parsad** (2005). Robustness of optimal block designs for triallel crosses experiments against interchange of a pair of cross. *Journal of Indian Society of Agricultural Statistics*, **59(1)**, 83-91.
3. Subhra Sarker, **Rajender Parsad** and V.K.Gupta (2005). Outliers in block designs for diallel crosses. *Metron-International Journal of Statistics*, **63(2)**, 177-191.
4. Abhijit Kar, Pitam Chandra, **Rajender Parsad** and S.K. Dash (2005). Mass transfer during osmotic dehydration of banana slices (*Dwarf Cavendish*). *Journal of Agricultural Engineering,* **42(3)**, 42-49.

Accepted for Publication

5. Abhishek Rathore, **Rajender Parsad** and V.K.Gupta. computer aided search of efficient block designs for making all possible pairwise treatment comparisons. *Journal of Statistics and Applications: A Publication of 'Forum of Interdisciplinary Mathematics'.*
6. **Rajender Parsad**, Sanpei Kageyama and V.K.Gupta. Use of complementary property of block designs in PBIB designs. *ARS Combinatoria.*
7. D.M. Kadam, D.V.K. Samuel and **Rajender Parsad**. Optimization of pre-treatments of solar dehydrated cauliflower. *Journal of Food Engineering.*

Communicated/Under Revision

8. **Rajender Parsad**, V.K.Gupta and R.Srivastava. Designs for cropping systems research *Journal of Statistical Planning and Inference. (Revised Version Submitted)*
9. Subrata Kumar Satpati, **Rajender Parsad** and V.K. Gupta. Efficient block designs for dependent observations: A computer aided search. *Communications-in-Statistics: Theory & Methods*.
10. Krishan Lal, **Rajender Parsad** and V.K. Gupta. Robustness of block designs on basis of pairwise treatment comparisons for the loss of observations. *Journal of Indian Statistical Association.*
11. Gunanand Choudhary, Jitendra Kumar, Suresh Walia, **Rajender Parsad** and Balraj S. Parmar. Development of controlled release formulations of Carbofuran and evaluation of their efficacy against Meloidogyne incognita**.** *Journal of Agricultural and Food Chemistry***.**

(B) Popular Article/Bulletin

1. A.K. Nigam, **Rajender Parsad** and V.K. Gupta. Design and Analysis of On-Station and On-Farm agricultural research experiments: A Revisit Workshop Series 0, IASRI, New Delhi.
2. R. Srivastava, **Rajender Parsad** and V.K. Gupta. Studies on Block Designs for Biological Assays. Workshop Series 2, IASRI, New Delhi.

(C) Book Chapters

1. V.K.Gupta and **Rajender Parsad**. Statistical Designing of Experiments with Emphasis on Hill Agriculture Research. Chapter 31 in Sustainable production from agricultural watersheds in NWH. Eds. HS Gupta.
2. **Rajender Parsad,** Anshu Dixit, P.K Malhotra and V.K. Gupta. Geoinformatics in Precision farming: An overview. In the Book entitled "*Geoinformatics Applications for sustainable development"*.

➢ Awards and Recognitions

- **Dr. D.N. Lal Memorial Lecture award** from Indian Society of Agricultural Statistics for the biennium 2004-2005.
- Nominated as **Member, Editorial Board** of the Journal of Indian Society of Agricultural Statistics.
- Nominated as **Member, Governing Body** of the Institute of Applied Statistics and Development Studies, Lucknow.
- **Special Invitee** to **Task Force on Balanced Use of Fertilizers**, Department of Agriculture and Co-operation, Ministry of Agriculture, Government of India. Discussed and presented the results obtained from the experiment conducted on farmers' fields to find out the response of nitrogen (N), phosphorus (P) and potassium (K) under different sub agro-ecological zones/NARP zones.
- **Member of the Committee** constituted **to finalize evaluation of Bt. Cotton entries into AICRP's**. The committee was chaired by Dr K.C.Jain, ADG and other members were Dr OP Dubey, ADG(OP), Dr BM Khadi, Direcor, CICR; Dr T P Rajendran, Project Co-ordinator of AICRP on Cotton; Dr Ananth Kumar, Principal Scientist, Indian Agricultural Research Institute. In this meeting the resolvable block designs (alpha, lattice, etc.) have been recommended for these trials. The spacing will depend upon the package of practices for the centre recommended and the experiment will be conducted in 3 replications with atmost 25 entries including 3 checks. There will be two separate experiments one for breeding trials (with regular spray) and other as plant protection trial (without any spray).
- **Chaired** a session of contributed Papers during the 8th Annual Conference of Society of Statistics, Computer and Applications held at Department of Statistics and Computer Science, Government Vidarbha Institute of Science and Humanities, Amravati during November 28-30, 2005.
- Prize for Special Contributions in Scientific Work Received during Hindi Chetna Mass on September 30, 2005.

➢ Conferences/ Workshops organized

- **Dissemination workshop** on the findings of the research project on **Studies on Block Designs for Biological Assays** at I.A.S.R.I., New Delhi on February 15, 2005.
- **Brain Storming Session** on **Statistical Issues in Rapeseed-Mustard Trials** in collaboration with National Research Centre on Rapeseed-Mustard on November 19, 2005 at IASRI, New Delhi.
- Symposium on **Role of Women in Rural Development** during 8th Annual Conference of the Society of Statistics, Computer and Applications held at Department of Statistics and Computer Science, Government Vidarbha Institute of Science and Humanities, Amravati during November 28-30, 2005.

➢ Teaching and Research Guidance

A) Teaching

Year	Trimester	Course	Taught Jointly with	Number of Lectures Taken
2004-05	Trimester III	AS 163: Statistical Inference (4 L + 1P)	Dr. L.M. Bhar	35(28L+7P)
2005-06	Trimester I	AS 200: Design of Experiments II (1 L+ 1P)	Dr. Cini Varghese	20(13L+7P)
2005-06	Trimester I	GEN 130: Introduction to Bio-informatics (3L+1P)	Dr. R.L. Sapra, Dr. K.V. Bhat and Dr. S.K. Lal	3 (3L+0P)

B) Research Guidance (P.G. Students Guided)

- **Chairman Advisory Committee**: 3 Ph.D. (Agricultural Statistics) : Continuing
- **Co-Chairman Advisory Committee:** 1 M.Sc. (Agricultural Statistics) student completed.
- **Member Advisory Committee:** 5 students (2 Ph.D. and 3 M.Sc.) completed their respective degrees.

➢ Participation/ Presentations in Conferences/ Symposia/ Workshop etc.

- Presented 10 research paspers (4 Invited papers by self, one invited paper by co-author, 3 contributed papers by co-authors and 2 in self organized workshop/symposia)

No.	Name of the Conference/ Sumposia/ Workshop	Organizing Institution/ Venue/ Duration	Papers Presented
1.	12th Annual Group Meeting of AICRP on Rapeseed and Mustard	GBPUA&T, Pantnagar during August 11-13, 2005.	Statistical designing and analytical techniques useful for varietal trials (**Rajender Parsad** and V.K.Gupta): **Invited Talk**
2.	59th Annual Conference of Indian Society of Agricultural Statistics	Shere-e-Kashmir, University of Agricultural Science & Technology of Jammu, Main Campus, Chatha, Jammu during November 11-13, 2005	1. Designing experiments for cropping systems research (**Rajender Parsad**): Dr. D.N. Lal Memorial Lecture. 2. Statistical designing of experiments for hill agriculture (**Rajender Parsad** and V.K. Gupta): **Invited Talk** in the Symposium on statistical and Computational issues in Rainfed Agriculture. 3. Statistical package for agricultural research: SPAR 2.0 (Sangeeta Ahuja*, P.K. Malhotra, V.K. Bhatia and **Rajender Parsad**)

			4. Trends of soil available nutrients and there predication in long-term fertilizer experiments (Ananta Sarkar*, **Rajender Parsad** and D.K. Mehta)
3.	8th Annual Conference of the Society of Statistics, Computer and Applications	Department of Statistics and Computer Science, Government Vidarbha Institute of Science and Humanities, Amravati during November 28-30, 2005	1. Current status of experimental designs in agricultural research-I. **(Invited Talk):** V.K. Gupta* and **Rajender Parsad** 2. Current status of experimental designs in agricultural research-II. **(Invited Talk): Rajender Parsad** and V.K. Gupta 3. Detection of multiple outliers in designed experiments (Lal Mohan Bhar*, **Rajender Parsad** and V.K. Gupta)
4.	International Biometric Society (Indian Region) conference	Department of Agricultural Statistics, University of Agricultural Sciences, Bangalore from October 04-07, 2005	Trend-free nested balanced incomplete block designs with applications in designs for diallel cross experiments (Krishan Lal*, **Rajender Parsad** and V. K. Gupta): **Lead paper**
5.	Dissemination Workshop on Studies on Block Designs for Biological Assays	I.A.S.R.I., New Delhi on February 15, 2005.	Biological Assays: Historical Perspective, Terminology and Analytical Techniques (**Rajender Parsad**)
6.	Dissemination Workshop on Sensitization cum Requirement Analysis for NISAGENET	I.A.S.R.I., New Delhi on June 07-08, 2005	——
7.	Dissemination Workshop on Modelling for forecasting of crop yield using weather parameters and agricultural inputs	I.A.S.R.I., New Delhi on May 20, 2005	——
8.	Brain Storming Session on Statistical Issues in Rapeseed-Mustard Trials	November 19, 2005 at IASRI, New Delhi	Statistical Issues in Rapeseed-Mustard Trials (**Rajender Parsad**)

(* represents the author who presented the paper)

Special Lectures Delivered

A) **Lectures Delivered in Training Programmes at IASRI, New Delhi**

- 13 lectures are delivered in the ad-hoc training programmes organized at IASRI.

