Modeling Agricultural Commodity Price Volatility using GARCH Model with Structural Break

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INTRODUCTION

A time series data has the usual assumption of constant mean and variance over time i.e., stationary. In real situation this assumption seems to be violated often. Mostly, the series dealing with financial aspects such as prices of various commodities hardly holds the assumption of constant variance. Moreover, few series shows an interesting behaviour of stationarity for some time, then suddenly the variability of the error term changes, it stays constant again for some time at this new value, until another change occurs (Ang and Bekaert, 2002). If in the analysis of a series, this factor is not accounted for, then we may land up having poor results (Clements and Hendry, 1998). The change in variance of the series at certain time epochs must be identified whether be a single or multiple. The very first work in this direction was started by Hsu *et al*. (1974), who used this method as an alternative to the Pareto distribution to model stock returns. There are many works aimed at identifying the point of changes in a sequence of independent random variables. The Bayes ratio was also used to find whether a series has a single change of variance at an unknown point or not. Further, the methods for detection of a variance shift at an unknown point in a sequence of independent observations, focusing on the detection of points of change one at a time were developed, because of the heavy computational burden involved in looking for several points of change simultaneously. The autoregressive moving average (ARMA) models were also studied by allowing for outliers and variance changes for finding the methods for detecting point of variance change in auto correlated observations. In recent years even the prediction of volatile series are being improved by identifying the proper points of change of variance present in it. It has been seen that researchers are increasingly collecting longer datasets. Let us consider, for example, recent work in genomics, looking at detecting changes in gene copy numbers or in the compositional structure of the genome and in finance where, for example, interest lies in detecting changes in the volatility of time series. Typically, such series will contain several chnagepoints. There is therefore a growing need to be able to search for such changes efficiently. Detection of chnagepoints in a series is now a researchable issue let that be single or multiple. For single point one can use Centred Cumulative Sum of Squares algorithm or the Atmost one Chnagepoint (AMOC) algorithm. The algorithm for detection of multiple changepoints in a series was first given by Inclan and Tiao (1994) in the form of Iterated Cumulative Sum of Squares (ICSS). In recent years using this algorithm efficient method for multiple changepoints detection has been developed as Pruned Exact Linear Time (PELT) by Killick *et al*., (2012).

The popular generalised autoregressive conditional heteroscedastic (GARCH) model is being applied to volatile time-series data for decades now. The superiority of this model has been found in agricultural data series as well (Lama, *et al*. 2015). But, in recent times the modelling of volatile series are not only done by GARCH model. The identification of chnagepoints in the variance of the series and incorporating it accordingly has also been started. This incorporation gives a better forecast efficiency of the model as well as proper reasoning of the abrupt change in the series. A structural break in the unconditional variance will result in a structural break in the GARCH process (Hillebrand, 2005).

MODEL DESCRIPTION

GARCH MODEL

The ARCH(q) model for the series $\{\varepsilon_t\}$ is defined by specifying the conditional distribution of ε_t given the information available up to time $t-1$. Let ψ_{t-1} denote this information. ARCH (q) model for the series $\{\varepsilon_t\}$ is given by

$$\varepsilon_t|\psi_{t-1} \sim N(0, h_t) \tag{1}$$

$$h_t = a_0 + \sum_{i=1}^{q} a_i \varepsilon_{t-i}^2 \tag{2}$$

Where, $a_0 > 0$, $a_i \geq 0$ for all i and $\sum_{i=1}^{q} a_i < 1$ are required to be satisfied to ensure nonnegative and finite unconditional variance of stationary $\{\varepsilon_t\}$ series.

