Trend Free Block Designs in Three Plots Per Block

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ABSTRACT

In this article, we have discussed some aspects of block designs incorporating systematic trend component. The necessary and sufficient condition for a block design to be trend free has been highlighted. A method of constructing of linear trend free block designs in three plots per block has been discussed. The characterization properties of such designs have been investigated. The performances of developed designs have also been evaluated under different correlation structure based on efficiency criteria as dependence among observations in agricultural experiments is a common phenomenon.

Keywords : Heterogeneity, Trend, Block designs, Association scheme, Mutually dependence, Correlated error structure, Efficiency.

1. INTRODUCTION

Heterogeneity in the experimental material is the most important problem to be taken care of while designing of scientific experiments. Block designs are the most commonly used designs when heterogeneity is present only in one direction. In block design known sources of variation are treatment and block. In agricultural experiments, particularly under block design setup, apart from the known source of variations, the response may also depend on the spatial position of the experimental unit, i.e. on systematic trend effects. For example, when plots occur in strips in a field, it is often the case that differing contiguous sets of plots within the same strip have different fertility gradient. Trend may occur in greenhouse experiments, where the source of heat is located on sides of the house and experimental units are kept in lines. In all such situations it is evident that the systematic trend can affect the response in the experimental material. For these situations, one can think of suitable designs which are orthogonal or nearly orthogonal to trend effects. Such design are known as trend resistant block designs. If the designs are completely orthogonal to trend effects, then they may be called as **trend-free block designs** (Bradley and Yeh, 1980). Trend free designs permit elimination of effects of low-order components of common trends over experimental units. Lot of work is available in literature which deals with different aspects of trend component in the field of experimental designs [for reference Bradley and Yeh (1980), Yeh and Bradley (1983), Jacroux *et al.* (1997), Majumdar and Martin (2002), Bhowmik (2013), Bhowmik *et al.* (2014, 2015), Bhowmik *et al*. (2017), Sarkar *et al*. (2017), Bhowmik *et al*. (2018) etc.]. Further, in designed experiments, one of the fundamental assumptions is that the observations are identically and independently distributed. However, there do occurs situations where the independence of observations are not ensured. In many experimental situations, due to environmental variations the observations may be mutually dependent or correlated. Thus, this information need to be considered for proper model specification. Thus, performance of trend free designs when observations are dependent or mutually correlated need to be investigates to evaluate the performances of the designs under such situations.

Here, we have discussed the experimental situations under block designs involving systematic trend component. The necessary and sufficient condition for a block design to be trend free have been highlighted. A class of linear trend free block designs in three plots per block have been obtained and their characterization properties have been studied. The performances of developed designs were also evaluated under Auto Regressive (1) [AR(1)] and Nearest Neighbour (NN) correlation structure based on efficiency criteria.

2. Experimental Setup and Model

Consider the following model in block design set-up for v treatments and b blocks of size 2 each incorporating trend component (within-block trend effects is represented by a linear orthogonal polynomials of degree one):

$$Y = \mu 1 + \Delta' \tau + D' \beta + Z\rho + e$$

where, Y is a n × 1 vector of observations, 1 is a n × 1 vector of ones, Δ' is a n × v matrix of observations versus treatment effects, τ is a v ×1 vector of treatment effects, D' is a n × b incidence matrix of observations versus block effects, β is a b × 1 vector of block effects, ρ representing the trend effects. The matrix Z, of order n × 1, is the matrix of coefficients given by $Z = 1_b \otimes F$ where F is a 2 × 1 matrix with columns representing the (normalized) orthogonal

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polynomials and e is a n × 1 vector of errors where errors follow normal distribution with zero mean and constant variance. Considering this set-up, the information matrix (C) for estimating the effect of treatments is obtained as

$$C = R - \frac{1}{k}NN' - \frac{1}{b}\Delta ZZ'\Delta'$$

where, $\mathbf{R} = \mathbf{\Delta\Delta'}$ and $\mathbf{N} = \mathbf{\Delta D'}$. Under the above model, a design will be trend free iff $\mathbf{\Delta Z} = \mathbf{0}$ which is actually the necessary and sufficient condition of a design to be trend free (Bradley and Yeh, 1980). Based on the above one can define trend free designs as follows:

Definition: A block design with v treatments, b blocks of size 3 each is called a linear trend-free block design if the adjusted treatment sum of square under the corresponding block model with trend component is same as the adjusted treatment sum of square under the usual block model without trend component.

Following is a method to obtain a class of trend free block designs of size three each for any number of treatment.