• Training programme on **Advances in Designing and Analysis of Agricultural Experiments** held at I.A.S.R.I., New Delhi under the aegis of Centre of Advanced Studies in Agricultural Statistics and Computer Applications during February 03 -23, 2005.	**9 Lectures**
- SAS: An Overview	1 Lecture
- Non-proper Variance Balanced Block Designs	1 Lecture
- Multiple Comparison Procedures	1 Lecture
- Transformation of Data	1 Lecture
- MS-EXCEL: Analysis of Experimental Data	1 Lecture
- Response Surface Methodology	1 Lecture
- Designs for test Treatments-Control Treatment(s) comparisons including Augmented Designs	1 Lecture
- Case studies related to Variance Components Estimation, Designs for Crop sequences, Post Harvest Experiments, Quality evaluation of RTS fruit beverages, Osmotic Dehydration of banana, intercropping experiments, etc.	2 Lecture
• Training programme on **Statistical Techniques for Agricultural Research with Emphasis on Use of Softwares** organized at IASRI under the aegis of Centre of Advanced Studies in Agricultural Statistics and Computer Applications during December 21, 2005 to January 10, 2006	**4 Lectures**
- SAS: An Overview	1 Lecture
- SAS: Statistical Procedures	1 Lecture
- Multiple Comparison Procedure	1 Lecture
- Statistical Packages on Designed Experiments Developed at IASRI	1 Lecture

B) Invited Lectures Delivered

16 Invited Lectures are delivered at M.D. University Rohtak, L.N. Hindu College, Rohtak, Allahabad Agricultural Institute, Allahabad; Ram Lal Anand College, New Delhi; Shri Shivaji Science College, Amravati; CCS HAU Hissar and Shaheed Rajguru College of Applied Sciences for Women, New Delhi.

- A lecture on **Block Designs** at Department of Statistics, M.D. University, Rohtak on February 05, 2005.
- A lecture on **Information Communication Technology and its Applications** at L.N. Hindu College, Rohtak (February 05, 2005).
- A lecture on **SPSS: Analysis of Experimental Data** to the participants of the training programme of **SPSS 10.0** under **Resources Generation Scheme** (June 15, 2005).
- Six lectures of 90 minutes duration each on **Design of Experiments** to the participants of the Summer School on Statistical Applications jointly organized by College of Basic Sciences, Allahabad Agricultural Institute-Deemed University and Forum for interdisciplinary Mathematics during July 04-25, 2005 (July 14-16, 2005).
- A lecture on **Statistics: Career and Prospects** at Ram Lal Anand College, South Campus, Delhi University during Random Walk-2005 organized by Statistical Society of Ram Lal Anand College. (November 21, 2005).
- A lecture on **Future Prospects of Statistics** at Shri Shivaji Science College, Amravati. (November 29, 2005).

- Two lectures on **1. Response Surface Designs and 2. Experiments with Mixtures** to the participants of the Refresher Course on **Design and Analysis of Agricultural Experiments** held at Academy of Agricultural Research and Education Management, Directorate of HRM, CCS HAU Hissar during November 19, 2005 to December 09, 2005 (December 07-08, 2005 and 8.12.2005).
- Three lectures on **SPSS: An Overview** to the participants **Application of Mathematical Techniques in Industries** held at Shaheed Rajguru College of Applied Sciences for Women during December 26, 2005 to January 09, 2006. (December 27-28, 2005).

19/4/06

सुखदेव शर्मा

Signature of Head of the Institute

निदेशक
Director
भा.कृ.सां.अ.सं., (भा.कृ.अ.प.)
I.A.S.R.I., (I.C.A.R.)
लाइब्रेरी एवेन्यू, नई दिल्ली-110012
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19/04/06

(Rajender Parsad)
Principal Investigator

Table 5.1.1: α-designs for $6 \leq v \leq 150$, $2 \leq r \leq 5$, $3 \leq k \leq 10$ and $2 \leq s \leq 15$ along with lower bounds to A- and D-efficieny (where $v = sk$).

S.No.	*v*	*b*	*r*	*k*	A-Efficiency	D-Efficiency	Remarks
1	6	4	2	3	0.8333	0.9253	α(0,1,2)$
2	6	6	3	3	0.9294	0.9651	α(0,1,2)$
3	6	8	4	3	0.9615	0.9801	α(0,2), SR19
4	6	10	5	3	0.9466	0.9737	α(0,2,3)
5	9	6	2	3	0.8889	0.9428	α(0,1), LS7, Square-Lattice
6	9	9	3	3	0.9697	0.9837	α(0,1), SR23, Square-Lattice
7	9	12	4	3	0.9412	0.9710	α(0,1,2)
8	9	15	5	3	0.9552	0.9768	α(0,1,2)
9	12	8	2	3	0.8176	0.9123	α(0,1), Rect-Lattice
10	12	12	3	3	0.9241	0.9628	α(0,1)
11	12	16	4	3	0.9513	0.9753	α(0,1,2)
12	12	20	5	3	0.9592	0.9788	α (0,1,2)$
13	15	10	2	3	0.7645	0.8914	α(0,1);
14	15	15	3	3	0.9067	0.9533	α(0,1)$;
15	15	20	4	3	0.9554	0.9767	α(0,1)$;
16	15	25	5	3	0.9800	0.9890	α(0,1);, SR28
17	18	12	2	3	0.7410	0.8799	α(0,1);
18	18	18	3	3	0.8915	0.9459	α(0,1);
19	18	24	4	3	0.9386	0.9689	α(0,1);
20	18	30	5	3	0.9644	0.9816	α(0,1);
21	21	14	2	3	0.7353	0.8741	α(0,1);
22	21	21	3	3	0.8856	0.9421	α(0,1);
23	21	28	4	3	0.9351	0.9660	α(0,1)**;
24	21	35	5	3	0.9559	0.9773	α(0,1)$;
25	24	16	2	3	0.7110	0.8651	α(0,1);
26	24	24	3	3	0.8712	0.9361	α(0,1);
27	24	32	4	3	0.9264	0.9619	α(0,1)***
28	24	40	5	3	0.9488	0.9738	α(0,1)***
29	27	18	2	3	0.6916	0.8580	α(0,1);
30	27	27	3	3	0.8607	0.9315	α(0,1)***
31	27	36	4	3	0.9243	0.9603	α(0,1);
32	27	45	5	3	0.9436	0.9711	α(0,1)***
33	30	20	2	3	0.6734	0.8519	α(0,1)***
34	30	30	3	3	0.8511	0.9276	α(0,1);
35	30	40	4	3	0.9170	0.9571	α(0,1)**;
36	30	50	5	3	0.9401	0.9692	α(0,1)***
37	33	22	2	3	0.6542	0.8462	α(0,1);
38	33	33	3	3	0.8471	0.9253	α(0,1);
39	33	44	4	3	0.9108	0.9544	α(0,1);
40	33	55	5	3	0.9364	0.9674	α(0,1)***
41	36	24	2	3	0.6509	0.8434	α(0,1)***
42	36	36	3	3	0.8403	0.9225	α(0,1)***
43	36	48	4	3	0.9052	0.9520	α(0,1);
44	36	60	5	3	0.9334	0.9658	α(0,1)***
45	39	26	2	3	0.6458	0.8406	α(0,1);
46	39	39	3	3	0.8343	0.9202	α(0,1)***
47	39	52	4	3	0.9011	0.9502	α(0,1)***

S.No.	*v*	*b*	*r*	*k*	A-Efficiency	D-Efficiency	Remarks
48	39	65	5	3	0.9327	0.9652	α(0,1)***
49	42	28	2	3	0.6342	0.8371	α(0,1);
50	42	42	3	3	0.8320	0.9187	α(0,1);
51	42	56	4	3	0.8969	0.9484	α(0,1)***
52	42	70	5	3	0.9307	0.9642	α(0,1)***
53	45	30	2	3	0.6242	0.8341	α(0,1)***
54	45	45	3	3	0.8285	0.9172	α(0,1)***
55	45	60	4	3	0.8935	0.9469	α(0,1);
56	45	75	5	3	0.9268	0.9627	α(0,1)***
57	8	4	2	4	0.9074	0.9571	α(0,1,2)
58	8	6	3	4	0.9608	0.9806	α(0,1,2), S6
59	8	8	4	4	0.9800	0.9898	α(0,2), SR36
60	8	10	5	4	0.9703	0.9856	α(0,2,3)$
61	12	6	2	4	0.8655	0.9388	α(0,1,2)
62	12	9	3	4	0.9380	0.9705	α(0,1,3)$
63	12	12	4	4	0.9644	0.9829	α(0,1,2)$
64	12	15	5	4	0.9747	0.9870	α(0,1,2)
65	16	8	2	4	0.8929	0.9473	α(0,1), LS28, Square-Lattice
66	16	12	3	4	0.9422	0.9717	α(0,1,2)$
67	16	16	4	4	0.9623	0.9816	α(0,1,2)
68	16	20	5	4	0.9760	0.9878	α(0,1,2)
69	20	10	2	4	0.8575	0.9333	α(0,1);, Rect-Lattice
70	20	15	3	4	0.9433	0.9715	α(0,1);, Rect.-Lattice
71	20	20	4	4	0.9735	0.9862	α(0,1);
72	20	25	5	4	0.9890	0.9941	α(0,1);, SR46
73	24	12	2	4	0.8307	0.9232	α(0,1);
74	24	18	3	4	0.9283	0.9649	α(0,1);
75	24	24	4	4	0.9619	0.9809	α (0,1);
76	24	30	5	4	0.9782	0.9890	α(0,1);
77	28	14	2	4	0.8182	0.9172	α(0,1)***
78	28	21	3	4	0.9205	0.9611	α(0,1)***
79	28	28	4	4	0.9555	0.9775	α(0,1)***
80	28	35	5	4	0.9730	0.9862	α(0,1);
81	32	16	2	4	0.8016	0.9113	α(0,1);
82	32	24	3	4	0.9123	0.9575	α(0,1)***
83	32	32	4	4	0.9496	0.9747	α(0,1);
84	32	40	5	4	0.9668	0.9833	α(0,1)***
85	36	18	2	4	0.7899	0.9069	α(0,1)***
86	36	27	3	4	0.9112	0.9563	α(0,1)***
87	36	36	4	4	0.9460	0.9729	α(0,1)***
88	36	45	5	4	0.9633	0.9815	α(0,1)***
89	40	20	2	4	0.7831	0.9039	α(0,1)***
90	40	30	3	4	0.9010	0.9525	α(0,1)***
91	40	40	4	4	0.9420	0.9710	α(0,1)***
92	40	50	5	4	0.9602	0.9800	α(0,1)***
93	44	22	2	4	0.7783	0.9016	α(0,1)***
94	44	33	3	4	0.9022	0.9523	α(0,1)***

