ON PLANS FOR FRACTIONAL FACTORIALS WITH BLOCKING

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ON PLANS FOR FRACTIONAL FACTORIALS WITH BLOCKING

By

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A C K N O W L E D G E M E N T

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C O N T E N T S

1. INTRODUCTION

It is well known that the asymmetrical factorial experiments are better suited to meet the needs of the experimenters as they are more flexible to accomodate factors with varying numbers of levels. But the designs for such experiments are not always easy to construct. Several methods of constructions and analysis of such designs are now available in literature (Das 1960) Kishen and Srivastava (1959). Further when the number of factors increases it becomes necessary to use fractional replicate of such experiments. Not much work has been done to obtain suitable fractional replicates of asymmetrical factorial designs. The purpose of the present investigation is to describe a method for the construction of fractions of asymmetrical factorial, to discuss the method of their analysis and also to present a number of layout plans, split into blocks of suitable sizes.

The present method is based on the technique of construction of confounded symmetrical factorial design given by Das and Rao (1967). There is actually no work available in literature in so far as fractions of asymmetrical factors of size 2^K are concerned.

2. METHOD

We have presented below the method of construction of fractional replicate of the asymmetrical factorials of the type $3^m \times 2^n$, the size of the fraction being always of the form $\frac{1}{2^k}$. The number of treatments retained in the fraction is also of the form 2^p. The method adopted is described below.

Given an asymmetrical factorial $3^m \times 2^n$, corresponding to each of the m factors each at 3 levels two pseudo factors each at two levels are taken. The combination of these pseudo factors as also the other n factors are written within the levels 1 and 0, We have thus got the symmetrical factorials 2^{2m+n}.

Next the required fraction of this factorial is obtained by using the ordinary method of fraction of the 2^n series of designs. As usual care has been taken to keep the identity group of interactions free from any interaction with less than five factors. However in some cases when the fraction size is $\frac{1}{2^3}$ designs with at the most one interaction with four factors in the identity group have also been considered. After the fraction has been obtained the first 2^n factors are made into m pairs and then the corresponding levels of the two factors in each such pair, (i.e. the two levels symbols against the two factors in the treatment combinations) are added in each of the t treatment combinations in the fraction. We shall now have in the fraction m factors each at three levels namely 0, 1, 2 while the remaining n factors at levels 1 and 0.

Against the two factors in any of the above pair the following combinations alone can occur.

0	0
0	1
1	0
1	1

In these when the corresponding levels of the above combination are added, as indicated above. We find that the levels of the factors of asymmetry at three levels

become -2, 0, 2. It will be seen that level 0 occurs twice. Let X_0 stand for the levels -2, X_1 for 0 and X_2 for 2, then the linear contrast $X_2 - X_0$ corresponds to the contrast (1,1)-(0,0) of the original pseudo factors. This contrast realy represents the main effect A or B in the $\frac{1}{2}$ fraction of 2^2 with identity group

$$I = AB$$

The quadratic contrast $X_0 - 2X_1 + X_2$ is obtained from the interaction of the pseudo factors $X_1 X_2$ namely (0,0)-(0,1)-(1,0)+(1,1). The remaining contrast namely (0,1)-(1,0) represents an error d.f as both the combinations (0,1) and (1,0) stands for the level zero.

Now any interaction in the symmetrical design can be put into correspondence with an interaction of the asymmetrical design. The rule is, if in any interaction of the symmetrical design, there are two factors belonging to the same group then these two factors are replaced by the quadratic component of the corresponding factor in the asymmetrical design. If again any one of the factors in a pair be present in the interaction then the factor is to be replaced by the linear component of the corresponding factor in the asymmetrical design. Following this rule the interactions in the identity group of the symmetrical design can be converted into an other set of interactions which are the interactions in the asymmetrical design.

<u>Blocking</u>:- So far we have described the method which is appropriate for getting a fraction. But often times blocking of the fraction is also necessary. We have therefore described below the method for spliting the

fraction into blocks saving as far as possible all the main effects and two factor interaction components.

For the purpose of blocking we have first made into blocks the fraction of the symmetrical design by following the method reported by Das, Shukla and Kartha (1967). Subsequently the blocks have been converted to a block of the asymmetrical design by following the method described earlier.

While getting these blocks first the key block is obtained. Next from this key block other blocks are generated by increasing one or more columns in the key block by unity and reducing the sum mod 2. This method has the draw back that often times the same block may be repeated. In order to remove this draw back the following method is helpful.

First the independent interactions in the identity group are written in a column Next the interactions confounded are also written in the same column, next an interaction with least possible number of factors is taken. The numbers of factors common between the interaction (which is written at the head of an other column) and each of the interactions present in the previous column that is formed of the interactions in the identity group and also those which are confounded are obtained. If the numbers of letters common is an even number, the number is taken as zero otherwise the number is 1. These numbers 0 and 1 are written against each of the interactions in the first column. It is necessary that against each of the interaction in the identity group the number should be zero. If this has not happend then the interaction taken to head the second

column should be suitably changed. Similarly an other interaction is taken to head the third column and an other set of numbers is obtained to fill this column as indicated earlier. This column should be different from the previous column subject to the condition that the number against each of the interactions in the identity group should be zero. Likewise if there are 2^q blocks in the fractions in all q columns are to be taken, taking care that each of these columns should be independent. From these q columns we have got q independent interactions and from these a total of 2^{q-1} interactions can be generated.

Next taking any of these interactions we increase by unity the columns against these factors in the interaction in the key block. Likewise we shall be getting a block for each of the above 2^{q-1} interaction. These blocks will not contain any repeated blocks.

METHOD OF ANALYSIS

The analysis of these designs does not offer any special problem. As a matter of fact they are analysed first as a fraction of the 2^n design by following the usual technique due to Yates. This analysis will give all the main effect and interactions of the factors at two levels together with the linear and quadratic components of the main effects and interactions of the factors at three levels. Subsequently the sum of squares due to linear and q-uadratic components can be added to get the sum of squares due to factors at 3 levels with 2 d.f. So far as the confounded interactions are concerned they are completely confounded and do not offer any difficulty.

These designs can also be analysed by forming

tables. As a matter of fact some of the designs were analysed by both the methods to demonstrate that the result obtained by the two methods agree.

SUMMARY.

In the present thesis a method of obtaining fractions of asymmetrical factorial of the type $3^m \times 2^n$ has been described. The fraction size is always of the form 2^k. The method of analysis of such fraction has also been described. The method adopted consists of taking the fractions of the asymmetrical factorial with a fractional of a suitable symmetrical factorial and then convert the fraction of the symmetrical factorial to an asymmetrical incomplete factorial. The second objective of the thesis is to provide a number of plan which might find application in ordinary situation. In all about 30 plans have been prepared and presented in the thesis. In these plans care has been taken to save main effect and two factor interaction in most of the designs. There may, however, be some exceptions to it. These designs have also been presented split into blocks. Two sizes of blocks viz. 2^3 and 2^4 have been presented in the case of all the designs. While blocking also care has been taken to save all main effects and two factor interactions as for as possible. The plans are all self explanatory and can be used as such. Only the key blocks have been presented, but a very convenient method of obtaining the other blocks has been described in the thesis.

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PLANS

Plan No.1.1

$\frac{1}{2}(3 \times 2^4)$ in blocks of size 2^3

$I = A\,A'\,B\,C\,D\,E$ $\qquad$ $A_2\,B\,C\,D\,E$

Key block.

Symmetrical						Asymmetrical				
A	A'	B	C	D	E	A	B	C	D	E
1	0	0	1	0	0	1	0	1	0	0
0	1	0	0	1	0	1	0	0	1	0
0	0	1	1	1	1	0	1	1	1	1
1	1	0	1	1	0	2	0	1	1	0
1	0	1	0	1	1	1	1	0	1	1
0	1	1	1	0	1	1	1	1	0	1
1	1	1	0	0	1	2	1	0	0	1
0	0	0	0	0	0	0	0	0	0	0

Confounded interaction

$A\,B\,C$, $A'\,B\,D$, $A\,A'\,C\,D$ $\qquad$ $A_1\,B\,C$, $A_1\,B\,D$, $A_2\,C\,D$

Alias of the confounded interaction in the asymmetrical design.

$$A_1\,B\,C = A_1\,D\,E$$
$$A_1\,B\,D = A_1\,C\,E$$
$$A_2\,C\,D = B\,E$$

where A_1 and A_2 stand respectively for the linear and quadratic components of the main effect A. Similar notation have been given for all other factors at 3 levels.

In the text, the levels of the factor of asymmetry were denoted by -2,0,2. These have been recorded in the plans as 0,1,2.