3. Method of construction of trend free block designs

For constructing a linear trend free block design of size 3 each, first construct an initial array of natural numbers of order v×1 where v (≥ 5) is the number of treatments. Then, by treating this array as initial column, and developing two more column with modulo v one can obtain a class of linear Trend Free Partially Balanced Incomplete Block Design (TF-PBIB) of size three each by treating all the row of the developed array as blocks. The parameters of the developed designs will be v = b, r = k = 3. The block contents (row wise) of the developed designs are

1	2	3
2	3	4
...		
v - 1	v	1
v	1	2

The class of linear TF-PBIB designs is equi-replicable, proper and connected class of block designs and can be obtained for any number of treatments. The association scheme for the class of the design is Varying Circular association scheme. Here, any treatment will have $\frac{v}{2}$ number of associates if v is even and $\frac{(v-1)}{2}$ number of associates if v is odd. The association scheme is given as follows:

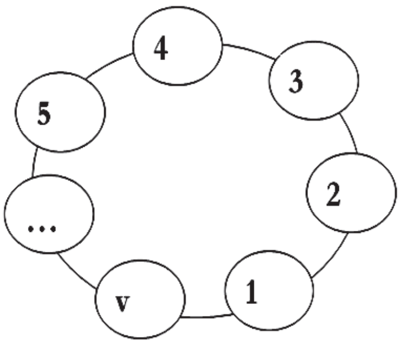

Figure 1: Varying circular association scheme with v treatment

Here, treatments at distance 1 on both the sides are first associates and the treatments at distance 2 are second associates and so on. The information matrix of the class of linear TF-PBIB designs as obtained above is given as

$$\mathbf{C}_\tau = a_{10}\mathbf{B}_0 + a_{11}\mathbf{B}_1 + \sum_{m=2}^{t} a_{1m}\mathbf{B}_m$$

For $t = \frac{(v-2)}{2}$ if v is even and $t = \frac{(v-3)}{2}$ if v is odd.

Here, $a_{10} = (k-1)$, $a_{11} = -\frac{(k-1)}{(k)}$ and $a_{1m} = -\frac{(k-2)}{(k-1)}$ for m = 2, 3, ..., t.

Here, $\mathbf{B}_0$ is an identity matrix of order v i.e. $\mathbf{B_0 = I_v}$ whereas $\mathbf{B}_1$ is the 1st association matrix of order v with elements $b^1_{\alpha\beta}$, where $b^1_{\alpha\beta} = 1$, if α and β which are mutually 1st associates of each other and $b^1_{\alpha\beta} = 0$, if α and β which are not mutually 1st associates of each other. $\mathbf{B}_m$ is the mth association matrix of order v with elements $b^m_{\alpha\beta}$, where $b^m_{\alpha\beta} = 1$, if α and β which are mutually mth associates of each other and $b^m_{\alpha\beta} = 0$, if α and β which are not mutually mth associates of each other for m = 2, 3, ..., t.

Remark: For v = 3, the above method results a complete block design whereas for v = 4, one can obtain a balanced incomplete block design where each pair of treatments will appear together twice number of times in the design.

Example 1: Following is an example of a linear TF-PBIB design obtained based on the above method with parameters v = 7 = b, r = 3 - k. First row of the design represents orthogonal trend component of degree one without normalization.

-1	0	1
1	2	3
2	3	4
3	4	5
4	5	6
5	6	7
6	7	1
7	1	2

The above design follows a varying circular association scheme with three associates. Association scheme for the above TF-PBIB design is as follows:

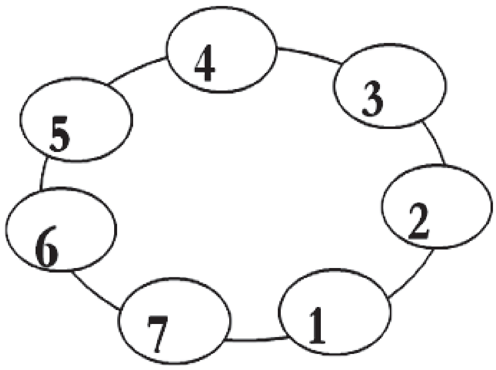

Figure 2: Varying circular association scheme with v treatment

Here, for any treatments, $n_1 = n_2 = n_3 = 2$ where n_i for i = 1, 2, 3 are the number of first, second and third associates for a particular treatment in the design. Table 1 highlights the first, second and third associates of each and every treatment for the above design based on the association scheme as given in Figure 2. Here, any two treatments which are mutually first associates of each other will appear together twice number of times in the design i.e. $\lambda_1 = 2$. Two treatments which are mutually second associates of each other will appear together only once in the design i.e. $\lambda_2 = 1$ whereas any two treatments which are mutually third associates of each other will not appear together in the design i.e. $\lambda_3 = 0$.

