SOME ASPECTS OF YIELD-SURVIVAL RELATIONSHIP IN DAIRY CATTLE

VIJAY KUMAR BHATIA

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(I.C.A.R.)
NEW DELHI - 12

INSTITUTE OF AGRICULTURAL RESEARCH STATISTICS
(I.C.A.R.)
LIBRARY AVENUE, NEW DELHI-12

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CERTIFICATE

This is to certify that the work incorporated in the dissertation entitled " Some Aspects of Yield Survival Relationship in Dairy Cattle " by Vijay Kumar Bhatia and submitted in fulfilment of the requirements for the award of DIPLOMA IN AGRICULTURAL STATISTICS of Institute of Agricultural Research Statistics (I.C.A.R.) was done under my guidance.

Prem Narain

(PREM NARAIN)
Professor of Statistics

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VKBhatia

(VIJAY KUMAR BHATIA)

C O N T E N T S

INTRODUCTION

The Institute of Agricultural Research Statistics collected breeding data from some important breeds of dairy cattle maintained on several livestock farms of the Central and the State Governments as well as private institutions. Some statistical investigations with the help of such data were undertaken by Amble et al (1967), Narain and Garg (1972) and others. The data collected included, records on milk yield for each lactation of a given cow. It is found that the maximum number of lactations of a cow, upto which the data were recorded is of the order of about 15. However the management practices in the herds are such that a cow is culled as soon as she becomes unproductive. Such culling has to be undertaken in order to allow for their replacement by heifers. As such if a particular cow continues to give milk upto a certain order of lactation, say k-th, then she can be said to have survived (k-1) culling i.e. during (k-1) proceeding lactations she was not removed from the herd for reasons of poor performance, on the assumption that the disposal of a cow for reasons unconnected with its productive performance is not very frequent. Since the herd owner judges the cow not only on the basis of milk yield but on all commercial characteristics, the length of useful life of a cow in the herd is an indicator of its commercial value. As shown by Robertson (1966), the culling of a cow can therefore be regarded on the basis of truncation selection using a culling variate of which one component is the milk yield of the cow. A high yielder cow is likely to survive several cullings and can therefore have a longer production life in the herd. From economic point of view also the longevity of a cow in a given herd is of vital importance. Rendel and Robertson (1950) showed that the increase in the average production life of a cow by one lactation is equal to that obtained by increasing milk yield by about 35 gallons. It is therefore, necessary to study the relationship between the longevity of a cow and its milk yield in the first lactation. A measure of this longevity can be taken as the proportion of cows in the first lactation which survive to various orders of lactation.

Only few studies on the yield-survival relationship in livestock have been conducted in the past. For instance, in cattle, Robertson and Barker (1966) studied the correlation between longevity and first lactation milk yield. In sheep, Bhatia and Narain (1973) studied the retention of a sheep in the flock and its relationship with its wool yield in the initial clip.

With the above end in view, the present investigation deals with the probability of survival of a cow upto a given lactation and its relation to its yield in the first lactation. An attempt has also been made to study the heritability of survival and its genetic correlation with milk yield in the first lactation.

2. MATERIAL AND METHODS

The Institute of Agricultural Research Statistics collected data from various organised Indian herds of dairy cattle spread over about 25 years. The data used in this study pertain to the following five herds: -

1. Red Sindhi at Hosur
2. Red Sindhi at Bangalore
3. Kangayam at Hosur
4. Tharparkar at Patna
5. Kankrej at Anand

A detailed description of the development of each of these herds and the nature of their records have been described earlier by Amble et al (1967). However in the case of Kankrej herd at Anand, the data utilised in the above study related only to the period 1944 to 1955. More data in respect of this herd for 1956 - 1963 were received subsequently at the Institute. The present investigation takes into account this additional data also.

2.1. Nature and Extent of data :

The analysis of data in the studies undertaken by Amble et al (1967) and Narain and Garg (1972) included as many as six characters of first lactation viz. lactation yield, lactation length, yield per day of lactation, calving interval, yield per day of calving interval and age at first calving. In the present investigation, however we confined to the character first lactation milk yield but extracted information about survival of the cow to various orders of lactation. It was found that data in regard to first lactation yield and first lactation length were available for cows upto about 15 lactations in the four herds mentioned at S.No. 1 to 4 above. In the case of Kankrej at Anand mentioned at S.No. 5 above, data were available only for first lactation yield for cows upto about 12 lactations. Data on progenies of sires with brand number zero in each of the herds was excluded from the study, since data on their dams were not available. The number of cows of each of the five herds for which survival-yield data were so available are given in Table - 1. In all the five herds these records could be sorted out according to sire groups. If a particular sire had less than 5 daughters, it was discarded. In each herd and for each sire so selected dam-daughter pairs were formed with respect to first lactation milk yield. The extent of data on the number of sires, number of daughters and number of dam-daughter pairs for each of the five herds is given in Table - 2.

2.2 <u>Processing of Survival Data</u>:

The processing of survival data was done according to the following procedure, taking into account the characters milk yield and lactation length in first lactation.

Take first the character milk yield and ascertain for each cow, upto what order of lactation the information is available. Put a tick mark in a table in the columns provided for lactations, wherever the information is available for the cow of a particular brand number.

By doing this for all the daughters of various sires in five different herds we get tables which would have a one-to-one correspondance with the original data regarding the character milk yield. Now take another character lactation length and put a tick mark in the above tables for those lactations of a cow which did not have information about milk yield but have now the information about lactation length. Do not put a tick mark wherever a tick mark is already there for a particular lactation. In this manner, tables become available which give us the probability of survival of a particular cow to various orders of lactation, starting from the first lactation.

2.3 Calculation of Proportion of Cows (q) surviving to various orders of lactation starting from the first lactation:

With the help of above tables, we find the numbers of cows which are available in 1st lactation for each of the five herds. In each of the herd and over all the sire groups, count the number of cows of the first lactation which have survived to 2nd lactation, 3rd lactation and so on upto the available order of lactation. The number of cows available in the first lactation is then taken as a divisor for all the lactations. The proportion in the 1st lactation is then obviously one and in the subsequent lactations are less than one.

2.4 Average Yield Characteristics of 1st lactation (y) and of their survivors to different order of lactations:

The average milk yield of cows in the 1st lactation is calculated by taking simply the average of the records of milk yields in the 1st lactation. Besides this average, the yields of the cows of the first lactation surviving to subsequent orders of lactation are also calculated. In a similar manner, the average lactation lengths of cows in 1st lactation and of their survivors to subsequent orders of lactation were also calculated. Further the average milk yield per day of lactation of a cow in 1st lactation was calculated by dividing the first lactation yield by its lactation length. The average milk yield per day of lactation of cows in the first lactation as well as of their survivors to subsequent orders of lactation were then worked out.

2.5. Calculation of Progeny test values (S) of sires based on dam-daughter comparison for 1st lactation yield as well as those based on only daughters first lactation yield.

In order to relate the probability of survival of a cow with its performance in the first lactation in regard to milk yield, a progeny test of each sire is required to be worked out. This can however be done in two ways. We can compare the daughters performance with the corresponding dams for the given sire and express the progeny test comparison as ($\bar{D} - \bar{M}$) where $\bar{D}$ is the average 1st lactation milk yield of the daughters of the sire.and $\bar{M}$ is the average 1st lactation milk yield of the corresponding mates of the sire which produced the daughters. Another way is to take average of milk yields of the daughters of a sire and express it as a deviation from the herd average viz. ($\bar{D} - A$) where A is the herd-average.

2.6 Regression of Proportion Surviving on the Progeny test (b_{qS}) :

The regression coefficients of the proportion surviving to a particular order of lactation on ($\bar{D} - \bar{M}$) as well as on ($\bar{D} - A$)

based on 1st lactation yield records were calculated in each of the five herds. This gives a series of regression coefficients corresponding to the survival to different orders of lactation. No regression coefficient was worked out for the first lactation, since the proportion in various sires group in this lactation are all unity.

2.7. Relative Survival Coefficient (R) :

In order to compare the above mentioned regression coefficients, it is necessary to divide them by the corresponding overall proportion surviving ($\bar{q}$). This gives us the relative survival coefficient (R) which is independent of units :

$$R = \left(\frac{b_{qS}}{\bar{q}} \right)$$

2.8. Heritability of Survival (h_q^2) :

2.8.1. First method :

Let there be s sires in a given herd with i-th sire having n_i daughters in the first lactation, such that $n_i \geqslant 5$ for all $i = 1, 2, \ldots, s$. Suppose the number of daughters of the i-th sire surviving to 2nd lactation is a_i for $i = 1, 2, \ldots, s$. This means that for i-th sire ($n_i - a_i$) daughters were culled after the first lactation. An estimate of the phenotypic value of the overall survival to 2nd lactation ($\bar{q}$) in this herd will then be given by

$$\bar{q} = \frac{\sum_{i=1}^{s} a_i}{\sum_{i=1}^{s} n_i}$$

Following the technique used by Robertson and Lerner (1947) in the case of poultry, we can consider, for the j-th daughter of the i-th sire ($i = 1, 2, \ldots, s$; $j = 1, 2, \ldots, n_i$) random variable y_{ij} which takes the value one if j-th daughter survives to the 2nd lactation and takes the value zero if she does not survive to the 2nd lactation. As such, we can perform an analysis of variance of y_{ij} values with two sources of variation as Between Sires (B) with $(s-1)$ d.f. and Within Sires (W) with $(\sum_{i=1}^{s} n_i - s)$ d.f. We then have,

$$\sum_{j=1}^{n_i} y_{ij} = 1 + 0 + 0 + 1 \ldots + \quad = a_i$$

$$\sum_{j=1}^{n_i} y_{ij}^2 = 1^2 + 0^2 + 0^2 + 1^2 \ldots + \quad = a_i$$

$$n_i y_{i.} = \sum_{j=1}^{n_i} y_{ij} = a_i$$

$$G = \sum_{i=1}^{s} \sum_{j=1}^{n_i} y_{ij} = \sum_{i=1}^{s} n_i y_{i.} = \sum_{i=1}^{s} a_i$$

$$y_{..} = \frac{G}{N}$$

where $N = \sum_{i=1}^{s} n_i$

Total sum of squares $= \sum_{i=1}^{s} \sum_{j=1}^{n_i} (y_{ij} - y_{..})^2$

$$= \sum_{i=1}^{s} \sum_{j=1}^{n_i} y_{ij}^2 - \frac{G^2}{N}$$

$$= \sum_{i=1}^{s} a_i - \frac{(\sum_{i=1}^{s} a_i)^2}{N}$$

$$B = \sum_{i=1}^{s} \sum_{j=1}^{n_i} (y_{i.} - y_{..})^2$$

$$= \sum_{i=1}^{s} \frac{a_i^2}{n_i} - \frac{G^2}{N}$$

$$W = \sum_{i=1}^{s} \sum_{j=1}^{n_i} (y_{ij} - y_{i.})^2$$

$$= \sum_{i=1}^{s} a_i - \sum_{i=1}^{s} \frac{a_i^2}{n_i}$$

Now we consider the following model,

$$y_{ij} = \mu + s_i + e_{ij}$$

In this model μ is the general mean representing overall survival and is treated as a fixed effect. s_i is the effect of the i-th sire affecting the survival of his daughters and is treated as a random effect with mean zero and variance as σ_s^2, e_{ij} is the random error for the survival of j-th daughter belonging to the i-th sire. It is a binomial variable with parameter q. That is, e_{ij} takes values 1 with probability q and takes the value zero with probability (1 - q). We then have

$$E (e_{ij}) = 1 \times q + 0 \times (1 - q) = q$$

$$V (e_{ij}) = E (e_{ij} - q)^2 = q - q^2 = q (1 - q)$$

It is further assumed in the model that s_i's and e_{ij}'s are independent as well as e_{ij}'s are independent amongest themselves.

i.e.

$$\text{Cov}(s_i, e_{ij}) = 0 \quad \text{for all } i, j$$

$$\text{Cov}(e_{ij}, e_{ik}) = 0 \quad \text{for } j \neq k = 1, 2, \ldots, n_i$$