Plan No. 1.2

$\frac{1}{2}$ ($3^2 \times 2^2$) in blocks of size 2^3

$I \equiv A\ A'\ B\ B'\ C\ D \qquad A_2\ B_2\ C\ D$

Key block

Symmetrical						Asymmetrical			
A	A'	B	B'	C	D	A	B	C	D
0	0	0	0	1	1	0	0	1	1
1	1	0	1	0	1	2	1	0	1
0	1	1	0	0	0	1	1	0	0
1	1	0	1	1	0	2	1	1	0
0	1	1	0	1	1	1	1	1	1
1	0	1	1	0	1	1	2	0	1
1	0	1	1	1	0	1	2	1	0
0	0	0	0	0	0	0	0	0	0

Confounded interactions

$A\ C\ D,\ A\ A'\ B,\ A\ B\ B'\ C \qquad A_1\ C\ D,\ A_2\ B_1,\ A_1\ B_2\ C$

Aliae of the confounded interactions in the asymmetrical design.

$A_1\ C\ D \equiv A_1\ B_2$

$A_2\ B_1 \equiv B_1\ C\ D$

$A_1\ B_2\ C \equiv A_1\ D$

Plan No. 1.3

$\frac{1}{4}$ (3×2^5) in blocks of size 2^3

I = A A' B C D E F A_2 B C D E F

Key block

Symmetrical							Asymmetrical					
A	A'	B	C	D	E	F	A	B	C	D	E	F
1	0	0	1	0	1	1	1	0	1	0	1	1
0	1	0	1	1	0	1	1	0	1	1	0	1
0	0	1	0	1	1	1	0	1	0	1	1	1
1	1	0	0	1	1	0	2	0	0	1	1	0
1	0	1	1	1	0	0	1	1	1	1	0	0
0	1	1	1	0	1	0	1	1	1	0	1	0
1	1	1	0	0	0	1	2	1	0	0	0	1
0	0	0	0	0	0	0	0	0	0	0	0	0

Confounded interactions

A A' C, A' B D, A B E A_2 C, A_2 B D, A_2 B E

ABCD, ABCE, AA'DE, CDE A_1BCD, A_1BCE, A_2DE, CDE

Alias of the confounded interactions in the asymmetrical design.

A_2 C = B D E F

A_2 B D = A_2 C E F

A_2 B E = A_2 C D F

A_1 BCD = A_1 E F

A_1 BCE = A_1 D F

A_2 DE = B C F

CDE = A_2 B F

Plan No. 1.4*

$\frac{1}{2}$ (3×2^5) in blocks of size 2^3

I = A A' B D E = A' B C E F = A C D F

Key block

Symmetrical							Asymmetrical					
A	A'	B	C	D	E	F	A	B	C	D	E	F
1	0	0	1	1	0	1	1	0	1	1	0	1
0	1	0	1	0	1	1	1	0	1	0	1	1
0	0	1	0	1	0	1	0	1	0	1	0	1
1	1	0	0	1	1	0	2	0	0	1	1	0
1	0	1	1	0	0	0	1	1	1	0	0	0
1	1	0	0	0	1	1	2	1	0	0	1	1
0	1	1	1	1	1	0	1	1	1	1	1	0
0	0	0	0	0	0	0	0	0	0	0	0	0

Confounded interactions

A A' C, A B D, A' B C D A_2 C, A_2 B D, A_1 B C D

Aliae of the confounded interactions in the asymmetrical design.

A_2 C = B C D E = A_1 B E F = A_1 D E

A_2 B D = A_2 E = A_2 C D E F = B C F

A_1 B C D = A_1 C E = D E F = A_2 B E

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* Indicates one four factor interaction in the identity group.

Plan No. 1.5

$\frac{1}{2}$ $(3^2 \times 2^3)$ in blocks of size 2^3

X = A A' B B' C D E

Key block

Symmetrical

A	A'	B	B'	C	D	E
0	1	0	1	0	1	1
1	1	1	0	0	1	0
0	1	1	0	1	0	1
1	0	1	1	0	0	1
1	0	0	0	1	1	1
0	0	1	1	1	1	0
1	1	0	1	1	0	0
0	0	0	0	0	0	0

Asymmetrical

A	B	C	D	E
1	1	0	1	1
2	1	0	1	0
1	1	1	0	1
1	2	0	0	1
1	0	1	1	1
0	2	1	1	0
2	1	1	0	0
0	0	0	0	0

Confounded interactions

B' A D, A B C, B' C E

B' B C D, A C E E,

A B B' E, B D E

A_1 B_1 D, A_1 B_1 C,

B_1 C E, B_2 C D, A_1 C D E,

A_1 B_2 E, B_1 D E

Alias of the confounded interactions in the asymmetrical design.

A_1 B_1 D = A_1 B_1 C E

A_1 B_1 C = A_1 B_1 D E

B_1 C E = A_2 B_1 D

B_2 C D = A_2 E

A_1 C D E = A_1 B_2

A_1 B_2 E = A_1 C D

B_1 D E = A_2 B_1 C

Plan No. 1.6*

$\frac{1}{2^2}$ ($3^2 \times 2^3$) in blocks of size 2^3

$I = A\,B\,B'\,D\,E = A'\,B'\,C\,D = A\,A'\,B\,C\,E$

Key block

Symmetrical							Asymmetrical				
A	A'	B	B'	C	D	E	A	B	C	D	E
0	0	1	1	0	1	1	0	2	0	1	1
1	1	0	0	0	1	0	2	0	0	1	0
0	0	1	0	1	1	0	0	1	1	1	1
1	1	1	1	0	0	1	2	2	0	0	1
0	0	0	1	1	0	1	0	1	1	0	1
1	1	1	0	1	0	0	2	1	1	0	0
1	1	0	1	1	1	1	2	1	1	1	1
0	0	0	0	0	0	0	0	0	0	0	0

Confounded interactions

$A\,B'\,C\,D$, $B\,B'\,C$, $A\,B\,D$ $\qquad$ $A\,B_1\,C\,D$, $B_2\,C$, $A_1\,B_1\,D$

Alias of the confounded interactions in the asymmetrical design.

$A_1 B_1\,C\,D = B_1\,C\,E = A_2 = A_1\,B_2\,D\,E$

$B_2\,C = A_1\,C\,D\,E = A_1\,B_1\,D = A_2\,B_1\,E$

$A_1\,B_1\,D = B_1\,E = A_2\,B_2\,C = A_1\,C\,D\,E$

Plan No. 1.7

$\frac{1}{2}(3^3 \times 2)$ in blocks of size of 2^3

$I = A\ A'\ B\ B'\ C\ C'\ D$

Key block

Symmetrical

A	A'	B	B'	C	C'	D
1	0	0	1	1	0	1
0	1	0	0	1	1	1
0	0	1	1	0	1	1
1	1	0	1	0	1	0
1	0	1	0	1	1	0
0	1	1	1	1	0	0
1	1	1	0	0	0	0
0	0	0	0	0	0	0

Asymmetrical

A	B	C	D
1	1	1	1
1	0	2	1
0	2	1	1
2	1	1	0
1	1	2	0
1	2	1	0
2	1	0	1
0	0	0	0

Confounded interactions

A B B', A A' C, A' B C',
A'B B' C, A A' B' C',
A B C C', B'C C'

$A_1 B_2$, $A_2 C_1$, $A_1 B_1 C_1$,
$A_1 B_2 C_2$, $A_2 B_1 C_1$,
$A_1 B_1 C_2$, $B_1 C_2$

Alias of the confounded interactions in the asymmetrical design.