Table 1: Associates of all the treatments for design in Example 1

Treatments	1st associates	2nd associates	3rd associates
1	2, 7	3, 6	4, 5
2	1, 3	4, 7	5, 6
3	2, 4	1, 5	6, 7
4	3, 5	2, 6	1, 7
5	4, 6	3, 7	1, 2
6	5, 7	1, 4	2, 3
7	1, 6	2, 5	3, 4

4. Performance of developed TF-PBIB designs under correlated error structure

The efficiencies of developed TF-PBIB designs have been computed for different values of correlation under some specific parametric combinations by considering AR(1) and NN correlation structure. The criteria for computing efficiencies of designs are well known A- and D-efficiency criteria. Since, the TF-PBIB designs based on the aforementioned method follows varying circular association scheme, thus, λ's will vary accordingly. The parameters (for $6 \leq v \leq 10$) excluding λ's and A- and D- efficiency of developed TF-PBIB designs have been highlighted in Table 2 for different values of correlation coefficient (ρ) under AR(1) and NN correlation structure for different parametric combinations.

It has been observed that, efficiencies remains quite high for the developed designs under both AR(1) and NN correlation structure even for higher values of the correlation coefficient. Under NN correlation structure, efficiencies increases as ρ increases in negative directions. The magnitude of decrease is more for positive values under NN correlation structure whereas for AR (1) it is in opposite direction. Designs are performing better under D-efficiency criteria.

Table 2: Efficiencies of developed TF-PBIB under AR(1) and NN correlation structure

Parameters $v = 6 = b, r = 3 = k$					
AR(1) correlation structure			NN correlation structure		
ρ	A-Efficiency	D-Efficiency	ρ	A-Efficiency	D-Efficiency
-0.9	0.5203	0.8058	-0.5	0.9615	0.9800
-0.7	0.8566	0.9375	-0.4	0.9606	0.9796
-0.5	0.9466	0.9737	-0.3	0.9575	0.9783
-0.3	0.9610	0.9798	-0.2	0.9519	0.9757
-0.1	0.9440	0.9720	-0.1	0.9428	0.9714
0	1	1	0	1	1
0.1	0.9120	0.9567	0.1	0.9099	0.9557
0.3	0.8723	0.9370	0.2	0.8827	0.9422
0.5	0.8287	0.9147	0.3	0.8445	0.9229
0.7	0.7833	0.8901	0.4	0.7909	0.8949
0.9	0.7372	0.8659	0.5	0.7142	0.8531
Parameters $v = 7 = b, r = 3 = k$					
-0.9	0.7750	0.8832	-0.5	0.9285	0.9638
-0.7	0.8960	0.9443	-0.4	0.9214	0.9622
-0.5	0.9321	0.9636	-0.3	0.9175	0.9595
-0.3	0.9255	0.9627	-0.2	0.9083	0.9557
-0.1	0.8973	0.9508	-0.1	0.8957	0.9501
0	1	1	0	1	1
0.1	0.8578	0.9325	0.1	0.8554	0.9314
0.3	0.8126	0.9104	0.2	0.8242	0.9162
0.5	0.7647	0.8860	0.3	0.7820	0.8949
0.7	0.7158	0.8601	0.4	0.7240	0.8645
0.9	0.6670	0.8334	0.5	0.6428	0.8198