Using the method of least squares for obtaining the estimates of μ and s_i's, we get the following normal equations:

$$N y_{..} = N \hat{\mu} + \sum_{i=1}^{s} n_i \hat{s}_i$$

$$n_i y_{i.} = n_i \hat{\mu} + n_i \hat{s}_i \qquad , i = 1, 2, \ldots, s.$$

Imposing the condition $\sum_{i=1}^{s} n_i \hat{s}_i = 0$ we get

$$\hat{\mu} = y_{..} = \frac{\sum_{i=1}^{s} a_i}{\sum_{i=1}^{s} n_i} = \bar{q} = \text{Overall survival}$$

$$\hat{s}_i = y_{i.} - \mu = \left(\frac{a_i}{n_i} - \bar{q} \right), \qquad i = 1, 2, \ldots, s.$$

We then have

$$\sum_{j=1}^{n_i} y_{ij} = n_i \mu + n_i s_i + \sum_{j=1}^{n_i} e_{ij}$$

$$n_i y_{i.} = n_i \mu + n_i s_i + \sum_{j=1}^{n_i} e_{ij}$$

or $$y_{i.} = \mu + s_i + \frac{\sum_{j=1}^{n_i} e_{ij}}{n_i}$$

Also $y_{..} = \dfrac{\sum_{i=1}^{s} n_i \, y_{i.}}{\sum_{i=1}^{s} n_i} = \mu + \dfrac{\sum_{i=1}^{s} n_i \, a_i}{\sum_{i=1}^{s} n_i} + \dfrac{\sum_{i=1}^{s} n_i \, e_{i.}}{\sum_{i=1}^{s} n_i}$

$$\cdot \quad y_{ij} - y_{i.} = e_{ij} - \frac{\sum_{j=1}^{n_i} e_{ij}}{n_i}$$

$$\cdot \quad (y_{ij} - y_{i.})^2 = e_{ij}^2 + \frac{1}{n_i^2} \left(\sum_{j=1}^{n_i} e_{ij} \right)^2 - \frac{2 e_{ij} \sum_{j=1}^{n_i} e_{ij}}{n_i}$$

This gives

$$\sum_{j=1}^{n_i} (y_{ij} - y_{i.})^2 = \sum_{j=1}^{n_i} e_{ij}^2 + \frac{n_i \left(\sum_{j=1}^{n_i} e_{ij} \right)^2}{n_i^2} - \frac{2 \left(\sum_{j=1}^{n_i} e_{ij} \right)^2}{n_i}$$

$$= \sum_{j=1}^{n_i} e_{ij}^2 - \frac{\left(\sum_{j=1}^{n_i} e_{ij} \right)^2}{n_i}$$

$$= \sum_{j=1}^{n_i} e_{ij}^2 - \frac{e_{i.}^2 \, n_i^2}{n_i}$$

$$= \sum_{j=1}^{n_i} e_{ij}^2 - n_i \, e_{i.}^2$$

where $e_{i.} = \dfrac{\sum_{j=1}^{n_i} e_{ij}}{n_i}$

Hence

$$W = \sum_{i=1}^{s} \sum_{j=1}^{n_i} (y_{ij} - y_{i.})^2 = \sum_{i=1}^{s} \sum_{j=1}^{n_i} e_{ij}^2 - \sum_{i=1}^{s} n_i \, e_{i.}^2$$

Now taking expectation of both the sides we get,

$$E(W) = E\left[\sum_{i=1}^{s}\sum_{j=1}^{n_i} e_{ij}^2\right] - E\left[\sum_{i=1}^{s} n_i e_{i.}^2\right]$$

$$= \sum_{i=1}^{s}\sum_{j=1}^{n_i} E(e_{ij}^2) - \sum_{i=1}^{s} n_i E(e_{i.}^2)$$

Since $E(e_{ij}) = q$, $E(e_{i.}) = q$

$$V(e_{i.}) = V\left(\frac{\sum_{j=1}^{n_i} e_{ij}}{n_i}\right) = \frac{1}{n_i^2}\sum_{j=1}^{n_i} q(1-q) = \frac{q(1-q)}{n_i}$$

and

$$E(e_{i.}^2) = V(e_{i.}) + q^2 = \frac{q(1-q)}{n_i} + q^2 \text{ , we get}$$

$$E(W) = \sum_{i=1}^{s}\sum_{j=1}^{n_i} q - \sum_{i=1}^{s} n_i \left[\frac{q(1-q)}{n_i} + q^2\right]$$

$$= Nq - sq(1-q) - Nq^2$$

$$= (N-s)\, q(1-q)$$

Now

$$y_{i.} - y_{..} = \mu + a_i + \frac{\sum_{j=1}^{n_i} e_{ij}}{n_i} - \left[\mu + \frac{\sum_{i=1}^{s} n_i a_i}{\sum_{i=1}^{s} n_i} + \frac{\sum_{i=1}^{s} n_i e_{i.}}{\sum_{i=1}^{s} n_i}\right]$$

$$= \left(a_i - \frac{\sum_{i=1}^{s} n_i a_i}{N}\right) + \left(e_{i.} - \frac{\sum_{i=1}^{s} n_i e_{i.}}{N}\right)$$

$$= (a_i - a_.) + (e_{i.} - e_{..})$$

$$(y_{i.} - y_{..})^2 = (a_i - a_.)^2 + (e_{i.} - e_{..})^2 + 2(a_i - a_.)(e_{i.} - e_{..})$$

Taking summation over j,

$$n_i (y_{i.} - y_{..})^2 = n_i (s_i - s_.)^2 + n_i (e_{i.} - e_{..})^2 + 2n_i (s_i - s_.)(e_{i.} - e_{..})$$

Now taking summation over i ,

$$B = \sum_{i=1}^{s} n_i (y_i - y_{..})^2 = \sum_{i=1}^{s} n_i (s_i - s_.)^2 + \sum_{i=1}^{s} n_i (e_{i.} - e_{..})^2$$

$$+ 2 \sum_{i=1}^{s} n_i (s_i - s_.) (e_{i.} - e_{..})$$

Taking expectation of both the sides.

$$E (B) = \sum_{i=1}^{s} n_i E (s_i - s_.)^2 + \sum_{i=1}^{s} n_i E (e_{i.} - e_{..})^2 + 2 \sum_{i=1}^{s} n_i E (s_i - s_.)(e_{i.} - e_{..})$$

$$= \sum_{i=1}^{s} n_i E (s_i^2 + s_.^2 - 2 s_i s_.) + \sum_{i=1}^{s} n_i E (e_{i.}^2 + e_{..}^2 - 2 e_{i.} e_{..}) ,$$

the product term/ is vanishing due to the independence of s_i's and e_{ij}'s .

Now

$$E (s_.^2) = E \left(\frac{\sum_{i=1}^{s} n_i s_i}{N} \right)^2$$

$$= \frac{1}{N^2} \sum_{i=1}^{s} n_i^2 \sigma_s^2$$

$$= \sigma_s^2 \frac{\sum_{i=1}^{s} n_i^2}{N^2}$$

$$E (s_i s_.) = E \left(s_i \frac{\sum_{i=1}^{s} n_i s_i}{N} \right)$$

$$= \frac{1}{N} n_i E (s_i^2)$$

$$= \frac{n_i}{N} \sigma_s^2$$

the product of s_i with other terms except s_i giving expectations zeros due to the expected value of each of s_i's being zero. Also

$$E(e_{..}^2) = V(e_{..}) + [E(e_{..})]^2$$

$$= V\left(\frac{\sum_{i=1}^{s} n_i e_{i.}}{N}\right) + q^2$$

$$= \sum_{i=1}^{s} \frac{n_i^2}{N^2} \frac{q(1-q)}{n_i} + q^2$$

$$= \frac{q(1-q)}{N} + q^2 ,$$

$$E(e_{i.} e_{..}) = E\left(e_{i.} \frac{\sum_{i=1}^{s} n_i e_{i.}}{N}\right)$$

$$= \frac{1}{N} [n_1 E(e_{i.} e_{1.}) + n_2 E(e_{i.} e_{2.})$$

$$+ \dots + n_i E(e_{i.}^2) + \dots + n_s E(e_{i.} e_{s.})]$$

$$= \frac{1}{N} \left[n_1 q^2 + n_2 q^2 + \dots + n_i \left(\frac{q(1-q)}{n_i} + q^2\right) + n_s q^2\right]$$

$$= \frac{1}{N} \left[\frac{n_i q(1-q)}{n_i} + \sum_{i=1}^{s} n_i q^2\right] = \frac{q(1-q)}{N} + q^2$$

Hence

$$E(B) = \sum_{i=1}^{s} n_i \left(\sigma_s^2 + \sigma_s^2 \frac{\sum_{i=1}^{s} n_i^2}{N^2} - 2 \sigma_s^2 \frac{n_i}{N}\right) + \sum_{i=1}^{s} n_i \left[\frac{q(1-q)}{n_i} + q^2\right.$$

$$\left. + \frac{q(1-q)}{N} + q^2 - \frac{2q(1-q)}{N} - 2q^2\right]$$

$$= \sigma_s^2 \left(\sum_{i=1}^{s} n_i + \frac{\sum_{i=1}^{s} n_i^2}{N} - \frac{2\sum_{i=1}^{s} n_i^2}{N} \right) + \left[sq(1-q) - q(1-q) \right]$$

$$= \sigma_s^2 \left(\sum_{i=1}^{s} n_i - \frac{\sum_{i=1}^{s} n_i^2}{N} \right) + (s-1)\,q(1-q)$$

$$= (s-1)\left[q(1-q) + k\,\sigma_s^2 \right]$$

where $k = \frac{1}{s-1}\left[\sum_{i=1}^{s} n_i - \frac{\sum_{i=1}^{s} n_i^2}{N} \right]$

Thus we get the expected value of the mean sum of squares between sires as

$$E\left(\frac{B}{s-1} \right) = q\,(1-q) + k\,\sigma_s^2$$

Taking an estimate of q by the overall survival $\bar{q} = \frac{\sum_{i=1}^{s} a_i}{\sum_{i=1}^{s} n_i}$, we can get an estimate of σ_s^2 as

$$\hat{\sigma}_s^2 = \frac{B - \bar{q}(1-\bar{q})(s-1)}{k\,(s-1)}$$

For determining the genetic variability for probability of survival, let the genotypic values for viability of N individual be $P_1, P_2, \ldots, P_N$ with mean $\bar{p} = \frac{\sum_{i=1}^{N} p_i}{N}$ and variance

$$\sigma_p^2 = \frac{\sum_{i=1}^{N} p_i^2}{N} - \bar{p}^2 .$$

Since mean genotypic value of the population is equal to mean phenotypic value, $\bar{p} = \bar{q}$. The phenotypic value for the viability of the j-th individual is $p_j + e_j$ where e_j is the environmental component. This phenotypic value lie between 0 and 1. The mean genotypic viability of survivors is then given by

$$\bar{w} = \frac{\sum_{j=1}^{N} p_j (p_j + e_j)}{\sum_{j=1}^{N} p_j}$$

$$= \frac{\sum_{j=1}^{N} p_j^2 + \sum_{j=1}^{N} p_j e_j}{\sum_{j=1}^{N} p_j}$$

$$= \frac{N (\bar{p}^2 + \sigma_p^2)}{N \bar{p}} = \frac{\bar{p}^2 + \sigma_p^2}{\bar{p}} = \frac{\bar{q}^2 + \sigma_p^2}{\bar{q}}$$

under the assumption $\sum_{j=1}^{N} p_j e_j = 0$ i.e. there is no correlation between genotype and environment and taking $\bar{q} = \bar{p}$. The expected genetic change in the viability over the previous generation is given by

$$\bar{w} - \bar{p} = \frac{\bar{q}^2 + \sigma_p^2}{\bar{q}} - \bar{q}$$

$$= \frac{\bar{q}^2 + \sigma_p^2 - \bar{q}^2}{\bar{q}} = \frac{\sigma_p^2}{\bar{q}}$$

The phenotypic selection differential, being the difference between the phenotypic viability of survivors (taken as unity) and mean phenotypic viability of the population is $(1 - \bar{q})$. As such, denoting the heritability of survival by h_q^2, the expected genetic change $(\bar{w} - \bar{p})$ is also given by

$$\bar{w} - \bar{p} = h_q^2 \times (1 - \bar{q})$$

Comparing the two expressions for the genetic gain, we get

$$h_q^2 = \frac{\sigma_p^2}{\bar{q}\,(1-\bar{q})}$$

It thus appears that the heritability of the survival can be obtained if we know the genotypic variance of survival as well as overall survival value. If we denote by r, the difference for the genetic relationship between and within sire groups, then we get the relation

$$\sigma_s^2 = r\,\sigma_p^2$$

using the relation for σ_s^2 and h_q^2 we get

$$h_q^2 = \frac{\left[\dfrac{B}{\bar{q}\,(1-\bar{q})} - (s-1)\right]}{r \cdot n_o}$$

where $n_o = k\,(s-1)$.