$A_1 B_2 = A_1 C_2 D$
$A_2 C_1 = B_2 C_1 D$
$A_1 B_1 C_1 = A_2 B_1 C_1 D$
$A_1 B_2 C_1 = A_1 C_1 D$
$A_2 B_1 C_1 = B_1 C_1 D$
$A_1 B_1 C_2 = A_1 B_1 D$
$B_1 C_2 = A_2 B_1 D$

Plan No.1.8*

$\frac{1}{2^2}$ (3^3 x 2) in blocks of size 2^3

$$I = A\,A'\,B\,B'\,C' = A'\,B\,B'\,C\,D = A\,C\,C'\,D$$

Key block

Symmetrical							Asymmetrical			
A	A'	B	B'	C	C'	D	A	B	C	D
1	0	0	0	1	1	1	1	0	2	1
0	1	0	1	0	0	0	1	1	0	0
0	0	1	1	1	0	1	0	2	1	1
1	1	0	1	1	1	1	2	1	2	1
0	1	1	1	0	1	0	1	2	1	0
0	1	1	0	1	0	1	1	1	1	1
1	1	1	0	0	1	0	2	1	1	0
0	0	0	0	0	0	0	0	0	0	0

Confounded interaction

A' B B', A B C, A A' B' C $\qquad$ $A_1\,B_2$, $A_1\,B_1\,C_1$, $A_2\,B_1\,C$

Alias of the confounded interaction in the asymmetrical design.

$$A_1\,B_2 = A_1\,C_1 = C_1\,D = A_2\,B_2\,C_2\,D$$

$$A_1\,B_1\,C_1 = A_1\,B_1\,C_2 = A_2\,B_1\,D = B_1\,C_1\,D$$

$$A_2\,B_1\,C_1 = B_1\,C_2 = A_1\,B_1\,D = A_1\,B_1\,C_1\,D$$

Plan No. 1.2

$\frac{1}{2}$ (3×2^6) in blocks of size 2^4

I = A A' B C D E F G

Key block

Symmetrical								Asymmetrical						
A	A'	B	C	D	E	F	G	A	B	C	D	E	F	G
1	0	0	0	1	0	0	0	1	0	0	1	0	0	0
0	1	0	0	0	1	1	1	1	0	0	0	1	1	1
0	0	1	0	1	0	1	1	0	1	0	1	0	1	1
0	0	0	1	1	1	1	0	0	0	1	1	1	1	0
1	1	0	0	1	1	1	1	2	0	0	1	1	1	1
1	0	1	0	0	0	1	1	1	1	0	0	0	1	1
1	0	0	1	0	1	1	0	1	0	1	0	1	1	0
0	1	1	0	1	1	0	0	1	1	0	1	1	0	0
0	1	0	1	1	0	0	1	1	0	1	1	0	0	1
0	0	1	1	0	1	0	1	0	1	1	0	1	0	1
1	1	1	0	0	1	0	0	2	1	0	0	1	0	0
1	1	0	1	0	0	0	1	2	0	1	0	0	0	1
0	1	1	1	0	0	1	0	1	1	1	0	0	1	0
1	0	1	1	1	1	0	1	1	1	1	1	1	0	1
1	1	1	1	1	0	1	0	2	1	1	1	0	1	0
0	0	0	0	0	0	0	0	0	0	0	0	0	0	0

Confounded interaction

A B C D, A' C E, A' B C F, A B C D, A_1 C E, A_1 B C F,
A A' B D E, A A' D F, A_2 B D E, A_2 D F, B E F,
B E F, A C D E F A_1 C D E F

Alias of the confounded interactions in the asymmetrical design.

A_1 B C D = A_1 E F G
A_1 C E = A_1 B D F G
A_1 B C F = A_1 D E G
A_2 B D E = C F G
A_2 D F = B C E G
B E F = A_2 C D G
A_1 C D E F = A_1 B G

Plan No. 110

$\frac{1}{2^2}$ (3×2^6) in blocks of size 2^4

I = A A' B D F = A A' C E G = B C D E F G

Key block

Symmetrical

A	A'	B	C	D	E	F	G
1	0	0	0	1	0	0	1
0	1	0	0	0	1	1	0
0	0	1	0	1	1	0	1
0	0	0	1	1	1	1	0
1	1	0	0	1	1	1	1
1	0	1	0	0	1	0	0
1	0	0	1	0	1	1	1
0	1	1	0	1	0	1	1
0	1	0	1	1	0	0	0
0	0	1	1	0	0	1	1
1	1	1	0	0	0	1	0
1	1	0	1	0	0	0	1
1	0	1	1	1	0	1	0
0	1	1	1	0	1	0	1
1	1	1	1	1	1	0	0
0	0	0	0	0	0	0	0

Asymmetrical

A	B	C	D	E	F	G
1	0	0	1	0	0	1
1	0	0	0	1	1	0
0	1	0	1	1	0	1
0	0	1	1	1	1	0
2	0	0	1	1	1	1
1	1	0	0	1	0	0
1	0	1	0	1	1	1
1	1	0	1	0	1	1
1	0	1	1	0	0	0
0	1	1	0	0	1	1
2	1	0	0	0	1	0
2	0	1	0	0	0	1
1	1	1	1	0	1	0
1	1	1	0	1	0	1
2	1	1	1	1	0	0
0	0	0	0	0	0	0

Confounded interactions

A B C D, A' B C E, A A' D E A_1 B C D, A_1 B C E, A_2 D E

Alias of the confounded interactions in the asymmetrical design.

A_1 B C D = A_1 C F = A_1 B D E G = A_1 E F G

A_1 B C E = A_1 C D E F = A_1 B G = A_1 D F G

A_2 D E = B E F = C D G = A_2 B C F G

Plan No. 1·11

$\frac{1}{2}$ ($3^2 \times 2^4$) in blocks of size 2^4

I = A A' B B' C D E F

Key block.

Symmetrical

A	A'	B	B'	C	D	E	F
1	0	0	0	1	0	1	1
0	1	0	0	1	1	0	1
0	0	1	0	1	1	1	0
0	0	0	1	0	1	1	1
1	1	0	0	0	1	1	0
1	0	1	0	0	1	0	1
1	0	0	1	1	1	0	0
0	1	1	0	0	0	1	1
0	1	0	1	1	0	1	0
0	0	1	1	1	0	0	1
1	1	1	0	1	0	0	0
1	1	0	1	0	0	0	1
1	0	1	1	0	0	1	0
0	1	1	1	0	1	0	0
1	1	1	1	1	1	1	1
0	0	0	0	0	0	0	0

Asymmetrical

A	B	C	D	E	F
1	0	1	0	1	1
1	0	1	1	0	1
0	1	1	1	1	0
0	1	0	1	1	1
2	0	0	1	1	0
1	1	0	1	0	1
1	1	1	1	0	0
1	1	0	0	1	1
1	1	1	0	1	0
0	2	1	0	0	1
2	1	1	0	0	0
2	1	0	0	0	1
1	2	0	0	1	0
1	2	0	1	0	0
2	2	1	1	1	1
0	0	0	0	0	0

Confounded interactions

A A' B C_0 A' B B' D_0 A B B' E_0
A B' C D_0 A' B' C E_0 A A' B E_0
B C D E

A_2 B_1 C_0 A_1 B_2 D_0 A_1 B_2 E_0
A_1 B_1 C D_0 A_1 B_1 C E_0
A_2 D E_0 B_1 C D E

Alias of the confounded interactions in the asymmetrical design.

A_2 B_1 C = B_1 D E F
A_1 B_2 D = A_1 C E F
A_1 B_2 E = A_1 C D F
A_1 B_1 C D = A_1 B_1 E F
A_1 B_1 C E = A_1 B_1 D F
A_2 D E = B_2 C F
B_1 C D E = A_2 B_1 F

Plan No. 1.12

$\frac{1}{2^2}(3^2 \times 2^4)$ in blocks of size of 2^4

$I = A\,A'\,B'\,C\,E = A'\,B\,C\,D\,F = A\,B\,B'\,D\,E\,F$

Key block

Symmetrical								Asymmetrical					
A	A'	B	B'	C	D	E	F	A	B	C	D	E	F
1	0	0	0	1	0	0	1	1	0	1	0	0	1
0	1	0	0	1	1	0	1	1	0	1	1	0	1
0	0	1	0	1	1	1	1	0	1	1	1	1	1
0	0	0	1	0	1	1	1	0	1	0	1	1	1
1	1	0	0	0	1	0	0	2	0	0	1	0	0
1	0	1	0	0	1	1	0	1	1	0	1	1	0
1	0	0	1	1	1	1	0	1	1	1	1	1	0
0	1	1	0	0	0	1	0	1	1	0	0	1	0
0	1	0	1	1	0	1	0	1	1	1	0	1	0
0	0	1	1	1	0	0	0	0	2	1	0	0	0
1	1	1	0	1	0	1	1	2	1	1	0	1	1
1	1	0	1	0	0	1	1	2	1	0	0	1	1
1	0	1	1	0	0	0	1	1	2	0	0	0	1
0	1	1	1	0	1	0	1	1	2	0	1	0	1
1	1	1	1	1	1	0	0	2	2	1	1	0	0
0	0	0	0	0	0	0	0	0	0	0	0	0	0

Confounded interactions

$A\,A'\,B\,C$, $A'\,E\,B'\,D$, $A\,B'\,C\,D$ $\quad A_2\,B_1\,C$, $A_1\,B_2\,D$, $A_1\,B_1\,C\,E$

Alias of the confounded interactions in the asymmetrical design.

