Designs Involving Varying Temporal Environments Under Factorial Treatment Structure

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Abstract

In agricultural or animal experiments, when a series of tasks are assigned to the experimental units under different environmental conditions, it is often a tedious job to alter the conditions at the end of each task period. Hence, each unit is put to perform all the assigned tasks under one set of conditions during one session (environment) and then the conditions are altered for the next session for the next series of tasks. The setting is such that there is a gap between each main session and hence it is assumed that no carry over effect transfers from a main session to another. But carry over effects are assumed to be present within main sessions, between sub-sessions. Designs involving sequences of treatment combinations of two factors, one nested within the other, are suitable for such situations. A general method of construction of such designs has been obtained here. The resultant designs are balanced in the sense that each combination is followed by every level of second factor same number of times. The method of construction has been illustrated by two series of designs wherein the second series is a rearrangement of the first one and used for comparative study. It was observed that the precision of estimation of direct and carry over effects is more with the design having more number of levels of the nested factor.

Key Words: direct effects, carry over effects, change over designs, nested factors

Mathematics Subject Classification: 62K99

1. INTRODUCTION

In many animal experiments, it is often required to measure the effect of response from two or more factors over various temporal environments, like studying the effect of simultaneous application of various fertilizers on a crop or, the effect of feeds and environments on milk yield of cows. Designs involving sequences of treatments, or more popularly known as change over designs (CODs), having two or more factors are suitable for such situations. Several researchers have contributed to the construction and related aspects of CODs for the simultaneous application of more than one factor to experimental units over periods (Fletcher, 1987; Fletcher and John, 1985; Fletcher *et al.*, 1990; Dwivedi *et al*., 2008 and Mason and Hinkelmann, 1971). Most of these studies assumed the interaction among the factors to be present. Some studies were made without considering the presence of residual effects (*i.e*., the carry over effects of treatments that persist even after the period of application of treatments) and some considered the situations where one of the factors exhibit residual effects.

Sometimes, different levels of two unrelated factors are to be applied to experimental units simultaneously and their joint effect after each period of application is measured, but as observations are taken over different time periods from the units, first order residual effects of the levels of both the factors may be present. For example, in an experiment to study the effect of different methods of shearing and various feeds on body weight of sheep, method of shearing is not related to type of

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feed. Here, observation (body weight) has to be taken from each unit during each period and both the factors may exhibit residual effects. Some classes of such factorial CODs assuming the absence of interactions were given by Lewis and Russell (1998) and Varghese *et al.* (2009).

In experimental situations, wherein experimental units are required to perform a series of tasks one after another under various environmental conditions such as different types of lighting or temperature or equipments, it is difficult to change the environmental conditions. Thus, each subject is required to perform all the assigned tasks under one set of conditions during one session. The conditions are altered from one session to another. Designs with nested structure having experimental conditions treated as levels of first factor and different tasks treated as levels of nested factor are suitable for such experiments. This experimental situation has been considered in literature by few researchers (Dean *et al*., 1999; Raghavarao and Xie; 2003). The experimental setting is such that there is a gap between each main session and hence it is assumed that no carry over effect transfers from a main session to another. But carry over effects are assumed to be present within main sessions (from sub-session to sub-session). Here, a class of designs involving sequences of treatment combinations with nested factors has been obtained.

2. DESIGNS INVOLVING SEQUENCE OF TREATMENT COMBINATIONS BALANCED FOR ONE FACTOR

We first give here some definitions of designs involving sequence of treatments having factorial treatment structure followed by the model and the class of designs proposed.

Uniform: A factorial COD with two factors F_1 and F_2 (where F_2 is nested within F_1) having levels f_1 and f_2 respectively, is called uniform on periods if every treatment combination occurs in each period the same number of times, say χ_1. A necessary condition for this to hold is that the number of units, n = $\chi_1 v$, v being the number of treatment combinations (= $f_1 f_2$). A factorial COD with two factors F_1 and F_2 having levels f_1 and f_2 respectively, is called uniform on units if every treatment combination is applied to each experimental unit the same number of times, say χ_2. This can occur only if the number of periods, p = $\chi_2 v$. A design is called uniform if it is uniform on both periods and units.

Balanced: A COD with two factors F_1 and F_2 having levels f_1 and f_2 respectively, with levels of F_2 nested within levels of F_1, is said to be balanced if every combination of the two factors is preceded by each level of the nested factor F_1 (except the level appearing in the combination) an equal number of times.

