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Diallel cross designs for test versus control comparisons

Sudhir Srivastava, Cini Varghese*, Seema Jaggi and Eldho Varghese

Indian Agricultural Statistics Research Institute, Library Avenue, New Delhi 110 012

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Abstract

Block designs for diallel crosses for comparing a group of test lines with a control line are suitable when the experimenters are interested to estimate test vs. control comparisons with respect to their general combining ability effects with a smaller variance. In this paper, some families of small block designs for diallel cross experiments for test vs. control comparisons have been obtained. Out of these, some designs are suitable when the experimenter cannot have blocks of equal size while the others yield designs with equal block size. As the number of lines increases, the variance of estimated contrasts pertaining to test vs. test lines as well as test vs. control lines decreases. Moreover, test vs. control comparisons are made with less variance as compared to test vs. test comparisons, in all the cases.

Key words: Association scheme, partial balance, partially balanced incomplete block design, partial diallel cross

Introduction

A major objective of plant and animal breeding programmes is to improve the genetic potential of plants and animals, respectively. The breeding trials involve two types of designs namely, mating designs and environmental designs in order to develop or raise the offsprings of parents/lines and to subject these progenies to the environmental conditions, in a systematic manner. A judicious choice of a combined mating-environmental design will solve both these goals of the breeder. Such designs for diallel crosses, where the interest of the experimenter lies in making all pair-wise comparisons of the general combining abilities (gca) for the lines, have been considered in the literature by several researchers.

There may be experimental situations where several new lines are developed in the initial stage of an experiment and it is expected that only a few of the new lines are worthy of further investigations. The new lines may first be compared with a control line that is already being used by the experimenter in order to screen out the best lot of new lines for further investigation. In such cases, the experimenter would like to make the comparisons between test and control lines with as much precision as possible whereas the comparisons within test lines may not be of much interest.

Das [1] derived sufficient conditions for completely randomized designs to be A and MV optimal for diallel cross experiments for comparing test line with a control line and gave some classes of designs along with their efficiencies. Choi *et al*. [2] studied diallel crosses for comparing a control line with test line under the model for completely randomized designs and listed designs that estimate control versus test comparisons with a minimum variance within a practical range of parameters. Type S designs with nested blocks were introduced and some series of type S block deigns were provided. Subsequently, Hsu and Ting [3] studied A-optimality of diallel cross experiments for comparing two or three test lines with a control line under the model for block designs. Das *et al*. [4] further investigated this problem and derived a sufficient condition for designs to be A-optimal.

The objective of the present study is to obtain some families of small and efficient mating-environment designs for complete diallel cross (CDC)/ partial diallel cross (PDC) experiments for comparing a set of test lines with a control line. We make use of partially balanced incomplete block (PBIB) designs

*Corresponding author's e-mail: cini_v@iasri.res.in
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and their association schemes (AS) [5] for this purpose.

Definition

A block design for diallel cross experiments for comparing t test lines with a control line is said to be partially balanced if all elementary contrasts pertaining to gca effects

- among test lines that are i^{th} (i = 1, 2, … , m) associates to each other are estimated with the same variance ($V_{t_i \times t_i}$) (= 1, 2, … , n_i where n_i is the number of i^{th} associates) as long as they remain i^{th} associates. Further, all the test lines are either 1^{st}, or 2^{nd} or …, m^{th} associates to each other.
- among test and control lines are estimated with the same variance ( ) as long as the test lines are i^{th} associates to each other.

Materials and methods

Method 1: Diallel cross designs for test vs. control comparisons using group divisible association scheme

Let there be t (= uv, u > 2) test lines and one control line (c = 1). These t test lines can be arranged in a group divisible association scheme having u rows and v columns where lines belonging to the same column are first associates and other lines are second associates. Pair each test line with each of its n_2 2^{nd} associates exactly once to form $tn_2/2$ pairs. Augment the control line to each one of these pairs to form $tn_2/2$ triplets. Treating each triplet as a column, we get $tn_2/2$ columns. Juxtaposing these columns to the v columns of the association scheme and making all possible distinct crosses within each column, a partially balanced diallel cross design in b blocks with equal/ unequal block sizes is obtained with parameters t = uv, c = 1, b = $tn_2/2$ + v, r_{tt} (number of times a test vs. test cross appears in the design) = 1, r_{tc} (number of times a test vs. control cross appears in the design) = n_2, k_1 = ${}^uC_2$, k_2 = 3, N_{total} (total number of crosses in the design) = v x ${}^uC_2$ + $3tn_2/2$. Depending on the column size of the association scheme chosen *i.e*., v = 3 or v > 3, the resultant design would be having equal block sizes or unequal block sizes respectively. The resultant design still follows the group divisible association scheme, for the test lines.

