ON STUDY OF OPTIMALITY AND ORTHOGONALITY IN A CLASS OF FRACTIONAL FACTORIALS

R.P. SINGH

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(I.C.A.R.)
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INSTITUTE OF AGRICULTURAL RESEARCH STATISTICS
(I.C.A.R.)
NEW DELHI-110012.

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R.P. SINGH

I.A.R.S.
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CONTENTS.

CHAPTER - I

INTRODUCTION

When different factors influence a character under study, it is always desirable to test different combinations of the factors at various levels. Such experiments are called factorial experiments and are widely used because of the wider inductive basis on the conclusions drawn from them and of estimating the effects on interactions between different factors. By this approach, the consistency of each effect may be tested, and when it is shown to be dependent on the other factors, any inter-relation may be studied.

With increasing number of factors or levels of factors, the size of thefactorial experiments becomes unmanageably large. In factorial set up, often higher order interactions are found to be insignificant and difficult to interpret. To rduce the size of the experiment, Finney (1945) suggested the use of fractional replication in factorial experiments. The device consists of selecting a fraction of all possible treatment combinations such that main effects of the factors and lower order interactions of interest are estimable, assuming certain higher order interactions to be negligible.

When interest of the experimenter is to estimate the main effects only, it is possible to select a sub-set of assemblies (treatment combinations) from a complete factorial design which enables the main effects to be estimated with a fair degree of precision. Such plans have been called main effect plans. Plackett and Burman (1946) introduced optimum plans for multi-factorial experiments which enable the estimation of the main effects with the same accuracy as if the attention had been concentrated on varying the levels of a single component throughout the experiment. A good survey on the work done on main effect plans for symmetrical factorial experiments have made by Davies and Hay (1950), Davies (1954), Fry (1956), Box and Hunter (1961), Addelman and Kempthorne (1961) and Addelman (1962a, 1962b, 1963). Rao (1946, 1947) developed the concept of orthogonal arrays and has shown that the orthogonal estimates of main effects and interactions of order k can be obtained from an orthogonal array of strength d $(d > k)$ when higher-order interactions are absent. Chakravarti (1957) observed that the main effects can be estimated from an orthogonal array after om-itting certain assemblies from it and defined a new type of array as partially balanced array. Hotelling (1944), Kishen (1945), Mood (1946) and Raghavarao (1959) also developed certain main effect plans for weighing designs.

Some main effect plans for asymmetrical factorial experiments were developed by Chakravarti (1956) and Morrison (1956) which require a large number of treatment combinations. Addelman (1962) obtained main effect plans for asymmetrical factorial from orthogonal main effect plans for symmetrical factorials by collapsing the factors.

Orthogonal main effect plans provide optimum designs. But since they do not exist for all k, the number of assemblies and, p, the No. of parameters to be estimated, there is a need to select a best design for a given situation. Since the purpose of any design is to estimate each of the individual effects with whatever accuracy we estimate the individual effects, it is reasonable to expect the design to give minimal average variance for all the estimated effects. This led Kishen (1945) and Kiefer (1959) to define A - optimal criterion for efficiency. Mood (1946) and Kiefer (1959) defined the D - optimal criterion for efficiency of the design which gives a minimum generalised variance for the estimated effects. Ehrenfeld (1955) and Kiefer (1959) defined the E - optimal criterion for efficiency. These have been discussed in detail in Chapter - III.

It is interesting to note that a design need not satisfy all the three aforesaid criteria simultaneously. Thus a compromise has to be made for providing the set of points which perform uniformly better with respect to all the three criteria of optimality. This led to a new concept of C - optimality which has been elaborated in Chapter - III.

The implications of these different criteria for optimality have been studied for the designs adopted for fertilizers trials on cultivator's fields. This has led us to show that the design being currently adopted by I.C.A.R. on cultivator's fields is of very low efficiency. Designs more efficient than this have been obtained and presented in Chapter - IV.

A situation may also arise when the experimenter may be interested to estimate the lth order effect of one factor and the mth order effect of the other factor with a fair degree of precision. In this case the design selected considering any criteria for optimality defined above need not fulfil the aim of the experimenter. For this the concept of C - orthogonality has been introduced and is discussed in details in Chapter - III.

The designs for cultivator's field trials have been also used to demonstrate the concept underlying C-orthogonality. A discussion of these appears in Chapter V. In the last Chapter VI presents summary and conclusions of the study.

CHAPTER-II

Cultivator's Field Trials.

The conclusions based on the results of experiments conducted at research stations can not be immediately recommended for general adoption under actual farming conditions in the country because of the fact that the number of experimental stations in the country is small and that the fertility of the soil and the level of management at the experimental stations are superior to those in the surrounding cultivator's fields, experimentation at research station cannot provide a reliable guide for general adoption in actual farming conditions on cultivator's fields. A satisfactory method of bridging the gulf between the results of research at experimental stations and their adoption by cultivators is to conduct experiments in the fields representative of the entire tract.

Since the experiments are to be conducted in normal cultivation practices of the farmers so in order to gain his confidence and co-operation it is necessary to ensure that the design of the experiment should be simple enough to be conducted within the limited resources available on a cultivator's field and such that it can easily be fitted into the normal routine of his work and that utmost care should be taken while choosing the treatments for the experimentation so that the farmer may not incur any loss through trying these treatments on his field.

Following are the problems which one come across while conducting the experiements on cultivator's field.

1. Selection of fields,

2. Choice of treatments,

3. Choice of design, and

4. the number of experimental fields and its distribution between and within places.

While each of the above considerations have their own dimensions, here we shall restrict ourselves with the choice of the design.

Choice of the design:-

Apart from the proper choice of the treatments, the design of an experiment on cultivator's fields must be extremely simple and possess a demonstration value for the success of the experiment.

The simplest of experimental designs for field experiments is the randomized block design. But even a R.B.D with its replication involving numerous small plots lying side by side in the field cannot fulfil the requirement of enabling the cultivator to carry out his normal field operation undistrubed. This objection can however, be met by repeating the experiment on another field. A design which might appeal to the cultivator would be to devide his field into as many portions as there are treatments, apply the treatment over the whole of each of these portions and harvest the plots of given dimensions at harvest time in the presence of the experimenter. At harvest time plots of given dimension would be marked in random positions within the different portions and the produce from these plots would be weighed and recorded. It is important to note that the treatments should be allotted at random to different portions of the field.

2.1. <u>Fertilizer trials on cultivator's field</u>:-

In fertilizers trial on cultivator's field, the aim is to study the responses of different factors (Nitrogen, phosphorus, potassium etc.) at different levels.

If the number of factors is 'n' each at 's' levels then the total number of assemblies is s^n. In order to have small number of assemblies (requirement of the experiment in cultivator's field), choose a faction 'k' (depending on the plan) of assemblies from s^n assemblies. The total number of designs each with 'k' assemblies is $\binom{s^n}{k}$. The problem arises which of the $\binom{s^n}{k}$ designs shall we choose so that it is most efficient, i.e. it gives mimimum variance for all the estimated effects. This has been done by studying various optimality criteria spelt out in Chapter-III.

On the other hand if the experimenter aims at minimising the extent of non-orthogonality of different order of effects of different factors in preference to optimality, one can adopt the tool of C-orthogonality.

CHAPTER-III

Optimality and Orthogonality Criteria.

3.1. Preliminaries:-

For any factorial set up the observational system can be represented by the matrix equation

$$\underset{\sim}{Y} = \underset{\sim}{X}\underset{\sim}{\beta} + \underset{\sim}{e}$$

Where $\underset{\sim}{X}$ is a k x p design matrix, β is a p column vector of single degree of freedom effects, $\underset{\sim}{e}$ is a k column vector of errors assumed to be normally and independently distributed around a mean of zero and with common variance σ^2 and $\underset{\sim}{Y}$ is a k column vector of observations.

A design X is optimum when the information matrix i.e. $(X'X)$ is kI_p where I_p is a p x p unit matrix. This led to the formulation of orthogonal plans. But such orthogonal plans do not exist for all k and p. Hence there is a need to select a best design for a given situation. For this purpose different optimal criteria have been defined for studying the efficiency of designs.

The least square estimates of different effects are given by

$$(X'X)\hat{\beta} = X'Y$$

If $(X'X)$ is non-singular, that is X is of rank p, then

$$\hat{\beta} = (X'X)^{-1} X'Y$$

and the variance co-variance matrix of the estimates is

$$(X'X)^{-1} \hat{\sigma}^2$$

Since the purpose of the design is to ostimate each of the individual effects, with whatever accuracy we estimate the individual effects, it is reasonable to expect the design to give minimal average variance for all the estimated effects. This led Kishen (1945) and Kiefer (1959) to define the following

Definition 3.1.1.:-

Of the class of all k x p designs, design X is A-optimal if it has the least value for the trace of $(X'X)^{-1}$.

