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ORIGINAL ARTICLE

EMPIRICAL COMPARISON OF THE PERFORMANCE OF LINEAR DISCRIMINANT FUNCTION UNDER MULTIVARIATE NON-NORMAL AND NORMAL DATA

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Abstract : Discriminant analysis is a multivariate technique concerned with classifying distinct set of objects (or set of observations) and with allocating new objects (or observations) to the previously defined groups. Fisher linear discriminant function is studied under multivariate normal as well as non-normal data. The different multivariate non-normal and normal populations are simulated by using distinct mean vectors and dispersion matrix for rice and maize data sets. Further fifty different independent samples each are simulated for different dimensions and sample sizes for maize and rice data to obtain empirical probabilities of misclassification in case of non-normal data. Taking into consideration the overall results of maize and rice, it has been noticed that D^2 values and discriminating power are higher in 58 per cent and 86 per cent cases, respectively irrespective of sample size and dimensions in case of normal data compared to non-normal data. The probabilities of misclassification are more in case of multivariate non-normal data compared to normal data.

Key words : Discriminant analysis, Discriminant power, Multivariate normal and non-normal distribution, Probability of misclassification.

1. Introduction

Discrimination is a multivariate technique concerned with separating distinct set of objects and with allocating new objects to previously defined groups. Discriminant analysis is the appropriate statistical technique when the dependent variable is categorical (Nominal or non-metric) and the independent variable is metric. The pioneering work by Fisher (1936) and Mahalanobis (1936) has been used by biologists to solve the classificatory problems involving multiple measurements in different context. The two basic assumptions required to be satisfied by the data for using discriminant function are :

(i) The conformation of multivariate normal distribution of the population under study.

(ii) The equality of dispersion matrices of the populations into which the observations are to be classified.

It is common to see the breeding data may not follow multivariate normal distribution. Hence, it will be of interest to study the performance of Fisher Linear Discriminant Function (LDF) when the data do not follow multivariate normal distribution. Reyment (1962) had studied the effect of unequalness of Dispersion matrices in the distance obtained on some biological data. He had tabulated the values obtained for Mahalanobis D^2 and that calculated by Anderson and Bhadur's (1962) method from Irish species for comparing the two methods. Simulation studies conducted by Marks and Dunn (1974) and subsequently by Wahl and Kronmal (1977) to make the comparison of the performance of three discriminant functions *i.e.*, the quadratic, best linear and Fisher's linear function in classifying individuals into two multivariate normal populations, when the dispersion matrices are unequal. These results indicated that for large samples from multivariate normal distributions, the quadratic is much better than Fisher's function. Wahi *et al.* (1986) used the best linear discriminant function for comparing the different grades of sheep in cross breeding programme. In 80 per cent of comparisons among the different grades of sheep the probability of misclassification by the best linear discriminant function were found to be

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either lower or equal to the probability of misclassification obtained by Fisher's linear discriminant function. Wahi and Bhatia (1995) used bootstrap technique for comparing the performance of linear discriminant functions. The coefficients of variation among the bootstrap estimates of D^2-statistics in case of best linear discriminant function were found lower than Fisher's linear discriminant function. These results further confirmed the superiority and consistency of best linear discriminant function over the Fisher's linear discriminant function. The bootstrap technique yielded higher values of D^2-statistics for both the linear function indicating the bias of the D^2-statistics. Minhajuddin *et al*. (2004) proposed a method to simulate the joint distributions, which have equal positive pair-wise correlations and the method was illustrated for the p-dimensional families of beta and gamma distributions.

Sever *et al*. (2005) compared Fisher's discriminant analysis under normal and skewed curved normal distribution based on the apparent error rates, which were used as a measure of classification performance, and found that Fisher's discriminant analysis to be highly robust under skewed curved normal distribution. Todorov and Pires (2007) studied the comparative performance of several robust linear discriminant analysis methods. Rausch and Kelley (2009) compared different methods for discriminant analysis with respect to classification accuracy under non-normality through Monte Carlo simulation. The methods compared were linear discriminant analysis based both on raw scores and on ranks; linear logistic discrimination; and mixture discriminant analysis. Linear discriminant analysis and linear logistic discrimination were suboptimal in a number of scenarios with skewed predictors. Linear discriminant analysis based on ranks yielded the highest rates of classification accuracy in only a limited number of situations and did not produce a practically important advantage over competing methods. There are very few studies done for comparison of the LDF under normal and non-normal condition. With this aim, the study has been under taken. Raman *et al*. (2012) studied the performance of LDF in case of non-normal data sets for maize and rice.