$A_2\,B_1\,C = B_2\,E = A_1\,D\,F = A_1\,B_2\,C\,D\,E\,F$

$A_1\,B_2\,D = A_1\,B_1\,C\,D\,E = B_1\,C\,F = A_2\,E\,F$

$A_1\,B_1\,C\,D = A_1\,D\,E = A_2\,B_2\,F = B_1\,C\,E\,F$

Plan No. 1.13

$\frac{1}{2^2}(3^2 \times 2^4)$ in blocks of size 2^3

$I = A\,A'\,B'\,C\,E = A'\,B\,C\,D\,F = A\,B\,B'\,D\,E\,F$

Key block

Symmetrical

A	A'	B	B'	C	D	E	F
1	0	0	1	1	0	1	1
0	1	0	1	0	1	0	0
0	0	1	0	1	1	1	1
1	1	0	0	1	1	1	1
1	0	1	1	0	1	0	0
0	1	1	1	1	0	1	1
1	1	1	0	0	0	0	0
0	0	0	0	0	0	0	0

Asymmetrical

A	B	C	D	E	F
1	1	1	0	1	1
1	1	0	1	0	0
0	1	1	1	1	1
2	0	1	1	1	1
1	2	0	1	0	0
1	2	1	0	1	1
2	1	0	0	0	0
0	0	0	0	0	0

Confounded interactions

$A\,A'\,B'$, $A\,B\,C$, $A'\,B\,D$, $A'\,B\,B'\,C$, $A\,B\,B'\,D$, $A'\,C\,D$, $B'\,C\,D$

$A_2\,B_2$, $A_1\,B_1\,C$, $A_1\,B_1\,D$, $A_1\,B_2\,C$, $A_1\,B_2\,D$, $A_2\,C\,D$, $B_1\,C\,D$

Alias of the confounded interactions in the asymmetrical design.

$A_2\,B_1 = C\,E = A_1\,C\,D\,F = A_1\,B_1\,D\,E\,F$

$A_1\,B_1\,C = A_1\,B_2\,E = A_2\,D\,F = B_1\,C\,D\,E\,F$

$A_1\,B_1\,D = A_1\,B_2\,C\,D\,E = C\,F = A_2\,B_1\,E\,F$

$A_1\,B_2\,C = A_1\,B_1\,E = B_1\,D\,F = A_2\,C\,D\,E\,F$

$A_1\,B_2\,D = A_1\,B_1\,C\,D\,E = A_2\,B_1\,C\,F = E\,F$

$A_2\,C\,D = B_1\,D\,E = A_1\,B_1\,F = A_1\,B_2\,C\,E\,F$

$B_1\,C\,D = A_2\,D\,E = A_1\,B_2\,F = A_1\,B_1\,C\,E\,F$

Plan No. 1·14

$\frac{1}{2}$ ($3^3 \times 2^2$) in blocks of size 2^4

$I = A\ A'\ B\ B'\ C\ C'\ D\ E$

Key block

Symmetrical								Asymmetrical				
A	A'	B	B'	C	C'	D	E	A	B	C	D	E
1	0	0	0	0	1	1	1	1	0	1	1	1
0	1	0	0	1	0	1	1	1	0	1	1	1
0	0	1	0	1	1	0	1	0	1	2	0	1
0	0	0	1	1	1	1	0	0	1	2	1	0
1	1	0	0	1	1	0	0	2	0	2	0	0
1	0	1	0	1	0	1	0	1	1	1	1	0
1	0	0	1	1	0	0	1	1	1	1	0	1
0	1	1	0	0	1	1	0	1	1	1	1	0
0	1	0	1	0	1	0	1	1	1	1	0	1
0	0	1	1	0	0	1	1	0	2	0	1	1
1	1	1	0	0	0	0	1	2	1	0	0	1
1	1	0	1	0	0	1	0	2	1	0	1	0
1	0	1	1	0	1	0	0	1	2	1	0	0
0	1	1	1	1	0	0	0	1	2	1	0	0
1	1	1	1	1	1	1	1	2	2	2	1	1
0	0	0	0	0	0	0	0	0	0	0	0	0

Confounded interactions

$A'\ B\ B'\ C$, $A\ B\ B'\ C$, $A\ A'\ B'\ D$, $A\ A'\ C\ C'$, $A\ B\ C\ D$, $A'\ B\ C'\ D$, $B'\ C\ C'\ D$

$A_1\ B_2\ C_1$, $A_1\ B_2\ C_1$, $A_2\ B_1\ D$, $A_2\ C_2$, $A_1\ B_1 C_1 D$, $A_1\ B_1\ C_1\ D$, $B_1\ C_2\ B$.

Alias of the confounded interactions in the asymmetrical design.

$A_1\ B_2\ C_1 = A_1\ C_1\ D\ E$

$A_1\ B_2\ C_1 = A_1\ C_2\ D\ E$

$A_2\ B_1\ D = B_1\ C_2\ B$

$A_2\ C_2 = B_2\ D\ E$

$A_1\ B_1\ C_1\ D = A_1\ B_1\ C_1\ E$

$A_1\ B_1\ C_1\ D = A_1\ B_1\ C_1\ E$

$B_1\ C_2\ D = A_2\ B_1\ E$

Plan No. 1.15.

$\frac{1}{2^2}$ ($3^3 \times 2^2$) in blocks of size 2^4

$I = A\ A'\ B'\ C\ D = A\ B\ B'\ C'\ E = A'\ B\ C\ C'\ D\ E$

Key block

Symmetrical								Asymmetrical				
A	A'	B	B'	C	C'	D	E	A	B	C	D	E
1	0	0	0	1	0	0	1	1	0	1	0	1
0	1	0	0	0	1	1	1	1	0	1	1	1
0	0	1	0	1	1	1	0	0	1	2	1	0
0	0	0	1	1	1	0	0	0	1	2	0	0
1	1	0	0	1	1	1	0	2	0	2	1	0
1	0	1	0	0	1	1	0	1	1	1	1	0
1	0	0	1	0	1	0	1	1	1	1	0	01
0	1	1	0	1	0	0	1	1	1	1	0	1
0	1	0	1	1	0	1	1	1	1	1	1	1
0	0	1	1	0	0	1	0	0	2	0	1	0
1	1	1	0	0	0	0	0	2	1	0	0	0
1	1	0	1	0	0	1	0	2	1	0	1	0
1	0	1	1	1	0	1	0	1	2	1	1	0
0	1	1	1	0	1	0	1	1	2	1	0	1
1	1	1	1	1	1	0	0	2	2	2	0	0
0	0	0	0	0	0	0	0	0	0	0	0	0

Confounded interactions

$A\ B\ B'\ C,\ A'\ B\ B'\ C',\ A\ A'\ C\ C'$ $\quad A_1\ B_2\ C_1,\ A_1\ B_2\ C_1,\ A_2\ C_2$

Alias of the confounded interactions in the asymmetrical design.

$A_1\ B_2\ C_1 = A_1\ B_1\ D = C_2\ E = A_2\ B_1\ C_1\ D\ E$

$A_1\ B_2\ C_1 = A_1\ B_1\ C_2\ D = A_2\ E = B_1\ C_1\ D\ E$

$A_2\ C_2 = B_1\ C_1\ D = A_1\ B_2\ C_1\ E = A_1\ B_1\ D\ E$

Plan No. 1.16

$\frac{1}{2^2}$ ($3^3 \times 2^2$) in blocks of size 2^3

$$I = A' B B' C E = A A' B C' D = A B' C C' D E$$

Key block

Symmetrical

A	A'	B	B'	C	C'	D	E
1	0	0	1	0	1	0	1
0	1	0	0	1	1	0	0
0	0	1	1	1	0	1	1
1	1	0	1	1	0	0	1
1	0	1	0	1	1	1	0
0	1	1	1	0	1	1	1
1	1	1	0	0	0	1	0
0	0	0	0	0	0	0	0

Asymmetrical

A	B	C	D	E
1	1	1	0	1
1	0	2	0	0
0	2	1	1	1
2	1	1	0	1
1	1	2	1	0
1	2	1	1	1
2	1	0	1	0
0	0	0	0	0

Confounded interactions

A B B', A' B C, A A' C', A A' B' C, A' B B' C', A B C C', B' C C'

$A_2 B_2$, $A_1 B_1 C_1$, $A_2 C_1$, $A_2 B_1 C_1$, $A_1 B_2 C_1$, $A_1 B_1 C_2$, $B_1 C_2$

Alias of the confounded interactions in the asymmetrical design.