Strongly balanced: A COD with two factors F_1 and F_2 having levels f_1 and f_2 respectively, with levels of F_2 nested within levels of F_1, is said to be strongly balanced if every combination of the two factors is preceded by every other level of the nested factor (excluding the level appearing in the combination) equally often say, Δ_1 times and by the level of the nested factor appearing in the combination Δ_2 times. Δ_1 and Δ_2 may or may not be equal.

Variance Balanced: A COD with two factors F_1 and F_2 is said to be variance balanced if all elementary contrasts pertaining to direct effects of various treatment combinations consisting of levels of both the factors are estimated with a constant variance.

3. DESIGNS INVOLVING SEQUENCES OF TREATMENT COMBINATIONS
WITH NESTED FACTORS

Let two factors F_1 and F_2 have number of levels f_1 (represented by 1, 2, 3,…) and f_2 (represented by a, b, c,…) giving rise to $f_1 \times f_2$ treatment combinations where the levels of F_2 are nested within the levels of F_1. First consider a balanced COD for f_1 levels of first factor in p_1 periods (main sessions) and n_1 experimental units. Within each cell (period-unit intersection) of this design, consider another balanced/strongly balanced COD for f_2 levels of second factor in p_2 periods (sub-sessions) and n_2 experimental units. The resultant design will have p_1p_2 periods and n_1n_2 units and each sub-cell receives a treatment combination out of f_1f_2 possible combinations. It is assumed that there is a gap between each main session and hence no carry over effect is assumed from one main session to another main session. But carry over effects are assumed to be present within main sessions (from sub-session to sub-session). The resultant design is balanced/ strongly balanced depending on the design considered for the nested factor.

To explain the general method described above, we take up two illustrations using two classes of designs wherein the second case is just the reverse arrangement of the first. In the first case, we make use of Williams (1949) Latin squares as the main session design and a two treatment design as the sub-session design. An easy method of obtaining Williams Latin squares was given by Sharma (1975). The steps involved are described below:

- Construct one (or two) $f_1 \times f_1$ table(s) in which columns refer to experimental units and rows to periods according to even (or odd) f_1.
- In both the squares, number the periods from 1 to f_1 successively.
- Assign the levels of first factor 1, 2, …, f_1 successively to the f_1 cells in the first column of both the squares by proceeding from top to bottom, entering only in odd-numbered cells in the first and even numbered cells in the second square, and then reversing the direction, filling in even-numbered cells in the first and odd numbered cells in the second square.
- Obtain the successive columns of the squares by adding integer 1 to each element of the previous column and reducing the elements, if necessary, by mod f_1.

It is to be noted that in each of the constructed squares every level occurs in each row and in each column precisely once. Moreover, when f_1 is even, each level is preceded exactly once by other level in either of the two squares. Thus, in this case either of the two squares may be used. This situation occurs in neither of the two squares if f_1 is odd. However, when both the squares are considered together, one after another horizontally, each level is preceded by every other level exactly twice. Consequently, both the squares must be used in this case.

The sub-session design considered is a two treatment COD involving 2 periods and 4 units obtained by taking all possible pairs (including the identical pairs) between the two symbols.

Example 3.1: Let there are two factors with 4 levels of main session factor denoted by (1, 2, 3, 4) and 2 levels of sub-session factor denoted by (a, b). Hence there are a total of 8 (= f_1f_2 = 4 × 2) treatment combinations. By the method of construction described above, using a Williams square design as the main session factor design and the above mentioned two period design as the sub-session factor design, we get a nested COD in 8 (= p_1p_2 = 4 × 2) periods and 16 (= n_1n_2 = 4 × 4) experimental units with 2 sub-sessions nested within 4 main sessions as given below:

		Experimental Unit															
		i	ii	iii	iv	v	vi	vii	viii	ix	x	xi	xii	xiii	xiv	xv	xvi
Period	1	1a	1a	1b	1b	2a	2a	2b	2b	3a	3a	3b	3b	4a	4a	4b	4b
	2	1a	1b	1a	1b	2a	2b	2a	2b	3a	3b	3a	3b	4a	4b	4a	4b
	3	4a	4a	4b	4b	1a	1a	1b	1b	2a	2a	2b	2b	3a	3a	3b	3b
	4	4a	4b	4a	4b	1a	1b	1a	1b	2a	2b	2a	2b	3a	3b	3a	3b
	5	2a	2a	2b	2b	3a	3a	3b	3b	4a	4a	4b	4b	1a	1a	1b	1b
	6	2a	2b	2a	2b	3a	3b	3a	3b	4a	4b	4a	4b	1a	1b	1a	1b
	7	3a	3a	3b	3b	4a	4a	4b	4b	1a	1a	1b	1b	2a	2a	2b	2b
	8	3a	3b	3a	3b	4a	4b	4a	4b	1a	1b	1a	1b	2a	2b	2a	2b