2. Materials and Methods

Data Descriptions

The secondary data on 77 maize genotypes grown at seven different locations data in the Annual progress report for the year 2005-06 of All India Coordinated Maize Improvement Project, Directorate of Maize Research, IARI Campus, New Delhi and seventy five genotypes consist of twenty five tall, twenty five medium and twenty five dwarf rice genotypes procured from Genetics Division of IARI, New Delhi have been used in the present investigation.

RANDDIRICHLET Function is used for simulation of multivariate non-normal data in SAS 9.2 [Kotz *et al*. (2000)].

For simulating the multivariate non-normal the following algorithm was used.

N is the number of desired observations sampled from the distribution.

Shape is a 1 × (p+1) vector of shape parameters for the distribution, shape[i] > 0.

The Dirichlet distribution is a multivariate generalization of the beta distribution. The RANDDIRICHLET Function returns an N × p matrix containing N random draws from the Dirichlet distribution.

If $X = (X_1, X_2, ..., X_p)$ with $\sum_{i=1}^{p} X_i < 1$ and $X_i > 0$ follows a Dirichlet distribution with shape parameter $\alpha = (\alpha_1, \alpha_2, ..., \alpha_{p+1})$, then

The probability density function of X is

$$f(x,\alpha) = \frac{\Gamma\left(\sum_{i=1}^{p+1}\alpha_i\right)}{\prod_{i=1}^{p+1}\Gamma(\alpha_i)} \prod_{i=1}^{p+1} X_i^{\alpha_i - 1}\left(1 - x_1, x_2, ..., x_p\right)^{\alpha_{p+1}-1}$$

If p = 1, the probability distribution is a beta distribution.

If $\alpha_0 = \sum_{i=1}^{p+1}\alpha_i$, then

The expected value of X_i is $\frac{\alpha_i}{\alpha_0}$, the variance of X_i is $\frac{\alpha_i(\alpha_0 - \alpha_i)}{\alpha_0^2(\alpha_0 + 1)}$, the covariance of X_i and X_j is $-\frac{\alpha_i\alpha_j}{\alpha_0^2(\alpha_0 + 1)}$.

RANDNORMAL Function is used for simulation of multivariate normal data in SAS 9.2

The multivariate normal data was generated by using the following algorithm.

The inputs are as follows:

N is the number of desired observations sampled from the distribution.

Mean is a 1 × p vector of means.

Cov is a p × p symmetric positive definite variance-covariance matrix.

The RANDNORMAL Function returns an N × p matrix containing N random draws from the multivariate normal distribution with mean vector Mean and covariance matrix Cov.

If X follows a multivariate normal distribution with mean vector μ and variance covariance matrix Σ, then the probability density function of X is

$$f(x;\mu,\Sigma)=\frac{1}{(2\pi)^{p/2}|\Sigma|^{1/2}}\exp\left(-(x-\mu)\Sigma^{-1}(x-\mu)^{T}/2\right)$$

If p = 1, the probability density function reduces to a univariate normal distribution .

The expected value of X_i is μ_i and the covariance of X_i and X_j is Σ_{ij}.

Discriminant Function Procedure

Let the classical Fisher's linear discriminant function between two populations have $\Sigma_1 = \Sigma_2$ and the function is of the form

$$Y=\sum_{i=1}^{p} b_i X_i \quad (1)$$

Where, b_i's are coefficients of linear discriminant function and are obtained by maximizing the ratio of between and within class variances *i.e.* $\frac{D^2}{S}$. (2)

Where, $D^2 = b_1d_1 + b_2d_2 + \dots + b_pd_p$ and $S=\sum_{i}^{p}\sum_{j}^{p} b_i b_j W_{ij}$, where W_{ij}'s are the elements of pooled dispersion matrix and d_i's are the elements of the vector of difference between the mean vector of the two populations. The solution for b_i's are obtained by differentiating (2) w.r.t., b_i's

$$\frac{D}{S^2}\left[2S\frac{\partial D}{\partial b}-\frac{D\partial S}{\partial b}\right]=0 \Rightarrow \frac{1}{2}\frac{\partial S}{\partial b}=\frac{S}{D}\frac{\partial D}{\partial b}$$

Since, S/D is a constant factor, the unknown b_i's are proportional to the solutions of the equations.

$$b_1W_{11} + b_2W_{12} + ... + b_pW_{1p} = d_1$$
$$b_2W_{21} + b_2W_{22} + ... + b_pW_{2p} = d_2$$
$$b_1W_{p1} + b_2W_{p2} + ... + b_pW_{pp} = d_p$$

So, $\hat{b}=d'W^{-1}$ and $D_p^2=d'W^{-1}d$.