Parameters $v = 8 = b, r = 3 = k$					
AR(1) correlation structure			NN correlation structure		
ρ	A-Efficiency	D-Efficiency	ρ	A-Efficiency	D-Efficiency
-0.9	0.5936	0.8289	-0.5	0.8870	0.9456
-0.7	0.8448	0.9247	-0.4	0.8812	0.9436
-0.5	0.8944	0.9465	-0.3	0.8731	0.9405
-0.3	0.8831	0.9443	-0.2	0.8622	0.9360
-0.1	0.8495	0.9306	-0.1	0.8477	0.9298
0	1	1	0	1	1
0.1	0.8058	0.9106	0.1	0.8032	0.9094
0.3	0.7575	0.8871	0.2	0.7698	0.8932
0.5	0.7076	0.8614	0.3	0.7255	0.8707
0.7	0.6574	0.8344	0.4	0.6657	0.8389
0.9	0.6077	0.8066	0.5	0.5833	0.7925
Parameters $v = 9 = b, r = 3 = k$					
AR(1) correlation structure			NN correlation structure		
ρ	A-Efficiency	D-Efficiency	ρ	A-Efficiency	D-Efficiency
-0.9	0.7063	0.8508	-0.5	0.8473	0.9294
-0.7	0.8396	09161	-0.4	0.8400	0.9269
-0.5	0.8621	0.9323	-0.3	0.8305	0.9233
-0.3	0.8423	0.9277	-0.2	0.8181	0.9183
-0.1	0.8041	0.9124	-0.1	0.8021	0.9116
0	1	1	0	1	1
0.1	0.7577	0.8913	0.1	0.7550	0.8900
0.3	0.7079	0.8668	0.2	0.7205	0.8731
0.5	0.6572	0.8403	0.3	0.6753	0.8499
0.7	0.6068	0.8126	0.4	0.6151	0.8172
0.9	0.5575	0.7841	0.5	0.5333	0.7698
Parameters $v = 10 = b, r = 3 = k$					
AR(1) correlation structure			NN correlation structure		
ρ	A-Efficiency	D-Efficiency	ρ	A-Efficiency	D-Efficiency
-0.9	0.6147	0.8237	-0.5	0.8085	0.9144
-0.7	0.8076	0.9026	-0.4	0.8003	0.9116
-0.5	0.8271	0.9183	-0.3	0.7899	0.9077
-0.3	0.8029	0.9125	-0.2	0.7766	0.9025
-0.1	0.7618	0.8962	-0.1	0.7597	0.8953
0	1	1	0	1	1
0.1	0.7138	0.8743	0.1	0.7111	0.8730
0.3	0.6634	0.8491	0.2	0.6761	0.8556
0.5	0.6128	0.8221	0.3	0.6308	0.8319
0.7	0.5630	0.7939	0.4	0.5712	0.7987
0.9	0.5145	0.7651	0.5	0.4909	0.7506

5. Discussion

In agricultural, animal and allied experiments under block design setup, there may be evidences of systematic trend affecting the response under consideration. The effect of trend although remote, still may have high influence on response and hence should be incorporated in to the model for proper model specification. The trend-free block design would nullify the effects of common trend effects and thus will increase the precision of the experiments. Further, agricultural experiments may often witness situations where observations may be correlated or mutually dependent. This information also need to be taken care while dealing with block design. The developed designs in the present study are linear trend free and processes good characterization properties. Further, they also remain highly efficient under AR(1) and NN correlation structure for higher values of correlation coefficient. Thus, the developed designs can be advantageously used in agricultural and allied experiments.

REFERENCES

Bhowmik, A. 2013. *Experimental designs involving treatments exhibiting interference effects*. Unpublished Ph.D. Thesis, IARI, New Delhi.

Bhowmik, A., Jaggi, S., Varghese, C. and Varghese, E. 2014. Trend free block designs balanced for interference effects from neighbouring experimental units. *Journal of Combinatorics, Information and System Sciences*, 39 (1 and 2) : 117-133.

Bhowmik, A., Jaggi, S., Varghese, C. and Varghese, E. 2015. Trend free second order neighbor balanced block designs. *Journal of the Indian Statistical Association*, 53 (1 and 2): 63-78.

Bhowmik, A., Jaggi, S., Varghese, E. and Yadav, S. K. 2017. Trend free design under two-way elimination of Heterogeneity. *RASHI*, 2 (1): 34-38.

Bhowmik, A., Varghese, E., Jaggi, S. and Yadav, S. K. 2018. Designs for animal experiments under two-way blocking structure in the presence of systematic trend. *Indian Journal of Animal Sciences*, 88 (1): 121–124.

Bradley, R. A. and Yeh, C. M. 1980. Trend-free block designs: theory. *Annals of Statistics*, 8: 883-93.

Jacroux, M., Majumdar, D. and Shah, K.R. 1997. On the determination and construction of optimal block designs in the presence of linear trend. *Journal of the American Statistical Association*, 92: 375-82.

Majumdar, D. and Martin, R. J. 2002. Finding optimal designs in the presence of trends. *Journal of Statistical Planning and Inference*, 106: 177-90.

Sarkar, K.A., Jaggi, S., Bhowmik, A., Varghese, E. and Varghese, C. 2017. Trend resistant neighbour balanced bipartite block designs. *Journal of the Indian Society of Agricultural Statistics*, 71: 53-59.

Yeh C M and Bradley R A. 1983. Trend-free block designs: existence and construction results. *Communication in Statistics-Theory and Methods*, 12 : 1-24.