The data on survival according to sire-group can also be looked upon as $2 \times s$ contigency table giving the number of survivors and non-survivors in each of the s sire-groups. The heterogeneity χ^2 for this table is obtained as

$$\chi^2 = \frac{B}{\bar{q}\,(1-\bar{q})}$$

with $(s-1)$ d.f.

In terms of this χ^2, the heritability can then be expressed as

$$h_q^2 = \frac{\chi^2 - (s-1)}{r \cdot n_o}.$$

Since $E(\chi^2)$ is $(s - 1)$, this shows that h_q^2 depends upon the excess of the observed χ^2 above its expected value. Cochran (1936) has shown that for small numbers the expected value of χ^2 is slightly greater than $(s - 1)$. The exact expression of this expected value with a constant number n in each of s groups is given by

$$E(\chi^2) = (s-1) + \frac{1}{n} \left[1 - \frac{1}{s} \left(1 - \frac{1}{n} \right) \right]$$

In the present case, however, n is required to be replaced by k in order to take into account the different number of observations in the various sire-groups.

2.8.2. <u>Second method</u> :

The above method involves conducting the analysis of variance for a binomial variable and is therefore not strictly valid, since in such case the variance is not independent of mean. As such it is desirable to use an angular transformation which makes the variance independent of mean and therefore justifies the use of analysis of variance technique.

Following Snedecor and Cochran, we first convert the ones and zeros in the various sire groups to proportion. For zero the proportion is $\frac{1}{4 n_i}$ and for one the proportion is $\frac{(n_i - \frac{1}{4})}{n_i}$. This empirical device suggested by Barlett (1947), improves the equality of variance in the angles. Using these proportions we look up Table - X of Fisher and Yates and transform the proportion for each sire to angle Θ where $\Theta = \sin^{-1}\sqrt{p}$. Denoting the proportion corresponding to one and zero p_i and p_i' respectively, we transform them to angles Θ_i and

Q_1' respectively. Having obtained this transformed values, the analysis of variance between and within sire groups is conducted in the usual way. Such an analysis of variance giving the expected value of the mean sum of squares in term σ_s^2 (between sire component of variance) and σ_o^2 (within sire component of variance) is given in Table - 3.

Equating mean sum of squares with its expected value and solving for σ_s^2 we get

$$\sigma_s^2 = \frac{\text{Between Sires} - \text{Within Sires}}{k} = \frac{B' - W'}{k}$$

$$\text{where } k = \frac{\sum_{i=1}^{s} n_i - \sum_{i=1}^{s} n_i^2/N}{s - 1}$$

This gives t, the intra-class correlation coefficient as

$$t = \frac{(B' - W')/k}{\frac{(B' - W')}{k} + W} = \frac{(B' - W')/k}{\frac{[(B' - W') + k W']}{k}}$$

$$= \frac{B' - W'}{B' - W' + kW'} = \frac{B' - W'}{B' + W'(k - 1)}$$

Now $h_q^2 = \frac{t}{r}$ where r is the genetic relationship. In our case with mixtures of half sibs and full sibs, r takes the value 0.285. As such

$$h_q^2 = \frac{B' - W'}{0.285 [B' + W'(k - 1)]}$$

The standard error of the heritability coefficient as given by Becker (1967) is

$$S.E.(h_q^2) = \frac{[1 + (k - 1)t](1 - t)}{0.285 \sqrt{\frac{1}{2} k(k - 1)(s - 1)}}$$

4.9 Genetic Correlation between survival to various orders of lactation and milk yield in the first lactation :

In order to estimate genetic correlation between the survival upto a given order say k-th lactation and milk yield in the first lactation, a relationship between the survival value of each sire group with its daughter's performance in terms of $(\bar{D} - \bar{M})$ is required to be established. In terms of notation previously used, this means a linear regression of the form

$$y_{i.} = \alpha + \beta(\bar{D}_i - \bar{M}_i)$$

is fitted. The sum of squares between sires viz. B gets therefore broken into two parts.

$$B = b \sum_{i=1}^{s} n_i (y_{i.} - y_{..})(\bar{D}_i - \bar{M}_i) + \sum_{i=1}^{s} n_i \left[y_{i.} - a - b(\bar{D}_i - \bar{M}_i) \right]^2$$

where a and b are least squares estimates of α and β respectively. The first expression on the right is also equal to $r^2 B$, where r is the genetic correlation between survival and milk yield in the first lactation. Now it has already been shown that $\frac{B}{\bar{q}(1-\bar{q})}$ is a χ^2 with (s-1)d.f. This χ^2 can be broken down into two parts as χ_1^2 and $\chi_{(s-2)}^2$ with 1 and (s - 2) d.f. respectively. Hence χ^2 with 1 d.f. corresponds to $\frac{r^2 B}{\bar{q}(1-\bar{q})}$ and χ^2 with (s - 2) d.f. corresponds to $\frac{\sum_{i=1}^{s} n_i \left[y_{i.} - a - b(\bar{D}_i - \bar{M}_i) \right]^2}{\bar{q}(1-\bar{q})}$

It is therefore obvious that the proportion of excess χ^2 removed by fitting of the regression of survival on initial milk yield provides with an estimate of the square of the genetic correlation coefficient. That is [illegible]

$$r^2 = \frac{\chi_1^2 - 1}{\chi^2_{(s-2)} - (s-2)} \ .$$

3. RESULTS AND DISCUSSION

In this section data collected from five important herds of Indian dairy cattle, relating to the longevity of cows and milk yield have been analysed. The relationship between the survival of a cow to different orders of lactation and its milk yield has also been studied.

3.1. Overall proportion ($\bar{q}$) of cows surviving to different orders of lactation :

The overall proportion of cows surviving to lactations upto the 15th order are shown graphically in Figure - 1 for the three herds Tharparkar, Red Sindhi (Hosur) and Red Sindhi (Bangalore) and in Figure - 2 for the two herds of Kangayam and Kankrej. It is apparent from both the figures that in a given herd the proportion of cows surviving to different order of lactations continues to decrease with the increase in the order of lactation. In the Tharparkar herd, the 50 per cent of survival occurs between 5th and 6th lactation. In the Red Sindhi (Bangalore) herd, it occurs between 4th and 5th lactation. In the Red Sindhi (Hosur) it occurs between 3rd and 4th lactation. From Figure - 2 it appears that for Kangayam herd, the 50 per cent of survival occurs between 3rd and 4th lactation. For the Kankrej, it is seen that it occurs between 2nd and 3rd lactation. It is, therefore, apparent that judged from 50 per cent survival, the cows in the Tharparkar herd survive longer than in the other herds whereas those in the Kankrej herd survive for a shorter period.

3.2. Average yield characteristics of 1st lactation as well as of their survivors to different orders of lactation :

3.2.1. Average milk yield :

The average milk yield of cows in the 1st lactation as well as of their survivors to different orders of lactation are shown graphically in Figure - 3 for Tharparkar, Red Sindhi (Hosur) and Red Sindhi (Bangalore) and in Figure - 4 for Kankrej and Kangayam herds. It is seen from Figure - 3 that for Tharparkar herd there is an initial increase in milk yield upto the 8th lactation and thereafter there is a decreasing trend upto the 15th order of lactation . It is also seen from Figure - 3 that there is rapid increase in the milk yield of cows surviving to 9th and subsequent lactations upto 14th lactation for Red Sindhi (Bangalore) herd. In the case of Red Sindhi (Hosur), this increase is noticed only for 9th and 10th lactation. Further, it is observed from Figure - 4 that in the case of Kangayam, there is an initial increase in milk yield upto 9th lactation and thereafter both the increasing as well as decreasing trends are observed upto the 14th order of lactation. In Kankrej herd, however, an increasing trend is observed with some fluctuations upto 8th lactation, sudden fall at the 9th lactation and rapid increase at the 10th lactation. It is, therefore, noticed that barring few cases of irregular fluctuations (probably due to difference in management and feeding etc.), the first lactation average milk yield of survivors to different orders of lactation shows an increasing trend. This points out that cows with a higher initial milk yield tend to survive longer in the herd in the sense that such cows are retained deliberately for several generations.

3.2.2. Average lactation length :

The average first lactation length of cows and of their survivors are shown in Figure - 5 for Red Sindhi (Hosur) and Tharparkar herds whereas the same are shown in Figure -6 for Red Sindhi (Bangalore) and Kangayam herds. It is clear from Figure - 5 that average first lactation length in the case of Tharparkar herd increases continuously upto 8th lactation and thereafter it starts decreasing upto the 15th lactation. But in the case of Red Sindhi (Hosur), it increases upto the 10th lactation with a small depression at the 6th lactation. It is also observed from Figure -6 that for Red Sindhi (Bangalore), it increases continuously upto the 9th lactation and thereafter it fluctuates. In the case of Kangayam herd it first increases upto the 6th lactation and decreases thereafter upto the 10th lactation. No definite trend is observed subsequently.

3.2.3. Average milk yield per day of lactation :

The average first lactation milk yield per day of lactation of starter cows and of their survivors to different orders of lactation are shown graphically in Figure -7 for Tharparkar and Red Sindhi (Hosur) herds whereas the same are shown in Figure - 8 for Red Sindhi (Bangalore) and Kangayam herds. It is apparent from Figure -7 that for Tharparkar herd, average milk yield per day of lactation of cows surviving increases upto the 8th lactation and no definite trend is observed subsequently. For Red Sindhi (Hosur) it increases almost continuously upto the 10th lactation with some depression at the 3rd and 4th lactation. It is further observed from Figure-8 that for Red Sindhi (Bangalore) it increases upto the 14th order of lactation with a little depression at the 6th order of lactation. In the case of Kangayam it almost increases upto the 14th order of lactation.

It is interesting to see from the above discussion that while the proportion surviving decreases, the yield characteristics increases with the increase in the order of lactation in each of the five herds. This indicates some sort of relationship between the survival and yield of a cow. The cows which survive several culling processes might, therefore, possess higher milk yield and hence more productive from a commercial point of view. The longevity of a cow in terms of its useful life on the herd is thus dependent on its milk yield in the first lactation.

3.3. <u>Progeny test values (S) of sires based on dam-daughter comparison for 1st lactation yield as well as those based on only daughters first lactation yield and proportion surviving to different orders of lactation for all the five herds</u> :

In order to study the regression of proportion surviving on ($\bar{D}$ - $\bar{M}$) as well as on ($\bar{D}$ - A), five tables for five different herds were prepared. These are presented in Table - 4 to 8. In each of these tables, ($\bar{D}$ - $\bar{M}$) as well as ($\bar{D}$ - A) based on first lactation yield records in different sire groups and also the corresponding proportion surviving to different orders of lactations are given.

3.4. <u>Regression of proportion surviving on the progeny test</u> (b_{qS}):

The regression coefficient of the proportion surviving to a particular order of lactation on ($\bar{D}$ - $\bar{M}$) as well as on ($\bar{D}$ - A) based on 1st lactation yield records were calculated in each of the five herds. Taking the progeny test as an independent variate and proportion surviving to a given lactation as the dependent variate, regression coefficients were worked out. The values of the regression coefficients are shown in Table - 9 (using $\bar{D}$ - $\bar{M}$) and in Table - 10 (using $\bar{D}$ - A).