The D_p^2 is the estimate of variance of Y and are the root of D_p^2 gives the discriminatory power of the linear discriminant function.

$$\hat{V}(Y)=\sum b_i b_j W_{ij}=\sum b_i d_i$$

The significance of D_p^2 can be tested for its significance by a F-test given by

$$F=\frac{(N_1+N_2-p-1)N_1N_2D_p^2}{p(N_1+N_2)(N_1+N_2-2)},$$

with p and $(N_1 + N_2 - p - 1)$ degrees of freedom. The approximate probabilities of misclassification for Fisher's linear discriminant function is given by $\Phi\left(-\frac{1}{2}D_p\right)$, where Φ is cumulative normal distribution and D_p is the square root of D_p^2.

In the present investigation, maize data with ten morphological characters and rice data with nine morphological characters with correlation matrices given in Tables 1 and 2, respectively, have been used. The normality of both the data sets are tested by Mardia's skewness and Kurtosis test. It is found that the probability for testing the kurtosis is 0.043 and 0.049 in maize and rice data, respectively which showed that both the data sets are not normal. Three distinct populations are simulated by using different mean vectors and dispersion matrices for both rice and maize data. Differences of mean vectors of these populations are tested by Hotelling's T^2 test, which are found to be significant. A pooled dispersion matrix is formed using these three populations in each of the data sets. Three distinct multivariate non-normal data sets for both maize and rice of different dimensions (*i.e.*, four, six and nine characters) and different sample sizes (*i.e.*, fifty, hundred and one hundred fifty) are simulated by using these three mean vectors and pooled dispersion matrix with the help of RANDDIRICHLET function in SAS package [SAS 9.2 (2009)]. Linear discriminant function

Table 1 : Correlation between ten morphological maize characters.

Characters	x_1	x_2	x_3	x_4	x_5	x_6	x_7	x_8	x_9	x_{10}
x_1	1.000	0.670	0.609	0.580	0.011	-0.397	-0.404	-0.416	0.502	0.511
x_2	0.670	1.000	0.984	0.876	0.100	-0.367	-0.347	-0.403	0.525	0.592
x_3	0.609	0.984	1.000	0.901	0.057	-0.415	-0.412	-0.397	0.535	0.601
x_4	0.580	0.876	0.901	1.000	0.073	-0.363	-0.443	-0.348	0.510	0.502
x_5	0.011	0.100	0.057	0.073	1.000	0.045	0.079	0.180	0.159	0.092
x_6	-0.397	-0.367	-0.415	-0.363	0.045	1.000	0.751	0.741	-0.412	-0.389
x_7	-0.404	-0.347	-0.412	-0.443	0.079	0.751	1.000	0.734	-0.329	-0.371
x_8	-0.416	-0.403	-0.397	-0.348	0.180	0.741	0.734	1.000	-0.313	-0.392
x_9	0.502	0.525	0.535	0.510	0.159	-0.412	-0.329	-0.313	1.000	0.884
x_{10}	0.511	0.592	0.601	0.502	0.092	-0.389	-0.371	-0.392	0.884	1.000

Table 2 : Correlation between morphological nine rice characters.

Characters	x_1	x_2	x_3	x_4	x_5	x_6	x_7	x_8	x_9
x_1	1.000	-0.413	-0.089	-0.364	-0.142	0.078	0.246	0.003	0.238
x_2	-0.413	1.000	0.585	0.557	0.132	0.687	-0.391	0.744	0.283
x_3	-0.089	0.585	1.000	0.378	0.194	0.583	-0.291	0.560	0.425
x_4	-0.364	0.557	0.378	1.000	0.371	0.473	-0.207	0.415	0.423
x_5	-0.142	0.132	0.194	0.371	1.000	0.190	0.059	0.128	0.264
x_6	0.078	0.687	0.583	0.473	0.190	1.000	-0.436	0.957	0.729
x_7	0.246	-0.391	-0.291	-0.207	0.059	-0.436	1.000	-0.356	0.259
x_8	0.003	0.744	0.560	0.415	0.128	0.957	-0.356	1.000	0.505
x_9	0.238	0.283	0.425	0.423	0.264	0.729	0.259	0.505	1.000

is fitted and D^2, discriminating power and probability of misclassification using normal approximation are obtained. Fifty samples each for different dimensions and sample sizes populations are simulated for obtaining empirical probability of misclassifications for non-normal data.