S.No.	*v*	*b*	*r*	*k*	A-Efficiency	D-Efficiency	Remarks
95	44	44	4	4	0.9390	0.9696	α(0,1)***
96	44	55	5	4	0.9575	0.9786	α(0,1);
97	48	24	2	4	0.7758	0.8999	α(0,1)***
98	48	36	3	4	0.8971	0.9503	α(0,1)***
99	48	48	4	4	0.9365	0.9683	α(0,1)***
100	48	60	5	4	0.9562	0.9779	α(0,1)***
101	52	26	2	4	0.7748	0.8987	α(0,1)***
102	52	39	3	4	0.8931	0.9487	α(0,1)***
103	52	52	4	4	0.9335	0.9671	α(0,1)**;
104	52	65	5	4	0.9539	0.9768	α(0,1)**;
105	56	28	2	4	0.7612	0.8952	α(0,1);
106	56	42	3	4	0.8898	0.9473	α(0,1)***
107	56	56	4	4	0.9317	0.9662	α(0,1)***
108	56	70	5	4	0.9517	0.9758	α(0,1)***
109	60	30	2	4	0.7629	0.8947	α(0,1)***
110	60	45	3	4	0.8868	0.9461	α(0,1)***
111	60	60	4	4	0.9305	0.9655	α(0,1)***
112	60	75	5	4	0.9501	0.9750	α(0,1)***
113	10	4	2	5	0.9067	0.9600	α(0,1,2)
114	10	6	3	5	0.9550	0.9792	α(0,1,2,3)$
115	10	8	4	5	0.9720	0.9867	α(0,2,4)$
116	10	10	5	5	0.9770	0.9889	α(0,1,2,3,4)$
117	15	6	2	5	0.8909	0.9515	α(0,1,2)$
118	15	9	3	5	0.9515	0.9766	α(0,1,3)
119	15	12	4	5	0.9726	0.9863	α(0,1,2)$
120	15	15	5	5	0.9834	0.9915	α(0,1,2)
121	20	8	2	5	0.8848	0.9479	α(0,1,2)
122	20	12	3	5	0.9492	0.9754	α(0,1,2)
123	20	16	4	5	0.9671	0.9840	α(0,1,2)$
124	20	20	5	5	0.9838	0.9918	α(0,1,2)
125	25	10	2	5	0.9000	0.9524	α(0,1);, LS 51, Square-Lattice
126	25	15	3	5	0.9600	0.9798	α(0,1);, LS52, square-Lattice
127	25	20	4	5	0.9818	0.9906	α(0,1);, LS53, LS61, Square-Lattice
128	25	25	5	5	0.9931	0.9964	α(0,1), Square-Lattice
129	30	12	2	5	0.8797	0.9448	α(0,1), Rect.-Lattice
130	30	18	3	5	0.9478	0.9745	α(0,1)
131	30	24	4	5	0.9690	0.9846	α(0,1,2)
132	30	30	5	5	0.9746	0.9874	α(0,1,2)
133	35	14	2	5	0.8639	0.9390	α(0,1)
134	35	21	3	5	0.9406	0.9712	α(0,1)
135	35	28	4	5	0.9658	0.9832	α(0,1)
136	35	35	5	5	0.9771	0.9887	α(0,1,2)
137	40	16	2	5	0.8536	0.9350	α(0,1)
138	40	24	3	5	0.9337	0.9683	α(0,1)
139	40	32	4	5	0.9610	0.9810	α(0,1)
140	40	40	5	5	0.9718	0.9860	α(0,1,2)
141	45	18	2	5	0.8439	0.9315	α(0,1)

S.No.	*v*	*b*	*r*	*k*	A-Efficiency	D-Efficiency	Remarks
142	45	27	3	5	0.9304	0.9666	α(0,1)
143	45	36	4	5	0.9592	0.9798	α(0,1)
144	45	45	5	5	0.9685	0.9843	α(0,1,2)
145	50	20	2	5	0.8378	0.9290	α(0,1)
146	50	30	3	5	0.9271	0.9651	α(0,1)
147	50	40	4	5	0.9552	0.9781	α(0,1)
148	50	50	5	5	0.9700	0.9851	α(0,1)
149	55	22	2	5	0.8341	0.9272	α(0,1)
150	55	33	3	5	0.9239	0.9637	α(0,1)
151	55	44	4	5	0.9538	0.9773	α(0,1)
152	55	55	5	5	0.9677	0.9840	α(0,1)
153	60	24	2	5	0.8279	0.9251	α(0,1)
154	60	36	3	5	0.9210	0.9625	α(0,1)
155	60	48	4	5	0.9509	0.9760	α(0,1)
156	60	60	5	5	0.9660	0.9832	α(0,1)
157	65	26	2	5	0.8227	0.9234	α(0,1)
158	65	39	3	5	0.9210	0.9621	α(0,1)
159	65	52	4	5	0.9493	0.9752	α(0,1)
160	65	65	5	5	0.9644	0.9824	α(0,1)
161	70	28	2	5	0.8190	0.9220	α(0,1)
162	70	42	3	5	0.9179	0.9609	α(0,1)
163	70	56	4	5	0.9484	0.9747	α(0,1)
164	70	70	5	5	0.9630	0.9817	α(0,1)
165	75	30	2	5	0.8158	0.9208	α(0,1)
166	75	45	3	5	0.9153	0.9599	α(0,1)
167	75	60	4	5	0.9463	0.9738	α(0,1)
168	75	75	5	5	0.9628	0.9815	α(0,1)
169	12	4	2	6	0.9308	0.9698	α(0,1,2)
170	12	6	3	6	0.9614	0.9823	α(0,1,2,3)
171	12	8	4	6	0.9758	0.9885	α(0,2,4)
172	12	10	5	6	0.9824	0.9915	α(0,1,2,3,4)$
173	18	6	2	6	0.9175	0.9628	α(0,1,2)
174	18	9	3	6	0.9633	0.9822	α(0,1,2)
175	18	12	4	6	0.9721	0.9866	α(0,1,2,3)$
176	18	15	5	6	0.9827	0.9915	α(0,1,2,3)
177	24	8	2	6	0.8975	0.9550	α(0,1,2)
178	24	12	3	6	0.9529	0.9777	α(0,1,2)
179	24	16	4	6	0.9699	0.9854	α(0,1,2,3)
180	24	20	5	6	0.9812	0.9907	α(0,1,2,3)
181	30	10	2	6	0.8981	0.9544	α(0,1,2)
182	30	15	3	6	0.9536	0.9778	α(0,1,3)
183	30	20	4	6	0.9734	0.9870	α(0,1,2,3)
184	30	25	5	6	0.9836	0.9919	α(0,1,2,3)$
185	36	12	2	6	0.9074	0.9571	α(0,1);, LS74, Square-Lattice
186	36	18	3	6	0.9550	0.9782	α(0,1,2);
187	36	24	4	6	0.9753	0.9877	α(0,1,2);
188	36	30	5	6	0.9826	0.9913	α(0,1,2)

S.No.	*v*	*b*	*r*	*k*	A-Efficiency	D-Efficiency	Remarks
189	42	14	2	6	0.8946	0.9524	α(0,1), Rect.-Lattice
190	42	21	3	6	0.9539	0.9776	α(0,1), Rect-Lattice
191	42	28	4	6	0.9721	0.9862	α(0,1,2);
192	42	35	5	6	0.9802	0.9901	α(0,1,2)
193	48	16	2	6	0.8844	0.9488	α(0,1)
194	48	24	3	6	0.9482	0.9752	α(0,1)
195	48	32	4	6	0.9682	0.9844	α(0,1,2)
196	48	40	5	6	0.9771	0.9888	α(0,1,2)
197	54	18	2	6	0.8769	0.9460	α(0,1)
198	54	27	3	6	0.9431	0.9731	α(0,1)
199	54	36	4	6	0.9671	0.9839	α(0,1)
200	54	45	5	6	0.9760	0.9881	α(0,1,2)
201	60	20	2	6	0.8707	0.9438	α(0,1)
202	60	30	3	6	0.9414	0.9722	α(0,1)
203	60	40	4	6	0.9648	0.9828	α(0,1)
204	60	50	5	6	0.9765	0.9884	α(0,1)
205	66	22	2	6	0.8667	0.9421	α(0,1)
206	66	33	3	6	0.9387	0.9710	α(0,1)
207	66	44	4	6	0.9621	0.9816	α(0,1)
208	66	55	5	6	0.9736	0.9871	α(0,1)
209	72	24	2	6	0.8616	0.9405	α(0,1)
210	72	36	3	6	0.9371	0.9702	α(0,1)
211	72	48	4	6	0.9576	0.9796	α(0,1,2)
212	72	60	5	6	0.9712	0.9859	α(0,1,2)
213	78	26	2	6	0.8577	0.9391	α(0,1)
214	78	39	3	6	0.9349	0.9693	α(0,1)
215	78	52	4	6	0.9585	0.9800	α(0,1)
216	78	65	5	6	0.9698	0.9853	α(0,1,2)
217	84	28	2	6	0.8550	0.9380	α(0,1)
218	84	42	3	6	0.9321	0.9682	α(0,1)
219	84	56	4	6	0.9575	0.9795	α(0,1)
220	84	70	5	6	0.9705	0.9855	α(0,1)
221	90	30	2	6	0.8529	0.9372	α(0,1)***
222	90	45	3	6	0.9312	0.9678	α(0,1)***
223	90	60	4	6	0.9564	0.9790	α(0,1)**
224	90	75	5	6	0.9694	0.9850	α(0,1)
225	14	4	2	7	0.9337	0.9722	α(0,1,2)
226	14	6	3	7	0.9684	0.9855	α(0,1,2,3)
227	14	8	4	7	0.9803	0.9906	α(0,2,4)$
228	14	10	5	7	0.9863	0.9933	α(0,1,2,3,4)
229	21	6	2	7	0.8993	0.9591	α(0,1,2)
230	21	9	3	7	0.9589	0.9809	α(0,1,2,3)
231	21	12	4	7	0.9758	0.9883	α(0,1,2,3)
232	21	15	5	7	0.9845	0.9924	α(0,1,2,3,4)
233	28	8	2	7	0.9135	0.9622	α(0,1,2)$
234	28	12	3	7	0.9603	0.9812	α(0,1,2)
235	28	16	4	7	0.9731	0.9871	α(0,1,2,3)$