Example 1: Consider t = 8 test lines (denoted by 1, 2, … , 8) and one control line. These test lines can be arranged in a group divisible (GD) association scheme having 4 rows and 2 columns as:

1	5
2	6
3	7
4	8

Here, lines in the same column are first associates and remaining are second associates. Form all possible distinct pairs of lines between the two columns (each line with its second associates once). Augmenting the control line (denoted by 0) to these pairs and juxtaposing these triplets to the above 4×2 array horizontally on the right hand side, we have the following array

$V_{t_i t_i}$

Columns

1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18
1	5	1	1	1	1	2	2	2	2	3	3	3	3	4	4	4	4
2	6	5	6	7	8	5	6	7	8	5	6	7	8	5	6	7	8
3	7	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
4	8																

Make all the possible distinct crosses among the lines in each column to get the following complete diallel cross design for comparing 8 test lines with a control line in 18 blocks with unequal block sizes 6 and 3 involving 60 crosses:

Blocks

1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18
1×2	5×6	1×5	1×6	1×7	1×8	2×5	2×6	2×7	2×8	3×5	3×6	3×7	3×8	4×5	4×6	4×7	4×8
1×3	5×7	1×0	1×0	1×0	1×0	2×0	2×0	2×0	2×0	3×0	3×0	3×0	3×0	4×0	4×0	4×0	4×0
1×4	5×8	5×0	6×0	7×0	8×0	5×0	6×0	7×0	8×0	5×0	6×0	7×0	8×0	5×0	6×0	7×0	8×0
2×3	6×7																
2×4	6×8																
3×4	7×8																

The parameters of the design are t = 8, c = 1, b = 18, r_{tt} = 1, r_{tc} = 4, k_1 = 6, k_2 = 3, N_{total} = 60.

Average variance of estimated contrasts pertaining to test vs. test lines as well as test vs. control lines were computed using a program (given in Appendix) written in PROC IML of SAS software [7]. The parameters of the designs obtained have been listed for t = 20 in Table 1 along with the computed variances.

Method 2: Diallel cross designs for test vs. control comparisons using group divisible design

Consider a group divisible design for t (= uv) lines replicated r' times in b' blocks of size k' each with blocks arranged in columns. Add the control line 0 to each column. The resultant array has k'+1 rows and b columns. To the right hand side of this array, juxtapose the corresponding group divisible association scheme arranged in u rows (u > 2) and v columns and if u = 2, juxtapose control augmented columns of the association scheme. Now, form all possible distinct crosses within each column to obtain a complete/ partial diallel cross design for test vs. control comparisons with equal/ unequal block sizes. The block size will be same if (i) k'+1 = u (where u $\geq$ 3), (ii) k'+1 = u+1 (where u=2) and different otherwise. The parameters of the design are t = uv (u > 2), c = 1, b = b' + v, r_{tc} = r', $k_1 = {}^{(k'+1)}C_2$, $k_2 = {}^{u}C_2$, $N_{total} = b' \times {}^{(k'+1)}C_2 + v \times {}^{u}C_2$. For u = 2, the parameters are t = 2v, c = 1, b = b'+v, r_{tc} = r'+1, $k_1 = {}^{(k'+1)}C_2$, k_2 = 3, $N_{total} = b' \times {}^{(k'+1)}C_2 + 3v$.

Example 2: Consider the following group divisible design for t = 12 lines having 9 blocks each of size 4:

Blocks

1	2	3	4	5	6	7	8	9
1	1	1	2	2	2	3	3	3
4	5	6	4	5	6	4	5	6
7	8	9	8	9	7	9	7	8
10	11	12	12	10	11	11	12	10

Augment the control line 0 to the blocks of this design. To the right hand side of the resultant array, juxtapose the corresponding association scheme (with lines belonging to same column are first associates to each other).

