Jour. Ind. Soc. Ag. Statistics
54(3), 2001 : 376-384

Robustness of Block Designs for Diallel Crosses Against Missing Observations

Amitava Dey, R. Srivastava and Rajender Parsad
Indian Agricultural Statistics Research Institute, New Delhi-110 012
(Received : June, 2001)

SUMMARY

Robustness aspects of block designs for diallel crosses against one missing observation have been investigated using connectedness and efficiency criterion. Robustness of binary balanced block designs against the missing observations pertaining to one block have also been studied.

Key words: Block designs for diallel crosses, General combining ability, Robustness, Connectedness, Efficiency.

1. Introduction

The diallel cross is a type of mating design used to study the genetic properties of a set of inbred lines. If we consider only the $\frac{p(p-1)}{2}$ – F_1 crosses of p inbred lines, it is known as Type IV mating design of Griffing [11].

The problem of generating optimal block designs for complete diallel cross experiments has been recently investigated by Gupta and Kageyama [12], Dey and Midha [4], Mukerjee [18] Das, *et al.* [2], and Parsad *et al.* [20]. All these studies have been made for the situations, where the experimenter is interested in estimating the general combining ability (*g.c.a*) effects and the specific combining ability (*s.c.a*) effects have been excluded from the model. The statistical procedures followed for the above designs, for making inductive inferences are based on ideal conditions. However, aberrations may occur due to some causes (human or natural) during experimentation. Loss of observation(s) is one such aberration. An optimal design may become non-optimal (lose its original properties) when some observation(s) is(are) missing. Therefore, there is a need to look for the designs that are insensitive or robust to this type of disturbances. Following Ghosh [9], *a block design for diallel crosses* (d_0) *is* *termed as robust against the loss of observation(s) if the resulting design* (d_m) *obtained after loss of observation(s) remains connected.* This criterion of robustness is called as *connectedness criterion.*

Even if the design remains connected, the efficiency of the resulting design may fall considerably. Hence, there is need to examine the efficiency of the resulting design relative to the original design. *A connected design is said to be robust against loss of observation(s) if the efficiency of the d_m relative to d_0 is high (*in this investigation high relative efficiency means that it is at least 0.9500). This criterion of robustness is called as *efficiency criterion.*

The robustness of general block designs against missing observation(s) has been investigated in abundance, see e.g., Hedayat and John [14]; John [15]; Ghosh [9]; Ghosh *et al.* [10]; Baksalary and Tabis [1]; Kageyama [16]; Mukerjee and Kageyama [19]; Srivastava *et al.* ([21], [22]); Dey *et al.* [6]; Gupta and Srivastava [13]; Dey [3]; Dey *et al.* [5]; Srivastava *et al.* [23]; Lal *et al.* [17], etc. In the context of block designs for diallel crosses, Ghosh and Desai ([7], [8]) have investigated the robustness aspects of complete diallel cross plans subject to the non-availability of observations pertaining to one of the blocks of the block designs obtained by taking all possible crosses of lines present in a block of a balanced incomplete block (BIB) design and Singular Group Divisible design. However, these designs are now known to be inefficient for estimation of *g.c.a.* effects. Therefore, in this investigation an attempt has been made to study the robustness of efficient block designs against one missing observation in Section 2 and against one missing block in Section 3 using both connectedness and efficiency criteria.

Throughout the present investigation, we use the following notations. $\mathbf{A}^-$ a generalized inverse of $\mathbf{A}$ i.e. $\mathbf{AA}^-\mathbf{A} = \mathbf{A}$, $\mathbf{A}'$ the transpose of $\mathbf{A}$. All vectors are column vectors, $\mathbf{1}_t$ being a $t \times 1$ vector of ones, $\mathbf{0}_t$ denotes a null matrix, and $\mathbf{I}_t$ denotes an identity matrix of order t.