S.No.	*v*	*b*	*r*	*k*	A-Efficiency	D-Efficiency	Remarks
236	28	20	5	7	0.9823	0.9913	α(0,1,2,3)
237	35	10	2	7	0.9064	0.9592	α(0,1,2)
238	35	15	3	7	0.9564	0.9795	α(0,1,2)
239	35	20	4	7	0.9740	0.9874	α(0,1,2,3)
240	35	25	5	7	0.9830	0.9916	α(0,1,2,3)
241	42	12	2	7	0.9082	0.9593	α(0,1,2)
242	42	18	3	7	0.9582	0.9801	α(0,1,2)
243	42	24	4	7	0.9742	0.9874	α(0,1,2,3)
244	42	30	5	7	0.9816	0.9909	α(0,1,2)
245	49	14	2	7	0.9143	0.9610	α(0,1);, LS84, Square-Lattice
246	49	21	3	7	0.9624	0.9817	α(0,1);, LS85, Square-Lattice
247	49	28	4	7	0.9796	0.9897	α(0,1);, Square-Lattice, LS86
248	49	35	5	7	0.9884	0.9941	α(0,1), Square-Lattice, LS87
249	56	16	2	7	0.9057	0.9580	α(0,1), Rect.-Lattice
250	56	24	3	7	0.9573	0.9796	α(0,1)
251	56	32	4	7	0.9743	0.9874	α(0,1,2)
252	56	40	5	7	0.9819	0.9910	α(0,1,2)
253	63	18	2	7	0.8987	0.9556	α(0,1)
254	63	27	3	7	0.9539	0.9781	α(0,1)
255	63	36	4	7	0.9715	0.9861	α(0,1,2)
256	63	45	5	7	0.9801	0.9902	α(0,1,2)
257	70	20	2	7	0.8928	0.9536	α(0,1)
258	70	30	3	7	0.9508	0.9768	α(0,1)
259	70	40	4	7	0.9703	0.9856	α(0,1)
260	70	50	5	7	0.9793	0.9897	α(0,1,2)
261	77	22	2	7	0.8880	0.9519	α(0,1)
262	77	33	3	7	0.9485	0.9758	α(0,1)
263	77	44	4	7	0.9688	0.9848	α(0,1)
264	77	55	5	7	0.9788	0.9896	α(0,1)
265	84	24	2	7	0.8852	0.9507	α(0,1)
266	84	36	3	7	0.9468	0.9750	α(0,1)
267	84	48	4	7	0.9670	0.9841	α(0,1)
268	84	60	5	7	0.9773	0.9889	α(0,1)
269	91	26	2	7	0.8817	0.9496	α(0,1);
270	91	39	3	7	0.9453	0.9744	α(0,1)***
271	91	52	4	7	0.9657	0.9835	α(0,1)***
272	91	65	5	7	0.9763	0.9884	α(0,1)
273	98	28	2	7	0.8788	0.9486	α(0,1);
274	98	42	3	7	0.9438	0.9737	α(0,1)**
275	98	56	4	7	0.9649	0.9831	α(0,1)***
276	98	70	5	7	0.9752	0.9879	α(0,1)
277	105	30	2	7	0.8765	0.9478	α(0,1)
278	105	45	3	7	0.9419	0.9730	α(0,1)
279	105	60	4	7	0.9636	0.9825	α(0,1)
280	105	75	5	7	0.9744	0.9875	α(0,1)
281	16	4	2	8	0.9454	0.9768	α(0,1,2)
282	16	6	3	8	0.9683	0.9859	α(0,1,2,3)

S.No.	*v*	*b*	*r*	*k*	A-Efficiency	D-Efficiency	Remarks
283	16	8	4	8	0.9794	0.9904	α(0,2,4)
284	16	10	5	8	0.9866	0.9936	α(0,1,2,3,4)
285	24	6	2	8	0.9286	0.9695	α(0,1,2)
286	24	9	3	8	0.9664	0.9843	α(0,1,2,3)
287	24	12	4	8	0.9772	0.9892	α(0,1,2,3)
288	24	15	5	8	0.9847	0.9926	α(0,1,2,3,4)
289	32	8	2	8	0.9144	0.9641	α(0,1,2)
290	32	12	3	8	0.9590	0.9812	α(0,1,2,3)
291	32	16	4	8	0.9753	0.9882	α(0,1,2,3)
292	32	20	5	8	0.9843	0.9923	α(0,1,2,3)
293	40	10	2	8	0.9170	0.9642	α(0,1,2)
294	40	15	3	8	0.9612	0.9818	α(0,1,2)
295	40	20	4	8	0.9766	0.9887	α(0,1,2)
296	40	25	5	8	0.9845	0.9923	α(0,1,2,3)
297	48	12	2	8	0.9144	0.9630	α(0,1,2)
298	48	18	3	8	0.9603	0.9813	α(0,1,2)
299	48	24	4	8	0.9737	0.9874	α(0,1,2,3)
300	48	30	5	8	0.9831	0.9917	α(0,1,2)
301	56	14	2	8	0.9162	0.9633	α(0,1,2)
302	56	21	3	8	0.9612	0.9816	α(0,1,2)
303	56	28	4	8	0.9754	0.9880	α(0,1,2)
304	56	35	5	8	0.9831	0.9917	α(0,1,2)
305	64	16	2	8	0.9205	0.9644	α(0,1);, LS102, Square-Lattice
306	64	24	3	8	0.9618	0.9818	α(0,1,2);
307	64	32	4	8	0.9778	0.9891	α(0,1,2)***
308	64	40	5	8	0.9824	0.9914	α(0,1,2)
309	72	18	2	8	0.9144	0.9623	α(0,1), Rect.-Lattice
310	72	27	3	8	0.9610	0.9814	α(0,1)
311	72	36	4	8	0.9740	0.9874	α(0,1,2)
312	72	45	5	8	0.9827	0.9915	α(0,1,2)
313	80	20	2	8	0.9093	0.9606	α(0,1)
314	80	30	3	8	0.9582	0.9802	α(0,1)
315	80	40	4	8	0.9740	0.9874	α(0,1,2)
316	80	50	5	8	0.9808	0.9906	α(0,1,2)
317	88	22	2	8	0.9052	0.9592	α(0,1)
318	88	33	3	8	0.9563	0.9794	α(0,1)
319	88	44	4	8	0.9724	0.9867	α(0,1,2)
320	88	55	5	8	0.9810	0.9907	α(0,1,2)
321	96	24	2	8	0.9020	0.9580	α(0,1);
322	96	36	3	8	0.9539	0.9785	α(0,1)***
323	96	48	4	8	0.9718	0.9864	α(0,1)***
324	96	60	5	8	0.9807	0.9906	α(0,1)
325	104	26	2	8	0.8990	0.9570	α(0,1)
326	104	39	3	8	0.9527	0.9780	α(0,1)
327	104	52	4	8	0.9702	0.9858	α(0,1)
328	104	65	5	8	0.9797	0.9901	α(0,1)
329	112	28	2	8	0.8966	0.9562	α(0,1)

S.No.	*v*	*b*	*r*	*k*	A-Efficiency	D-Efficiency	Remarks
330	112	42	3	8	0.9513	0.9774	α(0,1)
331	112	56	4	8	0.9684	0.9849	α(0,1,2)
332	112	70	5	8	0.9783	0.9895	α(0,1,2)
333	120	30	2	8	0.8947	0.9555	α(0,1)
334	120	45	3	8	0.9497	0.9768	α(0,1)
335	120	60	4	8	0.9689	0.9851	α(0,1)
336	120	75	5	8	0.9783	0.9894	α(0,1)
337	18	4	2	9	0.9482	0.9786	α(0,1,2)
338	18	6	3	9	0.9711	0.9872	α(0,1,2,3)
339	18	8	4	9	0.9811	0.9912	α(0,2,4)
340	18	10	5	9	0.9878	0.9942	α(0,1,2,3,4)$
341	27	6	2	9	0.9294	0.9709	α(0,1,2)
342	27	9	3	9	0.9626	0.9832	α(0,1,2,3)
343	27	12	4	9	0.9795	0.9903	α(0,1,2,3)
344	27	15	5	9	0.9859	0.9932	α(0,1,2,3)
345	36	8	2	9	0.9207	0.9673	α(0,1,2)
346	36	12	3	9	0.9634	0.9833	α(0,1,2,3)
347	36	16	4	9	0.9747	0.9881	α(0,1,2,4)
348	36	20	5	9	0.9843	0.9924	α(0,1,2,3)
349	45	10	2	9	0.9272	0.9687	α(0,1,2)
350	45	15	3	9	0.9660	0.9840	α(0,1,2)
351	45	20	4	9	0.9795	0.9900	α(0,1,2,3)$
352	45	25	5	9	0.9863	0.9932	α(0,1,2,3)
353	54	12	2	9	0.9221	0.9667	α(0,1,2)$
354	54	18	3	9	0.9632	0.9829	α(0,1,2)
355	54	24	4	9	0.9760	0.9886	α(0,1,2,3)
356	54	30	5	9	0.9840	0.9922	α(0,1,2,3)
357	63	14	2	9	0.9213	0.9662	α(0,1,2)
358	63	21	3	9	0.9628	0.9827	α(0,1,2)
359	63	28	4	9	0.9703	0.9862	α(0,1,2,3)
360	63	35	5	9	0.9845	0.9924	α(0,1,2)
361	72	16	2	9	0.9229	0.9665	α(0,1,2)
362	72	24	3	9	0.9641	0.9831	α(0,1,2)
363	72	32	4	9	0.9750	0.9881	α(0,1,2,3)
364	72	40	5	9	0.9804	0.9906	α(0,1,2,3)
365	81	18	2	9	0.9259	0.9673	α(0,1);, LS119, Square-Lattice
366	81	27	3	9	0.9662	0.9839	α(0,1);,Square-Lattice, LS120
367	81	36	4	9	0.9790	0.9897	α(0,1,2)***
368	81	45	5	9	0.9756	0.9886	α(0,1,2,3)
369	90	20	2	9	0.9215	0.9658	α(0,1)
370	90	30	3	9	0.9618	0.9821	α(0,1,2)
371	90	40	4	9	0.9707	0.9862	α(0,1,2,3)
372	90	50	5	9	0.9780	0.9895	α(0,1,2,3,4)
373	99	22	2	9	0.9177	0.9645	α(0,1);
374	99	33	3	9	0.9617	0.9820	α(0,1);
375	99	44	4	9	0.9761	09885	α(0,1,2)***
376	99	55	5	9	0.9801	0.9904	α(0,1,2)