If $(X'X)^{-1} = (c_{ij})$, then the efficiency of the design can be measured by the factor

$$\frac{p}{k \sum_{i=1}^{p} c_{ii}}$$

Instead of thinking of minimizing the average variance of all the estimated effects, one may consider as a best design the one that gives a minimum generalized variance of the estimated effects. Since $(X'X)^{-1}\sigma^2$ is the dispersion matrix of the estimated effects, one may be interested in minimizing det. $(X'X)^{-1}$ or consequently maximizing det. $(X'X)$. This led Mood (1946) and Kiefer (1959) to define the following

Definition 3.1.2.:-

Of the class of all k x p designs, design X is D-optimal if it has the minimum value of det. $(X'X)^{-1}$.

Ehrenfeld (1955) and Kiefer (1959) made the following definition

Definition 3.1.3.:-

Of the class of all k x p designs, design X is E-optimal if it has the least value for $\lambda_{max.}$ which is the maximum characteristic root of $(X'X)^{-1}$.

3.2 C-optimality:-

It is interesting to note that a design which is A-optimal need not be D-optimal, D-optimal need not be E-optimal and so on. In order to choose a design considering all the three aforesaid criteria simultaneously, a compromise has to be made for providing the set of points which perform uniformly better with respect to all the three criteria of optimality. This has led us to a new concept of C-optimality which can be defined as

Definition 3.2.1.:-

Of the class of all k x p designs, design X is C-optimal if it has the minimum value of the sum of percentage departures from the optimum points for the respective criteria defined earlier.

Thus if D_{ij} is the percentage departure for the jth criterion (j = 1,2,3 for A,D and E-optimality respectively) in the ith design then the C-optimal design is one for which ϕ_i is minimum where

$$\phi_i = \sum_{j=1}^{3} D_{ij}$$

3.3. C-orthogonality:-

Any design selected on the basis of C-optimality criterion formulated in section 3.2 will be an efficient design for a given situation. But the problem may arise when the experimenter may instead be interested to estimate the lth order effect of one factor orthogonal to the mth order effect of the other factor. Thus if

$$A_l = \sum_{r=1}^{h} a_r Y_r \text{ and}$$

$$B_m = \sum_{r=1}^{h} b_r Y_r$$

(l,m = 1,2,---.(s-1) where s is the level of each factor). are two linear functions for estimating the lth order effect of one factor and mth order effect of the other factor respectively where a_r and b_r are some constants then the correlation coefficient between lth order effect of one factor and mth order effect of the other factor is given by

$$R_{lm} = \frac{\text{Cov. } (a_r, b_r)}{\sqrt{\text{Var. } (a_r) \text{ Var. } (b_r)}}$$

Here R_{11} will indicate correlation between linear (first order) effect of one factor and linear effect of the other factor, R_{12} will indicate correlation between linear effect of one factor and quadratic (2nd order) effect of the other factor, and so on. If $R_{lm} = 0$ for any l and m, l,m = 1,2,--(s-1) then the lth and mth order effects of two factors are estimated independently of each other, i.e. the two effects are orthogonal. In case of irregular fractions, however , such orthogonality amongst the effects of different factors is not attained. The next best course will be to minimise R_{lm} for lower values of l and m. It is seen that A,D,E or C-optimal designs need not have minimum values of R_{lm} for lower values of l and m. Thus the choice between optimality or orthogonality will be guided by the specific need and objective of the experiment.

But, a design which is R_{11}-orthogonal may not be R_{12}-orthogonal, R_{12}-orthogonal may not be R_{13}-orthogonal, and so on. Thus, in order to choose a design considering R_{lm}-orthogonality $\forall$ $l,m = 1,2,---,(s-1)$ simultaneously we may define the following index of C(compromise)-orthogonality

$$I = \sum_{l} \sum_{m} W_{lm} |R_{lm}|$$

Where $|R_{lm}|$ is the modulus value of R_{lm} and W_{lm} are the weights to be assigned to different R_{lm}. One can then choose a design for which I is minimum.

Now the problem arises how to assign weights to different correlation coefficients R_{lm}. One of the simplest way is to assign arbitrary number to different effects according to their importances e.g. when s = 4

Effects		Numbers to be assigned.
Linear	(1)	(s-1)
Quadratic	(2)	(s-2)
Cubic	(3)	(s-3)

Here we assume that the importance of the effects decreases from linear to quadratic, quadratic to cubic and so on, although this may not always be the case. Thus if the specific requirement of any experiment is to prefer some higher order effect to lower order effect, the number can be assigned accordingly. Otherwise in usual situations, the weights W_{lm} will be given by the product of numbers attached with the lth and mth effects. For example, for any experiment whose factors have s levels each, we have

$$W_{11} = (s-1)\ (s-1)$$

$$W_{12} = W_{21} = (s-1)\ (s-2)$$

Thus under the assumption stated above the index will be

$$I = \sum_{l} \sum_{m} (s-l)\ (s-m) \left| R_{lm} \right| .$$

CHAPTER-IV

Optimal designs for cultivator's field trials.

4.1 Selection of treatment combinations:-

The design currently being used by I.C.A.R. in fertilizer trials on cultivator's field is

N	P	K
0	0	0
2	0	0
3	3	3
2	2	0
2	0	2
2	1	2
2	2	2
2	2	1
2	2	3
2	3	2

(three factors Nitrogen, phosphorus and potassium each at four levels)

Where 0,1,2 and 3 are the four levels of each factors. Here 0 indicate no application of fertilizer and level 2 indicates the base level.

The object of the experiment is to study the responses of N,P and K with a view to formulate fertilizer recommendations for different agro-climatic conditions in the country.

In the above design the treatment combinations 000,200 and 333 are included in order to study the responses of mean effect, effect of nitrogen at base level and combined effect of N,P igher levels.

rest of the soven treatment combinations the levels vary at the base level of nitrogen in order to responses of phosphorus and potassium at the base of itrogen.

e problem now reduces to study the responses of ach at four levels taking a fraction of seven treatment ons from the 4^2 design excluding the combination her in order to study the responses of P and K at l the combinations 20 and 02 have been included. erefore to choose five out of nine treatment combinations 21,22,23,31,32 and 33, so as to have $\binom{9}{5} = 126$ sets (designs) each with seven treatment combinations s 20 and 02.

Now the problem is to examine the optimality for all sets for estimating the vector β, consisting of mean effects with single degree of freedom (when the on that all the higher order effects are negligible is d).

<u>dy of optimality:-</u>

Here we are estimating mean and main effects and each fect has three degrees of freedom which can further tioned as

	1 Linear effect
Main effect	1 Quadratic effect
	1 Cubic effect

To estimate linear, quadratic and cubic effects the following three mutually orthogonal contrasts have been defined.

Effects	Contrasts
Linear:	$-3a_0 - a_1 + a_2 + 3a_3$
Quadratic:	$a_0 - a_1 - a_2 + a_3$
Cubic:	$-a_0 + 3a_1 - 3a_2 + a_3$

Where a_0, a_1, a_2 and a_3 denote the four levels of a factor.

On the basis of the above defined contrasts, we can generate the design matrix for each of the 126 sets and obtain the information matrix $(X'X)$. As an example, consider the following table.

Set No. 3 T	Design Matrix X						
02	1	-3	1	-1	1	-1	-3
20	1	1	-1	-3	-3	1	-1
11	1	-1	-1	3	-1	-1	3
12	1	-1	-1	3	1	-1	-3
13	1	-1	-1	3	3	1	1
21	1	1	-1	-3	-1	-1	3
31	1	3	1	1	-1	-1	3

Information Matrix $(X'X)$

$$\begin{bmatrix} 7 & -1 & -3 & 3 & -1 & -3 & 3 \\ & 23 & 1 & -9 & -13 & 1 & 19 \\ & & 7 & -3 & 1 & -1 & -3 \\ & & & 47 & 19 & -3 & 3 \\ & & & & 23 & 1 & -9 \\ & & & & & 7 & -3 \\ & & & & & & 47 \end{bmatrix}$$

All computations for evaluating the values of det. $(X'X)^{-1}$, characteristic roots of $(X'X)^{-1}$, etc. have been done on Electronic Computer IBM 1620.

Out of 126 sets, 45 sets were such for which the vector β Could not be estimated as the value of det. $(X'X)$ was zero.

Remaining 81 sets have been explored and presented in Appendix A along with the values of det. $(X'X)^{-1}$, maximum characteristic root and trace of $(X'X)^{-1}$ and correlation coefficients R_{lm} between various order of effects.