Multivariate normal data have been simulated from different populations with same parameters as in case of multivariate non-normal data, same sample sizes and same dimensions by using RANDNORMAL function in SAS package [SAS 9.2 (2009)]. D^2, discriminating power, probability of misclassification are obtained by fitting linear discriminant function. The performance of linear discriminant function for normal and non-normal multivariate data is compared on the basis of D^2, discriminating power and probability of misclassification.

3. Results

D^2, discriminating power and probability of misclassification are obtained by fitting linear discriminant function based on different dimensions (*i.e.*, four, six and nine) and different sample sizes (*i.e.*, fifty, hundred and one hundred fifty) for both multivariate normal and non-normal maize and rice and given in Table 3 for maize and Table 4 for rice.

4. Discussion

In case of four characters in maize data (Table 3), it has been observed from the results that D^2 values for different pairs of populations are found to be significant. Probabilities of misclassifications of linear discriminant function using normal data as compared to non-normal data are always found to be lower in all the pairs of population. Comparing the D^2 values of normal and non-normal data it is found that D^2 values are higher in 100% pairs of population for normal data sets. It has been observed that discriminating power in the case of normal data are mostly higher as compared to non-normal data. Probabilities of misclassification are always found to be lower in all the pairs of population for linear discriminant function based on 6 characters

Table 3 : Comparison of D^2, Discriminating power and probability of misclassification based on following four, six and nine characters between maize populations in normal and non-normal data.

	Four Characters between maize populations									
		Sample Size 50			**Sample Size 100**			**Sample Size 150**		
Pop		**D²**	**DP**	**POM**	**D²**	**DP**	**POM**	**D²**	**DP**	**POM**
1-2	N	0.701*	0.837	0.337	0.711*	0.843	0.336	0.754*	0.868	0.332
1-2	Nn	0.682*	0.826	0.402	0.652*	0.807	0.391	0.623*	0.789	0.387
1-3	N	0.632*	0.794	0.345	0.683*	0.826	0.339	0.703*	0.838	0.337
1-3	Nn	0.421*	0.649	0.412	0.413*	0.647	0.402	0.393*	0.627	0.391
2-3	N	0.721*	0.849	0.335	0.782*	0.884	0.329	0.801*	0.894	0.327
2-3	Nn	0.707*	0.841	0.431	0.692*	0.832	0.417	0.653*	0.808	0.427
	Six Characters between maize populations									
		Sample Size 50			**Sample Size 100**			**Sample Size 150**		
		D²	**DP**	**POM**	**D²**	**DP**	**POM**	**D²**	**DP**	**POM**
1-2	N	0.781*	0.883	0.329	0.523*	0.723	0.358	0.481*	0.693	0.364
1-2	Nn	0.712*	0.843	0.383	0.523*	0.723	0.373	0.573*	0.576	0.369
1-3	N	0.827*	0.909	0.324	0.592*	0.769	0.350	0.603*	0.776	0.348
1-3	Nn	0.717*	0.846	0.391	0.733*	0.856	0.387	0.941*	0.970	0.355
2-3	N	0.521*	0.721	0.359	0.302*	0.549	0.391	0.220*	0.469	0.407
2-3	Nn	0.731*	0.854	0.427	0.387*	0.622	0.419	0.291*	0.539	0.411
	Nine Characters between maize populations									
		Sample Size 50			**Sample Size 100**			**Sample Size 150**		
		D²	**DP**	**POM**	**D²**	**DP**	**POM**	**D²**	**DP**	**POM**
1-2	N	0.782*	0.884	0.329	0.566*	0.752	0.353	0.466*	0.682	0.366
1-2	Nn	0.823*	0.907	0.379	0.544*	0.738	0.371	0.522*	0.722	0.367
1-3	N	0.803*	0.896	0.327	0.573*	0.756	0.352	0.576*	0.758	0.352
1-3	Nn	0.907*	0.952	0.376	0.928*	0.963	0.361	0.876*	0.936	0.355
2-3	N	0.792*	0.889	0.328	0.420*	0.648	0.372	0.234*	0.483	0.404
2-3	Nn	0.791	0.889	0.421	0.417*	0.646	0.413	0.315*	0.561	0.406

Pop = Population, **DP** = Discriminating Power, **POM** = Probability of Misclassification, **(*)** represents the significance at 5% level, **N** = Normal and **Nn** = Non-normal.

of maize (Table 3). On the contrary D^2 values in 33% of comparisons in normal data are higher as compared non-normal data based on 9 characters of maize. In case of 9 characters similar trend in probabilities of misclassifications is seen as in case of four and six characters (Table 3). On comparing the D^2 values of normal and non-normal data, it is found that D^2 values in case of normal data is higher in case of 33% pairs of population.

