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Non-Linear Mixed Effect Models for Estimation of Growth Parameters in Goats

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SUMMARY

Modelling growth of an animal is a complex process, because it requires describing longitudinal measurements with few parameters with biological interpretation. With longitudinal data, the variance of observations may increase with time (age), and repeated measurements of an individual over time are correlated. The non-independence of data violates a key assumption underlying many statistical procedures and has been ignored in most traditional non-linear fixed effect models. A solution to this problem is the use of non-linear mixed effect models (NLMM). A NLMM makes it possible to account for random covariates before testing for fixed effects and control autocorrelation in repeated measures. In this study, growth data of Goat has been used. Attempt has been made to develop the Von-bertalanffy mixed model. Logistic, Gompertz and Von-bertalanffy fixed and mixed models have also been explored for these data. Comparison of the models i.e. between fixed and mixed type of the same model and among different fixed and mixed models has been attempted. The goodness of fit statistics like *i.e.* Mean Square Error (MSE) and Root Mean Square Error (RMSE) of the fitted models has been computed. The parameters of the best fitted models along with their corresponding standard error are estimated. The performance of mixed effect models was found to be better than the fixed effect model. Specifically, under the category of mixed effect model, the Logistic model out performed over the other types that were considered in the study.

Keywords: DM test, NLMM, Longitudinal data, Random covariates.

1. INTRODUCTION

In animal experiment, variation in growth rates of an animal is important to the input costs and returns for both producer and consumer. Growth is defined as the increase in body size per unit of time and is combination of hereditary and environmental effects. Broody (1995) defined that growth is a relatively irreversible time in the magnitude of measured dimension or function. Growth is a vital study for animals because it influences the different forms of production such as milk, meat etc. in later ages. The relationship between body weight and age is important particularly for the meat producing animal such as beef cattle, goat, pig, sheep etc. A polynomial model used to describe the data has large number of parameters and the parameters would not be easily interpretable (Pinheiro and Bates 2000). In animal growth data mostly the fixed effect models and polynomial models are used. These models have few drawbacks such as, in case of repeated measurements of body weight from the same animal are likely to be more closely correlated than measures made on different animals, and the measures made close in time on the same individual are likely to be highly correlated than the measures made further apart time (Littell *et al.* 2000). In case of longitudinal growth data, there are *within* and *between* individuals variation. The variability between individuals are not included in fixed effect model (Craig and Schinckel 2001). In case of longitudinal data, there exists correlation between the variance of observations along with increment of time (age). It affects many statistical procedures. NLMM deals with all of these shortcomings of the traditional nonlinear fixed

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effect model with the inclusion of a random factor. Mixed effects models for longitudinal data such as growth is important because of their flexible covariance structure and their ability to handle unbalanced data. A NLMM makes it possible to account for random covariates and control autocorrelation in repeated measures. Wang and Zuidhof (2004) used mixed Gompertz growth model to account for between-bird variation and heterogeneous variance. Schinckel *et al.* (2005) evaluated the alternative non-linear mixed effects models of duck growth. Eyduran *et al.* (2008) introduced the new approaches like Absolute Reduction Ratio (ARR) to determine non-linear growth model. Singh *et al.* (2009) studied the statistical properties of genetic parameter of growth curves on goat. Fu *et al.* (2013) conducted Non-linear mixed-effects crown width models for individual trees of Chinese fir (Cunninghamia lanceolata) in south-central China. A goat is useful to humans in alive and dead. We can use it as a renewable provider of milk and fibre and then as meat. Some charities provide Goats to impoverished people in poor countries because Goats are easier and cheaper to manage than cattle and have multiple uses. But genetic analysis of growth curve parameter of goat is not done so far. Hence it is necessary to estimate the growth curve parameters and body weight of different ages. The aim of this paper is to fit nonlinear mixed growth models that describe the growth pattern of goat and to compare empirically, the performance of non-linear mixed effect growth models with fixed effect models.

2. MATERIALS AND METHODS

In the present study growth data of 142 goats were used. The goat data were collected from goat farm of C.I.R.G., Makhdoom, Uttar Pradesh for the year 2005. The body weight data of goat were taken in following form:

W_t; $t = 1, ..., 12$

where, W_t is the observations of Body Weights (BW) at the t-th month ($t = 1, ..., 12$).

All the observations are taken in Kg. In this study, 3 nonlinear growth models-Gompertz, Logistic model and Von-Bertalanffy-were considered.

To estimate the growth parameters in goat, a Non-linear fixed effect model has to be taken and then a random function i.e. random deviation from the average mean has to be incorporated to convert the fixed effect model into mixed effect model. Then the parameters will be estimated from the newly formed mixed effect model.