Columns

1	2	3	4	5	6	**7**	**8**	**9**	**10**	**11**	**12**	**13**
1	1	1	2	2	2	3	3	3	1	4	7	10
4	5	6	4	5	6	4	5	6	2	5	8	11
7	8	9	8	9	7	9	7	8	3	6	9	12
10	11	12	12	10	11	11	12	10				
0	0	0	0	0	0	0	0	0				

The resultant arrangement has 13 columns. By making all possible distinct crosses among lines within each column, we get the following diallel cross design for test vs. control comparisons having block sizes 10 and 3, respectively:

Blocks

1	2	3	4	5	6	7	8	9	10	11	12	13
1×4	1×5	1×6	2×4	2×5	2×6	3×4	3×5	3×6	1×2	4×5	7×8	10×11
1×7	1×8	1×9	2×8	2×9	2×7	3×9	3×7	3×8	1×3	4×6	7×9	10×12
1×10	1×11	1×12	2×12	2×10	2×11	3×11	3×12	3×10	2×3	5×6	8×9	11×12
1×0	1×0	1×0	2×0	2×0	2×0	3×0	3×0	3×0				
4×7	5×8	6×9	4×8	5×9	6×7	4×9	5×7	6×8				
4×10	5×11	6×12	4×12	5×10	6×11	4×11	5×12	6×10				
4×0	5×0	6×0	4×0	5×0	6×0	4×0	5×0	6×0				
7×10	8×11	9×12	8×12	9×10	7×11	9×11	7×12	8×10				
7×0	8×0	9×0	8×0	9×0	7×0	9×0	7×0	8×0				
10×0	11×0	12×0	12×0	10×0	11×0	11×0	12×0	10×0				

The parameters of the design are t = 12, c = 1, b = 13, r_{tt} = 1, r_{tc} = 3, k_1 = 10, k_2 = 3, N_{total} = 102. Variances of designs using Method 2 have also been worked out and listed in Table 1.

Table 1. Diallel cross designs for test vs. control comparisons

S.No.	t	N_{total}	b	k_1	k_2	V_{txt}	V_{txc}	Association scheme/ design used	Method
1	8 (=4×2)	60	18	4	3	0.4388	0.2857	GD AS	1
2	10 (=5×2)	95	27	5	3	0.3251	0.2063	GD AS	1
3	12 (=6×2)	138	38	6	3	0.2576	0.1597	GD AS	1
4	14 (=7×2)	189	51	7	3	0.2130	0.1295	GD AS	1
5	16 (=8×2)	248	66	8	3	0.1814	0.1085	GD AS	1
6	18 (=9×2)	315	83	9	3	0.1579	0.0931	GD AS	1
7	20 (=10×2)	390	102	10	3	0.1397	0.0814	GD AS	1
8	8 (=2×4)	72	10	10	3	0.2379	0.1627	GD design (S6)*	2
9	8 (=2×4)	60	12	6	3	0.3547	0.2234	GD design (R54)*	2
10	12 (=3×4)	102	13	10	3	0.2380	0.1554	GD design (SR41)*	2

Method 3: PDC designs for test vs. control comparisons using triangular association scheme

Consider an array in n rows and n columns such that the diagonal positions are filled by the control line 0 and $^nC_2$ positions each above and below diagonal are occupied by t (= $^nC_2$) test lines in a symmetric manner, thus forming a symmetric arrangement of a two-class triangular association scheme. Making all possible distinct crosses among all the rows (columns) of this array and treating rows (columns) as block contents, a PDC design for test vs. control comparisons is obtained. The parameters of the design are t = $^nC_2$, c = 1, b = n, r_{tc} = 2, k = $^nC_2$, N_{total} = n x $^nC_2$.

Example 3: Let t = 10 test lines and 1 control line (denoted by 0) be arranged in the triangular scheme as shown below:

0	1	2	3	4
1	0	5	6	7
2	5	0	8	9
3	6	8	0	10
4	7	9	10	0

Making all possible crosses within each column, a PDC design comparing 10 test lines with a control line in blocks of size 10 is obtained.

Blocks

1	2	3	4	5
0×1	1×0	2×5	3×6	4×7
0×2	1×5	2×0	3×8	4×9
0×3	1×6	2×8	3×0	4×10
0×4	1×7	2×9	3×10	4×0
1×2	0×5	5×0	6×8	7×9
1×3	0×6	5×8	6×0	7×10
1×4	0×7	5×9	6×10	7×0
2×3	5×6	0×8	8×0	9×10
2×4	5×7	0×9	8×10	9×0
3×4	6×7	8×9	0×10	10×0

The parameters of the design are t = 10, c = 1, b = 5, r_{tc} = 2, k = 10, N_{total} = 50. These variances are calculated for the designs obtained and are given in Table 2.