S.No.	*v*	*b*	*r*	*k*	A-Efficiency	D-Efficiency	Remarks
377	108	24	2	9	0.9146	0.9635	$\alpha(0,1)$
378	108	36	3	9	0.9578	0.9805	$\alpha(0,1,2)$
379	108	48	4	9	0.9721	0.9868	$\alpha(0,1,2)$
380	108	60	5	9	0.9793	0.9900	$\alpha(0,1,2)$
381	117	26	2	9	0.9119	0.9626	$\alpha(0,1)$
382	117	39	3	9	0.9510	0.9779	$\alpha(0,1,2)$
383	117	52	4	9	0.9697	0.9857	$\alpha(0,1,2)$
384	117	65	5	9	0.9817	0.9911	$\alpha(0,1,2)$
385	126	28	2	9	0.9097	0.9618	$\alpha(0,1)$
386	126	42	3	9	0.9565	0.9800	$\alpha(0,1)$
387	126	56	4	9	0.9699	0.9859	$\alpha(0,1,2)$
388	126	70	5	9	0.9776	0.9893	$\alpha(0,1,2,3)$
389	135	30	2	9	0.9076	0.9611	$\alpha(0,1)$
390	135	45	3	9	0.9554	0.9796	$\alpha(0,1)$
391	135	60	4	9	0.9693	0.9856	$\alpha(0,1,2)$
392	135	75	5	9	0.9766	0.9888	$\alpha(0,1,2,3)$
393	20	4	2	10	0.9475	0.9792	$\alpha(0,1,2)$
394	20	6	3	10	0.9744	0.9886	$\alpha(0,1,2,3)$
395	20	8	4	10	0.9823	0.9919	$\alpha(0,1,2,3,4)$
396	20	10	5	10	0.9892	0.9948	$\alpha(0,1,2,3,4)$
397	30	6	2	10	0.9344	0.9734	$\alpha(0,1,2)$
398	30	9	3	10	0.9683	0.9857	$\alpha(0,1,2,3)$
399	30	12	4	10	0.9815	0.9913	$\alpha(0,1,2,3,4)$
400	30	15	5	10	0.9873	0.9939	$\alpha(0,1,2,3,5)$
401	40	8	2	10	0.9286	0.9707	$\alpha(0,1,2)$
402	40	12	3	10	0.9692	0.9858	$\alpha(0,1,2,3)$
403	40	16	4	10	0.9762	0.9889	$\alpha(0,1,2,4)$
404	40	20	5	10	0.9860	0.9932	$\alpha(0,1,2,3,4)$
405	50	10	2	10	0.9361	0.9724	$\alpha(0,1,2)$
406	50	15	3	10	0.9661	0.9844	$\alpha(0,1,2)$
407	50	20	4	10	0.9787	0.9899	$\alpha(0,1,2,3)$
408	50	25	5	10	0.9852	0.9928	$\alpha(0,1,2,3)$
409	60	12	2	10	0.9298	0.9701	$\alpha(0,1,2)$
410	60	18	3	10	0.9668	0.9846	$\alpha(0,1,2)$
411	60	24	4	10	0.9796	0.9902	$\alpha(0,1,2,3)$
412	60	30	5	10	0.9843	0.9924	$\alpha(0,1,2,3,4)$
413	70	14	2	10	0.9272	0.9691	$\alpha(0,1,2)$
414	70	21	3	10	0.9655	0.9840	$\alpha(0,1,2)$
415	70	28	4	10	0.9787	0.9898	$\alpha(0,1,2,3)$
416	70	35	5	10	0.9851	0.9927	$\alpha(0,1,2)$
417	80	16	2	10	0.9271	0.9689	$\alpha(0,1,2)$
418	80	24	3	10	0.9655	0.9840	$\alpha(0,1,2)$
419	80	32	4	10	0.9788	0.9898	$\alpha(0,1,2)$
420	80	40	5	10	0.9848	0.9926	$\alpha(0,1,2,3)$
421	90	18	2	10	0.9285	0.9692	$\alpha(0,1,2)$
422	90	27	3	10	0.9664	0.9843	$\alpha(0,1,2)$
423	90	36	4	10	0.9782	0.9895	$\alpha(0,1,2)$

S.No.	*v*	*b*	*r*	*k*	A-Efficiency	D-Efficiency	Remarks
424	90	45	5	10	0.9834	0.9920	α(0,1,2)
425	100	20	2	10	0.9308	0.9698	α(0,1);,LS137, Square- Lattice
426	100	30	3	10	0.9667	0.9843	α(0,1,2);
427	100	40	4	10	0.9790	0.9898	α(0,1,2)**
428	100	50	5	10	0.9811	0.9910	α(0,1,2,3)
429	110	22	2	10	0.9274	0.9686	α(0,1)
430	110	33	3	10	0.9661	0.9841	α(0,1), Rect.- Lattice
431	110	44	4	10	0.9780	0.9894	α(0,1,2)
432	110	55	5	10	0.9786	0.9899	α(0,1,2,3)
433	120	24	2	10	0.9245	0.9677	α(0,1)
434	120	36	3	10	0.9645	0.9834	α(0,1)
435	120	48	4	10	0.9783	0.9895	α(0,1)
436	120	60	5	10	0.9837	0.9921	α(0,1,2)
437	130	26	2	10	0.9220	0.9668	α(0,1)
438	130	39	3	10	0.9632	0.9829	α(0,1)
439	130	52	4	10	0.9744	0.9879	α(0,1,2)
440	130	65	5	10	0.9807	0.9907	α(0,1,2,3)
441	140	28	2	10	0.9195	0.9661	α(0,1)
442	140	42	3	10	0.9619	0.9824	α(0,1)
443	140	56	4	10	0.9721	0.9870	α(0,1,2)
444	140	70	5	10	0.9787	0.9899	α(0,1,2)
445	150	30	2	10	0.9109	0.9635	α(0,1,2)
446	150	45	3	10	0.9612	0.9821	α(0,1)
447	150	60	4	10	0.9714	0.9867	α(0,1,2)
448	150	75	5	10	0.9782	0.9897	α(0,1,2)

$ denotes that for corresponding parameters no resolvable PBIB (2) design exists.

** indicates that the design has been generated using the basic array or their dual of Patterson, Williams and Hunter (1978).

***indicates that the design generated is better than the design obtainable from the basic arrays of Patterson, Williams and Hunter (1978) or corresponding Resolvable PBIB (2) designs of Clatworthy (1973).

; indicates that the design generated has efficiency equal to that of the design obtainable from Basic array or dualization of basic array of Patterson, Williams and Hunter (1978).

Table 5.1.2: α-Arrays for Generating Designs with $v \le 150, s \le 15, k \le 10, r \le 5$.

k	***s***	**Array for *r* = 2**	**Array for *r* = 3**	**Array for *r* = 4**	**Array for *r* = 5**
3	2	0 0 0 1 0 1	0 0 0 0 1 1 0 1 0	0 0 0 0 0 1 1 0 0 1 0 1	0 0 0 0 0 0 1 1 0 1 0 1 0 1 1
3	3	0 0 0 2 0 1	0 0 0 0 2 1 0 1 2	0 0 0 0 0 2 1 1 0 1 2 0	0 0 0 0 0 0 2 1 1 0 0 1 2 0 1
3	4	0 0 0 2 0 3	0 0 0 0 2 3 0 3 1	0 0 0 0 0 2 3 1 0 3 1 2	0 0 0 0 0 0 2 3 1 0 0 3 1 2 3
3	5	0 0 0 4 0 3	0 0 0 0 4 3 0 3 1	0 0 0 0 0 4 3 2 0 3 1 4	0 0 0 0 0 0 4 3 2 1 0 3 1 4 2
3	6	0 0 0 3 0 5	0 0 0 0 3 2 0 5 3	0 0 0 0 0 3 2 5 0 5 3 4	0 0 0 0 0 0 3 2 5 4 0 5 3 4 2
3	7	0 0 0 5 0 4	0 0 0 0 5 2 0 4 3	0 0 0 0 0 5 2 6 0 4 3 2	0 0 0 0 0 0 5 2 6 4 0 4 3 2 1
3	8	0 0 0 2 0 7	0 0 0 0 2 1 0 7 3	0 0 0 0 0 2 1 4 0 7 3 5	0 0 0 0 0 0 2 1 4 5 0 7 3 5 1
3	9	0 0 0 4 0 6	0 0 0 0 4 1 0 6 8	0 0 0 0 0 6 1 7 0 3 8 2	0 0 0 0 0 0 3 7 2 8 0 8 5 6 1
3	10	0 0 0 9 0 7	0 0 0 0 9 7 0 5 4	0 0 0 0 0 9 7 6 0 5 4 8	0 0 0 0 0 0 9 7 6 1 0 5 4 8 2
3	11	0 0 0 5 0 2	0 0 0 0 6 9 0 4 8	0 0 0 0 0 2 8 1 0 3 4 6	0 0 0 0 0 0 4 9 6 10 0 3 4 7 2
3	12	0 0 0 9 0 1	0 0 0 0 5 6 0 9 8	0 0 0 0 0 10 5 1 0 7 8 6	0 0 0 0 0 0 10 2 1 7 0 5 11 7 3
3	13	0 0 0 6 0 8	0 0 0 0 7 12 0 9 3	0 0 0 0 0 8 2 12 0 3 4 11	0 0 0 0 0 0 6 7 3 8 0 9 2 1 12
3	14	0 0 0 10 0 13	0 0 0 0 8 3 0 1 13	0 0 0 0 0 12 13 10 0 9 6 1	0 0 0 0 0 0 1 5 6 11 0 13 10 7 8
3	15	0 0 0 8 0 6	0 0 0 0 14 4 0 13 7	0 0 0 0 0 14 12 5 0 13 3 7	0 0 0 0 0 0 1 10 7 12 0 13 2 11 6
4	2	0 0 0 0 0 1 0 1	0 0 0 0 0 1 0 1 0 0 1 1	0 0 0 0 0 0 1 1 0 1 0 1 0 1 1 0	0 0 0 0 0 0 0 1 1 0 0 1 0 1 0 0 1 1 0 1