For example, in Gompertz model, a fixed effect Gompertz growth function proposed by Kutner *et al.* (1996) is:

$$W_{it} = W_m \exp^{\left(-\exp\left(b\left(t-t^*\right)\right)\right)} + e_{it} \tag{1.1}$$

where W_m is the average mature BW of all individuals in the same group; b is rate of maturing; t* is the time in days at which growth rate is Max.; e_{it} is the residual BW of individual i at age t; e_{it} is normally distributed with mean 0 and constant variance σ^{ij} and W_{it} is the expected BW of individual i at age t days.

A random function *ui* is incorporated in the model. The random function is a random deviation of mature BW of the individual from average mature BW of its genotype W_m. Now Mixed effect model can also be written as:

$$W_{it} = f(t;\, \theta) + u_i\, g(t;\, \theta) + e_{it} \tag{1.2}$$

where $\theta = \{W_m, b\}$, such that the random effect u_i is multiplied by $g(t;\theta) = \exp^{(-\exp(b(t-t^*)))}$; and the mean, variance and covariance of the mixed effect growth model are: $E(W_{it}) = f(t;\theta)$, $V(W_{it}) = \sigma_u^2 g^2(t;\theta) + \sigma_e^2$ and $Cov(W_{it}, W_{it+j}) = \sigma_u^2 g(t;\theta) g(t+j;\theta)$.

Similarly the form of the mixed effect Logistic model is

$$W_{it} = \frac{(W_m + u)}{1 + e^{-\left(\frac{t-t^*}{b}\right)}} + e_{it} \tag{2}$$

The notations of the model are same as defined in above model.

The reduced form obtained of mixed effect Von-Bertalanffy model is

$$W_{it} = (W_m + u)(1 - Be^{-b(t-t^*)})^3 + e_{it} \tag{3}$$

where all the notations are same; B is the integrating function.

In the present study, the estimation of parameters was carried out by Levenberg-Marquardt method using SAS software package version 9.2.

Model Selection Criteria: For selection of best fit models, the following criterion was used (in this study)

i. ***Root Mean Squared Error (RMSE)***: Root Mean Squared Error (RMSE) is defined as

$$\text{RMSE} = \sqrt{\sum_{j=1}^{N}\sum_{i=1}^{t_j}\{W(t)-\hat{W}(t)\}^2 \Big/ \sum_{j=1}^{N}(t_j-p)} \quad (4)$$

where p is number of parameters fitted. N is number of animals, t_j is the number of weights for the j^{th} animal.

ii. ***Percent Prediction Error (PPE):***

$$\text{PPE} = \left|\frac{W(t)-\hat{W}(t)}{W(t)}\right| \times 100 \quad (5)$$

iii. ***Mean Absolute Error (MAE):***

$$\text{MAE} = \sum_{j=1}^{N}\sum_{i=1}^{t_j}\left|W(t)-\hat{W}(t)\right| \Big/ \sum_{j=1}^{N}(t_j-p) \quad (6)$$

2.1 Model Fitting Criteria

The results of the study will be compared empirically with the following criteria:

i. ***AIC (Akaike's Information Criterion):***

$$\text{AIC} = -\ln L + p \quad (7)$$

where L is the likelihood function for model with p parameters.

ii. ***Bayesian Information Criterion (BIC):***

$$\text{BIC} = -2L_p + p\ln n \quad (8)$$

where n is the sample size, L_p is the maximized log-likelihood of the model and p is the number of parameters in the model.

3. ACCURACY OF FIXED AND MIXED EFFECT MODEL

In practical situation, the traditional statistics such as Mean squared error (MSE), Root mean squared error (RMSE) etc. are used to evaluate forecasting results and make forecasting comparisons. In this study Diebold Mariano (DM) test (Diebold and Mariano 1995) has been used.

Let $\{y_t\}$ denote the series to be forecast and let $y^1_{t+h|t}$ and $y^2_{t+h|t}$ denote two competing forecasts of y_{t+h} based on information up to time t. The forecast errors from the two models are $\varepsilon^1_{t+h|t} = y_{t+h} - y^1_{t+h|t}$ and $\varepsilon^2_{t+h|t} = y_{t+h} - y^2_{t+h|t}$. The accuracy of each forecast is measured by a particular loss function

$$L\left(y_{t+h}, y^i_{t+h|t}\right) = L\left(\varepsilon^i_{t+h|t}\right), \quad i = 1, 2$$

Some popular loss functions are

Squared error loss: $L\left(\varepsilon^i_{t+h|t}\right) = \left(\varepsilon^i_{t+h|t}\right)^2$

Absolute error loss: $L\left(\varepsilon^i_{t+h|t}\right) = \left|\varepsilon^i_{t+h|t}\right|$