However, ARCH model has some drawbacks. Firstly, when the order of ARCH model is very large, estimation of a large number of parameters is required. Secondly, conditional variance of ARCH(q) model has the property that unconditional autocorrelation function (Acf) of squared residuals; if it exists, decays very rapidly compared to what is typically observed, unless maximum lag q is large. To overcome the weaknesses of ARCH model, Bollerslev (1986) proposed the Generalized ARCH (GARCH) model, in which conditional variance is also a linear function of its own lags and has the following form

$$\varepsilon_t = \xi_t h_t^{1/2}$$

$$h_t = a_0 + \sum_{i=1}^{q} a_i \varepsilon_{t-i}^2 + \sum_{j=1}^{p} b_j h_{t-j} \tag{3}$$

Where, ξ_t ~ IID $(0,1)$. A sufficient condition for the conditional variance to be positive is

$$a_0 > 0,\ a_i \geq 0,\ i = 1,2,\ldots,q.\ \ b_j \geq 0,\ \ j = 1,2,\ldots,p$$

The GARCH (p, q) process is weakly stationary if and only if

$$\sum_{i=1}^{q} a_i + \sum_{j=1}^{p} b_j < 1$$

GARCH MODEL WITH STRUCTURAL BREAKS

Lamoureux and Lastrapes (1990) showed that standard GARCH models overestimate the underlying volatility persistence and structural breaks should be incorporated into a GARCH model to get reliable parameter estimates. The augmented univariate GARCH model with structural breaks as:

$$h_t = a_0 + d_1 D_1 + d_2 D_2 + \ldots + d_n D_n \sum_{i=1}^{q} a_i\, \varepsilon_{t-i}^2 + \sum_{j=1}^{p} b_j\, h_{t-j}$$

Where, following Aggarwal, Inclan, and Leal (1999), $D_1\ldots$, D_n, are the set of dummy variables taking a value of one from each point of structural break in variance onwards and zero elsewhere. Following the above mentioned procedure Ewing and Malik (2013) modeled the series of crude oil prices with and without considering the changepoints. Their results clearly indicate the improvement in the estimates of the model after incorporating changepoints in the model.

DATA AND METHODOLOGY

Data used in this paper is International Cotlook A index which was collected from the commodity price bulletin, published by the United Nations Convention of Trade and Development (UNCTAD). The series contains 393 data points (April, 1982 to December, 2014). The time plot of the series is given in Figure 1.

We need to note that as we are interested to model the volatility of the series we keep the mean equation for both the GARCH models with and without structural break same. As volatility cannot be estimated directly hence the square of the residuals obtained from the mean equation is used as a proxy for volatility. Thus in detection of the structural break in the series this squared residual series has been used. After detection of the chnagepoints the series can be modeled in two different ways. Firstly, the most important single chnagepoint is to be identified and can split the series into two and model it differently Secondly, we can incorporate the chnagepoint and model the series with it. In this paper, we have employed both the approaches and observed the different behaviour of the series. In addition to it the series is also modeled directly with suitable GARCH model and the results are compared.

RESULTS AND DISCUSSION

The usual procedure for modelling the series with appropriate GARCH model was done first and the results are presented in Table 1. Then the single break point was identified and further analysis was carried out. The result of break point analysis is reported in Table 2 supported by Fig. 2. The identified break point was found to be a common point when multiple break points were identified using PELT algorithm. The result obtained from PELT algorithm are reported in Table 3 and further supported by Fig. 3. As mentioned earlier, after the identification of the changepoint the series is modeled by splitting it into two parts and also by incorporating the changepoint, the results are reported in Table 4 and Table 5 respectively. Further, the prediction of volatility by two models i.e. with and without structural break is depicted by Fig. 4.

The results from Table 1 clearly indicate the presence of high volatility in the series as the sum of ARCH and GARCH coefficients is 0.96. This also indicates weak stationarity of the series as the sum of the coefficients is close to 1. The values of ARCH and GARCH coefficients are 0.40 and 0.56 indicating slightly greater effect of conditional variance in the volatility process. But from this result we cannot clearly infer that among the ARCH and GARCH process which is dominant in the series for explaining its volatile nature.

From Table 2 we obtain the single changepoint in the 156 observation i.e. April, 1995. This indicates that the series has different structure from this point onwards in terms of its variance. This is an important finding as it clearly justifies the impact of formation of WTO (World Trade Organization) in 1995 on the prices of commodities in international markets. Thus, the analysis of the series pre and post WTO regime is important.