$A_1 B_2 = A_2 C_1 E = A_1 B_1 C_1 D = B_1 C_2 D E$

$A_1 B_1 C_1 = B_1 E = A_1 C_2 D = A_2 B_2 C_1 D E$

$A_2 C_1 = A_1 B_2 C_2 E = B_1 D = A_1 B_1 C_1 D E$

$A_2 B_1 C_1 = A_1 B_1 E = B_2 C_2 D = A_1 C_1 D E$

$A_1 B_2 C_1 = C_2 E = A_1 B_1 D = A_2 B_1 C_1 D E$

$A_1 B_1 C_2 = A_2 B_1 C_1 E = A_1 C_1 D = B_2 D E$

$B_1 C_2 = A_1 B_1 C_1 E = A_2 B_2 C_1 D = A_1 D E$

Plan No. 1.17

$\frac{1}{4}$ (3×2^7) in blocks of size 2^4

X = A A' B C D E F G H

Key block

Symmetrical									Asymmetrical								
A	B	C	D	A'	E	F	G	H	A	B	C	D	E	F	G	H
1	0	0	0	1	0	0	1	1	2	0	0	0	0	0	1	1
0	1	0	0	0	1	0	1	1	0	1	0	0	1	0	0	1
0	0	1	0	1	0	1	0	1	1	0	1	0	0	1	0	1
0	0	0	1	1	1	1	1	1	1	0	0	1	1	1	1	1
1	1	0	0	1	1	0	0	0	2	1	0	0	1	0	0	0
1	0	1	0	0	0	1	1	0	1	0	1	0	0	1	1	0
1	0	0	1	0	1	1	0	0	1	0	0	1	1	1	0	0
0	1	1	0	1	1	1	1	0	1	1	1	0	1	1	1	0
0	1	0	1	1	0	1	0	0	1	1	0	1	0	1	0	0
0	0	1	1	0	1	0	1	0	0	0	1	1	1	0	1	0
1	1	1	0	0	1	1	0	1	1	1	1	0	1	1	0	1
1	1	0	1	0	0	1	1	1	1	1	0	1	0	1	1	1
1	0	1	1	1	1	0	0	1	2	0	1	1	1	0	0	1
0	1	1	1	0	0	0	0	1	0	1	1	1	0	0	0	1
1	1	1	1	1	0	0	1	0	2	1	1	1	0	0	1	0
0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0

Confounded interactions

A' B C G, B C E F, A E G,
A E C F G, A A' B D E F,
A' C D E G, A' B D F G,
A C D E F G, A' E F G
AA'C D, B D E, C D F, A B D G,
A A' B C E, A A' F.

A_1B C G, B C E F, A_1 E G,
A_2B C F G, A_2 B D E F,
A_1C D E G, A_1B D F G,
A_1C D E F G, A_1 E F G,
A_2C D, C D E, C D F, A_1 B D G,
A_2 B C E, A_2 F

Alias of the confounded interactions in the asymmetrical design.

A_2 C D = B E F G H
B^2D E = A_2 C F G H
C D F = A_2 B E G H
A_1 B DG = A_1^2 C E F H
A_2 B C E = D^2F G H
A_2^2 F = B C D E G H
A_1^2 B C G = A_1D E F H
B^2C E F = A_1^2 D G H
A_1E G = A_2^2 B C D F H
A_1^2B C E G = A_1D E H
A_2B D E F = C G H
A_1 C D E G = A_1E F H
A_1 B D F G = A_1 C E H
A_1 C D E F G = A_1 B H
A_1 E F G = A_1 B C D H

Plan No. 118

$\frac{1}{2^2}(3 \times 2^7)$ in blocks of size 2^4

I = A A' B C D E F G = D E F G H = A A' B C H

Key block

Symmetrical	Asymmetrical
A A' B C D E F G H	A B C D E F G H
1 0 0 0 1 0 1 1 1	1 0 0 1 0 1 1 1
0 1 0 0 0 1 1 1 1	1 0 0 0 1 1 1 1
0 0 1 0 1 1 1 0 1	0 1 0 1 1 1 0 1
0 0 0 1 1 1 0 1 1	0 0 1 1 1 0 1 1
1 1 0 0 1 1 0 0 0	2 0 0 1 1 0 0 0
1 0 1 0 0 1 0 1 0	1 1 0 0 1 0 1 0
1 0 0 1 0 1 1 0 0	1 0 1 0 1 1 0 0
0 1 1 0 1 0 0 1 0	1 1 0 1 0 0 1 0
0 1 0 1 1 0 1 0 0	1 0 1 1 0 1 0 0
0 0 1 1 0 0 1 1 0	0 1 1 0 0 1 1 0
1 1 1 0 0 0 1 0 1	2 1 0 0 0 1 0 1
1 1 0 1 0 0 0 1 1	2 0 1 0 0 0 1 1
1 0 1 1 1 0 0 0 1	1 1 1 1 0 0 0 1
0 1 1 1 0 1 0 0 1	1 1 1 0 1 0 0 1
1 1 1 1 1 1 1 1 0	2 1 1 1 1 1 1 0
0 0 0 0 0 0 0 0 0	0 0 0 0 0 0 0 0

Confounded interactions

A B C D, A' B C E, A A' B F, A A' D E, A' C D F, A C E F, B D E F,

A_1 B C D, A_1 B C E, A_2 B F, A_2 D E, A_1 C D F, A_1 C E F, B D E F

Alias of the confounded interactions in the asymmetrical design.

A_1 B C D	= A_1 E F G	= A_1 B C E F G H	= A_1 D H
A_1 B C E	= A_1 D F G	= A_1 B C D F G H	= A_1 E H
A_2 B F	= C D E G	= A_2 B E G H	= C F H
A_2 D E	= B C F G	= A_2 F G H	= B C D E H
A_1 C D F	= A_1 B E G	= A_1 C E G H	= A_1 B D F H
A_1 C E F	= A_1 B D G	= A_1 C D G H	= A_1 B E F H
B D E F	= A_2 C G	= B G H	= A_2 C D E F H

Plan No. 1-19

$\frac{1}{2^3}(3 \times 2^7)$ in blocks of size 2^4

$I = A\,A'\,C\,E\,F = A'\,B\,C\,D\,E\,G = A\,A'\,C\,D\,G\,H = A\,B\,E\,H$
$= A\,B\,D\,F\,G = D\,E\,F\,G\,H = A'\,B\,C\,F\,H$

Key block

Symmetrical								Asymmetrical							
A	A'	B	C	D	E	F	G	A	B	C	D	E	F	G	H
1	0	0	0	1	1	0	0	1	0	0	1	1	0	0	0
0	1	0	0	1	0	1	0	1	0	0	1	0	1	0	0
0	0	1	0	1	1	1	1	0	1	0	1	1	1	1	0
0	0	0	1	1	1	0	1	0	0	1	1	1	0	1	1
1	1	0	0	0	1	1	0	2	0	0	0	1	1	0	0
1	0	1	0	0	0	1	1	1	1	0	0	0	1	1	0
1	0	0	1	0	0	0	1	1	0	1	0	0	0	1	1
0	1	1	0	0	1	0	1	1	1	0	0	1	0	1	0
0	1	0	1	0	1	1	1	1	0	1	0	1	1	1	1
0	0	1	1	0	0	1	0	0	1	1	0	0	1	0	1
1	1	1	0	1	0	0	1	2	1	0	1	0	0	1	0
1	1	0	1	1	0	1	1	2	0	1	1	0	1	1	1
1	0	1	1	1	1	1	0	1	1	1	1	1	1	0	1
0	1	1	1	1	0	0	1	1	1	1	1	0	0	0	1
1	1	1	0	1	0	0	1	2	1	1	0	1	0	0	1
0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0

Confounded interactions

A A' B C D, A B C E, A' D E

$A_2\,B\,C\,D$, $A_1\,B\,C\,E$, $A_1\,D\,E$

Alias of the confounded interactions in the asymmetrical design.

$A_2\,B\,C\,D = B\,D\,E\,F = A_1\,E\,G = B\,G\,H = A_1\,C\,F\,G =$
$= A_1\,C\,D\,E\,H = A_2\,B\,C\,E\,F\,G\,H = A_1\,B\,F\,H$

$A_1\,B\,C\,E = A_1\,B\,F = A_2\,D\,G = A_1\,B\,D\,E\,G\,H = C\,D\,E\,F\,G$
$= C\,H = A_1\,B\,C\,D\,F\,G\,H = A_2\,E\,F\,H$

$A_1\,D\,E = A_1\,C\,D\,F = B\,C\,G = A_1\,C\,E\,G\,H = A_2\,B\,E\,F\,G$
$= A_2\,B\,D\,H = A_1\,F\,G\,H = B\,C\,D\,E\,F\,H$