To determine if one model predicts better than another we may test null hypotheses

$$H_0 : E\left[L\left(\varepsilon^1_{t+h|t}\right)\right] = E\left[L\left(\varepsilon^2_{t+h|t}\right)\right]$$

against the alternative

$$H_1 : E\left[L\left(\varepsilon^1_{t+h|t}\right)\right] \neq E\left[L\left(\varepsilon^2_{t+h|t}\right)\right]$$

The Diebold-Mariano test is based on the loss differential $d_t = L\left(\varepsilon^1_{t+h|t}\right) - L\left(\varepsilon^2_{t+h|t}\right)$

The null of equal predictive accuracy is then H_0: $E[d_t] = 0$

The Diebold-Mariano test statistic is

$$S = \frac{\bar{d}}{\left(LRV_{\bar{d}}/T\right)^{1/2}}$$

where d_t the loss differential as is defined earlier; $\bar{d}$ is the mean of d_t and T is the length of the series.

$$LRV_{\bar{d}} = \gamma_0 + 2\sum_{j=1}^{\infty}\gamma_j, \qquad \gamma_j = \operatorname{cov}\left(d_t, d_{t-j}\right)$$

$LRV_{\bar{d}}$ is a consistent estimate of the asymptotic (long-run) variance of $\sqrt{T}\bar{d}$. Diebold and Mariano (1995) showed that under the null of equal predictive accuracy, S ~ $N(0, 1)$.

Detailed analysis of residuals is important for checking model adequacy. Normality of the residuals is tested with the help of many test such as Shapiro–Wilk test, Anderson-Darling Test and Kolmogorov-Smirnov test.

4. RESULTS AND DISCUSSION

A total of 1704 body weight observations from 142 goats (71 males and 71 females) were taken for this analysis. Gompertz, Logistic and Von-Bertalanffy models were fitted in both male and female body weight data sets. The same data has been fitted to both fixed and mixed effect growth models using the NLMIXED procedure in SAS software (SAS Institute 1999).

4.1 Initial Estimates

Linearization techniques are used to find out initial estimates. The initial estimate values of the growth curve parameters are given in Table1.

Table 1. Initial value of the parameters

Initial Values of Goat	Female	Male
W_m	1	1
b	17	17
σ_e^2	22.74	35.23
σ_u^2	1.5	1.5

4.2 Model Fitting Criteria

Model fitting criteria like MSE and RMSE are computed for the three models. The values are given in Tables 2 and 3.

Table 2. Model fitting criteria for female goats

Female	Gompertz Model		Logistic Model		Von Bertalanffy Model	
	Fixed Effect	Mixed Effect	Fixed Effect	Mixed Effect	Fixed Effect	Mixed Effect
MSE	5.344	4.270	6.270	1.190	29.637	26.745
RMSE	2.312	2.066	2.500	1.091	5.444	5.171

Table 3. Model fitting criteria for male goats

Male	Gompertz Model		Logistic Model		Von Bertalanffy Model	
	Fixed Effect	Mixed Effect	Fixed Effect	Mixed Effect	Fixed Effect	Mixed Effect
MSE	6.647	5.595	7.676	1.756	22.132	21.679
RMSE	2.578	2.365	2.772	1.313	4.704	4.656

Based on the above two tables it has been found that the mixed effect models are relatively better than the fixed models corresponding to each of the individual growth models. Further, under the mixed effect category, Logistic model has relatively low MSE and RMSE over the other two models (Gompertz and Von-Bertalanffy). i.e., the mixed effect Logistic models have highest precision.

4.3 Fit Statistics

The performance of fitted models *i.e.,* Gompertz, Logistic and Von-Bertalanffy model was compared by computing the four different fit statistics. These are: -2Loglikelihood, Akaike information criterion (AIC), Corrected Akaike information criterion (AICC) and Bayesian Information Criterion (BIC). The results of these criteria are summarized in Tables 4 and 5.

Table 4. Model fitting criteria for female goats

Criterion	Gompertz Model		Logistic Model		Von-Bertalanffy Model	
	Fixed Effect	Mixed Effect	Fixed Effect	Mixed Effect	Fixed Effect	Mixed Effect
-2 Log Likelihood (smaller is better)	5475.6	3496.9	3925.3	2873.9	6596.0	4648.7
AIC (smaller is better)	5483.6	3504.9	3933.3	2881.9	6604.0	4656.7
AICC (smaller is better)	5483.7	3505.0	3933.3	2882.0	6604.0	4656.8
BIC (smaller is better)	5502.6	3514.0	3952.2	2891.0	6622.9	4665.8

Table 5. Model fitting criteria for male goats

Criterion	Gompertz Model		Logistic Model		Von-Bertalanffy Model	
	Fixed Effect	Mixed Effect	Fixed Effect	Mixed Effect	Fixed Effect	Mixed Effect
-2 Log Likelihood (smaller is better)	4371.0	3965.5	4094.6	3158.7	4985.3	4984.8
AIC (smaller is better)	4381.0	3973.5	4102.6	3166.7	4993.3	4992.8
AICC (smaller is better)	4381.1	3973.6	4102.7	3166.7	4993.4	4992.8
BIC (smaller is better)	4404.7	3982.6	4121.5	3175.7	5012.3	5001.8

The fit statistics values of the mixed effect models are lower than those of the corresponding fixed effect model. These fit statistics indicated that the mixed effect models fit the data better than those of the fixed effect models. Specifically, under the mixed effects category, the Logistic model exhibited the best-fit over the Gompertz and Von-Bertalanffy model.