Table 4 clearly indicates the change in behaviour of the series pre and post WTO. It is found that in pre WTO period the series does not exhibit GARCH process in its volatility but in the later period it does exhibit the GARCH process. Moreover, Table 5 which shows the results of the GARCH model with changepoints incorporated reveals that the volatility present in the series is more due to the GARCH effect than the ARCH one. This result goes with the post WTO period which reveals the presence of GARCH in it. In, addition to it, sum of the coefficients of ARCH and GARCH is 0.83 which is much less than 0.96 obtained earlier for the series without chnagepoint. This indicates that the process is more stationary as compared to the earlier one. It is noteworthy that the GARCH coefficient is 0.70 as compared to 0.56 and the ARCH coefficient is 0.13 as compared to 0.40 of the model without changepoint incorporation. This result clearly indicates that the GARCH process is dominant in the series if the effect of changepoint is accounted for.

Finally, from Fig. 4 we can infer that the GARCH model with changepoints incorporated is able to model the volatility better as the conditional exhibited by it is more towards the end where the series is more volatile as compared to the rest of the part. This, finding adds to the support that the incorporation of changepoint has increased the efficiency of the model.

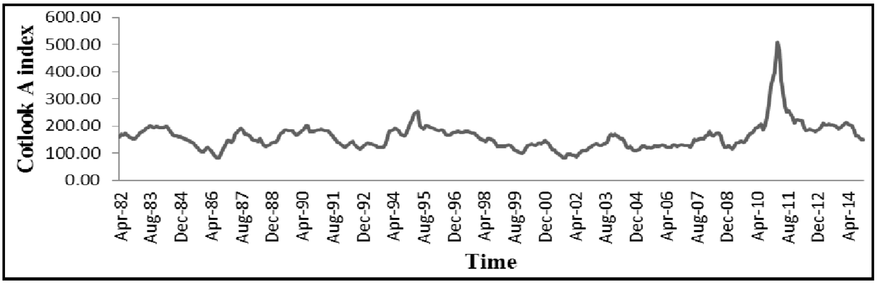

Fig. 1: Time Plot of the Series

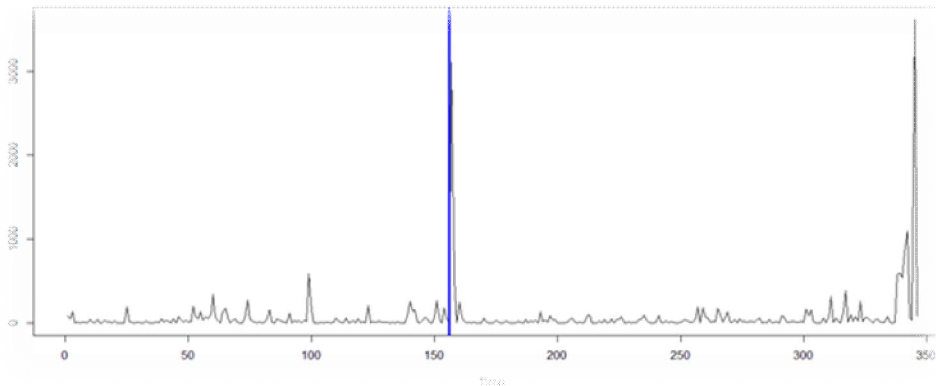

Fig. 2: Chnagepoint analysis using AMOC Algorithm

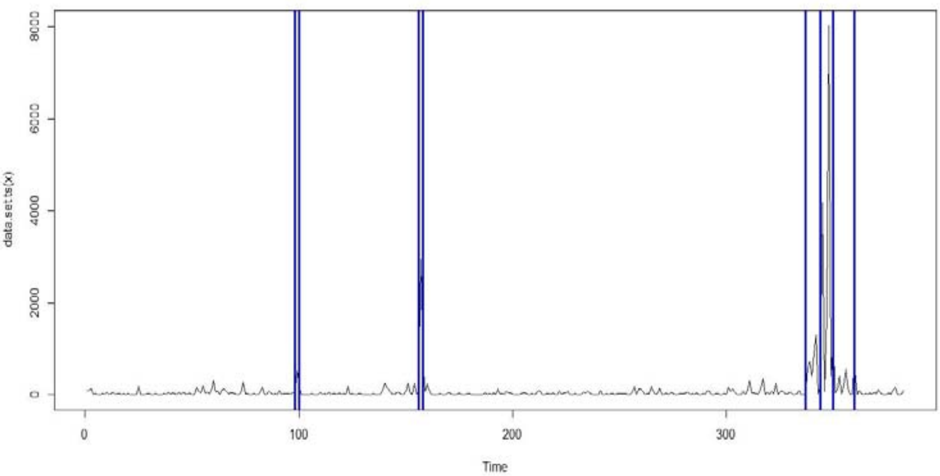

Fig. 3: Chnagepoints analysis using PELT Algorithm

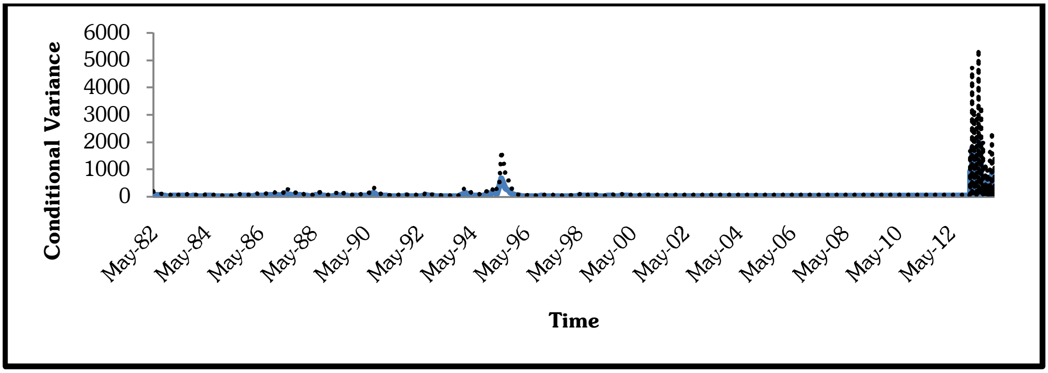

Fig. 3: Comparison between the Two GARCH Models

Table 1: Parameter Estimates of the GARCH Model