Method 4: PDC designs for test vs. control comparisons using PBIB designs

Let there be t test lines and one control line. Identify a

Table 2. PDC designs for test vs. control comparisons

S.No.	t	N_{total}	b	k	V_{txt}	V_{txc}	Association scheme/ Design used	Method
1	10	50	5	10	0.3968	0.2619	Triangular AS	3
2	15	90	6	15	0.2946	0.1875	Triangular AS	3
3	21	147	7	21	0.2333	0.1444	Triangular AS	3
4	28	224	8	28	0.1930	0.1167	Triangular AS	3
5	10	60	10	4	0.4242	0.2576	Triangular design (T9)*	4
6	15	120	20	4	0.3265	0.1857	Triangular design (T14)*	4
7	21	210	35	4	0.2647	0.1451	Triangular design (T20)*	4
8	28	336	56	4	0.2222	0.1191	Triangular design (T23)*	4

*Clatworthy [6]

PBIB design for t test lines with small block size k' with number of blocks and number of replications as b' and r', respectively. To each block of this PBIB design, augment the control line 0. Make all possible distinct crosses within each augmented block to get a PDC design for test vs. control comparisons in smaller blocks with parameters t, c = 1, b = b', r_{tc} = r', k = $^{(k'+1)}C_2$, N_{total} = b' x $^{(k'+1)}C_2$.

Example 4: Augmenting the control line 0 to each of these columns of a triangular design for 10 test lines in blocks of size 3 arranged in 10 columns, we get:

Columns

1	2	3	4	5	6	7	8	9	10
1	1	1	2	2	3	5	5	6	8
2	3	4	3	4	4	6	7	7	9
5	6	7	8	9	10	8	9	10	10
0	0	0	0	0	0	0	0	0	0

Making all possible crosses within each column, a PDC design comparing 10 test lines with a control line in blocks of size 6 is obtained.

Blocks

1	2	3	4	5	6	7	8	9	10
1x2	1×3	1×4	2×3	2×4	3×4	5×6	5×7	6×7	8×9
1x5	1×6	1×7	2×8	2×9	3×10	5×8	5×9	6×10	8×10
1x0	1×0	1×0	2×0	2×0	3×0	5×0	5×0	6×0	8×0
2x5	3×6	4×7	3×8	4×9	4×10	6×8	7×9	7×10	9×10
2x0	3×0	4×0	3×0	4×0	4×0	6×0	7×0	7×0	9×0
5×0	6×0	7×0	8×0	9×0	10×0	8×0	9×0	10×0	10×0

The parameters of the design are t = 10, c = 1, b = 10, r_{tc} = 3, k = 6, N_{total} = 60. Table 2 also lists some designs and estimated variances of interline comparisons of designs obtained using this method.

Results and discussion

When there are two distinct groups of lines in a breeding trial, one group consisting of test lines and the other group consisting of control lines, the comparisons between test lines with control lines are of prime interest to the breeder. In all the classes of designs obtained, test vs. control comparisons are made with more precision as compared to test vs. test comparisons. Further, there is a substantial amount of decrease in total number of crosses required for the trial as compared to a design giving equal importance to all possible pair-wise comparisons. Moreover, as the number of lines increases, the variance of estimated contrasts pertaining to test vs. test lines as well as test vs. control lines decreases. The first two methods yield designs to compare test lines with a control line in blocks of equal size/ unequal sizes. Designs in blocks of unequal sizes are particularly useful when the experimental situation does not permit to accommodate the same number of crosses in each block. Third and fourth methods give designs that are partially balanced involving t test lines and a control line in equi-sized blocks.