4.3. Discussion of results:-

It is seen from the Appendix A that when the criterion A-optimal is considered then the set numbers 3,7,9,37,48,60,69, 72,117 and 119 are the optimal designs and the sets upto ranks 10 are shown in table 1 of Appendix B. Set No. 79 is optimal with respect to criterion D-optimal and various sets upto ranks ten are shown in table 2 of Appendix B. Set numbers 117 and 119 are the optimal designs when the criterion E-optimal is considered and sets upto ranks ten are shown in table 3 of Appendix B.

As none of the set is optimal with respect to all the A,D and E-optimality so we go for C-optimality outlined in the Chapter-III. The sets having 1st ten ranks have been presented in table 4 of Appendix B. Set No. 117 comes out to be the optimal design when the criterion C-optimal is considered. The treatment combinations for this design are

P	K
0	2
2	0
1	3
2	2
2	3
3	1
3	3

A critical perusal of the design reveals that this design may not be practicable as it involves higher levels of P and K. Amongst all the first ten ranks set No. 3 may be preferable as it involves lower levels of P and K which may result in lowering the cost of experimentation.

The complete design is given below

N	P	K
0	0	0
2	0	0
3	3	3
2	2	0
2	0	2
2	1	1
2	1	2
2	1	3
2	2	1
2	3	1

Thus the design given above not only satisfies the requirements of a good design for cultivator's field but it is also near optimum for providing the best estimates of parameter β. The design is thus superior to the one currently adopted by I.C.A.R. for cultivator's field trial.

CHAPTER-V

C-Orthogonal designs for cultivator's field.trials

5.1. Study of orthogonality:-

In Chapter III we considered the concept of R_{lm} orthogonality clearly, if the experimenter is interested to estimate the lth order effect of one factor orthogonal to the mth order of the other factor then R_{lm} must be zero. For each of the 81 sets, which have been explored for optimality, correlation coefficients R_{lm} were computed between all possible combinations of linear (first order), quadratic (second order) and cubic (third order) effects of one factor with the linear, quadratic and cubic effects of the other factor i.e. 11,12,13, 21,22,23,31,32, and 33 (where 1,2 and 3 stands for linear, quadratic and cubic respectively) and are presented in Appendix-A.

5.2. Discussion of results:-

5.2.1. Orthogonal designs:-

Set number 77,100,104,105,108 and 114 are R_{11}-orthogonal designs and various sets upto ranks ten are shown in table 1 of Addpedix-C. It may be seen that R_{11}-orthogonal designs are neither C nor A,D or E-optimal. Set numbers 115,119 and 120 were R_{12}-orthogonal designs and various sets upto ranks ten are presented in table 2 of Appendix-C.

R_{13}-orthogonal designs were set numbers 4,45 and 54 and various sets upto ranks ten are shown in table-3 of Appendix-C.

Set numbers 88, 117 and 120 were R_{21}-orthogonal; 5, 9, 12, 22, 24, 25, 28, 30, 31, 39, 48, 58, 60, 61, 65, 69, 74, 77, 93, 95, 96, 100, 104 and 108 were R_{22}-orthogonal and 24, 60, 69 and 100 were R_{23}-orthogonal designs and various sets upto ranks ten are presented in tables from 4 to 6 of Appendix-C.

Set numbers 20, 30 and 70 were R_{31}-orthogonal; 5, 9, 48 and 77 were R_{32}-orthogonal and 5 and 24 were R_{33}-orthogonal designs and various sets upto ranks ten are shown in tables from 7 to 9 of Appendix-C.

From the above we see that not a single set is R_{lm}-orthogonal for all l, m = 1, 2----, (s-1). So in order to choose a design considering R_{lm}-orthogonality for all l, m = 1, 2, ---- (s-1) simultaneouslly the index I was complated for each of the sets giving weights to different R_{lm} as

Correlation Coefficients	Weights
R_{11}	(s-1) (s-1) = 9
R_{12} or R_{21}	(s-1) (s-2) = 6
R_{13} or R_{31}	(s-1) (s-3) = 3
R_{22}	(s-2) (s-2) = 4
R_{23} or R_{32}	(s-2) (s-3) = 2
R_{33}	(s-3) (s-3) = 1

Set numbers 47 and 66 were C-orthogonal designs as the index computed above has minimum value for these sets and the various sets upto ranks ten are presented in table-10 of Appendix-C.

Set number 92 which is the design currently being used by I.C.A.R. for cultivator's field has been shown in all tables of Appendix-B and C for the sake of comparison.

5.2.2. <u>Optimal and orthogonal designs:-</u>

Here we will consider the designs which are optimal as well as orthogonal. Amongst the first ten ranks we see that 117 and 119 were A-optimal and C-orthogonal designs. From table 1 of Appendix B and table-10 of Appendix-C.

From table-2 of Appendix-B and table-10 of Appendix-C we see that set number 52 is D-optimal and C-orthogonal design. From table-3 of Appendix B and table-10 of Appendix-C we see that set numbers 117 and 119 were E-optimal and C-orthogonal designs. From table 4 of Appendix B and table-10 of Appendix-C set number 117 comes out to be C-optimal and C-orthogonal design.

CHAPTER-VI

Summary and Conclusions:-

Kishen (1945) and Kiefer (1959) defined A-optimal criterion for efficiency of the design which will give minimal average variance for all the estimated effects. Mood (1946) and Kiefer (1959) defined D-optimal criterion which will give minimum generalized variance for all the estimated effects. Ehrenfeld (1955) and Kiefer (1959) defined E-optimal criterion for the efficiency of the design. As none of the design will satisfy all these criteria simultaneously so a compromise has to be made for providing the set of points which perform uniformly better with respect to all the three criteria of optimality. This led us to define a new concept of C-optimality.

On the other hand if the experimenter aims at minimizing the extent of non-orthogonality of different order effects in preference to optimality he can adopt the concepts of R_{lm} and C-orthogonality which have been introduced in Chapter-III.

The designs for cultivator's field trials have been used to demonstrate the above concepts of optimality and orthogonality.

When the criterion C-optimal was considered then amongst the 1st ten ranks set No. 3 comes out to be best design which not only satisfied the requirements of a good design for cultivator's field but it is also near optimum for providing the best estimates of parameters β . This design is superior to the one currently being adopted by I.C.A.R. for cultivator's field trials and also involves lower cost of experimentation.