Parameters	a_0	a_i	b_j	$a_i + b_j$
GARCH (1,1)	5.68 (1.63)	0.40 (0.08)	0.56 (0.06)	0.96

Note: Values in the parenthesis are the standard errors

Table 2: Change point Detection using AMOC

Method of analysis	AMOC
Test statistic	Normal
Type of penalty	SIC
Maximum number of chnagepoints	1
Chnagepoint locations	156, April 1995
No. of chnagepoints	1

Table 3: Change points Detection using PELT

Method of analysis	PELT
Test statistic	Normal
Type of penalty	SIC
Maximum number of chnagepoints	Infinity
Chnagepoint locations	98,100,156,158,337,344,350 and 360
No. of chnagepoints	8

Table 4: Parametrs of GARCH Model before and after Changepoint

GARCH Model	a_0	a_i	b_j	$a_i + b_j$
Before Changepoint	30.02 (3.45)	0.26 (0.13)	---	0.26
After Changepoint	11.76 (3.59)	0.68 (0.17)	0.26 (0.10)	0.94

Note: Values in the parenthesis are the standard errors

Table 5: Parameters of GARCH Model with Changepoint

Parameters	a_0	a_i	b_j	$a_i + b_j$
GARCH (1,1)	10.29 (14.59)	0.13 (0.01)	0.70 (0.03)	0.83

Note: Values in the parenthesis are the standard errors

CONCLUSION

The Cotlook A index series was modeled with GARCH model with and without changepoint incorporation. The results obtained in all the cases clearly indicates that the effect of structural break in the series needs to be incorporated for proper modeling and interpretation of the underlying processes. The identified chnagepoint is very encouraging as it goes with the impact of WTO on prices of the commodities in the international market. In this paper we have identified the single most important chnagepoint, one can also identify the multiple changepoints and carry on the analysis. But, it is upon the researchers to choose the changepoints judicially so that it can explain the different phenomenon governing the series.

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