Plan No. 1.20

$\frac{1}{4}(3^2 \times 2^5)$ in blocks of size 2^4

I = A A' B B' C D E G H

Key block

Symmetrical									Asymmetrical							
A	A'	B	B'	C	D	E	G	H	A	B	C	D	E	G	H
1	1	0	1	0	0	1	1	1	2	1	0	0	1	1	1
0	1	1	0	0	0	1	0	1	1	1	0	0	1	0	1
0	0	0	0	1	0	1	1	1	0	0	1	0	1	1	1
0	1	0	0	0	1	0	1	1	1	0	0	1	0	1	1
1	0	1	1	0	0	0	1	0	1	2	0	0	0	1	0
1	1	0	1	1	0	0	0	0	2	1	1	0	0	0	0
1	0	0	1	0	1	1	0	0	1	1	0	1	1	0	0
0	1	1	0	1	0	0	1	0	1	1	1	0	0	1	0
0	0	1	0	0	1	1	1	0	0	1	0	1	1	1	0
0	1	0	0	1	1	1	0	0	1	0	1	1	1	0	0
1	0	1	1	1	0	1	0	1	1	2	1	0	1	0	1
1	1	1	1	0	1	0	0	1	2	2	0	1	0	0	1
1	0	0	1	1	1	0	1	1	1	1	1	1	0	1	1
0	0	1	0	1	1	0	0	1	0	1	1	1	0	0	1
1	1	1	1	1	1	1	1	0	2	2	1	1	1	1	0
0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0

Confounded interactions

A B C E, A A' B D, A C D G,
A B C D H, A' C D E, B D E G,
D E H, A' B C G, A' C H, B G H,
A A' E G, A A' D G H, A E G H,
A' B C D E G H

A_1 B_1 C E, A_2 B_1 D, A_1 C D G,
A_1 B_1 C D H, A_1 C D E,
B_1 D E G, D E H, A_1 B_1 C G,
A_1 C H, B_1 G H, A_2 E G,
A_2 D G H, A_1 E G H,
A_1 B_1 C D E G H

Alias of the confounded interactions in the asymmetrical design.

A_1 B_1 C E = A_1 B_1 D G H
A_2 B_1 D = B_1 C E G H
A_1 C D G = A_1 B_2 E H
A_1 B_1 C D H = A_1 B_1 E G
A_1 C D E = A_1 B_2 G H
B_1 D E G = A_2 B_1 C G H
D E H = A_2 B_2 C E G

A_1 B_1 C G = A_1 B_1 D E H
A_1 C H = A_1 B_2 D E G
B_1 G H = A_2 B_1 C D E
A_2 E G = B_2 C D H
A_2 D G H = B_1 C D G
A_1 E G H = A_1 B_2 C D
A_1 B_1 C D E G H = A_1 B_1
A_2 C D E H = B_1 G

Plan No. 121

$\frac{1}{2^2}(3^2 \times 2^5)$ in blocks of size 2^4

$$I = A\,A'\,B\,C\,D\,E\,G = A\,B'\,C\,E\,F = A'\,B\,B'\,D\,F\,G$$

Key block

Symmetrical									Asymmetrical						
A	A'	B	B'	C	D	E	F	G	A	B	C	D	E	F	G
1	0	0	1	0	0	1	1	0	1	1	0	0	1	1	0
0	1	1	1	0	0	1	0	1	1	2	0	0	1	0	1
0	1	0	1	1	0	1	1	0	0	1	1	0	1	1	0
0	1	0	1	0	1	0	1	0	1	1	0	1	0	1	0
1	1	1	0	0	0	0	1	1	2	1	0	0	0	1	1
1	1	0	0	1	0	0	0	0	2	0	1	0	0	0	0
1	1	0	0	0	1	1	0	0	2	0	0	1	1	0	0
0	0	1	0	1	0	0	1	1	0	1	1	0	0	1	1
0	0	1	0	0	1	1	1	1	0	1	0	1	1	1	1
0	0	0	0	1	1	1	0	0	0	0	1	1	0	0	0
1	0	1	1	1	0	1	0	1	1	2	1	0	1	0	1
1	0	1	1	0	1	0	0	1	1	2	0	1	0	0	1
1	0	0	1	1	1	0	1	0	1	1	1	1	0	1	0
0	1	1	1	1	1	0	0	1	1	2	1	1	0	0	1
1	1	1	0	1	1	1	1	1	2	1	1	1	1	1	1
0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0

Confounded interactions

$A\,B\,C\,E$, $A'\,B\,C\,D$, $A\,C\,D\,G$, $A\,A'\,D\,E$, $B\,D\,E\,G$, $A\,A'\,B\,G$, $A'\,C\,E\,G$

$A_1\,B_1\,C\,E$, $A_2\,B_2\,C\,D$, $A_1\,C\,D\,G$, $A_2\,D\,E$, $B_1\,D\,E\,G$, $A_2\,B_1\,G$, $A_1\,C\,E\,G$

Alias of the confounded interactions in the asymmetrical design.

$$A_1\,B_1\,C\,E = A_1\,D\,G = B_2\,F = A_2\,B_1\,C\,D\,E\,F\,G$$
$$A_1\,B_1\,C\,D = A_1\,E\,G = A_2\,B_2\,D\,E\,F = B_1\,C\,F\,G$$
$$A_1\,C\,D\,G = A_1\,B_1\,E = B_1\,D\,E\,F\,G = A_2\,B_2\,C\,F$$
$$A_2\,E\,D = B_1\,C\,G = A_1\,B_1\,C\,D\,F = A_1\,B_2\,E\,F\,G$$
$$B_1\,D\,E\,G = A_2\,C = A_1\,B_2\,C\,D\,F\,G = A_1\,B_1\,E\,F$$
$$A_2\,B_1\,G = C\,D\,E = A_1\,B_2\,C\,E\,F\,G = A_1\,B_1\,D\,F$$
$$A_1\,C\,E\,G = A_1\,B_1\,D = A_2\,B_1\,F\,G = B_2\,C\,D\,E\,F$$

Plan No. 122

$\frac{1}{2^3}(3^2 \times 2^5)$ in blocks of size 2^4

$I = ABCA'F = ACDEG = A'B'CDE = A'BDEFG$
$= ABB'DEF = AA'B'G = BB'CFG$

Key block

Symmetrical									Asymmetrical						
A	A'	B	B'	C	D	E	F	G	A	B	C	D	E	F	G
1	0	0	1	0	0	1	1	0	1	1	0	0	1	1	0
0	1	1	0	0	0	1	0	1	1	1	0	0	1	0	1
0	1	0	1	1	0	1	0	0	1	1	1	0	1	0	0
0	1	0	0	0	1	0	1	1	1	0	0	1	0	1	1
1	1	1	1	0	0	0	1	1	2	2	0	0	0	1	1
1	1	0	0	1	0	0	1	0	2	0	1	0	0	1	0
1	1	0	1	0	1	1	0	1	2	1	0	1	1	0	1
0	0	1	1	1	0	0	0	1	0	2	1	0	0	0	1
0	0	1	0	0	1	1	1	0	0	1	0	1	1	1	0
0	0	0	1	1	1	1	1	1	0	1	1	1	1	1	1
1	0	1	0	1	0	1	1	1	1	1	1	0	1	1	1
1	0	1	1	0	1	0	0	0	1	2	0	1	0	0	0
1	0	0	0	1	1	0	0	1	1	0	1	1	0	0	1
0	1	1	1	1	1	0	1	0	1	2	1	1	0	1	0
1	1	1	0	1	1	100		0	2	1	1	1	1	0	0
0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0

Confounded interactions

A B C E, A' B C D, A A' D E

$A_1 B_1 CE$, $A_1 B_1 CD$, $A_2 DE$

Alias of the confounded interactions in the asymmetrical design.

$A_1 B_1 CE = A_1 EF = B_1 DG = A_2 B_2 D = A_2 CDFG$
$= B_1 CDF = A_1 B_2 CDG = A_1 B_1 EFG$

$A_1 B_1 CD = A_1 DF = A_2 B_1 EG = B_2 E = CEFG$
$= A_2 B_1 CEF = A_1 B_2 CDG = A_1 B_1 DFG$

$A_2 DE = B_1 CDEF = A_1 CG = A_1 B_1 C = A_1 B_1 F G$
$= A_1 B_2 F = B_1 DEG = A_2 B_2 CDEFG$

Plan No. 123

$\frac{1}{4}$ ($3^3 \times 2^3$) in blocks of size 2^4

$I = A\ A'\ B'\ C\ C'\ D\ E\ F$

Key block.