In case of four characters in rice data in Table 4, it can be seen that D^2 values for different pairs of populations are found to be significant. Probabilities of misclassifications of normal data as compared to non-normal data are always found to be lower in all the pairs of population. Comparing the D^2 values of normal and non-normal data, it is found that D^2 values are higher in 89% pairs of population. It is observed discriminating power in case of normal data are higher as compared to non-normal data. Probabilities of misclassification are always found to be lower in all the pairs of population for linear discriminant function based on 6 characters of rice (Table 4). On the contrary D^2 values in 89% of comparisons in normal data are higher as compared non-normal data as in case of 4 characters of rice. In case of 9 characters similar trend in probabilities of misclassifications is seen as in case of four and six characters (Table 4). On comparing the D^2 values of normal and non-normal data, it is found that D^2 values in case of normal data is higher in case of 78% pairs of population.

Taking into consideration the overall results of maize and rice, it has been noticed that in that D^2 values and

Table 4 : Comparison of D^2, Discriminating power and probability of misclassification based on following four, six and nine characters between rice populations in normal and non-normal data.

		Four Characters between rice populations								
		Sample Size 50			Sample Size 100			Sample Size 150		
Pop		**D^2**	**DP**	**POM**	**D^2**	**DP**	**POM**	**D^2**	**DP**	**POM**
1-2	N	1.091*	1.044	0.312	1.170*	1.081	0.294	1.252*	1.118	0.288
1-2	Nn	0.626*	0.791	0.391	0.586*	0.766	0.342	0.544*	0.738	0.345
1-3	N	1.632*	1.276	0.261	1.861*	1.363	0.247	1.956*	1.396	0.242
1-3	Nn	0.356*	0.597	0.412	0.311*	0.558	0.406	0.273*	0.522	0.392
2-3	N	0.597*	0.768	0.351	0.613*	0.774	0.349	0.632*	0.787	0.346
2-3	Nn	0.663*	0.826	0.384	0.577*	0.760	0.376	0.493*	0.702	0.362
		Six Characters between rice populations								
		Sample Size 50			**Sample Size 100**			**Sample Size 150**		
		D^2	**DP**	**POM**	**D^2**	**DP**	**POM**	**D^2**	**DP**	**POM**
1-2	N	1.650*	1.284	0.260	1.540*	1.240	0.267	1.220*	1.104	0.290
1-2	Nn	1.323*	1.150	0.387	1.151*	1.072	0.346	1.181*	1.086	0.331
1-3	N	1.560*	1.249	0.266	1.090*	1.044	0.300	0.674*	0.820	0.340
1-3	Nn	0.741*	0.860	0.405	0.462*	0.679	0.371	0.428*	0.654	0.378
2-3	N	2.150*	1.466	0.231	2.080*	1.442	0.235	1.834*	1.354	0.249
2-3	Nn	1.456*	1.206	0.318	1.521*	1.233	0..335	1.847*	1.359	0.293
		Nine Characters between rice populations								
		Sample Size 50			**Sample Size 100**			**Sample Size 150**		
		D^2	**DP**	**POM**	**D^2**	**DP**	**POM**	**D^2**	**DP**	**POM**
1-2	N	2.116*	1.454	0.233	0.932*	0.965	0.314	0.786*	0.886	0.328
1-2	Nn	2.077*	1.441	0.365	1.663*	1.288	0.334	1.130*	1.061	0.323
1-3	N	2.151*	1.466	0.231	1.293*	1.137	0.284	1.217*	1.103	0.290
1-3	Nn	0.856*	0.925	0.398	0.712*	0.842	0.357	0.551*	0.741	0.343
2-3	N	2.574*	4.604	0.211	2.284*	1.511	0.224	2.170*	1.473	0.320
2-3	Nn	1.562*	1.249	0.288	2.287*	1.509	0.268	2.075*	1.438	0.265

Pop = Population, **DP =** Discriminating Power, **POM =** Probability of Misclassification, **(*)** represents the significance at 5% level, **N =** Normal and **Nn =** Non-normal.

discriminating power are higher in 58% and 86%, respectively irrespective of sample size and number of characters. It can also be seen that probability of misclassifications in general is more in multivariate non-normal as compared to multivariate normal data in both maize and rice. Probabilities of misclassification decreases with increase in number of characters in majority of cases. The difference in probabilities of misclassification of multivariate non-normal and multivariate normal data decreases with increase in number of sample size and increase in number of characters. It can be seen that the difference in probabilities of misclassification in multivariate non-normal and normal data decreases with increase in sample size and increase in dimensions for both maize and rice.

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