4	3	0 0 0 2 0 1 0 1	0 0 0 0 1 2 0 2 1 0 2 1	0 0 0 0 0 2 1 0 0 1 2 1 0 0 1 2	0 0 0 0 0 0 1 0 2 1 0 2 1 2 0 0 2 2 1 1
4	4	0 0 0 1 0 3 0 2	0 0 0 0 3 2 0 1 3 0 2 1	0 0 0 0 0 1 2 3 0 2 3 1 0 1 3 2	0 0 0 0 0 0 3 1 2 2 0 3 2 0 1 0 2 3 1 3
4	5	0 0 0 4 0 3 0 1	0 0 0 0 2 3 0 3 2 0 4 1	0 0 0 0 0 1 4 2 0 4 1 3 0 3 2 1	0 0 0 0 0 0 1 2 4 3 0 2 4 3 1 0 3 1 2 4
4	6	0 0 0 1 0 2 0 3	0 0 0 0 3 4 0 1 5 0 4 3	0 0 0 0 0 3 5 2 0 2 1 3 0 1 4 5	0 0 0 0 0 0 4 1 5 2 0 2 3 4 5 0 5 4 3 1
4	7	0 0 0 5 0 2 0 1	0 0 0 0 3 2 0 6 1 0 5 3	0 0 0 0 0 1 6 2 0 6 3 4 0 4 5 3	0 0 0 0 0 0 4 2 3 6 0 2 1 5 3 0 6 3 1 2
4	8	0 0 0 5 0 1 0 3	0 0 0 0 3 5 0 2 6 0 5 4	0 0 0 0 0 3 2 6 0 6 7 4 0 4 1 3	0 0 0 0 0 0 7 6 3 4 0 5 1 4 3 0 1 7 6 2
4	9	0 0 0 5 0 1 0 3	0 0 0 0 4 7 0 2 1 0 3 5	0 0 0 0 0 1 5 2 0 3 8 6 0 7 6 5	0 0 0 0 0 0 2 1 5 8 0 3 7 8 4 0 8 5 3 1
4	10	0 0 0 6 0 1 0 3	0 0 0 0 1 4 0 3 2 0 7 5	0 0 0 0 0 7 8 2 0 6 2 3 0 1 4 9	0 0 0 0 0 0 8 1 2 4 0 9 3 7 8 0 4 2 5 1
4	11	0 0 0 1 0 9 0 5	0 0 0 0 5 1 0 7 10 0 2 4	0 0 0 0 0 7 8 3 0 10 1 9 0 8 6 2	0 0 0 0 0 0 7 2 6 10 0 8 1 9 4 0 3 10 1 8
4	12	0 0 0 7 0 1 0 9	0 0 0 0 10 8 0 1 5 0 6 7	0 0 0 0 0 11 6 7 0 9 1 10 0 3 8 6	0 0 0 0 0 0 1 6 10 5 0 6 10 11 8 0 9 4 3 7
4	13	0 0 0 5 0 4 0 7	0 0 0 0 11 5 0 9 6 0 2 1	0 0 0 0 0 12 10 4 0 9 5 11 0 3 2 12	0 0 0 0 0 0 12 10 4 1 0 9 5 11 3 0 3 2 12 7
4	14	0 0 0 13 0 6 0 4	0 0 0 0 6 13 0 9 12 0 8 4	0 0 0 0 0 11 1 7 0 12 5 13 0 6 3 4	0 0 0 0 0 0 7 6 10 4 0 11 3 13 2 0 13 7 8 9

4	15	0 0 0 2 0 14 0 6	0 0 0 0 2 5 0 3 11 0 10 9	0 0 0 0 0 12 9 1 0 10 11 5 0 8 4 6	0 0 0 0 0 0 2 7 14 12 0 13 2 6 5 0 8 9 3 11
5	2	0 0 0 1 0 1 0 0 0 1	0 0 0 0 0 1 0 1 0 0 1 1 0 1 0	0 0 0 0 0 0 1 1 0 1 0 1 0 0 1 1 0 1 1 0	0 0 0 0 0 0 1 1 1 0 0 1 0 0 1 0 0 1 0 1 0 0 1 1 1
5	3	0 0 0 1 0 1 0 2 0 2	0 0 0 0 2 1 0 1 2 0 1 2 0 2 1	0 0 0 0 0 1 2 1 0 1 0 2 0 2 1 1 0 0 1 2	0 0 0 0 0 0 2 1 2 1 0 0 2 1 1 0 1 1 0 2 0 2 0 1 2
5	4	0 0 0 1 0 3 0 2 0 2	0 0 0 0 3 1 0 3 2 0 1 3 0 2 3	0 0 0 0 0 2 1 3 0 1 3 2 0 3 2 1 0 3 1 2	0 0 0 0 0 0 3 1 2 0 0 3 2 1 2 0 1 2 0 3 0 2 3 3 1
5	5	0 0 0 3 0 1 0 4 0 2	0 0 0 0 3 2 0 2 3 0 4 1 0 1 4	0 0 0 0 0 4 1 2 0 1 4 3 0 2 3 1 0 3 2 4	0 0 0 0 0 0 3 4 2 1 0 1 3 4 2 0 2 1 3 4 0 4 2 1 3
5	6	0 0 0 2 0 3 0 5 0 1	0 0 0 0 5 3 0 3 2 0 4 5 0 1 4	0 0 0 0 0 2 5 1 0 4 5 2 0 3 1 5 0 1 3 4	0 0 0 0 0 0 5 3 2 1 0 1 3 4 2 0 2 5 3 0 0 1 2 5 4
5	7	0 0 0 1 0 5 0 6 0 3	0 0 0 0 6 3 0 1 4 0 4 2 0 5 6	0 0 0 0 0 4 2 1 0 5 4 6 0 2 3 4 0 3 6 2	0 0 0 0 0 0 6 4 1 2 0 1 3 2 5 0 2 5 6 1 0 4 1 3 2
5	8	0 0 0 7 0 4 0 1 0 2	0 0 0 0 7 1 0 2 6 0 1 4 0 5 2	0 0 0 0 0 4 5 7 0 3 2 1 0 2 6 3 0 6 1 5	0 0 0 0 0 0 2 7 6 3 0 4 6 1 7 0 6 2 7 4 0 5 3 4 2
5	9	0 0 0 5 0 7 0 6 0 4	0 0 0 0 8 6 0 5 7 0 4 5 0 1 4	0 0 0 0 0 6 8 4 0 8 5 3 0 3 1 2 0 4 7 6	0 0 0 0 0 0 2 5 7 6 0 1 2 8 4 0 6 3 7 8 0 5 1 4 2
5	10	0 0 0 8 0 2 0 9 0 5	0 0 0 0 4 8 0 2 7 0 3 6 0 9 1	0 0 0 0 0 1 9 5 0 9 6 8 0 6 2 7 0 3 5 6	0 0 0 0 0 0 8 6 9 4 0 7 3 4 6 0 5 9 7 8 0 2 5 1 7

5	11	0 0 0 9 0 6 0 2 0 1	0 0 0 0 10 1 0 7 4 0 8 3 0 2 9	0 0 0 0 0 9 4 1 0 10 8 6 0 6 2 3 0 5 10 4	0 0 0 0 0 0 6 1 4 3 0 3 2 10 7 0 2 5 7 9 0 7 4 8 2
5	12	0 0 0 10 0 11 0 8 0 5	0 0 0 0 4 9 0 6 1 0 11 3 0 10 8	0 0 0 0 0 11 2 9 0 1 6 5 0 6 4 8 0 10 7 1	0 0 0 0 0 0 6 1 8 7 0 9 6 2 11 0 4 3 7 10 0 1 2 11 4
5	13	0 0 0 12 0 2 0 3 0 7	0 0 0 0 10 12 0 3 11 0 2 6 0 1 8	0 0 0 0 0 1 6 10 0 10 11 7 0 2 12 5 0 12 3 1	0 0 0 0 0 0 12 1 8 9 0 8 11 12 2 0 2 10 3 8 0 7 3 5 12
5	14	0 0 0 5 0 9 0 6 0 2	0 0 0 0 1 12 0 4 8 0 11 13 0 5 10	0 0 0 0 0 4 6 10 0 8 7 3 0 9 3 2 0 11 1 12	0 0 0 0 0 0 2 13 6 9 0 4 3 5 1 0 9 2 12 11 0 12 6 11 3
5	15	0 0 0 8 0 12 0 9 0 13	0 0 0 0 3 9 0 2 13 0 12 5 0 10 14	0 0 0 0 0 8 2 4 0 7 12 6 0 3 11 1 0 9 13 10	0 0 0 0 0 0 12 13 14 2 0 7 9 5 6 0 14 10 3 11 0 9 6 4 13
6	2	0 0 0 0 0 1 0 0 0 1 0 1	0 0 0 0 1 0 0 1 0 0 1 1 0 0 1 0 1 1	0 0 0 0 0 1 1 0 0 1 1 0 0 0 1 1 0 1 0 1 0 0 1 1	0 0 0 0 0 0 1 0 0 1 0 1 0 1 0 0 0 1 1 0 0 0 1 0 1 0 0 0 1 1
6	3	0 0 0 0 0 2 0 2 0 1 0 1	0 0 0 0 1 2 0 2 1 0 1 1 0 0 2 0 2 0	0 0 0 0 0 1 0 2 0 1 2 0 0 1 2 2 0 2 1 2 0 2 2 1	0 0 0 0 0 0 2 1 0 2 0 2 2 1 1 0 1 2 2 0 0 2 1 2 0 0 0 1 2 1
6	4	0 0 0 3 0 1 0 2 0 1 0 2	0 0 0 0 1 2 0 2 3 0 3 2 0 2 1 0 1 3	0 0 0 0 0 3 1 2 0 2 3 1 0 2 1 3 0 1 0 3 0 1 2 3	0 0 0 0 0 0 3 1 2 2 0 2 3 3 1 0 1 2 1 3 0 0 3 2 1 0 1 0 2 3
6	5	0 0 0 3 0 4 0 2 0 1 0 1	0 0 0 0 1 3 0 4 2 0 3 4 0 2 1 0 1 3	0 0 0 0 0 1 3 2 0 2 1 4 0 4 2 3 0 4 3 1 0 3 4 1	0 0 0 0 0 0 2 4 3 1 0 3 4 2 1 0 1 3 4 2 0 2 1 3 4 0 4 2 1 3