APPENDIX- A

```
SET NO. =  3     DESIGN    VALUE OF DET.      LAMBDA MAX.   TRACE
************     ******    *************      **********    *****
                   02      0.24414270E-07       0.5246      1.050
                   20
                   11      CORRELATION  COEFFICIENTS
                   12                L        Q        C
                   13      L    -.5749    .0500    .6010
                   21      Q     .0500   -.3999   -.1060
                   31      C     .6010   -.1060    .0375
*-*-*-*-*-*-*-*-*-*-*-*-*-*-*-*-*-*-*-*-*-*-*-*-*-*-*-*-*-*-*-*-*-*-*
SET NO. =  4     DESIGN    VALUE OF DET.      LAMBDA MAX.   TRACE
************     ******    *************      **********    *****
                   02      0.24414243E-07       0.9181      1.375
                   20
                   11      CORRELATION  COEFFICIENTS
                   12                L        Q        C
                   13      L    -.3000    .0500    .0176
                   21      Q     .2999   -.3999   -.6363
                   32      C     .6363   -.1060   -.0375
*-*-*-*-*-*-*-*-*-*-*-*-*-*-*-*-*-*-*-*-*-*-*-*-*-*-*-*-*-*-*-*-*-*-*
SET NO. =  5     DESIGN    VALUE OF DET.      LAMBDA MAX.   TRACE
************     ******    *************      **********    *****
                   02      0.24414305E-07       0.6208      1.175
                   20
                   11      CORRELATION  COEFFICIENTS
                   12                L        Q        C
                   13      L    -.0219    .5477    .4410
                   21      Q     .4823    .0912   -.3067
                   33      C     .5891   -.0322    .0135
*-*-*-*-*-*-*-*-*-*-*-*-*-*-*-*-*-*-*-*-*-*-*-*-*-*-*-*-*-*-*-*-*-*-*
SET NO. =  7     DESIGN    VALUE OF DET.      LAMBDA MAX.   TRACE
************     ******    *************      **********    *****
                   02      0.24414202E-07       0.5246      1.050
                   20
                   11      CORRELATION  COEFFICIENTS
                   12                L        Q        C
                   13      L    -.4750    .0500    .3889
                   22      Q    -.0500   -.3999    .1060
                   31      C     .3889   -.1060    .4874
*-*-*-*-*-*-*-*-*-*-*-*-*-*-*-*-*-*-*-*-*-*-*-*-*-*-*-*-*-*-*-*-*-*-*
SET NO. =  8     DESIGN    VALUE OF DET.      LAMBDA MAX.   TRACE
************     ******    *************      **********    *****
                   02      0.24414329E-07       0.5502      1.125
                   20
                   11      CORRELATION  COEFFICIENTS
                   12                L        Q        C
                   13      L    -.2051    .0500   -.2208
                   22      Q     .2051   -.3999   -.4819
                   32      C     .4352   -.1060    .4685
*-*-*-*-*-*-*-*-*-*-*-*-*-*-*-*-*-*-*-*-*-*-*-*-*-*-*-*-*-*-*-*-*-*-*
SET NO. =  9     DESIGN    VALUE OF DET.      LAMBDA MAX.   TRACE
************     ******    *************      **********    *****
                   02      0.24414355E-07       0.4114      1.050
                   20
                   11      CORRELATION  COEFFICIENTS
                   12                L        Q        C
                   13      L     .0684    .5477    .2208
                   22      Q     .4107    .0912   -.0803
                   33      C     .4195   -.0322    .5253
*-*-*-*-*-*-*-*-*-*-*-*-*-*-*-*-*-*-*-*-*-*-*-*-*-*-*-*-*-*-*-*-*-*-*
SET NO. = 10     DESIGN    VALUE OF DET.      LAMBDA MAX.   TRACE
************     ******    *************      **********    *****
                   02      0.24414292E-07       0.9052      1.375
                   20
                   11      CORRELATION  COEFFICIENTS
                   12                L        Q        C
                   13      L    -.3288    .2282    .5752
                   23      Q    -.1315   -.5477   -.0383
                   31      C     .1550   -.4841    .2033
*-*-*-*-*-*-*-*-*-*-*-*-*-*-*-*-*-*-*-*-*-*-*-*-*-*-*-*-*-*-*-*-*-*-*
SET NO. = 11     DESIGN    VALUE OF DET.      LAMBDA MAX.   TRACE
************     ******    *************      **********    *****
                   02      0.24414238E-07       1.0484      1.575
                   20
                   11      CORRELATION  COEFFICIENTS
                   12                L        Q        C
                   13      L    -.0912    .2282   -.0602
                   23      Q     .0912   -.5477   -.6425
                   32      C     .1936   -.4841    .1277
*-*-*-*-*-*-*-*-*-*-*-*-*-*-*-*-*-*-*-*-*-*-*-*-*-*-*-*-*-*-*-*-*-*-*
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SET NO. = 12	DESIGN	VALUE OF DET.	LAMBDA MAX.	TRACE
	02 20 11 12 13 23 33	0.24414281E-07	0.6346	1.250

CORRELATION COEFFICIENTS

	L	Q	C
L	.1479	.7302	.4088
Q	.2958	-.0912	-.2581
C	.2091	-.4195	.1977

SET NO. = 20	DESIGN	VALUE OF DET.	LAMBDA MAX.	TRACE
	02 20 11 12 21 23 31	0.24414379E-07	0.9181	1.375

CORRELATION COEFFICIENTS

	L	Q	C
L	-.3000	.2999	.6363
Q	.0500	-.3999	-.1060
C	.0176	-.6363	-.0375

SET NO. = 21	DESIGN	VALUE OF DET.	LAMBDA MAX.	TRACE
	02 20 11 12 21 23 32	0.24414219E-07	1.3954	1.825

CORRELATION COEFFICIENTS

	L	Q	C
L	-.0499	.2999	.1060
Q	.2999	-.3999	-.6363
C	.1060	-.6363	-.2249

SET NO. = 22	DESIGN	VALUE OF DET.	LAMBDA MAX.	TRACE
	02 20 11 12 21 23 33	0.24414321E-07	0.6850	1.300

CORRELATION COEFFICIENTS

	L	Q	C
L	.1754	.7302	.4985
Q	.4823	.0912	-.3067
C	.1705	-.4195	-.1084

SET NO. = 24	DESIGN	VALUE OF DET.	LAMBDA MAX.	TRACE
	02 20 11 12 21 31 33	0.24414296E-07	0.6208	1.175

CORRELATION COEFFICIENTS

	L	Q	C
L	-.0219	.4823	.5891
Q	.5477	.0912	-.0322
C	.4410	-.3067	.0135

SET NO. = 25	DESIGN	VALUE OF DET.	LAMBDA MAX.	TRACE
	02 20 11 12 21 32 33	0.24414353E-07	0.6850	1.300

CORRELATION COEFFICIENTS

	L	Q	C
L	.1754	.4823	.1705
Q	.7302	.0912	-.4195
C	.4985	-.3067	-.1084

SET NO. = 26	DESIGN	VALUE OF DET.	LAMBDA MAX.	TRACE
	02 20 11 12 22 23 31	0.24414332E-07	0.9180	1.375

CORRELATION COEFFICIENTS

	L	Q	C
L	-.2249	.2999	.4772
Q	-.0500	-.3999	.1060
C	-.1414	-.6363	.3000

SET NO. = 27	DESIGN	VALUE OF DET.	LAMBDA MAX.	TRACE
	02 20 11 12 22 23 32	0.24414310E-07	0.7152	1.325

CORRELATION COEFFICIENTS

	L	Q	C
L	.0256	.2999	-.0602
Q	.2051	-.3999	-.4819
C	-.0544	-.6363	.1277

```
SET NO. = 28    DESIGN   VALUE OF DET.     LAMBDA MAX.   TRACE
************    ******   *************     **********    *****
                  02     0.24414276E-07      0.6346      1.250
                  20
                  11     CORRELATION  COEFFICIENTS
                  12              L       Q       C
                  22     L     .2510   .7302   .3413
                  23     Q     .4107   .0912  -.0803
                  33     C     .0322  -.4195   .2697
*-*-*-*-*-*-*-*-*-*-*-*-*-*-*-*-*-*-*-*-*-*-*-*-*-*-*-*-*-*-*-*-*
SET NO. = 30    DESIGN   VALUE OF DET.     LAMBDA MAX.   TRACE
************    ******   *************     **********    *****
                  02     0 .24414355E-07     0.4350      1.100
                  20
                  11     CORRELATION  COEFFICIENTS
                  12              L       Q       C
                  22     L     .0219   .4823   .4961
                  31     Q     .4107   .0912   .2581
                  33     C     .2300  -.3067   .4609
*-*-*-*-*-*-*-*-*-*-*-*-*-*-*-*-*-*-*-*-*-*-*-*-*-*-*-*-*-*-*-*-*
SET NO. = 31    DESIGN   VALUE OF DET.     LAMBDA MAX.   TRACE
************    ******   *************     **********    *****
                  02     0.24414284E-07      0.4469      1.125
                  20
                  11     CORRELATION  COEFFICIENTS
                  12              L       Q       C
                  22     L     .2249   .4823   .0880
                  32     Q     .6087   .0912  -.1466
                  33     C     .2950  -.3067   .3850
*-*-*-*-*-*-*-*-*-*-*-*-*-*-*-*-*-*-*-*-*-*-*-*-*-*-*-*-*-*-*-*-*
SET NO. = 33    DESIGN   VALUE OF DET.     LAMBDA MAX.   TRACE
************    ******   *************     **********    *****
                  02     0.24414290E-07      1.0964      1.550
                  20
                  11     CORRELATION  COEFFICIENTS
                  12              L       Q       C
                  23     L     .0576   .5204   .6054
                  31     Q     .2401  -.1666   .0700
                  33     C     .0168  -.6651   .1764
*-*-*-*-*-*-*-*-*-*-*-*-*-*-*-*-*-*-*-*-*-*-*-*-*-*-*-*-*-*-*-*-*
SET NO. = 34    DESIGN   VALUE OF DET.     LAMBDA MAX.   TRACE
************    ******   *************     **********    *****
                  02     0.24414387E-07      0.9192      1.500
                  20
                  11     CORRELATION  COEFFICIENTS
                  12              L       Q       C
                  23     L     .2401   .5204   .1585
                  32     Q     .4166  -.1666  -.3666
                  33     C     .0700  -.6651   .0462
*-*-*-*-*-*-*-*-*-*-*-*-*-*-*-*-*-*-*-*-*-*-*-*-*-*-*-*-*-*-*-*-*
SET NO. = 37    DESIGN   VALUE OF DET.     LAMBDA MAX.   TRACE
************    ******   *************     **********    *****
                  02     0.24414202E-07      0.5246      1.050
                  20
                  11     CORRELATION  COEFFICIENTS
                  13              L       Q       C
                  21     L    -.4750  -.0500   .3889
                  22     Q     .0500  -.3999  -.1060
                  31     C     .3889   .1060   .4874
*-*-*-*-*-*-*-*-*-*-*-*-*-*-*-*-*-*-*-*-*-*-*-*-*-*-*-*-*-*-*-*-*
SET NO. = 38    DESIGN   VALUE OF DET.     LAMBDA MAX.   TRACE
************    ******   *************     **********    *****
                  02     0.24414256E-07      0.9180      1.375
                  20
                  11     CORRELATION  COEFFICIENTS
                  13              L       Q       C
                  21     L    -.2249  -.0500  -.1414
                  22     Q     .2999  -.3999  -.6363
                  32     C     .4772   .1060   .3000
*-*-*-*-*-*-*-*-*-*-*-*-*-*-*-*-*-*-*-*-*-*-*-*-*-*-*-*-*-*-*-*-*
SET NO. = 39    DESIGN   VALUE OF DET.     LAMBDA MAX.   TRACE
************    ******   *************     **********    *****
                  02     0.24414328E-07      0.4350      1.100
                  20
                  11     CORRELATION  COEFFICIENTS
                  13              L       Q       C
                  21     L     .0219   .4107   .2300
                  22     Q     .4823   .0912  -.3067
                  33     C     .4961   .2581   .4609
*-*-*-*-*-*-*-*-*-*-*-*-*-*-*-*-*-*-*-*-*-*-*-*-*-*-*-*-*-*-*-*-*
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SET NO. = 43