Symmetrical									Asymmetrical					
A	A'	B	B'	C	C'	D	E	F	A	B	C	D	E	F
1	0	0	0	1	1	1	1	0	1	0	2	1	1	0
0	1	0	0	1	0	1	1	0	1	0	1	1	1	0
0	0	1	0	1	1	0	1	1	0	1	2	0	1	1
0	0	0	1	0	1	1	1	0	0	1	1	1	1	0
1	1	0	0	0	1	0	0	0	2	0	1	0	0	0
1	0	1	0	0	0	1	0	1	1	1	0	1	0	1
1	0	0	1	1	0	0	0	0	1	1	1	0	0	0
0	1	1	0	0	1	0	0	1	1	1	1	0	0	1
0	1	0	1	1	1	0	0	0	1	1	2	0	0	0
0	0	1	1	1	0	1	0	1	0	2	1	1	0	1
1	1	1	0	1	0	0	1	1	2	1	1	0	1	1
1	1	0	1	0	0	1	1	0	2	1	0	1	1	0
1	0	1	1	0	1	0	1	1	1	2	1	0	1	1
0	1	1	1	0	0	0	1	1	1	2	0	0	1	1
1	1	1	1	1	1	1	0	1	2	2	2	1	0	1
0	0	0	0	0	0	0	0	0	0	0	0	0	0	0

Confounded interactions

A A' B C, A C' B B', A A' B' D,
A A' B B' E, A' C C' B, B B' C D,
B C E, A' B C' D, A' C' E,
B D E, A C C' D, A B' C C' E,
A A' C D E, A B' C' D E, A' C C' D E

$A_2\ B_1\ C_1$, $A_1\ B_2\ C_1$,
$A_2\ B_1\ D$, $A_1\ B_1\ C_2$,
$B_2\ C_1\ D$, $A_2\ B_2\ E$,
$B_1\ C\ E$, $A_1\ B_1\ C_1\ D$,
$A_1\ C_1\ E$, $B_1\ D\ E$, $A_1\ C_2\ D$,
$A_1\ B_2\ C_2\ E$, $A_2\ C_1\ D\ E$,
$A_1\ B_1\ C_1\ D\ E$, $A_1\ C_2\ D\ E$

Alias of the confounded interactions in the asymmetrical design.

$A_2\ B_1\ C_1 = B_2\ C_1\ D\ E\ F$
$A_1\ B_2\ C_1 = A_1\ B_1\ C_1\ D\ E\ F$
$A_2\ B_1\ D = C_2\ E\ F$
$A_2\ B_2\ E = C_2\ D\ F$
$A_1\ B_1\ C_2 = A_1\ D\ E\ F$
$B_2\ C_1\ D = A_2\ B_1\ C_1\ E\ F$
$B_1\ C_1\ E = A_2\ B_1\ C_1\ D\ F$
$A_1\ B_1\ C_1\ D = A_1\ B_1\ C_1\ E\ F$

$A_1\ C_1\ E = A_1\ B_2\ C_1\ D\ F$
$B_1\ D\ E = A_2\ B_1\ C\ F$
$A_1\ C_2\ D = A_1\ B_2\ E\ F$
$A_2\ B_1\ C_2\ E = A_1\ B_1\ D\ F$
$A_2\ C_1\ D\ E = B_2\ F$
$A_1\ B_1\ C_1\ D\ E = A_1 B_1 C_1\ F$
$A_1\ C_2\ D\ E = A_1\ B_2\ F$

Plan No. 1-24

$\frac{1}{2^2}$ ($3^3 \times 2^3$) in blocks of size 2^3

$$I = A\,A'\,B'\,C'\,D\,E = A'\,B\,C\,C'\,F = A\,B\,B'\,C\,D\,E\,F$$

Key block

Symmetrical									Asymmetrical					
A	A'	B	B'	C	C'	D	E	F	A	B	C	D	E	F
1	0	0	0	1	0	1	0	1	1	0	1	1	0	1
0	1	0	0	1	1	1	1	1	1	0	2	1	1	1
0	0	1	0	1	1	0	1	1	0	1	2	0	1	1
0	0	0	1	0	1	1	1	0	0	1	1	1	1	0
1	1	0	0	0	1	0	1	0	2	0	1	0	1	0
1	0	1	0	0	1	1	1	0	1	1	1	1	1	0
1	0	0	1	1	1	0	1	1	1	1	2	0	1	1
0	1	1	0	0	0	1	0	0	1	1	0	1	0	0
0	1	0	1	1	0	0	0	1	1	1	1	0	0	1
0	0	1	1	1	0	1	0	1	0	2	1	1	0	1
1	1	1	0	1	0	0	0	1	2	1	1	0	0	1
1	1	0	1	0	0	1	0	0	2	1	0	1	0	0
1	0	1	1	0	0	0	0	0	1	2	0	0	0	0
0	1	1	1	0	1	0	1	0	1	2	1	0	1	0
1	1	1	1	1	1	1	1	1	2	2	2	1	1	1
0	0	0	0	0	0	0	0	0	0	0	0	0	0	0

Confounded interactions

$A\,A'\,B\,C$, $A'B\,B'C'$, $A\,A'B'D$, $A\,B'C\,C'$, $B\,B'C\,D$, $A\,B\,C'D$, $A'C\,C'D$

$A_2\,B_1\,C_1$, $A_1\,B_2\,C_1$, $A_2\,B_1D$, $A_1\,B_1\,C_2$, $B_2\,C_1\,D$, $A_1B_1C_1D$, $A_1\,C_2\,D$

Alias of the confounded interactions in the asymmetrical design.

$$A_2\,B_1\,C_1 = B_2\,C_2\,D\,E = A_1\,C_1\,F = A_1\,B_1\,D\,E\,F$$
$$A_1\,B_2\,C_1 = A_1\,B_1\,C_2D\,E = B_1\,C_1\,F = A_2\,D\,E\,F$$
$$A_2\,B_1\,D = C_1\,E = A_1\,B_2\,C_2\,D\,E\,F = A_1\,B_1\,C_1\,E\,F$$
$$A_1\,B_1\,C_2 = A_1\,C_1\,D\,E = A_2\,B_2\,F = B_1\,C_1D\,E\,F$$
$$B_2\,C_1\,D = A_2\,B_1\,C_2\,E = A_1\,B_1\,C_1\,F = A_1\,E\,F$$
$$A_1\,B_1\,C_1\,D = A_1\,B_2\,D\,E = A_2\,C_1\,F = B_1\,C_2\,E\,F$$
$$A_1\,C_2\,D = A_1\,B_1\,C_1\,E = B_1\,D\,F = A_2\,B_2\,C_1\,E\,F$$

Plan No. 125

$\frac{1}{2^3}(3^3 \times 2^3)$ in blocks of size 2^4

$$I = A\,A'\,B'\,C\,D = A\,B\,B'\,C'\,E = A\,A'\,B\,C\,C'\,F$$
$$= A'\,B\,C\,C'\,D\,E = B\,B'\,C'\,D\,F = A'\,B'\,C\,E\,F = A\,D\,E\,F$$

Key block

Symmetrical

A	A'	B	B'	C	C'	D	E	F
1	0	0	0	1	0	0	1	0
0	1	0	0	0	1	1	1	0
0	0	1	0	1	1	1	0	1
0	0	0	1	1	1	0	0	0
1	1	0	0	1	1	1	0	0
1	0	1	0	0	1	1	1	1
1	0	0	1	0	1	0	1	0
0	1	1	0	1	0	0	1	1
0	1	0	1	1	0	1	1	0
0	0	1	1	0	0	1	0	1
1	1	1	0	0	0	0	0	1
1	1	0	1	0	0	1	0	0
1	0	1	1	1	0	1	1	1
0	1	1	1	0	1	0	1	1
1	1	1	1	1	1	0	0	1
0	0	0	0	0	0	0	0	0

Asymmetrical

A	B	C	D	E	F
1	0	1	0	1	0
1	0	1	1	1	0
0	1	2	1	0	1
0	1	2	0	0	0
2	0	2	1	0	0
1	1	1	1	1	1
1	1	1	0	1	0
1	1	1	0	1	1
1	1	1	1	1	0
0	2	0	1	01	1
2	1	0	0	0	1
2	1	0	1	0	0
1	2	1	1	1	1
1	2	1	0	1	1
2	2	2	0	0	1
0	0	0	0	0	0

Confounded interaction

$A\,B\,B'\,C$, $A'\,B\,B'\,C'$, $A\,A'\,C\,C'$ $\quad A_1\,B_2\,C_1$, $A_1\,B_2\,C_1$, $A_2\,C_2$

Alias of the confounded interaction in the asymmetrical design.

$$A_1\,B_2\,C_1 = A_1\,B_1\,D = C_2\,E = A_1\,B_1\,C_1\,F = A_2\,B_1\,C_1\,D\,E$$
$$= A_1\,C_2\,D\,F = A_2\,B_1\,E\,F = B_2\,C\,D\,E\,F$$
$$A_1\,B_2\,C_1 = A_1\,B_1\,C_2\,D = A_2\,E = A_1\,B_1\,C_1\,F = B_1\,C_1\,D\,E$$
$$= A_1\,D\,F = B_1\,C_2\,E\,F = A_2\,B_2\,C_1\,D\,E\,F$$
$$A_2\,C_2 = B_1\,C_1\,D = A_1\,B_2\,C_1\,E = B_1\,F = A_1\,B_1\,D\,E = A_2\,B_2\,C_1\,E\,1$$
$$= A_1\,B_1\,C_1\,E\,F = A_1\,C_2\,D\,E\,F$$