2. *Robustness of Block Designs for Diallel Crosses Against One Missing Observation*

Consider a connected block design d_0 with p lines, b blocks such that there are k_j experimental units in the j^{th} block and $n = \sum_{j=1}^{b} k_j$ is the total number of experimental units. We take $(i \times i')$ as a cross between lines i and i' in diallel cross such that $i < i'$ and i, $i' = 1, 2, ..., p$. Let one of the n observations be missing. Without loss of generality we may assume that this observation pertains to the first observation of the first block and also this observation belongs to the cross (1 × 2). Let the resulting design obtained by deleting this observation from d_0, be called d_m. For the data obtained from a block design for diallel crosses, the model involving general combining ability (*g.c.a.*) is given by

$$\mathbf{y} = \mu \mathbf{1}_n + \Delta' \mathbf{g} + \mathbf{D}' \beta + \mathbf{e} \quad (2.1)$$

where $\mathbf{y}$ is (n × 1) vector of observations, μ is the general mean effect, $\mathbf{g}$ is the vector of p *g.c.a.* effects, β is the vector of b block effects, Δ' is the (n × p) observations vs lines design matrix. $(s, t)^{th}$ element of Δ' is 1 if s^{th} observation pertains to the t^{th} line and zero, otherwise. $\mathbf{D}'$ is the (n × b) observations vs block design matrix. $(s, t)^{th}$ element of $\mathbf{D}'$ is 1 if s^{th} observation pertains to the t^{th} block and zero, otherwise. $\mathbf{e}$ is the vector of normally distributed random errors with $E(\mathbf{e}) = 0$ and $Cov(\mathbf{e}) = \sigma^2 \mathbf{I}_n$.

Let $\mathbf{C}_0$ denote the coefficient matrix of the reduced normal equations for estimating linear functions of *g.c.a.* effects using design d_0 and $\mathbf{C}_m$ denote the coefficient matrix of the reduced normal equations for estimating linear functions of *g.c.a.* effects using design d_m. It can easily be seen that

$$\mathbf{C}_m = \mathbf{C}_0 - \mathbf{u}\mathbf{u}' \quad (2.2)$$

where $\mathbf{u} = [k_1(k_1 - 1)]^{-1/2} (k_1 \mathbf{a}_1 - \mathbf{n}_1)$ with $\mathbf{a}_1$ as the first column of Δ, $\mathbf{n}_1$ as the first column of $\mathbf{N}$ (the lines vs blocks incidence matrix of the original design d_0), k_1 is the size of the block 1. Following Dey [3], we have the following result.

Theorem 2.1: The design d_0 is robust as per connectedness criterion against the loss of single observation if and only if

$$\mathbf{u}' \mathbf{C}_0^{-} \mathbf{u} < 1$$

where $\mathbf{u}$ is as given in (2.2).

The above result is quite general in nature and holds for all block designs for diallel crosses (complete as well as partial). We shall, hereafter, study the robustness aspects of block designs for complete diallel crosses only.

Using Theorem 2.1, one can see that (i) a binary balanced block design for diallel crosses d_0 is robust as per connectedness criterion against the loss of one observation if and only if the non-zero eigenvalue of $\mathbf{C}_0$ is strictly larger than two and (ii) a balanced block design for diallel crosses d_0 in which each line appears twice in each block and is obtained from Family 5 of Das *et al.* [2] is robust as per connectedness criterion against loss of one observation if and only if number of lines (p) is greater than three. As mentioned earlier, a design which remains connected may lose efficiency. Therefore, we have computed the efficiency of the resulting design d_m relative to the original design d_0 using the expression

$$E = \frac{\text{Harmonic mean of the non-zero eigenvalues of } \mathbf{C}_m}{\text{Harmonic mean of the non-zero eigenvalues of } \mathbf{C}_0} \quad (2.3)$$

The information matrices $\mathbf{C}_0$, $\mathbf{uu}'$, eigenvalues of $\mathbf{C}_0(\mathbf{C}_m)$ and expressions for the efficiencies are given in Table 1 of Appendix.