DESIGN	VALUE OF DET.	LAMBDA MAX.	TRACE
02, 20, 11, 13, 21, 31, 32	0.24414284E-07	0.9052	1.375

CORRELATION COEFFICIENTS

	L	Q	C
L	-.3288	-.1315	.1550
Q	.2282	-.5477	-.4841
C	.5752	-.0383	.2039

SET NO. = 45

DESIGN	VALUE OF DET.	LAMBDA MAX.	TRACE
02, 20, 11, 13, 21, 32, 33	0.24414285E-07	1.0964	1.550

CORRELATION COEFFICIENTS

	L	Q	C
L	.0576	.2401	.0168
Q	.5204	-.1666	-.6651
C	.6054	.0700	.1764

SET NO. = 46

DESIGN	VALUE OF DET.	LAMBDA MAX.	TRACE
02, 20, 11, 13, 22, 23, 31	0.24414262E-07	0.5297	1.100

CORRELATION COEFFICIENTS

	L	Q	C
L	-.2850	.0912	.3643
Q	-.1315	-.5477	-.0383
C	.0620	-.1936	.6507

SET NO. = 47

DESIGN	VALUE OF DET.	LAMBDA MAX.	TRACE
02, 20, 11, 13, 22, 23, 32	0.24414366E-07	0.8327	1.375

CORRELATION COEFFICIENTS

	L	Q	C
L	-.0684	.0912	-.2208
Q	.0912	-.5477	-.6425
C	.1452	-.1936	.4685

SET NO. = 48

DESIGN	VALUE OF DET.	LAMBDA MAX.	TRACE
02, 20, 11, 13, 22, 23, 33	0.24414362E-07	0.4114	1.050

CORRELATION COEFFICIENTS

	L	Q	C
L	.1479	.5477	.1936
Q	.2958	-.0912	-.2581
C	.2091	-.0322	.6542

SET NO. = 49

DESIGN	VALUE OF DET.	LAMBDA MAX.	TRACE
02, 20, 11, 13, 22, 31, 32	0.24414274E-07	0.5297	1.100

CORRELATION COEFFICIENTS

	L	Q	C
L	-.2850	-.1315	.0620
Q	.0912	-.5477	-.1936
C	.3643	-.0383	.6507

SET NO. = 51

DESIGN	VALUE OF DET.	LAMBDA MAX.	TRACE
02, 20, 11, 13, 22, 32, 33	0.24414256E-07	0.4695	1.100

CORRELATION COEFFICIENTS

	L	Q	C
L	.1000	.2401	-.0880
Q	.4166	-.1666	-.3666
C	.4375	.0700	.6930

SET NO. = 52

DESIGN	VALUE OF DET.	LAMBDA MAX.	TRACE
02, 20, 11, 13, 23, 31, 32	0.24414230E-07	1.5177	1.950

CORRELATION COEFFICIENTS

	L	Q	C
L	-.2115	-.0400	.1345
Q	-.0400	-.7499	-.4200
C	.1345	-.4200	.3823

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SET NO. = 54   DESIGN   VALUE OF DET.     LAMBDA MAX.   TRACE
************   ******   *************     **********    *****
                 02     0.24414338E-07      0.7746      1.375
                 20
                 11     CORRELATION  COEFFICIENTS
                 13              L       Q       C
                 23     L     .1297   .3202  -.0188
                 32     Q     .2700  -.4166  -.6285
                 33     C     .2268  -.3150   .3795
*-*-*-*-*-*-*-*-*-*-*-*-*-*-*-*-*-*-*-*-*-*-*-*-*-*-*-*-*-*-*-*
SET NO. = 56   DESIGN   VALUE OF DET.     LAMBDA MAX.   TRACE
************   ******   *************     **********    *****
                 02     0.24414335E-07      0.5502      1.125
                 20
                 11     CORRELATION  COEFFICIENTS
                 21              L       Q       C
                 22     L    -.2051   .2051   .4352
                 23     Q     .0500  -.3999  -.1060
                 31     C    -.2208  -.4819   .4685
*-*-*-*-*-*-*-*-*-*-*-*-*-*-*-*-*-*-*-*-*-*-*-*-*-*-*-*-*-*-*-*
SET NO. = 57   DESIGN   VALUE OF DET.     LAMBDA MAX.   TRACE
************   ******   *************     **********    *****
                 02     0.24414315E-07      0.7152      1.325
                 20
                 11     CORRELATION  COEFFICIENTS
                 21              L       Q       C
                 22     L     .0256   .2051  -.0544
                 23     Q     .2999  -.3999  -.6363
                 32     C    -.0602  -.4819   .1277
*-*-*-*-*-*-*-*-*-*-*-*-*-*-*-*-*-*-*-*-*-*-*-*-*-*-*-*-*-*-*-*
SET NO. = 58   DESIGN   VALUE OF DET.     LAMBDA MAX.   TRACE
************   ******   *************     **********    *****
                 02     0.24414354E-07      0.4469      1.125
                 20
                 11     CORRELATION  COEFFICIENTS
                 21              L       Q       C
                 22     L     .2249   .6087   .2950
                 23     Q     .4823   .0912  -.3067
                 33     C     .0880  -.1466   .3850
*-*-*-*-*-*-*-*-*-*-*-*-*-*-*-*-*-*-*-*-*-*-*-*-*-*-*-*-*-*-*-*
SET NO. = 60   DESIGN   VALUE OF DET.     LAMBDA MAX.   TRACE
************   ******   *************     **********    *****
                 02     0.24414285E-07      0.4114      1.050
                 20
                 11     CORRELATION  COEFFICIENTS
                 21              L       Q       C
                 22     L     .0684   .4107   .4195
                 31     Q     .5477   .0912  -.0322
                 33     C     .2208  -.0803   .5253
*-*-*-*-*-*-*-*-*-*-*-*-*-*-*-*-*-*-*-*-*-*-*-*-*-*-*-*-*-*-*-*
SET NO. = 61   DESIGN   VALUE OF DET.     LAMBDA MAX.   TRACE
************   ******   *************     **********    *****
                 02     0.24414326E-07      0.6346      1.250
                 20
                 11     CORRELATION  COEFFICIENTS
                 21              L       Q       C
                 22     L     .2510   .4107   .0322
                 32     Q     .7302   .0912  -.4195
                 33     C     .3413  -.0803   .2697
*-*-*-*-*-*-*-*-*-*-*-*-*-*-*-*-*-*-*-*-*-*-*-*-*-*-*-*-*-*-*-*
SET NO. = 62   DESIGN   VALUE OF DET.     LAMBDA MAX.   TRACE
************   ******   *************     **********    *****
                 02     0.24414246E-07      1.0483      1.575
                 20
                 11     CORRELATION  COEFFICIENTS
                 21              L       Q       C
                 23     L    -.0912   .0912   .1936
                 31     Q     .2282  -.5477  -.4841
                 32     C    -.0602  -.6425   .1277
*-*-*-*-*-*-*-*-*-*-*-*-*-*-*-*-*-*-*-*-*-*-*-*-*-*-*-*-*-*-*-*
SET NO. = 64   DESIGN   VALUE OF DET.     LAMBDA MAX.   TRACE
************   ******   *************     **********    *****
                 02     0.24414349E-07      0.9192      1.500
                 20
                 11     CORRELATION  COEFFICIENTS
                 21              L       Q       C
                 23     L     .2401   .4166   .0700
                 23     Q     .5204  -.1666  -.6651
                 33     C     .1585  -.3666   .0462
*-*-*-*-*-*-*-*-*-*-*-*-*-*-*-*-*-*-*-*-*-*-*-*-*-*-*-*-*-*-*-*
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SET NO. = 65	DESIGN	VALUE OF DET.	LAMBDA MAX.	TRACE
	02 20 11 21 31 32 33	0.24414324E-07	0.6346	1.250