It can be seen that this design is uniform and combinatorially strongly balanced. The design is partially variance balanced following the rectangular association scheme given by Vartak (1955) as described below:

Association Scheme: Two treatment combinations $\phi\varphi$ and $\phi'\varphi'$ ($\phi \neq \phi'=1,2,\dots, f_1$; $\varphi \neq \varphi' = 1,2,\dots, f_2$) are said to be first associates if $\phi = \phi'$ i.e., the combinations with same level of first factor and different levels of second factor are first associates. Two treatment combinations $\phi\varphi$ and $\phi'\varphi'$ are said to be second associates if $\varphi = \varphi'$ i.e., the combinations with same level of second factor and different levels of first factor are second associates, and remaining are third associates. For the Example 3.1, the arrangement of 8 treatment combinations is as follows:
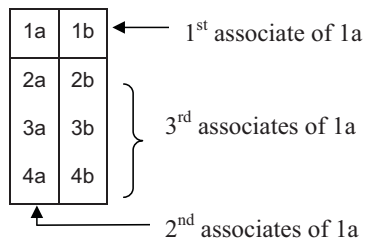
For the given association scheme for $f_1 \times f_2$ treatment combinations, number of first associates = $f_2 -1$, number of second associates = $f_1 -1$, and number of third associates = $f_1 f_2 - f_1 - f_2 + 1$.

Reverse Arrangement: By reversing the roles of F_1 and F_2 in the above example, we get another design belonging to a different class of designs involving sequences of treatment combinations with nested factors with same number of experimental periods and units.

Example 3.2: Let there are two factors with 2 levels of main session factor denoted by (1, 2) and 4 levels of sub-session factor denoted by (a, b, c, d). Hence there are 8 (= $f_1 \times f_2 = 2 \times 4$) treatment combinations. By the method of construction described above, using the above two period design and Williams square design as the main session factor design and nested factor design respectively, we get a nested COD in 8 (= $p_1 \times p_2 = 2 \times 4$) periods and 16 (= $n_1 \times n_2 = 4 \times 4$) experimental units as given below:

		Experimental Unit														
		i	ii	iii	iv	v	vi	vii	viii	ix	x	xi	xii	xiii	xiv	xv	xvi
	1	1a	1b	1c	1d	1a	1b	1c	1d	2a	2b	2c	2d	2a	2b	2c	2d
	2	1d	1a	1b	1c	1d	1a	1b	1c	2d	1a	2b	2c	2d	1a	2b	2c
	3	1b	1c	1d	1a	1b	1c	1d	1a	2b	2c	2d	2a	2b	2c	2d	2a
Period	4	1c	1d	1a	1b	1c	1d	1a	1b	2c	2d	2a	2b	2c	2d	2a	2b
	5	1a	1b	1c	1d	2a	2b	2c	2d	1a	1b	1c	1d	2a	2b	2c	2d
	6	1d	1a	1b	1c	2d	1a	2b	2c	1d	1a	1b	1c	2d	1a	2b	2c
	7	1b	1c	1d	1a	2b	2c	2d	2a	1b	1c	1d	1a	2b	2c	2d	2a
	8	1c	1d	1a	1b	2c	2d	2a	2b	1c	1d	1a	1b	2c	2d	2a	2b

This design is uniform and combinatorially balanced. In terms of variance of estimate of contrasts pertaining to direct as well as residual effects of treatment combinations, the design was seen to be partially variance balanced following rectangular association scheme. The precision of estimation of direct and residual effects is more in the design having more number of levels of the sub-session factor.

4. CONCLUSION

Series of designs involving varying temporal environments under factorial treatment structure balanced for one factor have been obtained. These designs are uniform, combinatorially balanced and in terms of variance of estimated contrasts pertaining to direct as well as residual effects of treatment combinations, partially variance balanced following rectangular association scheme. These designs find application in experiments involving more than one factor applied sequentially under different environmental conditions.

5. ACKNOWLEDGMENTS

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