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Appendix

SAS code for computing variances pertaining to interline comparisons of diallel cross designs for test vs. control comparisons

```
%let t=10;/*number of tests*/

%let cc=1;/*number of controls*/

/* Partial diallel cross design obtained using Triangular design (T9); Clatworthy, 1973
```

Blocks

1	2	3	4	5	6	7	8	9	10
1×2	1×3	1×4	2×3	2×4	3×4	5×6	5×7	6×7	8×9
1×5	1×6	1×7	2×8	2×9	3×10	5×8	5×9	6×10	8×10
1×0	1×0	1×0	2×0	2×0	3×0	5×0	5×0	6×0	8×0
2×5	3×6	4×7	3×8	4×9	4×10	6×8	7×9	7×10	9×10
2×0	3×0	4×0	3×0	4×0	4×0	6×0	7×0	7×0	9×0
5×0	6×0	7×0	8×0	9×0	10×0	8×0	9×0	10×0	10×0

```
*/

data pdc;

input block line1 line2 @@;

1 1 2
1 1 5
1 1 11
1 2 5
1 2 11
1 5 11
2 1 3
2 1 6
2 1 11
2 3 6
2 3 11
2 6 11
3 1 4
3 1 7
3 1 11
3 4 7
3 4 11
3 7 11
4 2 3
4 2 8
4 2 11
4 3 8
4 3 11
4 8 11
5 2 4
5 2 9
5 2 11
5 4 9
5 4 11
5 9 11
6 3 4
6 3 10
6 3 11
6 4 10
6 4 11
6 10 11
7 5 6
7 5 8
7 5 11
7 6 8
7 6 11
7 8 11
8 5 7
8 5 9
```

```
    8     5     11
    8     7     9
    8     7     11
    8     9     11
    9     6     7
    9     6     10
    9     6     11
    9     7     10
    9     7     11
    9     10    11
    10    8     9
    10    8     10
    10    8     11
    10    9     10
    10    9     11
    10    10    11
;
cards;
;
run;
proc iml;
use pdc;
read all into xx;
/*print xx;*/
block=xx[ ,1];
cross=xx[ ,2]||xx[ ,3];
m=j(nrow(cross),1,1);
/*print cross;*/
x1=j(nrow(cross),max(cross),0);
k=1;
do i=1 to nrow(cross);
do j=1 to ncol(cross);
if cross[i,j]>0 then
x1[k,cross[i,j]]=1;
end;
k=k+1;
end;
/*print x1;*/
x2=j(nrow(block),max(block),0);
k=1;
do i=1 to nrow(block);
if block[i, ]>0 then
x2[k,block[i, ]]=1;
k=k+1;
end;
/*print x2;*/
x22=m||x2;
x=m||x1||x2;
/*print x;*/
c=(x1“*x1)-(x1“*x22)*ginv(x22“*x22)*(x22“*x1);
g_invc=ginv(c);
print c;
/*print g_invc;*/
k=1;
tcont=j(comb(&t,2),(&t+&cc),0);
do i=1 to &t;
do j=i+1 to &t;
tcont[k,i]=1;
tcont[k,j]=-1;
k=k+1;
end;
end;
k=1;
if &cc>1 then do;
cccont=j(comb(&cc,2),(&t+&cc),0);
do i=&t+1 to (&t+&cc);
do j=i+1 to (&t+&cc);
cccont[k,i]=1;
cccont[k,j]=-1;
k=k+1;
end;
end;
end;
else do;
cccont=j(1,(&t+&cc),0);
end;
k=1;
totcont=j(comb((&t+&cc),2),(&t+&cc),0);
```

```
do i=1 to (&t+&cc);
do j=i+1 to (&t+&cc);
totcont[k,i]=1;
totcont[k,j]=-1;
k=k+1;
end;
end;
/*print tcont cccont totcont;*/
var_t=vecdiag(tcont*g_invc*tcont,);
if &cc>1 then do;
var_c=vecdiag(cccont*g_invc*cccont“);
end;
else do;
var_c=0;
end;
var_tot=vecdiag(totcont*g_invc*totcont“);
print var_t var_c var_tot;
avar_t=sum(var_t)/nrow(var_t);
avar_c=sum(var_c)/nrow(var_c);
avar_tot=sum(var_tot)/nrow(var_tot);
if &cc>1 then do;
avar_tvsc=(sum(var_tot)-(sum(var_t)+sum(var_c)))/(nrow(var_tot)-(nrow(var_t)+nrow(var_c)));
end;
else do;
avar_tvsc=(sum(var_tot)-sum(var_t))/(nrow(var_tot)-nrow(var_t));
end;
print avar_t avar_tvsc avar_c avar_tot;
```