CORRELATION COEFFICIENTS

	L	Q	C
L	.1479	.2958	.2091
Q	.7302	-.0912	-.4195
C	.4088	-.2581	.1977

SET NO. = 66	DESIGN	VALUE OF DET.	LAMBDA MAX.	TRACE
	02 20 11 22 23 31 32	0.24414379E-07	0.8327	1.375

CORRELATION COEFFICIENTS

	L	Q	C
L	-.0684	.0912	.1452
Q	.0912	-.5477	-.1936
C	-.2208	-.6425	.4685

SET NO. = 67	DESIGN	VALUE OF DET.	LAMBDA MAX.	TRACE
	02 20 11 22 23 31 33	0.24414278E-07	0.4695	1.100

CORRELATION COEFFICIENTS

	L	Q	C
L	.1000	.4166	.4375
Q	.2401	-.1666	.0700
C	-.0880	-.3666	.6930

SET NO. = 69	DESIGN	VALUE OF DET.	LAMBDA MAX.	TRACE
	02 20 11 22 31 32 33	0.24414304E-07	0.4114	1.050

CORRELATION COEFFICIENTS

	L	Q	C
L	.1479	.2958	.2091
Q	.5477	-.0912	-.0322
C	.1936	-.2581	.6542

SET NO. = 70	DESIGN	VALUE OF DET.	LAMBDA MAX.	TRACE
	02 20 11 23 31 32 33	0.24414333E-07	0.7746	1.375

CORRELATION COEFFICIENTS

	L	Q	C
L	.1297	.2700	.2268
Q	.3202	-.4166	-.3150
C	-.0188	-.6285	.3795

SET NO. = 72	DESIGN	VALUE OF DET.	LAMBDA MAX.	TRACE
	02 20 12 13 21 22 31	0.24414270E-07	0.5246	1.050

CORRELATION COEFFICIENTS

	L	Q	C
L	-.5749	-.0500	.6010
Q	-.0500	-.3999	.1060
C	.6010	.1060	.0375

SET NO. = 73	DESIGN	VALUE OF DET.	LAMBDA MAX.	TRACE
	02 20 12 13 21 22 32	0.24414289E-07	0.5502	1.125

CORRELATION COEFFICIENTS

	L	Q	C
L	-.3334	-.0500	.0803
Q	.2051	-.3999	-.4819
C	.7073	.1060	-.1703

SET NO. = 74	DESIGN	VALUE OF DET.	LAMBDA MAX.	TRACE
	02 20 12 13 21 22 33	0.24414306E-07	0.6354	1.175

CORRELATION COEFFICIENTS

	L	Q	C
L	-.0684	.4107	.4819
Q	.4107	.0912	-.0803
C	.7100	.2581	-.0283

SET NO. = 75

DESIGN	VALUE OF DET.	LAMBDA MAX.	TRACE
02, 20, 12, 13, 21, 23, 31	0.24414264E-07	0.5297	1.100

CORRELATION COEFFICIENTS

	L	Q	C
L	-.4385	.0912	.7669
Q	-.1315	-.5477	-.0383
C	.3876	-.1936	-.2033

SET NO. = 76

DESIGN	VALUE OF DET.	LAMBDA MAX.	TRACE
02, 20, 12, 13, 21, 23, 32	0.24414266E-07	1.1791	1.625

CORRELATION COEFFICIENTS

	L	Q	C
L	-.2282	.0912	.2008
Q	.0912	-.5477	-.6425
C	.4841	-.1936	-.4259

SET NO. = 77

DESIGN	VALUE OF DET.	LAMBDA MAX.	TRACE
02, 20, 12, 13, 21, 23, 33	0.24414337E-07	0.6354	1.175

CORRELATION COEFFICIENTS

	L	Q	C
L	0.0000	.5477	.6454
Q	.2958	-.0912	-.2581
C	.5229	-.0322	-.3042

SET NO. = 78

DESIGN	VALUE OF DET.	LAMBDA MAX.	TRACE
02, 20, 12, 13, 21, 31, 32	0.24414304E-07	0.5297	1.100

CORRELATION COEFFICIENTS

	L	Q	C
L	-.4385	-.1315	.3876
Q	.0912	-.5477	-.1936
C	.7669	-.0383	-.2033

SET NO. = 79

DESIGN	VALUE OF DET.	LAMBDA MAX.	TRACE
02, 20, 12, 13, 21, 31, 33	0.24414201E-07	0.6839	1.150

CORRELATION COEFFICIENTS

	L	Q	C
L	-.2115	.2401	.7231
Q	.2401	-.1666	.0700
C	.7231	.0700	-.1323

SET NO. = 84

DESIGN	VALUE OF DET.	LAMBDA MAX.	TRACE
02, 20, 12, 13, 22, 31, 32	0.24414274E-07	0.5485	1.125

CORRELATION COEFFICIENTS

	L	Q	C
L	-.4049	-.1315	.3346
Q	-.0468	-.5477	.1099
C	.5704	-.0383	.2772

SET NO. = 85

DESIGN	VALUE OF DET.	LAMBDA MAX.	TRACE
02, 20, 12, 13, 22, 31, 33	0.24414295E-07	0.5015	1.100

CORRELATION COEFFICIENTS

	L	Q	C
L	-.1801	.2401	.6516
Q	.1249	-.1666	.4032
C	.5601	.0700	.3696

SET NO. = 87

DESIGN	VALUE OF DET.	LAMBDA MAX.	TRACE
02, 20, 12, 13, 23, 31, 32	0.24414307E-07	1.0306	1.700

CORRELATION COEFFICIENTS

	L	Q	C
L	-.3202	-.0400	.4050
Q	-.1666	-.7499	-.1099
C	.3150	-.4200	-.0616

```
SET NO. = 88    DESIGN    VALUE OF DET.     LAMBDA MAX.   TRACE
************    ******    *************     **********    *****
                  02      0.24414274E-07      0.5897      1.175
                  20
                  12      CORRELATION  COEFFICIENTS
                  13            L       Q       C
                  23      L   -.1297  .3202   .7737
                  31      Q   0.0000 -.4166   .1964
                  33      C    .3403 -.3150   .0330
*-*-*-*-*-*-*-*-*-*-*-*-*-*-*-*-*-*-*-*-*-*-*-*-*-*-*-*-*-*-*-*-*
SET NO. = 91    DESIGN    VALUE OF DET.     LAMBDA MAX.   TRACE
************    ******    *************     **********    *****
                  02      0.24414335E-07      0.5502      1.125
                  20
                  12      CORRELATION  COEFFICIENTS
                  21            L       Q       C
                  22      L   -.3334  .2051   .7073
                  23      Q   -.0500 -.3999   .1060
                  31      C    .0803 -.4819  -.1703
*-*-*-*-*-*-*-*-*-*-*-*-*-*-*-*-*-*-*-*-*-*-*-*-*-*-*-*-*-*-*-*-*
SET NO. = 92    DESIGN    VALUE OF DET.     LAMBDA MAX.   TRACE
************    ******    *************     **********    *****
                  02      0.24414276E-07      0.8137      1.325
                  20
                  12      CORRELATION  COEFFICIENTS
                  21            L       Q       C
                  22      L   -.1052  .2051   .2472
                  23      Q    .2051 -.3999  -.4819
                  32      C    .2472 -.4819  -.5806
*-*-*-*-*-*-*-*-*-*-*-*-*-*-*-*-*-*-*-*-*-*-*-*-*-*-*-*-*-*-*-*-*
SET NO. = 93    DESIGN    VALUE OF DET.     LAMBDA MAX.   TRACE
************    ******    *************     **********    *****
                  02      0.24414315E-07      0.4564      1.125
                  20
                  12      CORRELATION  COEFFICIENTS
                  21            L       Q       C
                  22      L    .1170  .6087   .6180
                  23      Q    .4107  .0912  -.0803
                  33      C    .3666 -.1466  -.3225
*-*-*-*-*-*-*-*-*-*-*-*-*-*-*-*-*-*-*-*-*-*-*-*-*-*-*-*-*-*-*-*-*
SET NO. = 95    DESIGN    VALUE OF DET.     LAMBDA MAX.   TRACE
************    ******    *************     **********    *****
                  02      0.24414268E-07      0.6354      1.175
                  20
                  12      CORRELATION  COEFFICIENTS
                  21            L       Q       C
                  22      L   -.0684  .4107   .7100
                  31      Q    .4107  .0912   .2581
                  33      C    .4819 -.0803  -.0283
*-*-*-*-*-*-*-*-*-*-*-*-*-*-*-*-*-*-*-*-*-*-*-*-*-*-*-*-*-*-*-*-*
SET NO. = 96    DESIGN    VALUE OF DET.     LAMBDA MAX.   TRACE
************    ******    *************     **********    *****
                  02      0.24414324E-07      0.4564      1.125
                  20
                  12      CORRELATION  COEFFICIENTS
                  21            L       Q       C
                  22      L    .1170  .4107   .3666
                  32      Q    .6087  .0912  -.1466
                  33      C    .6180 -.0803  -.3225
*-*-*-*-*-*-*-*-*-*-*-*-*-*-*-*-*-*-*-*-*-*-*-*-*-*-*-*-*-*-*-*-*
SET NO. = 97    DESIGN    VALUE OF DET.     LAMBDA MAX.   TRACE
************    ******    *************     **********    *****
                  02      0.24414344E-07      1.1791      1.625
                  20
                  12      CORRELATION  COEFFICIENTS
                  21            L       Q       C
                  23      L   -.2282  .0912   .4841
                  31      Q    .0912 -.5477  -.1936
                  32      C    .2008 -.6425  -.4259