Using (2.3) and Table 1, the efficiencies for all the universally optimal binary balanced block designs for diallel crosses for $p \leq 30$ given by Dey and Midha [4] and Parsad *et al.* [20] were computed against the loss of one observation. The efficiencies of the designs were more than 0.9500 except for the designs in Table 2 (efficiency -A) of Appendix. The efficiencies of balanced block designs for diallel crosses for $p \leq 30$ obtainable from Family 5 of Das *et al.* [2] were also computed. It is observed that the relative efficiencies are greater than 0.9500 except for the designs given in Table 3 of Appendix. Therefore, this class of designs with $p > 7$ are fairly robust against one missing observation according to efficiency criterion as well. All the 9 universally optimal balanced block designs with $2k > p$ given in Table 1 of Das *et al.* [2] obtained from triangular partially balanced incomplete block designs are also found to be robust according to connectedness and efficiency criteria {except the design with parameters $p = 5, b = 10, k = 4$ that is connected but has the relative efficiency 0.9330} against one missing observation.

3. *Robustness of Proper Binary Balanced Block Designs for Diallel Crosses Against a Complete Block Missing*

Let d_0 be a proper binary balanced block design with parameters p, b, k. The information matrix for d_0 is $\frac{2b(k-1)}{(p-1)}\left(\mathbf{I}_p - p^{-1}\mathbf{1}_p\mathbf{1}_p'\right)$ with unique non-zero eigenvalue as $2b(k-1)/(p-1)$ with multiplicity $(p-1)$. Let all the observations pertaining to a block of the design d_0 be missing. Without loss of generality we may assume that these observations pertain to the first block and these observations are the crosses of first 2k consecutive lines, *i.e.* the first block which is missing contains k crosses of the following type

$$(1 \times 2), (3 \times 4), \ldots, (2k-1 \times 2k)$$

Let $\mathbf{C}_b$ be the information matrix of design d_b, the resulting block design after missing all the observations from the first block of design d_0. Now, it is easy to see that

$$\mathbf{C}_b = \mathbf{C}_0 - \mathbf{A} \tag{3.1}$$

where $\mathbf{A} = \begin{bmatrix} \mathbf{H} & \mathbf{0} \\ \mathbf{0}' & \mathbf{0} \end{bmatrix}$ where, $\mathbf{H} = \mathbf{I}_k \otimes (\mathbf{M} - \mathbf{N}) + \mathbf{1}_k\mathbf{1}_k' \otimes \mathbf{N}$

$$\mathbf{M} = \left(1 - \frac{1}{k}\right)\mathbf{1}_2\mathbf{1}_2'; \mathbf{N} = \left(-\frac{1}{k}\right)\mathbf{1}_2\mathbf{1}_2'$$

The non-zero eigenvalues of $\mathbf{C}_b$ in (3.1) are

(i) $\left[\frac{2b(k-1)}{p-1} - 2\right]$ with multiplicity $(k-1)$

(ii) $\frac{2b(k-1)}{p-1}$ with multiplicity $(p-k)$ (3.2)

From these eigenvalues, one can conclude that the design d_0 is robust as per connectedness criterion against the loss of all the observations from a block if and only if the non-zero eigenvalue of $\mathbf{C}_0$ is strictly larger than two. Using (2.3) and (3.2), the relative efficiency of design d_b is

$$E = \frac{p-1}{(p-k) + b(k-1)^2[b(k-1)-(p-1)]^{-1}} \tag{3.3}$$

The relative efficiencies of all proper binary balanced block designs catalogued in Dey and Midha [4] and Parsad *et al.* [20] for $p \le 30$ against one block missing were computed using (3.3). The relative efficiencies were greater than 0.9500 except for the designs given in Table 2 (Efficiency-B) of Appendix.

ACKNOWLEDGEMENTS

Discussions held with Dr. V. K. Gupta are gratefully acknowledged. The research work is supported by the IASRI, Junior Research Fellowship to the first author. The authors would also like to thank the referee for useful comments that have led to a considerable improvement in the presentation of results.

REFERENCES

[1] Baksalary, J.K. and Tabis, Z. (1987). Conditions for robustness of block designs against unavailability of data. *J. Statist. Plann. Inf.*, **16**, 49-54.

[2] Das, A., Dey, A. and Dean, A. (1998). Optimal designs for diallel cross experiments. *Statist. Prob. Letters*, **36**, 427-436.