From Table - 9 it is seen that for Red Sindhi (Hosur) herd, the regression coefficients are negative at the 2nd, 5th and 7th lactations and positive at 3rd, 4th and 6th lactation. The values of the regression coefficients pass through a maximum at the third lactation. The negative regression coefficient indicates that cows were not probably culled on the basis of the milk yield. The maximum value of regression coefficient in the 3rd lactation viz. 17.88×10^{-5} means that an increase of 100 kg. in milk yield (at progeny test level) would result in an increase in the proportion surviving by about 0.02 approximately. For Red Sindhi (Bangalore) herd, it is observed that all the regression coefficients are negative, indicating that milk yield did not affect the proportion surviving. Further, it is observed that for Kangayam herd, the regression coefficients are positive at the 2nd and the 3rd lactation and after that it is negative. It is also observed that maximum value of regression coefficient occurs at the second lactation itself. This shows that cows were initially culled on the basis of milk yield and more so after the first lactation than after the second. For Tharparkar herd, it is positive at the 2nd, 3rd, 4th and 7th lactation and negative at 5th and 6th lactation. It is observed that it achieves a maximum values at the 3rd lactation. At this lactation increase in milk yield by 100 kgs. is expected to result in the increase in the proportion surviving by 0.04. For Kankrej herd, all the regression coefficients are negative except that at the 8th lactation.

From Table - 10, it is interesting to find that for all the herds except Kangayam, regression coefficients behave qualitatively almost in the same manner as those in Table - 9. In the case of Red Sindhi (Hosur) herd, it appears that culling on the basis of yield was done from 3rd lactation onwards till the 6th lactation. The trend in the values of regression coefficients indicated that, on an average, it increased and reached maximum at the 6th lactation. For the Red Sindhi (Bangalore) herd, however the regression coefficients were all negative. For Kangayam herd, all the regression coefficients were found to be positive. It decreased continuously upto 7th lactation with some fluctuations at the 3rd and 5th lactation. The maximum was observed at the 3rd lactation. For Tharparkar herd, it is observed that amongst the positive regression coefficients the maximum occured at the 3rd lactation. For an increase of milk yield by 100 kgs. the proportion surviving is expected to increase by 0.03. For the Kankrej herd, all the regression coefficients were found to be negative.

It is thus seen that on an average the milk yield in the first lactation, as judged by the progeny test values, either on the basis of ($\bar{D}$ - $\bar{M}$) or on the basis of ($\bar{D}$ - A), significantly affected the chances of survival of a cow in the first few orders of lactation. By and large the maximum survival was noticed at the 3rd lactation. Quite a large number of negative regression coefficients in later lactation indicated that there were factors other than the first lactation yield which affected the chances of surviving the culling process.

3.5. Relative survival coefficient (R):

The various regression coefficients obtained above are, however, not directly comparable since the proportion surviving also changes along with the regression coefficients with the increase in the order of lactation. As such if we take the ratio of the regression coefficient and the overall proportion surviving, we get a coefficient which is independent of units and can therefore serve better for comparison purposes. Such a coefficient is termed as "Relative Survival Coefficient (R)". The values of R have been presented in Table - 11 corresponding to the regression coefficients calculated by using ($\bar{D}$ - $\bar{M}$) as the progeny test values and in Table - 12 corresponding to the regression coefficients obtained by using ($\bar{D}$ - A) as the progeny test values.

From Table - 11 and 12, it is apparent that the relative survival coefficients behave almost in the same fashion as that of regression coefficients given in Table - 9 and Table - 10. The values of R are however, different from those of the b_{qS}. Such differences become very prominent in the later lactations due to a considerable decrease in the proportion surviving.

3.6. Heritability of survival :

The heritability of survival has been calculated by two methods; (a) by using angular transformation and (b) by using heterogeneity χ^2. Along with the heritability of survival to different orders of lactation by the two methods their standard errors have also been worked out. These are presented in Table -13 to Table - 17 respectively for the five herds. For the Red Sindhi (Hosur) herd (Table -13), it is seen that the heritability of survival calculated by both the methods increases upto the 5th order of lactation and thereafter it continuously decreases upto the 10th order of lactation. For a particular order of lactation, the values of heritabilities obtained by the two methods do not differ markedly. The standard errors of the estimates are, however, considerable in the first few lactation but become smaller subsequently. It is seen from Table -14 that for Red Sindhi (Bangalore) herd, no definite trend is observed. It is seen from Table -15 that for Kangayam herd, it first increases upto the 4th order of lactation and thereafter inadmissible values are obtained for the three consecutive lactations. From Table -16 it is observed that for Tharparkar herd, it increases upto the 8th order of lactation except for small fluctuations at the 3rd and the 7th order of lactation. It is noticed that, on an average, the estimates of heritability of survival in this herd are not that high as noticed in the other herds. Since survival is related to fitness, the heritability is not expected to be very high. As such probably some reliance can be placed on its estimates from this herd. From Table -17 it is observed that for Kankrej herd, the coefficient of heritability calculated by the angular transformation method decreases upto the 4th lactation and increases at the 5th lactation and thereafter it continuously decreases upto the 9th order of lactation. No definite trend is observed for the coefficients of heritability calculated by the heterogeneity Chi-square method. Inadmissible estimates are obtained at the 2nd and 4th order of lactations.

3.7. Genetic correlation between survival and milk yield in the first lactation :

Genetic correlation coefficients between survival to a given lactation and milk yield in first lactation have been worked out, using the procedure given in Section 2.9 for each of the five herds. These are presented in Table - 18. It is seen from Table - 18 that for Red Sindhi (Hosur), the genetic correlation coefficients are of the order of 0.3 to 0.4 except at the 10th lactation, it assumes a very high value of 0.8. Since the estimated values at 3rd, 4th and 5th lactation are inadmissible, no conclusion about the trend in the estimates can be drawn. So far as Red Sindhi (Bangalore) herd is concerned the genetic correlation coefficients estimated from the 3rd lactation to the 9th lactation, indicates values between 0.2 to 0.5. There is an almost decreasing trend in these estimates as the order of lactation increases. In the case of Kangayam herd, the estimates are lying between 0.2 to 0.6 and there is an again a decreasing trend for 2nd lactation onwards. but at the 6th lactation the estimate shoots upto 0.6. For the Tharparkar herd, the majority of estimates are inadmissible. In this herd while the estimate at the 4th lactation is of the order of 0.5, the estimate at the next lactation falls to as low as value 0.1. The estimate of genetic correlation coefficients in the case of Kankrej herd, are comparatively appear to be at a higher level, the range of the values being 0.4 to 0.9.

From the above discussion it appears that the genetic relationship between milk yield in the first lactation and the survival to 2nd lactation is of moderate magnitude, the coefficients being the order of 0.4. While no definite conclusions can be drawn about the relationship of milk yield in first lactation with the survival to 3rd and subsequent lactations, it appears that the values of the corresponding genetic correlation coefficients increase in magnitude.

4. SUMMARY

The data collected by the Institute of Agricultural Research Statistics from five organised Indian herds of dairy cattle spread over about 25 years were processed and analysed in an attempt to study the probability of survival of a cow upto a given lactation and its relationship with first lactation milk yield. An attempt has also been made to study the heritability of survival and its genetic correlation with milk yield in the first lactation.

2. The results pertaining to proportion of cows surviving upto a particular order of lactation, the average milk yield characteristics of the first lactation and of their survivors, the regression of proportion surviving on the progeny test values based on dam-daughter records, relative survival coefficient, heritability of survival and genetic correlation coefficient were obtained in each of the five herds of Indian cattle viz. Red Sindhi (Hosur), Red Sindhi (Bangalore), Kangayam, Tharparkar and Kankrej.

3. It was found that proportion of cows surviving to various orders of lactation continues to decrease with the increase in the order of lactation for all the five herds. Judged from 50 per cent survival, the cows in the Tharparkar herd survive longer than in the other herds whereas those in the Kankrej herd survive for a shorter period.

4. The average first lactation yield, average first lactation length and average milk yield per day of lactation of cows surviving to various orders of lactation showed almost an increasing trend with the increase

in the order of lactation in each of the five herds. This points out that cows with a higher initial milk yield tend to survive longer in the herd in the sense that such cows are retained deliberately for several generations.

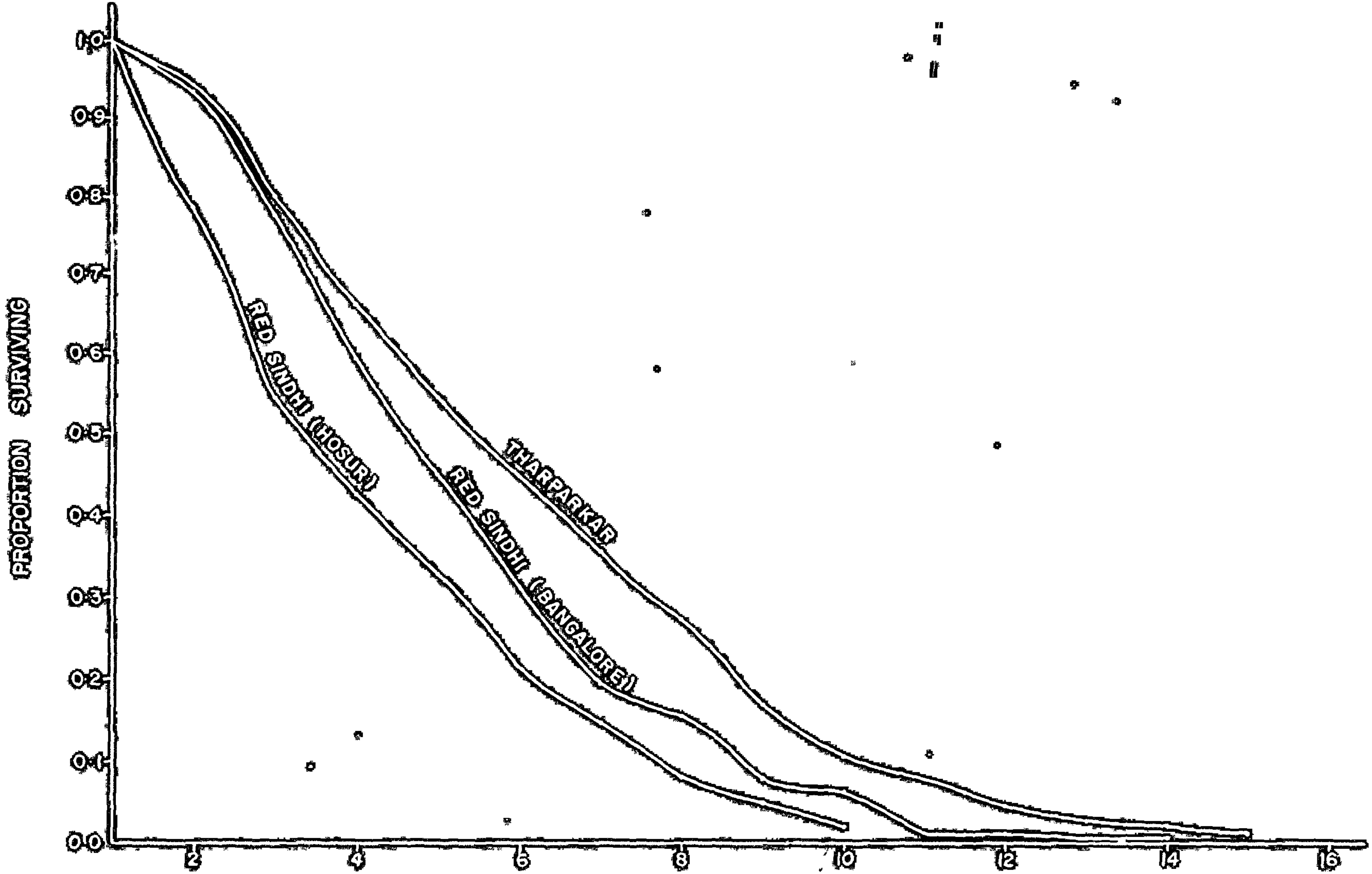
5. It was observed that on an average the milk yield in the first lactation, as judged by the progeny test values, based on dam-daughter records, significantly affected the chances of survival of a cow in the first few orders of lactation. By and large, the maximum survival was noticed at the 3rd lactation. Quite a large number of negative regression coefficients in later lactation indicated that there were factors other than the first lactation yield which affected the chances of surviving the culling process. The relative survival coefficients showed almost a similar trend as that in regression coefficients. Their values were however, different from those of regression coefficient. Such differences became very prominent in later lactations due to a considerable decrease in the proportion surviving.