*-*-*-*-*-*-*-*-*-*-*-*-*-*-*-*-*-*-*-*-*-*-*-*-*-*-*-*-*-*-*-*-*
SET NO. = 99    DESIGN    VALUE OF DET.     LAMBDA MAX.   TRACE
************    ******    *************     **********    *****
                  02      0.24414282E-07      0.9258      1.425
                  20
                  12      CORRELATION  COEFFICIEVTS
                  21            L       Q       C
                  23      L    .1250  .4166   .4032
                  32      Q    .4166 -.1666  -.3666
                  33      C    .4032 -.3666  -.5806
*-*-*-*-*-*-*-*-*-*-*-*-*-*-*-*-*-*-*-*-*-*-*-*-*-*-*-*-*-*-*-*-*
```

```
SET NO. =100    DESIGN   VALUE OF DET.     LAMBDA MAX.   TRACE
***********     ******   *************     **********    *****
                  02     0.24414298E-07      0.6354      1.175
                  20
                  12     CORRELATION  COEFFICIEOTS
                  21             L        Q        C
                  31     L   0.0000   .2958    .5229
                  32     Q    .5477  -.0912   -.0322
                  33     C    .6454  -.2581   -.3042
*-*-*-*-*-*-*-*-*-*-*-*-*-*-*-*-*-*-*-*-*-*-*-*-*-*-*-*-*-*-*-*-*-*-*
SET NO. =101    DESIGN   VALUE OF DET.     LAMBDA MAX.   TRACE
***********     ******   *************     **********    *****
                  02     0.24414271E-07      0.7332      1.325
                  20
                  12     CORRELATION  COEFFICIEOTS
                  22             L        Q        C
                  23     L   -.2107   .0912    .4949
                  31     Q   -.0468  -.5477    .1099
                  32     C    .0412  -.6425   -.0967
*-*-*-*-*-*-*-*-*-*-*-*-*-*-*-*-*-*-*-*-*-*-*-*-*-*-*-*-*-*-*-*-*-*-*
SET NO. =102    DESIGN   VALUE OF DET.     LAMBDA MAX.   TRACE
***********     ******   *************     **********    *****
                  02     0.24414331E-07      0.5287      1.175
                  20
                  12     CORRELATION  COEFFICIEOTS
                  22             L        Q        C
                  23     L   -.0208   .4166    .7882
                  31     Q    .1249  -.1666    .4032
                  33     C    .1466  -.3666    .0967
*-*-*-*-*-*-*-*-*-*-*-*-*-*-*-*-*-*-*-*-*-*-*-*-*-*-*-*-*-*-*-*-*-*-*
SET NO. =104    DESIGN   VALUE OF DET.     LAMBDA MAX.   TRACE
***********     ******   *************     **********    *****
                  02     0.24414357E-07      0.4564      1.125
                  20
                  12     CORRELATION  COEFFICIEOTS
                  22             L        Q        C
                  31     L   0.0000   .2958    .5939
                  32     Q    .3746  -.0912    .4032
                  33     C    .4415  -.2581    .1728
*-*-*-*-*-*-*-*-*-*-*-*-*-*-*-*-*-*-*-*-*-*-*-*-*-*-*-*-*-*-*-*-*-*-*
SET NO. =105    DESIGN   VALUE OF DET.     LAMBDA MAX.   TRACE
***********     ******   *************     **********    *****
                  02     0.24414296E-07      0.7986      1.375
                  20
                  12     CORRELATION  COEFFICIEOTS
                  23             L        Q        C
                  31     L   0.0000   .2700    .5939
                  32     Q    .1666  -.4166    .1099
                  33     C    .1964  -.6285   -.1728
*-*-*-*-*-*-*-*-*-*-*-*-*-*-*-*-*-*-*-*-*-*-*-*-*-*-*-*-*-*-*-*-*-*-*
SET NO. =106    DESIGN   VALUE OF DET.     LAMBDA MAX.   TRACE
***********     ******   *************     **********    *****
                  02     0.24414241E-07      0.5485      1.125
                  20
                  13     CORRELATION  COEFFICIENTS
                  21             L        Q        C
                  22     L   -.4049  -.0468    .5704
                  23     Q   -.1315  -.5477   -.0383
                  31     C    .3346   .1099    .2772
*-*-*-*-*-*-*-*-*-*-*-*-*-*-*-*-*-*-*-*-*-*-*-*-*-*-*-*-*-*-*-*-*-*-*
SET NO. =107    DESIGN   VALUE OF DET.     LAMBDA MAX.   TRACE
***********     ******   *************     **********    *****
                  02     0.24414276E-07      0.7332      1.325
                  20
                  13     CORRELATION  COEFFICIENTS
                  21             L        Q        C
                  22     L   -.2107  -.0468    .0412
                  23     Q    .0912  -.5477   -.6425
                  32     C    .4949   .1099   -.0967
*-*-*-*-*-*-*-*-*-*-*-*-*-*-*-*-*-*-*-*-*-*-*-*-*-*-*-*-*-*-*-*-*-*-*
SET NO. =108    DESIGN   VALUE OF DET.     LAMBDA MAX.   TRACE
***********     ******   *************     **********    *****
                  02     0.24414331E-07      0.4564      1.125
                  20
                  13     CORRELATION  COEFFICIENTS
                  21             L        Q        C
                  22     L   0.0000   .3746    .4415
                  23     Q    .2958  -.0912   -.2581
                  33     C    .5939   .4032    .1728
*-*-*-*-*-*-*-*-*-*-*-*-*-*-*-*-*-*-*-*-*-*-*-*-*-*-*-*-*-*-*-*-*-*-*
```

SET NO. =110	DESIGN	VALUE OF DET.	LAMBDA MAX.	TRACE
	02 20 13 21 22 31 33	0.24414301E-07	0.5015	1.100

CORRELATION COEFFICIENTS

	L	Q	C
L	-.1801	.1249	.5601
Q	.2401	-.1666	.0700
C	.6516	.4032	.3696

SET NO. =111	DESIGN	VALUE OF DET.	LAMBDA MAX.	TRACE
	02 20 13 21 22 32 33	0.24414297E-07	0.5287	1.175

CORRELATION COEFFICIENTS

	L	Q	C
L	-.0208	.1249	.1466
Q	.4166	-.1666	-.3666
C	.7882	.4032	.0967

SET NO. =112	DESIGN	VALUE OF DET.	LAMBDA MAX.	TRACE
	02 20 13 21 23 31 32	0.24414273E-07	1.2061	1.700

CORRELATION COEFFICIENTS

	L	Q	C
L	-.3202	-.1666	.3150
Q	-.0400	-.7499	-.4200
C	.4050	-.1099	-.0616

SET NO. =114	DESIGN	VALUE OF DET.	LAMBDA MAX.	TRACE
	02 20 13 21 23 31 33	0.24414315E-07	0.7986	1.375

CORRELATION COEFFICIENTS

	L	Q	C
L	0.0000	.1666	.1964
Q	.2700	-.4166	-.6285
C	.5939	.1099	-.1728

SET NO. =115	DESIGN	VALUE OF DET.	LAMBDA MAX.	TRACE
	02 20 13 21 31 32 33	0.24414328E-07	0.5897	1.175

CORRELATION COEFFICIENTS

	L	Q	C
L	-.1297	0.0000	.3403
Q	.3202	-.4166	-.3150
C	.7737	.1964	.0330

SET NO. =116	DESIGN	VALUE OF DET.	LAMBDA MAX.	TRACE
	02 20 13 22 23 31 32	0.24414274E-07	0.8940	1.450

CORRELATION COEFFICIENTS

	L	Q	C
L	-.3124	-.1666	.2749
Q	-.1666	-.7499	-.1099
C	.2749	-.1099	.3225

SET NO. =117	DESIGN	VALUE OF DET.	LAMBDA MAX.	TRACE
	02 20 13 22 23 31 33	0.24414258E-07	0.3837	1.050

CORRELATION COEFFICIENTS

	L	Q	C
L	-.1350	.1666	.6088
Q	0.0000	-.4166	.1964
C	.3563	.1099	.5530

SET NO. =119	DESIGN	VALUE OF DET.	LAMBDA MAX.	TRACE
	02 20 13 22 31 32 33	0.24414297E-07	0.3837	1.050

CORRELATION COEFFICIENTS

	L	Q	C
L	-.1350	0.0000	.3563
Q	.1666	-.4166	.1099
C	.6088	.1964	.5530

SET NO. =120	DESIGN	VALUE OF DET.	LAMBDA MAX.	TRACE
***********	******	************	**********	*****
	02	0.24414300E-07	0.8940	1.450
	20			
	13	CORRELATION COEFFICIENTS		
	23	L Q C		
	31	L -.1249 0.0000 .3818		
	32	Q 0.0000 -.7499 -.1964		
	33	C .3818 -.1964 .2222		