[3] Dey, A. (1993). Robustness of block designs against missing data. *Statistica Sinica*, **3**, 219-231.

[4] Dey, A. and Midha, C. K. (1996). Optimal block designs for diallel crosses. *Biometrika*, **83(2)**, 484-489.

[5] Dey, A., Midha, C.K. and Buchathal, D.C. (1996). Efficiency of the residual designs under the loss of observations in a block. *Jour. Ind. Soc. Agril. Stat.*, **49**, 237-248.

[6] Dey, A., Srivastava, R. and Gupta, V.K. (1991). Robust designs- a review and a bibliography. *Cahiers du CERO*, **33**, 51-61.

[7] Ghosh, D. K. and Desai, N. R. (1998). Robustness of a complete diallel crosses plans to the unavailability of one block. *Journal of Applied Statistics*, **25**, 827-837.

[8] Ghosh, D. K. and Desai, N. R. (1999). Robustness of a complete diallel crosses plan with an unequal number of crosses to the unavailability of one block. *Journal of Applied Statistics*, **26**, 563-577.

[9] Ghosh, S. (1982). Robustness of BIB designs against non-availability of data. *J. Statist. Plann. Inf.*, **6**, 29-32.

[10] Ghosh, S., Rao, S.B. and Singhi, N.M. (1983). On robust property of PBIB designs. *J. Statist. Plann. Inf.*, **8**, 119-129.

[11] Griffing, B. (1956). Concepts of general and specific combining ability in relation to diallel crossing systems. *Aust. J. Biol. Sci.*, **9**, 463-493.

[12] Gupta, S. and Kageyama, S. (1994). Optimal complete diallel crosses. *Biometrika*, **81**, 420-424.

[13] Gupta, V.K. and Srivastava, R. (1992). Investigations on robustness of block designs against missing observations. *Sankhya*, **B54**, 100-105.

[14] Hedayat, A. and John, P.W.M. (1974). Resistant and susceptible BIB designs. *Ann. Statist.*, **2**, 148-158.

[15] John, P.W.M. (1976). Robustness of balanced incomplete block designs. *Ann. Statist.*, **4**, 960-962.

[16] Kageyama, S. (1987). Some characterizations of locally resistant BIB designs of degree one. *Ann. Inst. Statist. Math.*, **39**, 661-667.

[17] Lal, K., Gupta, V.K. and Bhar, L.M. (2001). Robustness of designed experiments against missing data. *J. Applied Statist.*, **28**, 63-79.

[18] Mukerjee, R. (1997). Optimal partial diallel crosses. *Biometrika*, **84(4)**, 939-948.

[19] Mukerjee, R. and Kageyama, S. (1990). Robustness of group divisible designs. *Commu. Stat. —Theory and Methods*, **19**, 3189-3203.

[20] Parsad, R., Gupta, V.K. and Srivastava. R. (1999). Universally optimal block designs for diallel crosses. *Statistics and Applications*, **1**, 35-52.

[21] Srivastava, R., Gupta, V.K. and Dey, A. (1990). Robustness of some designs against missing observations. *Commu. Stat.—Theory and Methods*, **19**, 121-126.

[22] Srivastava, R., Gupta, V.K. and Dey, A. (1991). Robustness of some designs against missing data. *J. Applied Statist.*, **18**, 313-318.

[23] Srivastava, R., Parsad, R. and Gupta, V.K. (1996). Robustness of block designs for making test treatments-control comparisons against a missing observation. *Sankhya*, **B58(3)**, 407-413.

APPENDIX

Table 1. Expressions for $\mathbf{C}_0$, $\mathbf{uu}'$, eigenvalues of $\mathbf{C}_0$ $(\mathbf{C}_m)$ and expressions for the efficiencies for binary balanced block designs for diallel crosses obtained from Family 5 of Das, Dey and Dean (1998).