6. The heritability of survival was calculated by two different methods; one by using angular transformation and the other by using heterogeneity Chi-square. It was observed that for a particular order of lactation, the values of heritability obtained by the two methods did not differ markedly. It was also noticed that, on an average, the estimates of heritability of survival in Tharparkar herd were not that high as noticed in the other herds.

7. The genetic correlation coefficient between milk yield in the

...tion and the survival to 2nd lactation was found to be of ... 0.4. The genetic relationship of the milk yield in the first ... with survival to 3rd and subsequent lactations was found to ... in magnitude.

FIGURES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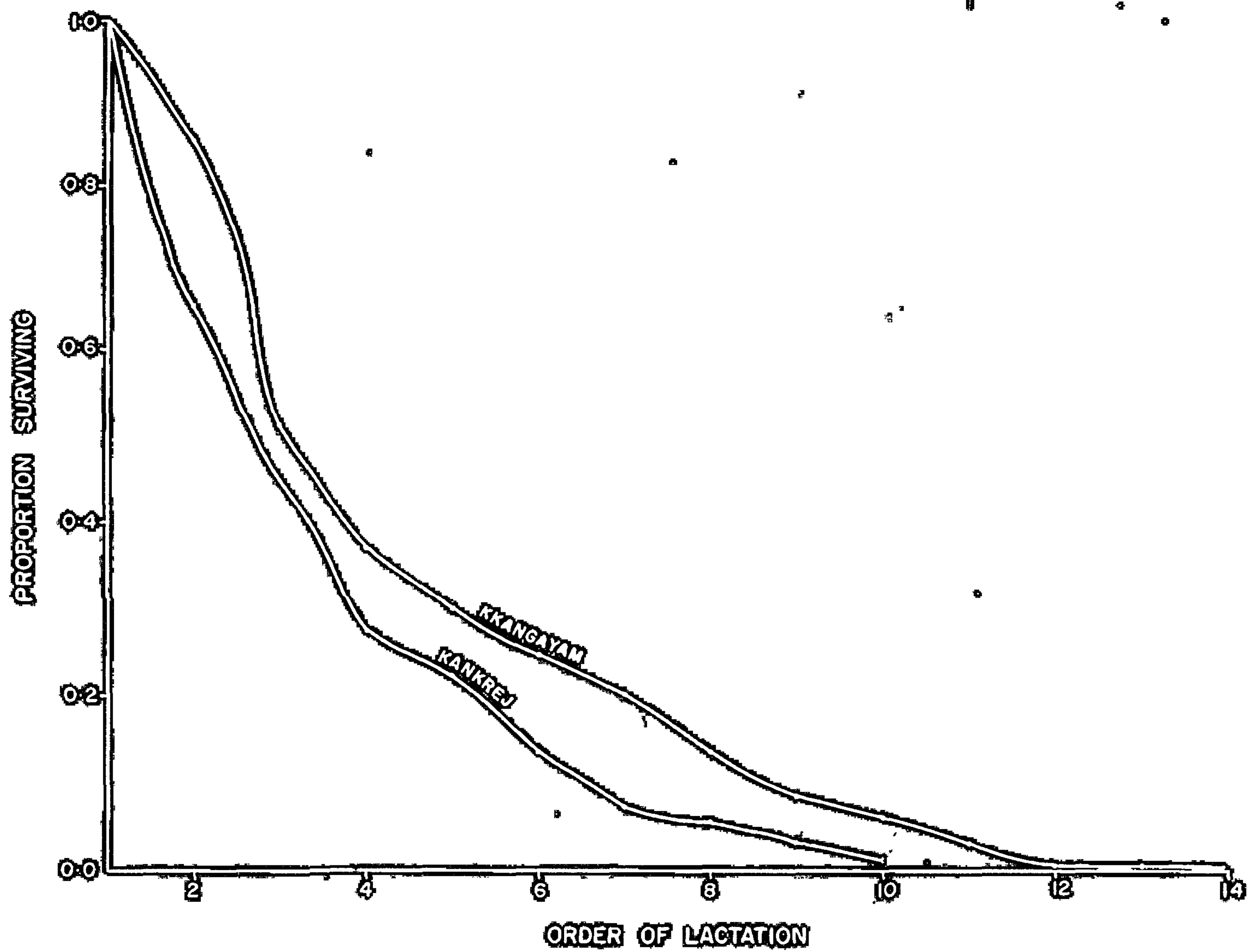
1·0
0·9
0·8
0·7
0·6
0·5
0·4
0·3
0·2
0·1
0·0
2
4
6
8
10
12
14
16
RED SINDHI (HOSUR)
RED SINDHI (BANGALORE)
THARPARKAR
ORDER OF LACTATION

(Fig. 1)



PROPORTION SURVIVING
1·0
0·8
0·6
0·4
0·2
0·0
KKANGAYAM
KANKREJ
2
4
6
8
10
12
14
ORDER OF LACTATION

Fig-2

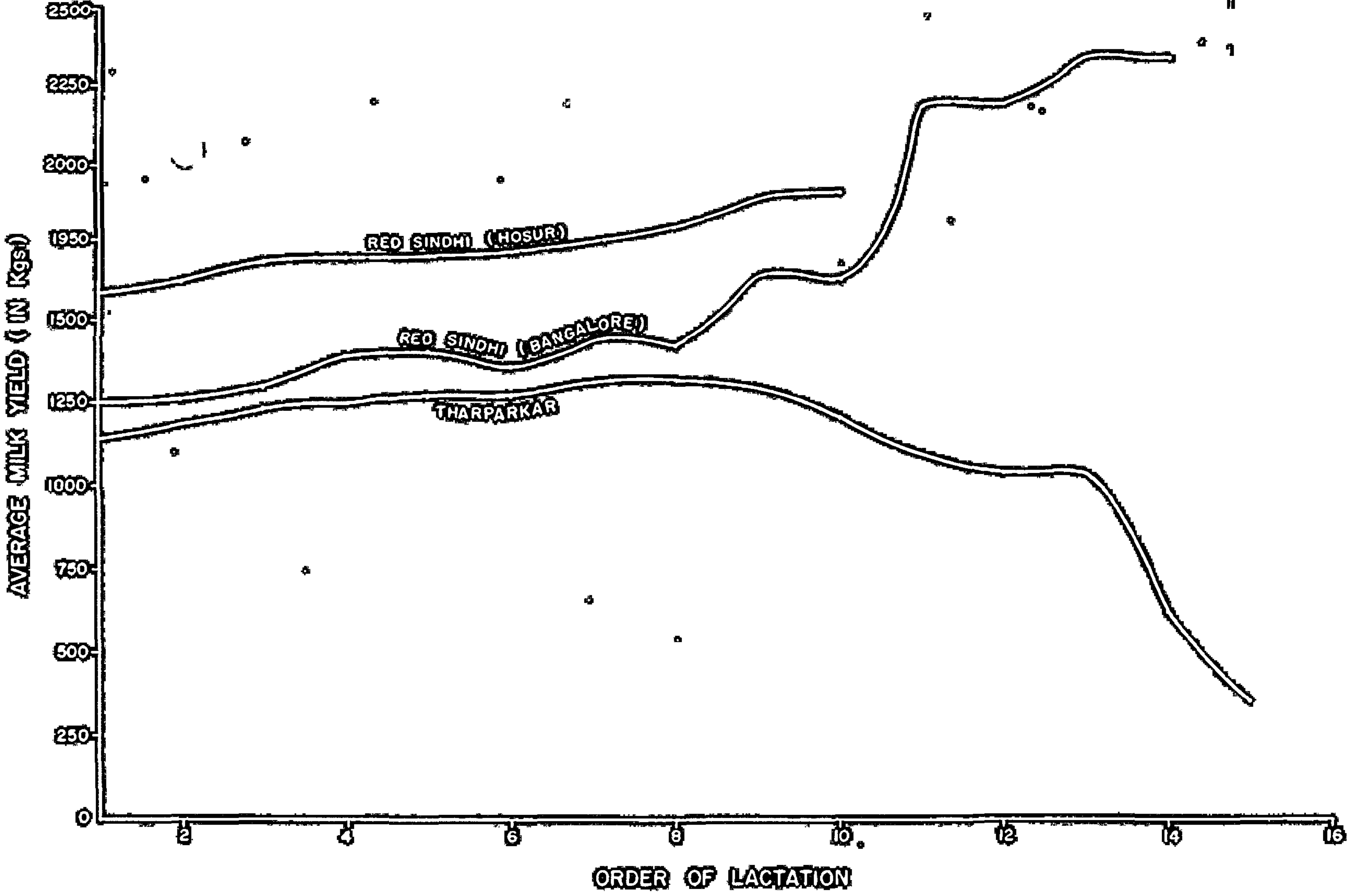
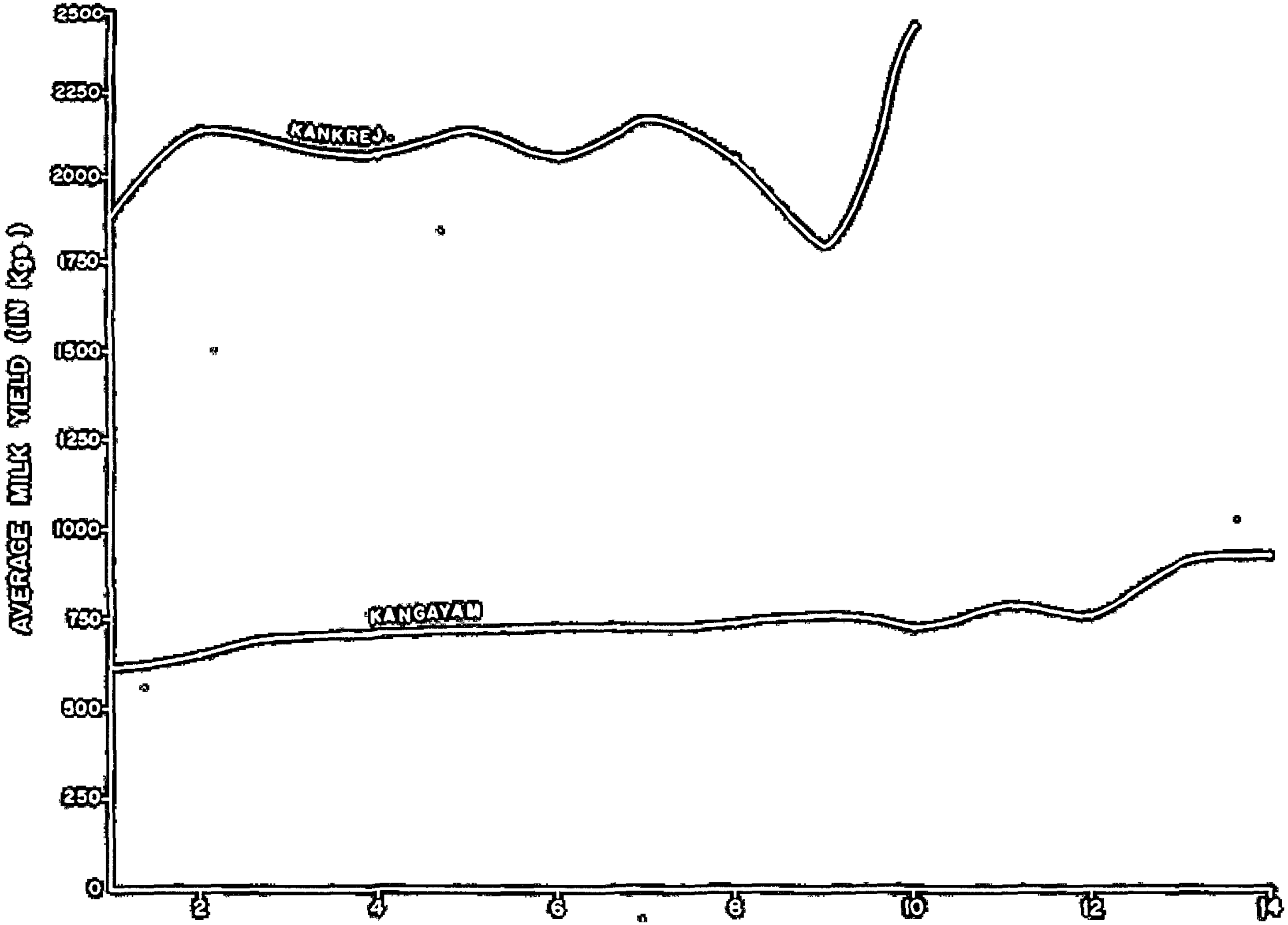
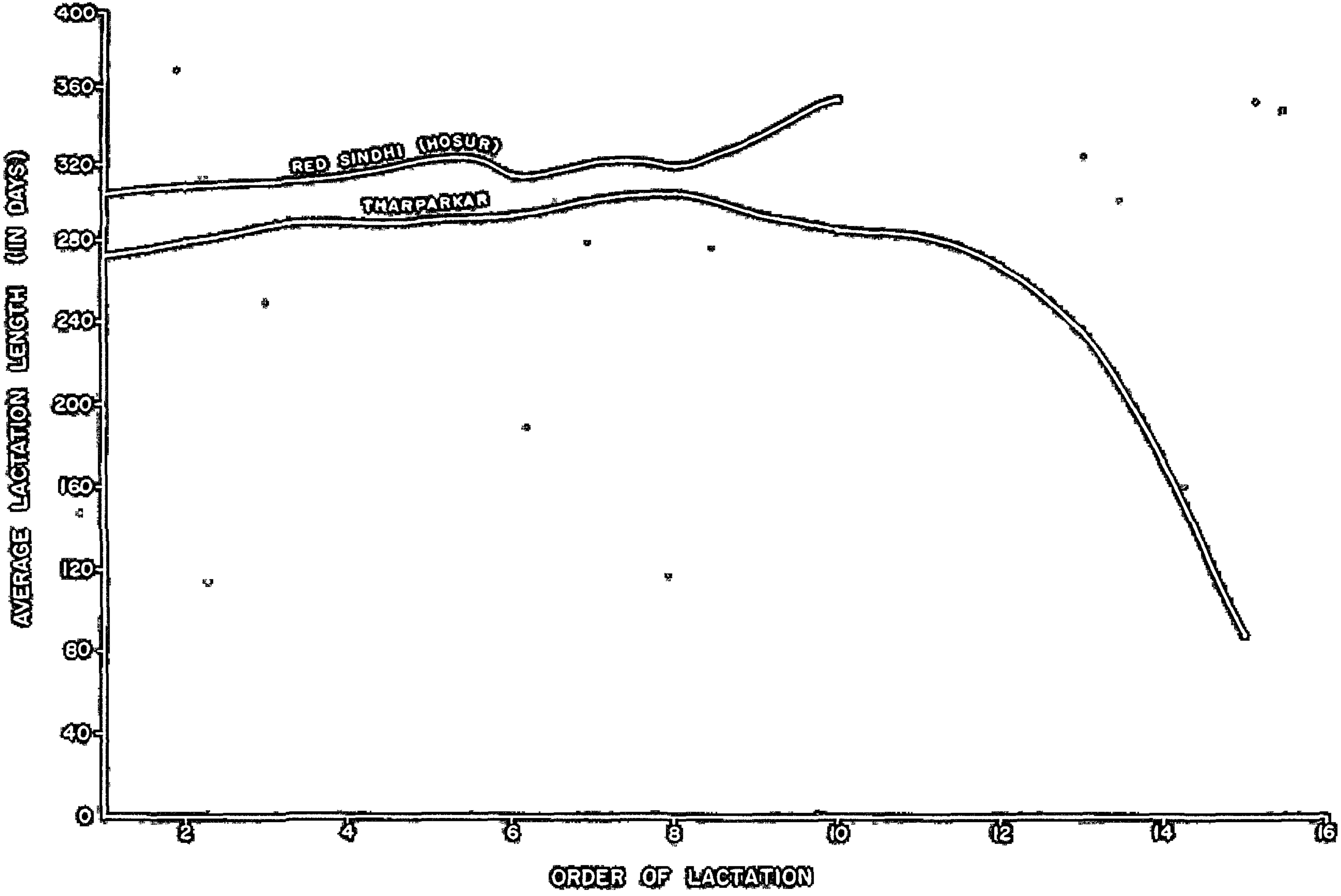