4.4 Parameter Estimates

On the basis of the model fitting criteria and fit statistics, it may be concluded that the Logistic model is most appropriate for the given data. Therefore, the parameters of the Logistic model have been estimated. The estimates of the growth parameters obtained with fixed and mixed effects models are summarized in Table 6.

Table 6. Parameter estimates-logistic model

Parameter	Female				Male			
	Fixed Effect Model		Mixed Effect Model		Fixed Effect Model		Mixed Effect Model	
	Estimate	S.E.	Estimate	S.E.	Estimate	S.E.	Estimate	S.E.
b	7.857	0.067	7.815	0.148	7.028	0.056	7.015	0.115
W_m	41.793	0.353	41.633	0.776	49.925	0.397	49.763	0.865
σ_e^2	6.265	0.305	1.297	0.066	7.665	0.374	1.855	0.094
σ_u^2	-	-	40.681	7.007	-	-	50.062	8.684

It can be observed that the estimates of the mature body weight (W_m) and maturity rate (b) are similar for both models because the expected means are same. It has also been found that the mean estimated asymptotic body weight (W_m) in case of Logistic growth model is 41.63 kg in mixed and 41.79 kg in fixed model for female data. In case of male data, the body weight value is 49.76 kg in mixed and 49.92 kg in fixed effect model. It is indicating that male having higher mature body weight (W_m) as compared to female. The mean estimated maximum growth rate (b) for female is 7.8579 ± 0.0676 months (fixed) and 7.8152 ± 0.1484 months (mixed); whereas these are 7.0282 ± 0.0564 months (fixed) and 7.0155 ± 0.1152 months (mixed) in male data. This shows that the females have high maturity rate.

The performance of fixed and mixed effects model with regard to the Logistic Model was also evaluated in terms of accuracy by applying the Diebold Mariano test.

The test is done using the forecast package of R software. The results of the test are given in Table 7.

Table 7. Diebold mariano test for goat data–logistic model

Female		Male	
DM Value	*p* Value	DM Value	*p* Value
3.160	0.004	3.812	0.001

The results implied that the mixed effect model performed better than the fixed effect models. The mixed effect models predict the body weight closer to the actual body weights. On the basis of above results it may be concluded that the mixed effect Logistic model is relatively the better choice for such type of body weight data.

Shapiro-Wilk test (Shapiro and Wilk, 1965) is used for checking normality of the fixed and mixed Logistic models in both female and male body weight data. The results of the test are summarized in Table 8.

Table 8. Shapiro test for goat data–logistic model

Female				Male			
Fixed model		Mixed model		Fixed model		Mixed model	
Test Value (W)	p Value	Test Value (W)	p Value	Test Value (W)	p Value	Test Value (W)	p Value
0.935	0.446	0.963	0.825	0.915	0.247	0.880	0.089

It can be observed that the p value is greater than 0.05 for both fixed and mixed effect model in both male and female case. The results clearly indicate that both the models hold the normality assumptions. So the fitted fixed and mixed Logistic models were validated for the data.

5. CONCLUSION

Growth studies in animal data and goats in particular are conducted to identify the goats that grow fast when the growth rate of the goats is to be evaluated in a relatively short test period. Additionally, utilization of growth curves allows recognition of goats that can grow to relatively greater slaughter weights and produce carcasses that remain lean. It is in this context, the study was conducted on total of 1704 body weight observations from 142 goats (71 males and 71 females). Gompertz, Logistic and Von-Bertalanffy non-linear growth models have been applied for representing the body weight data of Goats over time. Analysis of variance revealed that mixed effects models had significant influence on the productive performance of goat. Animal growth curve may be well explained by nonlinear mixed effect model as compared to the simple nonlinear fixed effect model. It has been explored that the

Logistic model has exhibited relatively best fit to the goat data. Further, the accuracy of mixed effect logistic model was found to be relatively higher than the fixed effect Logistic model. In general, it is found that the accuracy is relatively more in female data as compared to the male data as far as modelling of body weight is concerned. The residuals of the fitted models were examined for model adequacy by using Shapiro Wilk test. It has been found that in all the cases residuals were normally distributed.

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