6	6	0	0	0	0	0	0	0	0	0	0	0	0	0	0
		0	1	0	3	5	0	3	2	1	0	3	4	1	2
		0	5	0	2	3	0	5	1	4	0	2	1	5	3
		0	2	0	4	1	0	1	5	3	0	4	1	3	2
		0	4	0	5	4	0	1	4	2	0	1	5	2	4
		0	3	0	4	2	0	2	3	5	0	5	3	4	1
6	7	0	0	0	0	0	0	0	0	0	0	0	0	0	0
		0	1	0	3	6	0	6	1	3	0	1	2	6	4
		0	6	0	5	2	0	4	5	2	0	3	6	5	2
		0	3	0	6	4	0	2	6	5	0	3	1	4	5
		0	5	0	2	1	0	3	2	5	0	2	4	1	3
		0	4	0	4	5	0	5	3	4	0	4	3	5	1
6	8	0	0	0	0	0	0	0	0	0	0	0	0	0	0
		0	1	0	7	5	0	3	6	1	0	2	3	6	1
		0	5	0	3	2	0	4	7	6	0	6	1	5	3
		0	2	0	5	1	0	1	5	2	0	5	2	7	6
		0	7	0	4	7	0	7	1	3	0	1	7	2	4
		0	3	0	2	4	0	2	3	7	0	7	3	4	5
6	9	0	0	0	0	0	0	0	0	0	0	0	0	0	0
		0	8	0	2	5	0	2	7	3	0	6	1	4	7
		0	6	0	3	7	0	7	6	4	0	1	3	2	7
		0	3	0	1	6	0	6	4	8	0	4	2	6	3
		0	2	0	5	4	0	5	2	1	0	2	7	5	6
		0	7	0	8	1	0	1	3	5	0	6	5	1	8
6	10	0	0	0	0	0	0	0	0	0	0	0	0	0	0
		0	1	0	7	4	0	4	7	6	0	4	6	1	3
		0	7	0	4	9	0	3	2	8	0	5	1	3	6
		0	2	0	1	2	0	1	5	9	0	3	2	9	8
		0	5	0	3	7	0	2	8	3	0	7	5	8	4
		0	9	0	8	6	0	7	4	1	0	1	8	4	9
6	11	0	0	0	0	0	0	0	0	0	0	0	0	0	0
		0	10	0	10	8	0	9	3	1	0	6	9	8	5
		0	6	0	3	9	0	5	1	7	0	1	5	2	3
		0	4	0	1	3	0	7	4	8	0	2	4	5	9
		0	9	0	8	4	0	10	2	4	0	7	6	1	10
		0	7	0	6	5	0	8	6	5	0	8	3	7	2
6	12	0	0	0	0	0	0	0	0	0	0	0	0	0	0
		0	11	0	6	5	0	7	2	8	0	11	9	3	5
		0	4	0	1	2	0	11	9	1	0	3	8	9	1
		0	1	0	11	4	0	8	3	5	0	1	2	9	4
		0	9	0	8	3	0	9	10	3	0	2	1	4	7
		0	7	0	2	10	0	1	5	6	0	5	11	10	9
6	13	0	0	0	0	0	0	0	0	0	0	0	0	0	0
		0	11	0	1	6	0	1	9	3	0	7	11	12	9
		0	5	0	7	9	0	4	11	2	0	10	2	3	8
		0	6	0	5	12	0	9	10	5	0	11	6	8	10
		0	2	0	2	5	0	3	7	6	0	8	1	9	4
		0	1	0	10	11	0	6	12	10	0	2	9	5	3

6	14	0 0 0 9 0 5 0 6 0 3 0 7	0 0 0 0 4 7 0 12 11 0 6 1 0 9 2 0 10 8	0 0 0 0 0 5 10 1 0 8 12 2 0 1 7 3 0 13 2 11 0 4 3 9	0 0 0 0 0 0 12 8 3 4 0 5 4 11 9 0 3 6 4 1 0 2 3 9 11 0 6 11 10 13
6	15	0 0 0 1 0 10 0 2 0 12 0 4	0 0 0 0 12 6 0 3 8 0 2 1 0 4 11 0 5 7	0 0 0 0 0 1 8 7 0 3 12 14 0 7 2 5 0 10 13 11 0 14 3 8	0 0 0 0 0 0 1 7 2 5 0 10 8 9 4 0 7 12 10 2 0 4 13 6 12 0 11 6 3 14
7	2	0 0 0 0 0 1 0 0 0 1 0 1 0 1	0 0 0 0 1 1 0 0 1 0 0 1 0 1 1 0 1 0 0 1 0	0 0 0 0 0 1 0 1 0 1 1 0 0 0 1 1 0 1 0 1 0 1 1 0 0 0 1 1	0 0 0 0 0 0 0 1 1 0 0 1 1 0 1 0 1 1 1 0 0 1 0 1 1 0 0 1 0 1 0 0 0 1 1
7	3	0 0 0 2 0 2 0 1 0 2 0 1 0 1	0 0 0 0 1 1 0 1 2 0 2 1 0 0 2 0 1 2 0 2 1	0 0 0 0 0 2 2 1 0 0 2 1 0 2 1 2 0 1 2 0 0 0 1 2 0 1 0 2	0 0 0 0 0 0 1 2 0 1 0 2 2 1 0 0 2 1 2 1 0 1 0 2 2 0 2 0 1 2 0 0 1 1 2
7	4	0 0 0 3 0 2 0 1 0 2 0 3 0 1	0 0 0 0 1 3 0 2 3 0 3 1 0 2 1 0 1 2 0 3 2	0 0 0 0 0 2 1 3 0 0 1 2 0 3 1 2 0 3 2 1 0 2 3 1 0 1 3 2	0 0 0 0 0 0 1 2 1 3 0 0 3 1 2 0 1 2 3 2 0 2 0 3 1 0 2 1 0 3 0 3 1 2 0
7	5	0 0 0 4 0 1 0 2 0 2 0 3 0 3	0 0 0 0 1 3 0 1 4 0 4 1 0 3 2 0 2 3 0 2 1	0 0 0 0 0 1 3 4 0 1 4 3 0 3 4 2 0 4 2 1 0 2 1 3 0 4 1 2	0 0 0 0 0 0 2 4 3 1 0 1 2 4 3 0 2 3 1 4 0 1 4 3 2 0 4 3 1 2 0 3 1 2 4
7	6	0 0 0 2 0 4 0 3 0 1 0 5 0 5	0 0 0 0 2 3 0 3 5 0 1 5 0 1 2 0 4 1 0 5 4	0 0 0 0 0 3 4 5 0 2 1 5 0 5 1 3 0 1 5 2 0 4 3 1 0 5 2 4	0 0 0 0 0 0 4 3 2 5 0 2 5 3 4 0 3 4 5 1 0 5 4 1 2 0 3 1 4 2 0 1 3 5 4

7	7	0 0 0 5 0 1 0 3 0 4 0 6 0 2	0 0 0 0 3 6 0 5 4 0 2 3 0 6 1 0 4 2 0 1 5	0 0 0 0 0 1 3 2 0 2 6 4 0 3 2 6 0 4 5 1 0 5 1 3 0 6 4 5	0 0 0 0 0 0 1 3 2 5 0 2 6 4 3 0 3 2 6 1 0 4 5 1 6 0 5 1 3 4 0 6 4 5 2
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10	12	0 0 0 11 0 9 0 1 0 7 0 3 0 5 0 10 0 6 0 2	0 0 0 0 3 9 0 10 5 0 7 10 0 9 1 0 8 7 0 6 2 0 2 4 0 1 6 0 11 8	0 0 0 0 0 3 8 5 0 10 2 3 0 5 7 1 0 6 4 7 0 8 11 6 0 4 10 8 0 11 6 10 0 9 5 4 0 2 3 11	0 0 0 0 0 0 6 1 11 5 0 11 3 5 2 0 10 11 2 7 0 10 9 6 4 0 9 3 10 1 0 3 5 1 8 0 4 2 3 11 0 7 10 4 3 0 9 5 4 6
10	13	0 0 0 6 0 11 0 12 0 1 0 9 0 3 0 4 0 7 0 8	0 0 0 0 12 10 0 4 5 0 5 2 0 2 6 0 7 3 0 6 9 0 1 8 0 11 4 0 8 7	0 0 0 0 0 6 11 5 0 11 5 2 0 12 2 7 0 1 6 4 0 9 10 1 0 3 1 8 0 4 3 11 0 7 8 3 0 8 4 6	0 0 0 0 0 0 6 1 11 5 0 11 3 5 2 0 12 11 2 7 0 1 9 6 4 0 9 3 10 1 0 3 5 1 8 0 4 2 3 11 0 7 12 8 3 0 8 5 4 6