2500
2250
2000
1950
1500
1250
1000
750
500
250
0
AVERAGE MILK YIELD (IN Kgs)
RED SINDHI (HOSUR)
RED SINDHI (BANGALORE)
THARPARKAR
2
4
6
8
10
12
14
16
ORDER OF LACTATION

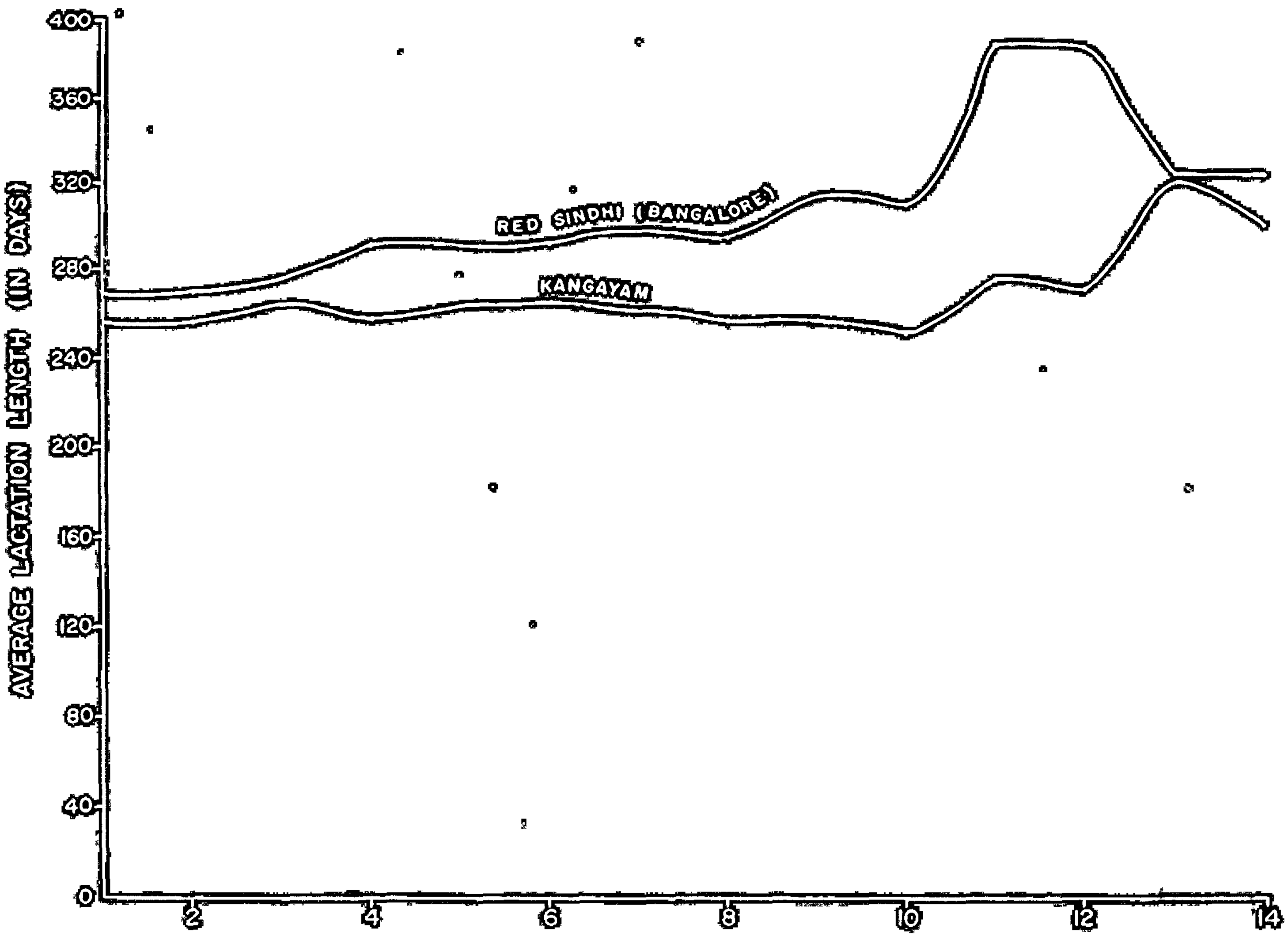
Fig-3


AVERAGE MILK YIELD (IN Kgs)
2500
2250
2000
1750
1500
1250
1000
750
500
250
0
KANKREJ.
KANGAYAM
2
4
6
8
10
12
14
ORDER OF LACTATION

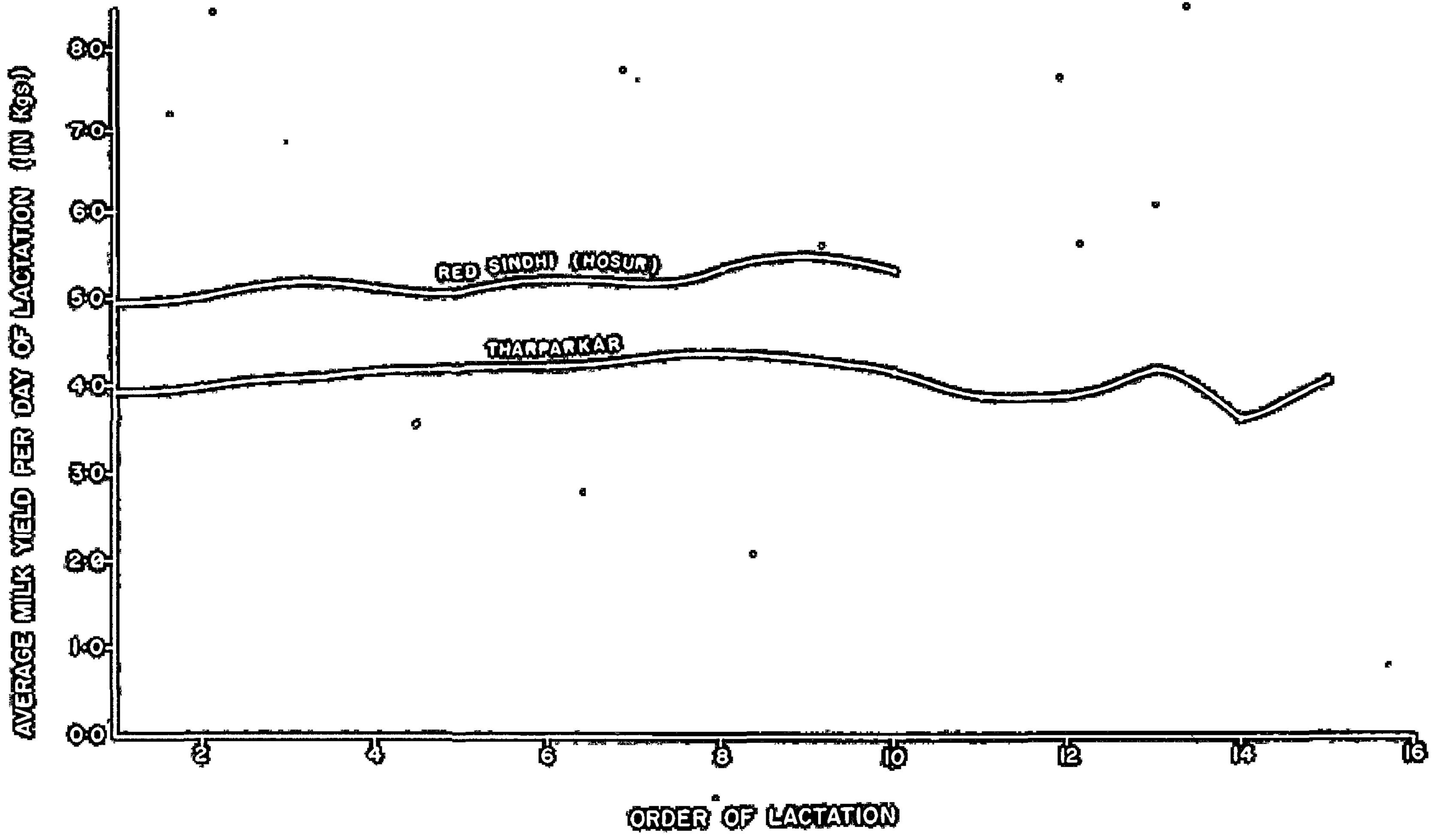
(Fig-4)