Design considered→ Properties ↓	Binary balanced block design for complete diallel crosses	Balanced block designs for diallel crosses obtained from Family 5 of Das, Dey and Dean (1998)
$\mathbf{C}_0$	$\frac{2(n-b)}{(p-1)}\left(\mathbf{I}_p - p^{-1}\mathbf{1}_p\mathbf{1}_p'\right)$	$(p-2)\left(\mathbf{I}_p - p^{-1}\mathbf{1}_p\mathbf{1}_p'\right)$
$\mathbf{uu}'$	$\begin{bmatrix}\mathbf{A} & \mathbf{B}' & \mathbf{0}' \\ \mathbf{B} & \mathbf{D} & \mathbf{0}' \\ \mathbf{0} & \mathbf{0} & \mathbf{0}\end{bmatrix}$ where $\mathbf{A} = \frac{k_1 - 1}{k_1}\mathbf{1}_2\mathbf{1}_2'$, $\mathbf{B} = \left(-\frac{1}{k_1}\right)\mathbf{1}_{2(k_1-1)}\mathbf{1}_2'$ and $\mathbf{D} = \frac{1}{k_1(k_1-1)}\mathbf{1}_{2(k_1-1)}\mathbf{1}_{2(k_1-1)}'$	$\begin{bmatrix}\mathbf{S} & \mathbf{T}' \\ \mathbf{T} & \mathbf{Z}\end{bmatrix}$ where $\mathbf{S} = \frac{(p-2)^2}{p(p-1)}\mathbf{1}_2\mathbf{1}_2'$, $\mathbf{T} = -\frac{2(p-2)}{p(p-1)}\mathbf{1}_{(p-2)}\mathbf{1}_2'$ and $\mathbf{Z} = \frac{4}{p(p-1)}\mathbf{1}_{(p-2)}\mathbf{1}_{(p-2)}'$
Non-zero eigenvalues of $\mathbf{C}_0$	$\frac{2(n-b)}{(p-1)}$ with multiplicity $(p-1)$	$(p-2)$ with multiplicity $(p-1)$
Non-zero eigenvalues of $\mathbf{C}_m$	(i) $\frac{2[(n-b)-(p-1)]}{(p-1)}$ with multiplicity one (ii) $\frac{2(n-b)}{(p-1)}$ with multiplicity $(p-2)$	(iii) $\frac{(p-2)(p-3)}{(p-1)}$ with multiplicity one (iv) $(p-2)$ with multiplicity $(p-2)$
Relative Efficiency	$E = \frac{p-1}{\left[\frac{(n-b)}{(n-b-p+1)} + (p-2)\right]}$	$E = \frac{p^2 - 4p + 3}{p^2 - 4p + 5}$

Table 2. Universally optimal binary balanced block designs for diallel crosses with Relative Efficiency-A (after one missing observation) and Efficiency-B (after one block missing) are less than 0.9500

Sl. No.	p	b	k	n	Efficiency A	Efficiency B
1.	5	5	2	10	0.5000	0.5000
2.	5	10	2	20	0.8571	0.8571
3.	5	15	2	30	0.9167	0.9167
4.	6	5	3	15	0.8333	0.7143
5.	6	15	3	45	-	0.9259
6.	7	7	3	21	0.8889	0.8000
7.	7	21	2	42	0.9375	0.9375
8.	8	7	4	28	0.9333	0.8235
9.	9	9	4	36	-	0.8636
10.	9	18	2	36	0.9091	0.9091
11.	10	9	5	45	-	0.8710
12.	11	11	5	55	-	0.8947
13.	12	11	6	66	-	0.8980
14.	13	13	6	78	-	0.9138
15.	14	13	7	91	-	0.9155
16.	15	15	7	105	-	0.9268
17.	16	15	8	120	-	0.9278
18.	17	17	8	136	-	0.9364
19.	18	17	9	153	-	0.9370
20.	19	19	9	171	-	0.9437
21.	20	19	10	190	-	0.9441
22.	21	21	10	210	-	0.9494
23.	22	21	11	231	-	0.9497

'–' denotes that the efficiency-A is greater than or equal to 0.9500

Table 3. Universally optimal balanced block designs for diallel crosses developed from Family-5 of Das, Dey and Dean (1998) with Relative Efficiencies (after missing one observation) are less than 0.9500

Sl. No.	p	b	k	n	Efficiency
1.	5	2	5	10	0.8000
2.	7	3	7	21	0.9231