Plan No. 1·26

$\frac{1}{2}$ (3^4 x 2) in blocks of size 2^4

I = A A' B B' C C' D D' E

Key block

Symmetrical

A	A'	B	B'	C	C'	D	D'	E
1	0	0	1	0	1	0	1	0
0	0	1	1	0	1	1	0	0
0	1	0	0	0	1	0	1	1
0	0	0	1	1	0	1	1	0
1	0	1	0	0	0	1	1	0
1	1	0	1	0	0	0	0	1
1	0	0	0	1	1	1	0	0
0	1	1	1	0	0	1	1	1
0	0	1	0	1	1	0	1	0
0	1	0	1	1	1	1	0	1
1	1	1	0	0	1	1	0	1
1	0	1	1	1	0	0	0	0
1	1	0	0	1	0	1	1	1
0	1	1	0	1	0	0	0	1
1	1	1	1	1	1	0	1	1
0	0	0	0	0	0	0	0	0

Asymmetrical

A	B	C	D	E
1	1	1	1	0
0	2	1	1	0
1	0	1	1	1
0	1	1	2	0
1	1	0	2	0
2	1	0	0	1
1	0	2	1	0
1	2	0	2	1
0	1	2	1	0
1	1	2	1	1
2	1	1	1	1
1	2	1	0	0
2	0	1	2	1
1	1	1	0	1
2	2	2	1	1
0	0	0	0	0

Confounded interactions

A B B' C, A A' C D', A A' B C',
B C D, A'B B'D', A'B'C C',
A B' D, B C C' D', A A'B D D',
A A'C C'D, A B'C' D, A'B'C D D',
A'B B'C'D, C'D D', A B B'C C'D D'

$A_1B_2C_1$, $A_2C_1D_1$, $A_2B_1C_1$,
$B_1C_1D_1$, $A_1B_2D_1$, $A_1B_1C_2$,
$A_1B_1D_1$, $B_1C_2D_1$, $A_2B_1D_2$,
$A_2C_2D_1$, $A_1B_1C_1D_1$, $A_1B_1C_1D_2$,
$A_1B_2C_1D_1$, C_1D_2, $A_1B_2C_2D_2$

Alias of the confounded interaction in the asymmetrical design.

$A_1B_2C_1 = A_1\ C_1D_2E$
$A_2C_1D_1 = B_2C_1D_1E$
$A_2B_1C_1 = B_1C_1D_2E$
$B_1C_1D_1 = A_2B_1C_1D_1E$
$A_1B_2D_1 = A_1C_2D_1E$
$A_1B_1C_2 = A_1B_1D_2E$
$A_1B_1D_1 = A_1B_1C_2D_1E$
$B_1C_2D_1 = A_2B_1D_1E$

$A_2\ B_1D_2 = B_1C_2E$
$A_2C_2D_1 = B_2D_1E$
$A_1B_1C_1D_1 = A_1B_1C_1D_1E$
$A_1B_1C_1D_2 = A_1B_1C_1E$
$A_1B_2C_1D_1 = A_1C_1D_1E$
$C_1D_2 = A_2B_2C_1E$
$A_1B_2C_2D_2 = A_1\ E$

Plan No. 127

$\frac{1}{2^2}(3^4 \times 2)$ in blocks of size 2^4

$$I = A\,A'\,B\,B'\,D\,D' = A'\,B\,C\,C'\,D'\,E = A\,B'\,C\,C'\,D\,E$$

Key block

Symmetrical

A	A'	B	B'	C	C'	D	D'	E
1	0	0	1	0	0	1	1	1
0	0	1	1	0	1	1	1	1
0	1	0	0	0	1	0	1	1
0	0	0	1	1	1	1	0	0
1	0	1	0	0	1	0	0	0
1	1	0	1	0	1	1	0	0
1	0	0	0	1	1	0	1	1
0	1	1	1	0	0	1	0	0
0	0	1	0	1	0	0	1	1
0	1	0	1	1	0	1	1	1
1	0	1	0	0	0	0	1	1
1	1	0	0	1	0	0	0	0
0	1	1	0	1	1	0	0	0
1	1	1	1	1	0	1	0	0
1	1	1	1	1	1	1	1	1
0	0	0	0	0	0	0	0	0

Asymmetrical

A	B	C	D	E
1	1	0	2	1
0	2	1	2	1
1	0	1	1	1
0	1	2	1	0
1	1	1	0	0
2	1	1	1	0
1	0	2	1	1
1	2	0	1	0
0	1	1	1	1
1	1	1	2	1
1	1	0	1	1
2	0	1	0	0
1	1	2	0	0
2	2	1	1	0
2	2	2	2	1
0	0	0	0	0

Confounded interactions

A A' B'C, A'BC C', A A'C D,
A B B'C', B'D, A B C'D, A'B B'C C' D

$A_2B_1C_1$, $A_1B_1C_2$, $A_2C_1D_1$,
$A_1B_2C_1$, B_1D_1, $A_1B_1C_1D_1$,
$A_1B_2C_2D_1$

Alias of the confounded interaction in the asymmetrical design.

$A_2B_1C_1 = B_1C_1D_2 = A_1B_2C_1D_1E = A_1C_1D_1E$
$A_1B_1C_2 = A_1B_1C_2D_2 = D_1E = B_2D_1E$
$A_2C_1D_1 = B_2C_2D_1 = A_1B_1C_1D_2E = A_1B_1C_1E$
$A_1B_2C_1 = A_1C_1D_2 = A_2B_1C_1D_1E = B_1C_1D_1E$
$B_1D_1 = A_2B_1D_1 = A_1B_2C_2D_2E = A_1B_2C_1E$
$A_1B_1C_1D_1 = A_1B_1C_1D_1 = A_2C_1D_2E = B_2C_1E$
$A_1B_2C_2D_1 = A_1C_2D_1 = B_1D_2E = A_2B_1E$

Plan No. 128

$\frac{1}{2^3}(3^4 \times 2)$ in blocks of size 2^4

$I = A\,A'\,B\,B'\,C'\,D' = A\,A'\,B\,B'\,C\,D = A\,B\,C\,D'\,E$
$= C\,C'\,D\,D' = A'\,B'\,C\,C'\,E = A'\,B'\,D\,D'\,E = A\,B\,C'\,D\,E$

Key block

Symmetrical

A	A'	B	B'	C	C'	D	D'	E
1	0	0	1	0	1	0	1	0
0	0	1	0	0	0	1	1	0
0	1	0	1	0	1	0	1	1
0	0	0	1	1	1	0	0	1
1	0	1	1	0	1	1	0	0
1	1	0	0	0	0	0	0	1
1	0	0	0	1	0	0	1	1
0	1	1	1	0	1	1	0	1
0	0	1	1	1	1	1	1	1
0	1	0	0	1	0	0	1	0
1	1	1	0	0	0	1	1	1
1	0	1	0	1	0	1	0	1
1	1	0	1	1	1	0	0	0
0	1	1	0	1	0	1	0	0
1	1	1	1	1	1	1	1	0
0	0	0	0	0	0	0	0	0

Asymmetrical

A	B	C	D	E
1	1	1	1	0
0	1	0	2	0
1	1	1	1	1
0	1	2	0	1
1	2	1	1	0
2	0	0	0	1
1	0	1	1	1
1	2	1	1	1
0	2	2	2	1
1	0	1	1	0
2	1	0	2	1
1	1	1	1	1
2	1	2	0	0
1	1	1	1	0
2	2	2	2	0
0	0	0	0	0

Confounded interactions

$A\,A'B'C$, $A\,B\,A'\,D'$, $C\,B\,B'\,D'$ $\quad A_2\,B_1\,C_1$, $A_2\,B_1\,D_1$, $C_1\,B_2\,D_1$,

Alias of the confounded interactions in the asymmetrical design.

$$A_2B_1C_1 = B_1C_2D_1 = B_1D_1 = A_1B_2D_1E = A_2B_2C_1D_2 = A_1C_1E$$
$$= A_1C_1D_2E = A_1B_2C_2D_1E$$
$$A_2B_1D_1 = B_1C_1 = B_1C_1D_2 = A_1C_1E = A_2B_1C_2D_1 = A_1B_2C_2D_1E$$
$$= A_1B_2D_1E = A_1C_1D_2E$$
$$B_2C_1D_1 = A_2C_2 = A_2D_2 = A_1B_1E = B_2C_1D_1 = A_1B_1C_1D_1E$$
$$= A_1B_2C_1D_1E = A_1B_1C_2D_2E$$