AVERAGE LACTATION LENGTH (IN DAYS)
0
40
80
120
160
200
240
280
320
360
400
RED SINDHI (HOSUR)
THARPARKAR
2
4
6
8
10
12
14
16
ORDER OF LACTATION

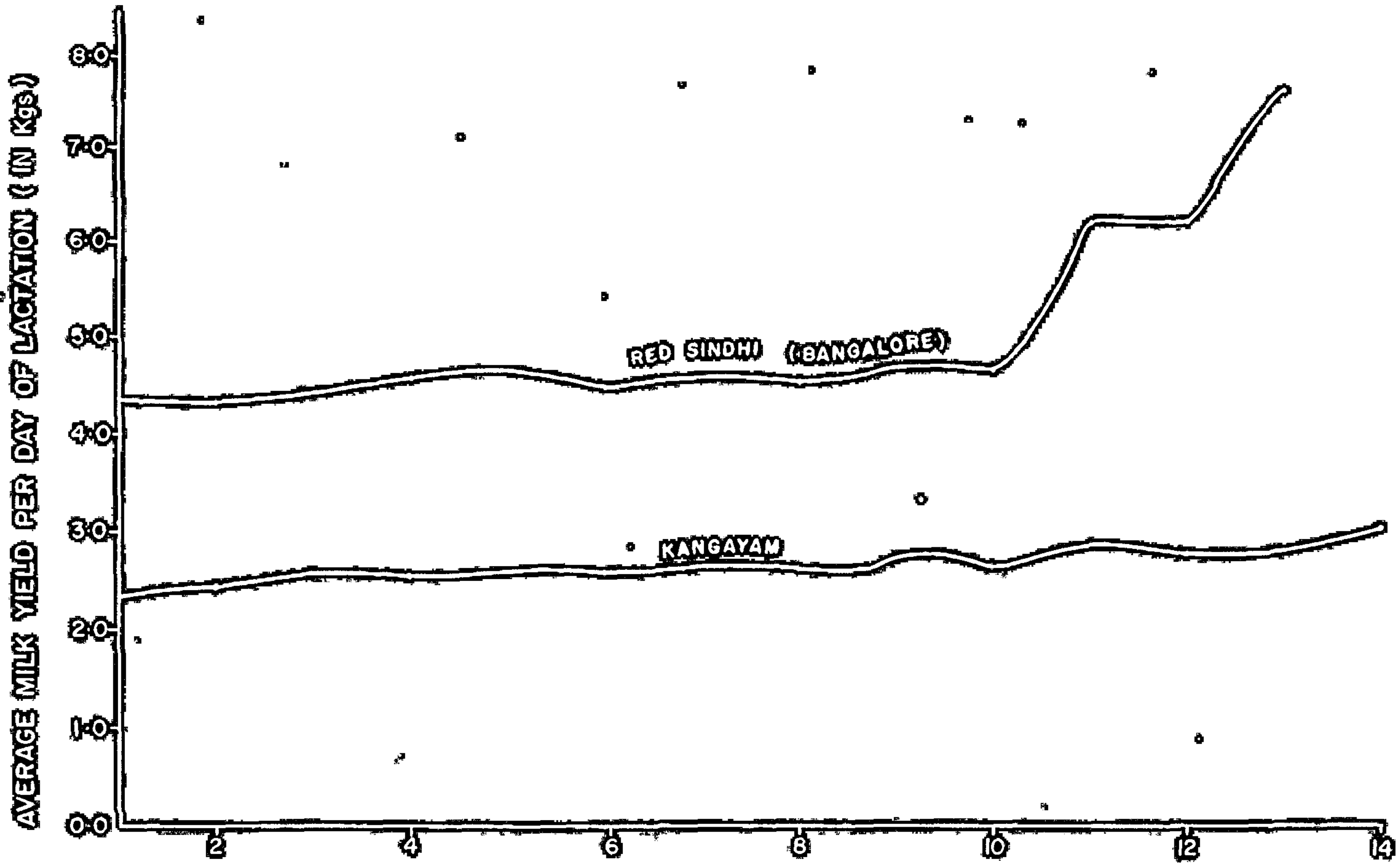
(Fig-5)


AVERAGE LACTATION LENGTH (IN DAYS)
0
40
80
120
160
200
240
280
320
360
400
RED SINDHI (BANGALORE)
KANGAYAM
2
4
6
8
10
12
14
ORDER OF LACTATION

(Fig.-6)


AVERAGE MILK YIELD PER DAY OF LACTATION (IN Kgs)
8·0
7·0
6·0
5·0
4·0
3·0
2·0
1·0
0·0
RED SINDHI (HOSUR)
THARPARKAR
2
4
6
8
10
12
14
16
ORDER OF LACTATION

(Fig.-7)


AVERAGE MILK YIELD PER DAY OF LACTATION (IN Kgs)
8·0
7·0
6·0
5·0
4·0
3·0
2·0
1·0
0·0
RED SINDHI (BANGALORE)
KANGAYAM
2
4
6
8
10
12
14
ORDER OF LACTATION

(Fig-8)

TABLES

TABLE - 1

Number of cows for which survival-yield data are available according to lactation

Order of lactation	Red Sindhi (Hosur)		Red Sindhi (Bangalore)		Kangayam		Tharparkar		Kankrej	
	Survival	Milk yield and lactation length	Survival	Milk yield and lactation length	Survival	Milk yield and lactation length	Survival	Milk yield and lactation length	Survival	Milk Yield
1	342	282	192	186	390	315	451	344	299	206
2	270	249	180	174	334	245	426	323	195	154
3	200	182	148	147	200	178	362	273	136	116
4	147	132	114	114	144	127	299	218	83	80
5	113	92	87	87	117	100	247	179	61	51
6	72	64	60	60	97	85	202	144	41	35
7	50	44	37	37	75	67	161	114	25	21
8	28	25	25	25	53	47	123	88	17	13
9	17	15	15	15	34	29	76	53	10	8
10	7	7	12	12	20	19	47	31	4	3
11	-	-	2	2	12	12	35	23	-	-
12	-	-	2	2	2	2	21	16	-	-
13	-	-	1	1	2	2	12	10	-	-
14	-	-	1	1	1	1	9	5	-	-
15	-	-	-	-	-	-	6	2	-	-

TABLE - 2

Number of sires, daughters and dam-daughter pairs for which survival yield data are available in the first lactation according to different herds.

Sl.No.	Name of the Herd	Survival		Milk Yield (dam-daughter basis)	
		No. of sires	No. of cows	No. of sires	No. of pairs
1	Red Sindhi (Hosur)	20	293	13	163
2	Red Sindhi (Bangalore)	16	178	13	110
3	Kangayam	21	370	13	127
4	Tharparkar	19	421	14	168
5	Kankrej	11	283	6	48

TABLE - 3

ANALYSIS OF VARIANCE

Source	d.f.	Sum of Squares	Mean sum of Squares	E (MSS)
Between Sires	$(s-1)$	$B = \sum_{i=1}^{s} \frac{[a_i\Theta_i + (n_i - a_i)\Theta_i']^2}{n_i} - \frac{[\sum_{i=1}^{s}(a_i\Theta_i + (n_i - a_i)\Theta_i')]^2}{\sum_{i=1}^{s} n_i}$	$\frac{B}{(s-1)} = B'$	$\sigma_o^2 + k\sigma_s^2$
Within Sires	$\sum_{i=1}^{s} n_i - s$	$W = \sum_{i=1}^{s}[a_i\Theta_i^2 + (n_i - a_i)\Theta_i'^2] - \sum_{i=1}^{s} \frac{[a_i\Theta_i + (n_i - a_i)\Theta_i']^2}{n_i}$	$\frac{W}{\sum_{i=1}^{s} n_i - s} = W'$	σ_o^2
Total	$\sum_{i=1}^{s} n_i - 1$	$\sum_{i=1}^{s}[a_i\Theta_i^2 + (n_i - a_i)\Theta_i'^2] - \frac{[\sum_{i=1}^{s}(a_i\Theta_i + (n_i - a_i)\Theta_i')]^2}{\sum_{i=1}^{s} n_i}$		

TABLE - 4

Progeny test values (S) of sires based on dam-daughter comparison for first lactation yield as well as those based on only daughters first lactation yield and proportion surviving to different orders of lactation for Red Sindhi (Hosur) herd.

Sire No.	($\bar{D}$ - $\bar{M}$) in kgs.	($\bar{D}$ - A) in kgs.	Proportion Surviving to different orders of lactation						
			1	2	3	4	5	6	7
2	333.92	157.13	1.000	0.863	0.681	0.545	0.454	0.409	0.318
8	46.83	- 84.22	1.000	0.941	0.882	0.588	0.588	0.470	0.294
14	319.26	400.56	1.000	0.857	0.714	0.571	0.571	0.428	-
15	- 236.61	- 210.58	1.000	0.818	0.636	0.545	0.454	0.181	0.181
21	- 158.11	- 78.10	1.000	1.000	0.857	0.857	0.714	0.571	0.428
25	261.72	- 105.66	1.000	0.777	0.666	0.111	0.111	-	-
32	- 257.06	157.85	1.000	0.909	0.818	0.636	0.545	0.090	-
34	- 41.11	- 152.59	1.000	0.842	0.736	0.631	0.315	0.105	0.052
39	507.72	445.78	1.000	0.933	0.733	0.466	-	-	-
40	- 79.96	252.90	1.000	0.857	0.714	0.142	0.142	-	-
41	- 307.66	- 224.71	1.000	0.909	0.363	0.045	-	-	-
43	- 108.86	- 377.89	1.000	1.000	1.000	0.750	0.250	-	-
44	- 200.56	- 154.54	1.000	0.833	0.166	0.083	-	-	-
	Overall		1.000 (163)	0.883	0.656	0.435	0.300	0.177	0.110

Figure in bracket shows the total number of daughters in a herd.

TABLE - 5

Progeny test values (S) of sires based on dam-daughter comparison for first lactation yield as well as those based on only daughters first lactation yield and proportion surviving to different orders of lactation for Red Sindhi (Bangalore) herd.

Sire No.	(D - M) in kgs.	($\bar{D}$ - A) in kgs.	Proportion surviving to different orders of lactation						
			1	2	3	4	5	6	7
3	- 496.09	- 287.22	1.000	1.000	1.000	0.846	0.692	0.384	0.153
5	- 74.45	231.12	1.000	0.933	0.866	0.800	0.666	0.466	0.200
7	- 810.69	- 194.60	1.000	1.000	1.000	0.714	0.714	0.571	0.428
9	999.71	341.40	1.000	1.000	0.750	0.750	0.500	0.250	-
11	- 161.73	- 184.58	1.000	0.888	0.666	0.444	0.444	0.333	0.222
15	274.68	358.33	1.000	1.000	0.714	0.571	0.428	0.285	-
19	- 887.81	- 425.93	1.000	0.812	0.375	0.187	0.125	-	-
20	- 58.40	634.98	1.000	0.750	0.750	0.500	0.500	0.500	0.250
21	- 373.57	- 323.86	1.000	0.800	0.800	0.800	0.400	0.400	-
22	- 190.14	412.77	1.000	0.800	0.800	0.400	0.200	-	-
23	89.94	- 81.69	1.000	1.000	0.900	0.500	0.500	0.100	-
24	- 860.27	- 85.45	1.000	1.000	0.800	0.400	-	-	-
25	1094.65	449.10	1.000	0.800	0.300	0.100	-	-	-
	Overall		1.000 (110)	0.909	0.727	0.527	0.409	0.245	0.100

Figure in bracket shows the total number of daughters in a herd.