-

APPENDIX-B

TABLE-1

(A-OPTIMAL)

RANK	SET NOS.	TRACE OF $(X'X)^{-1}$
1	3, 7, 9, 37, 48, 60, 69, 72, 117, 119	1.050
2	30, 39, 46, 49, 51, 67, 75, 78, 85, 110	1.100
3	8, 31, 56, 58, 73, 84, 91, 93, 96, 104, 106, 108	1.125
4	79	1.150
5	5, 24, 74, 77, 88, 95, 100, 102, 111, 115	1.175
6	12, 28, 61, 65	1.250
7	22, 25	1.300
8	27, 57, 92, 101, 107	1.325
9	4, 10, 20, 26, 38, 43, 47, 54, 66, 70, 105, 114	1.375
10	99	1.425

TABLE-2

(D-OPTIMAL)

RANK	SET NOS.	VALUE OF DET. $(X'X)^{-1}$
1	79	0.24414201E-07
2	7, 37	0.24414204E-07
3	21	0.24414219E-07
4	52	0.24414230E-07
5	11	0.24414238E-07
6	106	0.24414241E-07
7	4	0.24414243E-07
8	62	0.24414246E-07
9	38, 51	0.24414256E-07
10	117	0.24414258E-07
19	92	0.24414276E-07

TABLE-3

(E-OPTIMAL)

RANK	SET NOS.	MIN. OF LAMDA MAX.
1	117, 119	0.3837
2	9, 48, 60, 69	0.4114
3	30, 39	0.4350
4	31, 58, 93	0.4469
5	98, 104, 108	0.4564
6	51, 67	0.4695
7	85, 110	0.5015
8	3, 7, 37, 72	0.5246
9	46, 49, 75, 79, 102, 111	0.5297
10	8, 56, 73, 84, 91, 106	0.5502
21	92	0.8137

TABLE-4

(C-OPTIMAL)

RANK	SET NOS.	$\phi_i = \sum D_{ij}$
1	117	28.35
2	7, 37	37.34
3	119	47.76
4	60	49.68
5	51	55.80
6	69	59.13
7	67	64.75
8	31	66.38
9	3	71.16
10	72	71.17
49	92	166.18

APPENDIX-C

TABLE-1

(R_{11}-ORTHOGONAL)

RANK	SET NOS.	$\|R_{11}\|$
1	77,100,104,105,108,114	0.0000
2	5, 24, 27, 30, 39, 57,102,111	0.0219
3	21	0.0499
4	33, 45, 60	0.0576
5	9, 47, 66, 74, 95	0.0684
6	11, 62	0.0912
7	51, 67, 92	0.1000
8	93, 96, 99,120	0.1170
9	54, 70, 88,115	0.1297
10	117,119	0.1350

TABLE-2

(R_{12}-ORTHOGONAL)

RANK	SET NOS.	$\|R_{12}\|$
1	115,119,120	0.0000
2	52, 87	0.0400
3	3, 4, 7, 8, 37, 38, 72, 73,106,107	0.0500
4	46, 47, 62, 66, 75, 76, 97,101	0.0912
5	110,111	0.1249
6	43, 49, 78, 84	0.1315
7	112,114,116,117	0.1666
8	56, 57, 91, 92	0.2051
9	10, 11	0.2282
10	51, 79, 85	0.2401

TABLE-3

(R_{13}-ORTHOGONAL)

RANK	SET NOS.	$\|R_{13}\|$
1	4, 45, 54	0.0176
2	61	0.0322
3	107	0.0412
4	57	0.0544
5	11, 27, 49	0.0602
6	64	0.0700
7	73	0.0803
8	31, 51	0.0880
9	21	0.1060
10	52	0.1345
20	92	0.2472

TABLE-4

(R_{21}-ORTHOGONAL)

RANK	SET NOS.	$\|R_{21}\|$
1	88,117	0.0000
2	52,112	0.0400
3	3, 7, 20, 26, 37, 56, 72, 84, 91,101	0.0500
4	11, 47, 49, 66, 76, 78, 97,107	0.0912
5	85,102	0.1249
6	10, 46, 75,106	0.1315
7	87,105,116,119	0.1666
8	8, 27, 73, 92	0.2051
9	43, 62	0.2282
10	33, 67, 79,110,120	0.2401

TABLE-5

(R_{22}-ORTHOGONAL)

RANK	SET NOS.	$IR_{22}I$
1	5, 9, 12, 22, 24, 25, 28, 30, 31, 39, 48, 58, 60, 61, 65, 69, 74, 77, 93, 95, 96, 100, 104, 108	0.0912
2	33, 34, 45, 51, 64, 67, 79, 85, 99, 102, 110, 111	0.1666
3	3, 4, 7, 8, 20, 21, 26, 27, 37, 38, 56, 57, 66, 72, 73, 91, 92,	0.3999
4	54, 70, 88, 105, 114, 115, 117, 119	0.4166
5	10, 11, 43, 46, 47, 49, 62, 75, 76, 78, 84, 97, 101, 106, 107	0.5477
6	52, 87, 112, 116, 120	0.7499

TABLE-6

(R_{23}-ORTHOGONAL)

RANK	SET NOS.	$IR_{23}I$
1	24, 60, 69, 100	0.0322
2	10, 46, 75, 106	0.0383
3	33, 67, 79, 110	0.0700
4	9, 28, 74, 92	0.0803
5	3, 7, 20, 26, 37, 56, 72, 84, 87, 91, 101, 105, 116, 119	0.1060
6	31, 96	0.1466
7	49, 66, 78, 97	0.1936
8	88, 117, 120	0.1964
9	12, 30, 48, 77, 95, 108	0.2581
10	5, 22, 39, 58	0.3067
15	92	0.4819

TABLE-7

(P_{31}-ORTHOGONAL)

RANK	SET NOS.	$IR_{31}I$
1	20, 33, 70	0.0176
2	28	0.0322
3	101	0.0412
4	27	0.0544
5	46, 57, 62	0.0620
6	34	0.0700
7	91	0.0803
8	58, 67	0.0880
9	21	0.1060
10	52	0.1345
20	92	0.2472

TABLE-8

(R_{32}-ORTHOGONAL)

RANK	SET NOS.	$IR_{32}I$
1	5, 9, 48, 77	0.0322
2	43, 49, 78, 84	0.0383
3	45, 51, 79, 85	0.0700
4	60, 61, 95, 96	0.0803
5	3, 4, 7, 8, 37, 38, 72, 73, 106, 107, 112, 114, 116, 117	0.1060
6	58, 93	0.1466
7	46, 47, 75, 76	0.1936
8	115, 119, 120	0.1964
9	39, 65, 69, 74, 100, 104	0.2581
10	24, 25, 30, 31	0.3067
16	92	0.4819

TABLE-9

(R_{33}-ORTHOGONAL)

RANK	SET NOS.	$\|R_{33}\|$
1	5, 24	0.0135
2	74, 88, 95, 115	0.0283
3	3, 4, 20, 72	0.0375
4	34, 64	0.0462
5	87, 112	0.0612
6	101, 102, 107, 111	0.0967
7	22, 25,	0.1084
8	11, 27, 57, 62, 79	0.1277
9	91, 104, 105, 108, 114	0.1703
10	33, 45	0.1764
26	92	0.5806

TABLE-10

(C -ORTHOGONAL)

RANK	SET NOS.	I
1	47, 66	712
2	120	738
3	27, 57	748
4	117	786
5	119	804
6	11, 62	805
7	101, 107	812
8	52	821
9	105	831
10	114	834
15	92	890

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