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A Note on Semi-Latin Squares

Rajender Parsad
Indian Agricultural Statistics Research Institute, New Delhi
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SUMMARY

A new method of construction of $(n \times n)/2$ semi-Latin squares based on initial column solution has been developed. This method yields semi-regular group divisible designs after ignoring the row and column classifications. Preece and Freeman (1983) reported that for $k = 2, n = 6,8,10$ could not be obtained by rearrangement in semi-regular group divisible designs. These three semi-Latin squares can be obtained from the proposed method of construction.

Key words : Semi-Latin squares, Semi regular group divisible design.

1. INTRODUCTION

In many agricultural experiments, there are two sources of heterogeneity in experimental units that may influence the response variable. To deal with such experimental situations, row-column designs such as Latin square designs (LSD), Youden square designs (YSD), generalized Youden designs (GYD), Youden type designs, etc. have been developed in the literature. For details on these designs, a reference may be made to Shah and Sinha (1996).

Most of the row-column designs developed have only one unit in each row-column intersection. There, however, do occur situations, where there are more than one units in each row-column intersection. For such experimental situations, semi-Latin square designs were introduced. These are defined in the sequel.

Definition 1.1: A semi-Latin square is an arrangement of $v = nk$ symbols in n^2k units arranged in n rows and n columns, each row-column intersection contains $k > 1$ units and each symbol occurs exactly once in each row and each column.

A semi-Latin square with above parameters is represented by $(n \times n)/k$. A design based on semi-Latin square is called a semi-Latin square design. In fact semi-Latin squares are also a type of designs with nested structures. Here, the row-column intersection is called the blocks and the treatments are orthogonal to the rows and columns strata in semi-Latin squares. Therefore, it is important that the block design at the bottom stratum (ignoring row-column) classifications has some optimal properties.

Semi-Latin squares are useful for situations when the number of levels of both the nuisance factors (factors of heterogeneity) is same. A large number of such experimental situations in consumer testing, glass house crops, residual effect experiments, sugar beet trials, food industry, etc. have been given by Bailey (1992) and Bailey and Monod (2001). Some of these experimental situations are described in the sequel.

Example 1: (Organoleptic Evaluation of Food Products). Consider a food sensory experiment where 12 food items are to be compared. The experiment will be conducted in 4 sessions. There are 4 panelists and each of them will taste 3 food items at each session. In this case a semi-Latin square with 4 rows, 4 columns with each row-column intersection having size 3 can be used. Here the rows, columns and symbols represent sessions, panelists and food items respectively. This ensures that each of the 12 food items are organoleptically evaluated by each panelists, each food item is evaluated in each of the sessions and food items are orthogonal with respect to panelists and sessions.

Example 2: (Residual Effects: Bailey 1992). The effects of some treatments may persist during the next experiment. This is particularly true if the experimental units are long lived such as trees, animals, but it may also occur on arable crops if the treatments affect the soil directly, for example by inhibiting or encouraging nematode growth. Suppose that in last year 5 varieties of potato were compared in 5 replicates. This year, a single standard variety is grown and ten chemicals are tested for their ability to control nematodes. Last year's varieties will affect the number of nematodes in the soil, but it is assumed that there is no interaction between those varieties and this year's chemicals.

Chemicals can be applied to smaller areas of land than varieties, so each plot from last year is split into two for the application of chemicals. Use of a semi-Latin square, with rows representing old replicates, columns varieties and symbols chemicals, ensures that each chemical occurs once on soil that had each variety last year.

The semi-Latin squares can be useful for crop sequence experiments, wherein treatments applied on the first season crop has a residual effect on the soil and in turn affect the effects of the treatments applied in the subsequent season.

A semi-Latin square may also be used for obtaining fractional factorial plans with three factors say R (rows), C (columns) and S (symbols) with n, n and nk levels respectively. Then a one to one correspondence between a semi-Latin square and a fractional replication plan gives a fractional factorial plan for a $n \times n \times nk$ factorial in n^2k runs.

In the present investigation, we have given a new method of construction of semi-Latin squares.

2. METHOD OF CONSTRUCTION

Preece and Freeman (1983), Bailey (1988, 1992), Bedford and Whitaker (2001) have given several methods of construction of semi-Latin squares. Preece and Freeman (1983) have given a relationship between double resolvability of semi-regular group divisible designs and semi-Latin squares. They have also given a catalogue of semi regular group divisible designs given in Clatworthy (1973) that can be converted into semi-Latin squares. They have indicated that by rearrangement of treatments in a semi regular group divisible design, no semi-Latin square can be obtained for $k = 2, n = 6,8,10$ and $k = 3, n = 6,9,10$. In the present investigation, we have given a method of construction of semi-Latin square designs whose bottom stratum is a semi regular group divisible design and the semi-Latin square for $k = 2$, $n = 6,8,10$ can also be obtained through this method of construction.

Method 2.1: A $(n \times n)/2$ semi-Latin square with $v = 2n$, $R = n$, $C = n$, $k = 2$ can be constructed by developing the initial column

$$\left\{\begin{bmatrix} (1,2n) \\ (2,2n-1) \\ \vdots \\ (n,n+1) \end{bmatrix} \quad \begin{matrix} \text{mod } (2n) \\ \text{with steps as } 2 \end{matrix}\right\}$$

The bottom stratum design is a doubly resolvable semi-regular group divisible design with parameters $v = 2n, b = n^2, r = n, k = 2, \lambda_1 = 0, \lambda_2 = 1, m = 2, n = n$ {symbols have their usual meaning as in Clatworthy (1973)}.

Example 2.1: For $v = 12$ i.e., $n = 6$ we get a $(6\times6)/2$ semi-Latin square as

$$\begin{bmatrix} (1,12) & (3,2) & (5,4) & (7,6) & (9,8) & (11,10) \\ (2,11) & (4,1) & (6,3) & (8,5) & (10,7) & (12,9) \\ (3,10) & (5,12) & (7,2) & (9,4) & (11,6) & (1,8) \\ (4,9) & (6,11) & (8,1) & (10,3) & (12,5) & (2,7) \\ (5,8) & (7,10) & (9,12) & (11,2) & (1,4) & (3,6) \\ (6,7) & (8,9) & (10,11) & (12,1) & (2,3) & (4,5) \end{bmatrix}$$

The method of construction is very general in nature and gives doubly resolvable semi-regular group divisible designs and semi-Latin squares. Three new semi-Latin squares with $k = 2, n = 6$; $k = 2, n = 8$ and $k = 2, n = 10$ which cannot be constructed by rearrangement of the designs given in Clatworthy (1973) can be obtained. Further, the semi-Latin squares with $k = 2$ and $n > 10$ can also be obtained.

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