TABLE - 6

Progeny test values (S) of sires based on dam-daughter comparison for first lactation yield as well as those based on only daughters first lactation yield and proportion surviving to different orders of lactation for Kangayam herd.

Sire No.	($\bar{D}$ - $\bar{M}$) in kgs.	($\bar{D}$ - A) in kgs.	Proportion surviving to different orders of lactation						
			1	2	3	4	5	6	7
8	- 1777.96	-1236.89	1.000	0.750	0.750	0.750	0.500	0.500	0.500
10	726.97	452.48	1.000	0.823	0.764	0.647	0.411	0.411	0.352
14	- 395.20	601.39	1.000	0.857	0.714	0.571	0.428	0.428	0.428
15	- 572.78	- 329.43	1.000	0.888	0.777	0.555	0.555	0.555	0.555
17	249.73	295.77	1.000	0.888	0.666	0.555	0.444	0.444	0.444
18	92.08	211.68	1.000	0.833	0.777	0.666	0.555	0.333	0.166
22	- 567.21	406.46	1.000	0.750	0.750	0.500	0.500	0.500	0.500
23	- 570.90	- 189.84	1.000	1.000	0.875	0.500	0.250	0.250	0.250
24	865.79	781.12	1.000	1.000	0.750	0.250	0.250	-	-
25	179.85	- 365.04	1.000	0.875	0.375	0.125	-	-	-
26	- 155.77	- 75.18	1.000	0.642	0.357	0.035	0.035	-	-
29	- 472.91	- 798.93	1.000	0.600	0.200	0.200	-	-	-
30	- 1081.89	- 778.73	1.000	0.166	-	-	-	-	-
	Overall		1.000 (127)	0.771	0.590	0.393	0.291	0.244	0.212

Figure in bracket shows the total number of daughters in a herd.

TABLE - 7

Progeny test values(S) of sires based on dam-daughter comparison for first lactation yield as well as those based on only daughters first lactation yield and proportion surviving to different orders of lactation for Tharparkar herd.

Sire No.	($\bar{D}$ - $\bar{M}$) in kgs.	($\bar{D}$ - A) in kgs.	Proportion surviving to different orders of lactation						
			1	2	3	4	5	6	7
1	- 486.24	- 414.12	1.000	1.000	0.818	0.727	0.727	0.727	0.454
4	- 475.81	- 118.93	1.000	1.000	0.884	0.692	0.692	0.576	0.576
5	- 51.89	- 8.43	1.000	1.000	0.888	0.777	0.500	0.277	0.222
8	- 129.23	72.98	1.000	1.000	0.809	0.714	0.619	0.428	0.333
11	- 142.33	- 17.55	1.000	1.000	0.900	0.700	0.600	0.500	0.400
12	- 459.98	128.82	1.000	1.000	0.666	0.666	0.666	0.166	0.166
13	- 48.62	253.74	1.000	1.000	0.933	0.733	0.400	0.133	0.133
14	- 777.22	- 402.56	1.000	0.888	0.666	0.555	0.333	0.333	0.333
16	- 266.75	204.61	1.000	0.923	0.923	0.923	0.692	0.692	0.461
17	- 125.32	62.46	1.000	1.000	0.500	0.500	0.250	0.250	0.250
18	-1081.63	- 625.95	1.000	1.000	0.333	0.333	0.333	0.333	-
19	- 2.76	110.13	1.000	1.000	0.833	0.583	0.416	0.333	0.250
20	- 77.74	310.75	1.000	1.000	0.714	0.357	0.214	0.142	0.071
22	- 308.07	- 202.84	1.000	1.000	1.000	0.666	0.500	0.166	0.166
	Overall		1.000 (168)	0.976	0.886	0.672	0.529	0.392	0.315

Figure in bracket shows the total number of daughters in a herd.

TABLE - 8

Progeny test values(S) of sires based on dam-daughter comparison for first lactation yield as well as those based on only daughters first lactation yield and proportion surviving to different orders of lactation for Kankrej herd.

Sire No.	$(\bar{D} - \bar{M})$ in kgs.	$(\bar{D} - A)$ in kgs	Proportion surviving to different orders of lactation						
			1	2	3	4	5	6	7
4	- 547.93	140.22	1.000	0.750	0.250	0.250	-	-	-
6	- 475.05	- 115.09	1.000	0.916	0.583	0.333	0.291	0.208	0.083
7	- 107.95	- 24.36	1.000	0.857	0.571	0.142	-	-	-
19	-1666.37	- 937.39	1.000	1.000	1.000	0.750	0.250	0.250	0.250
25	1780.52	1019.92	1.000	0.600	0.200	0.200	-	-	-
28	871.34	255.44	1.000	0.750	0.750	0.750	0.750	0.250	-
	Overall		1.000 (48)	0.854	0.562	0.354	0.229	0.145	0.062

Figure in bracket shows the total number of daughters in a herd.

TABLE - 9

Regression coefficients ($x 10^{-5}$) of proportion surviving to a given lactation on progeny test ($\bar{D} - \bar{M}$) for five different herds.

Name of the herd	Order of lactation					
	2	3	4	5	6	7
Red Sindhi (Hosur)	- 3.76	17.88	5.16	- 5.55	14.33	- 0.05
Red Sindhi (Bangalore)	- 0.79	-11.60	-4.17	- 2.74	- 4.52	- 7.31
Kangayam	14.21	7.31	-3.30	- 1.28	- 7.24	- 9.17
Tharparkar	3.74	34.53	18.42	-0.64	- 11.62	3.81
Kankrej	-10.56	-14.51	-6.79	1.79	- 3.87	- 6.18

TABLE - 10

Regression coefficients ($x 10^{-5}$) of proportion surviving to a given lactation on progeny test ($\bar{D}$ - A) for five different herds

Name of the herd	Order of lactation					
	2	3	4	5	6	7
Red Sindhi (Hosur)	-2.44	8.78	0.11	7.49	13.69	-7.26
Red Sindhi (Bangalore)	-6.92	- 9.74	-10.86	-6.85	-1.46	-3.02
Kangayam	18.43	21.70	7.82	12.33	6.05	4.25
Tharparkar	2.07	27.30	16.37	-1.94	-20.04	-1.80
Kankrej	-21.52	-38.88	-22.57	-7.46	-11.72	-13.14

TABLE - 11

Relative survival coefficients (x 10^{-5}) = $b_{qS}/\bar{q}$ corresponding to a progeny test ($\bar{D} - \bar{M}$) to a given lactation for five different herds.

Name of the herd	Order of lactation					
	2	3	4	5	6	7
Red Sindhi (Hosur)	- 4.25	27.25	11.86	-18.50	80.96	- 0.45
Red Sindhi (Bangalore)	- 0.86	- 15.95	- 7.91	- 6.69	- 18.44	-73.10
Kangayam	18.43	12.38	- 8.39	- 4.39	- 29.67	-43.25
Tharparkar	3.83	38.97	27.41	- 1.20	- 29.64	12.09
Kankrej	- 12.36	-25.81	-19.18	7.81	- 26.68	-99.67

TABLE - 12

Relative survival coefficients (x 10^{-5}) = $b_{qS}/\bar{q}$ corresponding to a progeny test ($\bar{D}$ - A) to a given lactation for five different herds.

Name of the herd	Order of lactation					
	2	3	4	5	6	7
Red Sindhi (Hosur)	- 2.76	13.38	2.82	24.96	77.34	-66.00
Red Sindhi (Bangalore)	- 7.61	- 13.39	- 20.60	- 16.74	- 5.95	-30.20
Kangayam	23.90	36.76	19.89	42.37	24.79	20.04
Tharparkar	2.12	30.81	24.36	- 3.66	-51.12	- 5.71
Kankrej	-35.19	-69.18	- 63.75	-32.57	-80.82	-211.93

TABLE - 13

Heritabilities of survival to a given lactation along with their respective standard errors for Red Sindhi (Hosur) herd.

Order of lactation	Angular Transformation		Hetrogeneity χ^2	
	h_q^2	S. E. (h_q^2)	h_q^2	S. E. (h_q^2)
2	0.5150	0.2075	0.5661	0.2128
3	0.5626	0.2167	0.6024	0.2197
4	0.5916	0.2221	0.6240	0.2235
5	0.8977	0.2701	0.9010	0.2666
6	0.8390	0.2622	0.8200	0.2555
7	0.5814	0.2202	0.5438	0.2085
8	0.3341	0.1680	0.3010	0.1557
9	0.4287	0.1898	0.4520	0.1898
10	0.1137	0.1138	0.0607	0.0935

TABLE - 14

Heritabilities of survival to a given lactation along with their respective standard error for Red Sindhi (Bangalore) herd.

Order of lactation	Angular Transformation		Hetrogeneity χ^2	
	h_q^2	S. E. (h_q^2)	h_q^2	S. E. (h_q^2)
2	0.2457	0.1939	0.0971	0.1525
3	0.5698	0.2691	0.5220	0.2596
4	0.5507	0.2653	0.5570	0.2669
5	0.4436	0.2423	0.4490	0.2436
6	0.5713	0.2694	0.6310	0.2812
7	0.6420	0.2830	0.7280	0.2982
8	0.3502	0.2204	0.0293	0.1321
9	0.3250	0.2142	0.3190	0.2126

TABLE - 15

Heritabilities of survival to a given lactation along with their respective standard errors for Kangayam herd.

Order of lactation	Angular Transformation		Hetrogeneity χ^2	
	h_q^2	S. E. (h_q^2)	h_q^2	S. E. (h_q^2)
2	0.3605	0.1593	0.4450	0.1776
3	0.6856	0.2229	0.9450	0.2607
4	0.9460	0.2600	0.9630	0.2629
5	1.0323*	0.2707	1.0301*	0.2701
6	1.1261*	0.2800	1.1310*	0.2804
7	1.1744*	0.2841	1.1640*	0.2833
8	0.8446	0.2475	0.8100	0.2426
9	0.7361	0.2286	0.6420	0.2155
10	0.7503	0.2335	0.6310	0.2136

* Indicate inadmissible values.

TABLE - 16

Heritabilities of survival to a given lactation along with their respective standard errors for Tharparkar herd.

Order of lactation	Angular Transformation		Hetrogeneity χ^2	
	h_q^2	S. E. (h_q^2)	h_q^2	S. E. (h_q^2)
2	0.1078	0.0882	0.0426	0.0687
3	0.0167	0.0610	0.0125	0.0596
4	0.1475	0.0995	0.1609	0.1032
5	0.1693	0.1057	0.1876	0.1107
6	0.2171	0.1187	0.3068	0.1421
7	0.1592	0.1028	0.2401	0.1249
8	0.2423	0.1254	0.3084	0.1425

TABLE - 17

Heritabilities of survival to a given lactation along with their respective standard errors for Kankrej herd.

Order of lactation	Angular Transformation		Hetrogeneity χ^2	
	h^2_q	S.E. (h^2_q)	h^2_q	S.E. (h^2_q)
2	1.1354*	0.3804	1.1920*	0.3884
3	0.9301	0.3472	0.9850	0.3575
4	0.5863	0.2681	1.1570*	0.3838
5	0.7176	0.3018	0.7780	0.3165
6	0.6336	0.2808	0.3435	0.1943
7	0.5295	0.2522	0.5330	0.2536
8	0.2439	0.1600	0.2360	0.1573
9	0.4592	0.2314	0.3520	0.1975

* Indicate inadmissible values.

TABLE - 18

Genetic correlation coefficients between survival to a given lactation and milk yield in the first lactation for five different herds.

Herd	Order of lactation								
	2	3	4	5	6	7	8	9	10
Red Sindhi (Hosur)	0.339	*	*	*	0.386	0.378	0.363	0.441	0.855
Red Sindhi (Bangalore)	*	0.520	0.484	0.382	0.266	0.181	0.224	0.243	-
Kangayam	0.463	0.349	0.238	0.293	0.652	*	0.184	*	*
Tharparkar	*	*	0.506	0.134	*	*	*	-	-
Kankrej	0.381	0.509	0.487	0.722	*	0.948	*	0.877	-

* Indicate inadmissible values.

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