10	14	0 0 0 1 0 2 0 3 0 4 0 5 0 6 0 7 0 8 0 9	0 0 0 0 9 12 0 2 11 0 11 7 0 12 6 0 5 10 0 4 8 0 3 2 0 13 1 0 7 13	0 0 0 0 0 1 3 5 0 2 5 1 0 3 7 6 0 4 9 7 0 5 12 13 0 6 13 8 0 7 8 10 0 8 10 3 0 9 6 2	0 0 0 0 0 0 1 3 5 2 0 2 5 1 9 0 3 7 6 4 0 4 9 7 6 0 5 12 13 5 0 6 13 8 10 0 7 8 10 13 0 8 10 3 1 0 9 6 2 8
10	15	0 0 0 3 0 5 0 6 0 9 0 12 0 1 0 8 0 10 0 6	0 0 0 0 11 6 0 14 2 0 12 4 0 10 12 0 7 1 0 13 9 0 3 11 0 8 5 0 1 14	0 0 0 0 0 3 14 5 0 5 8 1 0 6 2 9 0 9 11 7 0 12 1 13 0 1 3 8 0 8 13 10 0 10 5 7 0 6 4 2	0 0 0 0 0 0 1 3 14 5 0 2 5 8 1 0 3 7 2 9 0 4 9 11 7 0 5 12 1 13 0 14 1 3 8 0 7 8 13 10 0 8 10 5 7 0 9 6 4 2

Table 5.2.1: **NBIB designs with** $r \le 30$, **obtainable from Method 5.2.1 and Method 5.2.2**

Sl. No.	v	b_1	b_2	r	k_1	k_2	λ_1	λ_2	Method
1	17	17	68	16	16	4	15	3	5.2.1
2	17	34	68	16	8	4	7	3	5.2.2
3	18	51	153	17	6	2	5	1	5.2.2
4	19	19	57	9	9	3	4	1	5.2.1
5	19	19	57	18	18	6	17	5	5.2.1
6	25	25	75	24	24	8	23	7	5.2.1
7	25	25	100	24	24	6	23	5	5.2.1
8	25	25	150	24	24	4	23	3	5.2.1
9	25	50	150	24	12	4	11	3	5.2.2
10	25	75	150	24	8	4	7	3	5.2.2
11	29	29	116	28	28	7	27	6	5.2.1
12	29	29	203	28	28	4	27	3	5.2.1
13	29	58	116	28	14	7	13	6	5.2.2
14	31	31	93	15	15	5	7	2	5.2.2
15	31	31	155	15	15	3	7	1	5.2.2
16	31	31	93	30	30	10	29	9	5.2.1
17	31	31	155	30	30	6	29	5	5.2.1
18	31	31	186	30	30	5	29	4	5.2.1
19	31	93	186	30	10	5	9	4	5.2.2
20	37	74	222	18	9	3	4	1	5.2.2
21	43	43	129	21	21	7	10	3	5.2.2
22	43	43	301	21	21	3	10	1	5.2.2

Table 5.2.2: Catalogue of the NPBIB Designs with GD Association Scheme with $r \le 15$ **obtainable from Methods 5.2.3 and 5.2.4.**

S/N	m	n	v	b_1	b_2	r	k_1	k_2	λ_{11}	λ_{12}	λ_{21}	λ_{22}	Solution
1	2	3	6	9	18	6	4	2	3	4	0	2	M 5.2.3; 2-Reso
2	2	3	6	18	36	12	4	2	6	8	0	4	Rep02(1)
3	2	3	6	18	36	12	4	2	6	8	3	2	M 5.2.4 (1+Disc. Set); 2-Reso
4	2	4	8	20	40	10	4	2	6	3	2	1	M 5.2.4; 1-Reso
5	2	4	8	24	48	12	4	2	4	6	0	3	M 5.2.3; 3-Reso
6	2	4	8	26	52	13	4	2	5	6	3	1	M 5.2.4; 1-Reso
7	2	4	8	26	52	13	4	2	9	3	3	1	M 5.2.4; 1-Reso
8	2	4	8	30	60	15	4	2	7	6	1	3	M 5.2.4; 3-Reso
9	2	4	8	16	48	12	6	2	8	9	0	3	M 5.2.3; 3-Reso
10	3	2	6	12	24	8	4	2	4	5	0	2	Trial and Error; 2-Reso
11	3	2	6	12	24	8	4	2	4	5	4	1	Trial and Error; 2-Reso
12	3	2	6	12	24	8	4	2	8	4	4	1	Trial And Error; 2-Reso
13	3	2	6	15	30	10	4	2	10	5	6	1	Trial and Error; 2-Reso
*14	3	2	6	18	36	12	4	2	8	7	0	3	SRP1: T 3.2:KPB (BIB^R(4,2)) +10; 2-Reso

S/N	m	n	v	b_1	b_2	r	k_1	k_2	λ_{11}	λ_{12}	λ_{21}	λ_{22}	Solution
*15	3	2	6	18	36	12	4	2	8	7	4	2	SRP1: T 3.2:KPB(BIB^R(4,2)) +11; 2-Reso
*16	3	2	6	21	42	14	4	2	10	8	2	3	SRP3: Method 2.4(S1 & NB1)+10; 2-Reso
*17	3	2	6	21	42	14	4	2	10	8	6	2	SRP3: Method 2.4(S1 & NB1) +11; 2-Reso
*18	3	2	6	21	42	14	4	2	14	7	2	3	SRP3: Method 2.4(S1 & NB1) +2 copies of SRP1: T 3.2:KPB(BIB^R(4,2)) ; 2-Reso
*19	3	2	6	21	42	14	4	2	14	7	6	2	SRP3: Method 2.4(S1 & NB1)+12; 2-Reso
*20	3	3	9	9	27	6	6	2	3	4	0	1	Trial and Error (SR65 & R34); 2-Reso
*21	3	3	9	9	27	6	6	2	6	3	0	1	Trial and Error (S23 & R34); 2-Reso
*22	3	3	9	15	30	10	6	3	7	6	4	2	M 5.2.4; 2-Reso
*23	3	3	9	18	36	12	6	3	9	7	6	2	M 5.2.4; 2-Reso
24	3	3	9	18	54	12	6	2	6	8	0	2	Rep02(20) ; 2-Reso
*25	3	3	9	18	54	12	6	2	9	7	0	2	20+21; 2-Reso
*26	3	3	9	18	54	12	6	2	12	6	0	2	Rep02(21) ; 2-Reso
*27	3	3	9	21	42	14	6	3	11	8	8	2	M 5.2.4; 2-Reso
*28	3	4	12	12	48	8	8	2	8	4	0	1	M 5.2.4 (S56 & R38); 2-Reso
29	4	2	8	12	24	6	4	2	0	3	0	1	Half(SRP8), Trial and Error; 2-Reso
30	4	2	8	12	24	6	4	2	6	2	0	1	Half(SRP6); Trial and Error; 1-Reso
*31	4	2	8	24	48	12	4	2	6	5	0	2	29+30; (1,2)-Reso

T 3.2: KPB, denotes Theorem 3.2: Kageyama, Philip and Banerjee (1995)

Method 2.4(X # & Y#), denotes the designs obtained by Method 2.4 of Satpati and Parsad (2005) using the X type of designs of Clatworthy [3] of serial number # and Y #, denotes NB(I)B design #.

Method 5.2.4(X # & Y#), denotes the designs obtained by Method 5.2.4 using the X type of designs of Clatworthy [3] of serial number # and Y #, denotes NB(I)B design #.

NB1, denotes nested balanced block design with parameters $v=4$, $b_1=3$, $b_2=6$, $r=3$, $k_1=4$, $k_2=2$, $\lambda_1=3$, $\lambda_2=1$.

BIB (x,y), denotes the irreducible BIB design with x treatments and block size as y

M#, denotes the designs obtained by the Method #.

SRP #, denotes the designs catalogued in Satpati and Parsad (1995).

Table 5.3.1: Catalogue of Triangular Partially Balanced Treatment block designs for $n = m + 1$ and $v = m(m + 1)$.

Sl. No.	v	b	k	r	r_0	λ_0	λ_1	AE
1	6	4	4	2	4	2	1	0.9985
2	10	5	5	2	5	2	1	0.9790
3	15	6	6	2	6	2	1	0.9730
4	21	7	7	2	7	2	1	0.9679
5	28	8	8	2	8	2	1	0.9646
6	36	9	9	2	9	2	1	0.9626
7	45	10	10	2	10	2	1	0.9614
8	56	11	11	2	11	2	1	0.9611
9	66	12	12	2	12	2	1	0.9610
10	78	13	13	2	13	2	1	0.9613
11	91	14	14	2	14	2	1	0.9617
12	105	15	15	2	15	2	1	0.9623

Table 5.8.1: Response ratios for different crop groups (All India)

Crop groups	Area 000 ha (2000-01)	Average Control yield (kg/ha)	Average Response Ratio (kg/ha)							
			N	NP	NK	NPK	P over		K over	
			Over Control				N	NK	N	NP
Cereal	99757	1803	8.56	8.97	8.66	8.63	10.02	11.29	9.16	10.85
Oilseed	23250	897	8.53	5.19	6.91	5.37	4.48	5.48	6.02	7.88
Pulses	20026	586	8.11	7.53	8.97	7.12	7.22	5.95	12.09	5.32
Foodgrains		1485	8.50	8.15	8.42	7.89	8.73	9.60	9.06	9.59

Table 5.8.2: Percentage increase in yield for different crop groups (All India)

Crop groups	Area 000 ha (2000-01)	Average Control yield (kg/ha)	% increase in yield due to							
			N	NP	NK	NPK	P over		K over	
			Over Control				N	NK	N	NP
Cereal	99757	1803	46.76	74.27	65.80	96.34	18.38	18.09	12.74	12.51
Oilseed	23250	897	30.74	63.39	50.59	87.57	24.02	24.17	14.62	14.48
Pulses	20026	586	33.38	99.24	58.08	116.97	48.23	37.75	18.62	9.70
Foodgrains		1485	42.28	75.99	62.25	97.80	23.48	21.83	13.87	12.44