आई.ए.एस.आर.आई./पी.आर.-09/2020
I.A.S.R.I./P.R.-09/2020

संस्थान परियोजना कोड: AGENIASRISIL201700300089

हर कदम, हर डगर
किसानों का हमसफर
भारतीय कृषि अनुसंधान परिषद
Agrisearch with a human touch

परियोजना रिपोर्ट
Project Report

फसल एवं पशु प्रजनन कार्यक्रमों के लिए त्रि-पथ और चार-पथ आनुवांशिक क्रॉस संबंधित अभिकल्पनाएं

Designs Involving Three-way and Four-way Genetic Crosses for Crop and Animal Breeding Programmes

मोहम्मद हारून	Mohd Harun
अनिंदिता दत्ता	Anindita Datta
सिनी वर्गीस	Cini Varghese
सीमा जग्गी	Seema Jaggi

परीक्षण अभिकल्पना प्रभाग
Divison of Design of Experiments

2020

भा.कृ.अ.प.–भारतीय कृषि सांख्यिकी अनुसंधान संस्थान
ICAR- Indian Agricultural Statistics Research Institute
An ISO 9001:2008 Certified Institute

आमुख

प्रजनन तकनीकों का उपयोग व्यवसायिक वर्ण-संकरों के विकास के साधन की तरह किया जाता है जिसमें पादप एवं पशु प्रजनकों का मुख्य उद्देश्य अनुवांशिक क्षमता बढ़ाना होता है। कोई भी प्रजनन परीक्षण अलग-अलग लाइनों की सामान्य संयोजन क्षमता प्रभावों एवं वर्ण-संकरों की विशिष्ट संयोजन क्षमता प्रभावों के बारे में सूचना प्राप्त करने हेतु केन्द्रित होता है। सामान्य संयोजन क्षमता एवं विशिष्ट संयोजन क्षमता प्रभावों पर एकत्रित सूचनाएं सर्वश्रेष्ठ अभिभावक लाइनों के दोष-रहित चुनाव करने हेतु एक आधार का निर्माण करती है। विशिष्ट संयोजन क्षमता प्रभाव न्यून होकर भी प्रजनकों के लिए सामान्य संयोजन क्षमता प्रभावों से अधिक महत्वपूर्ण होते हैं एवं इनपर प्राप्त सूचनाएं आनुवांशिक एवं आर्थिक महत्व की विभिन्न विशेषताओं को समाविष्ट करने हेतु आधार प्रदान करती हैं।

इस शोध कार्य में उच्च वर्ण-संकरों जैसे की त्रि-पथ एवं चार-पथ वर्ण-संकरों के अध्ययन हेतु शोध को एक महत्वपूर्ण विषय माना गया है क्योंकि प्रजनक सामान्य संयोजन क्षमता प्रभावों के साथ–साथ विशिष्ट संयोजन क्षमता प्रभावों पर अधिक से अधिक सूचनाएं एकत्र कर, जो कि त्रि-पथ एवं चार-पथ वर्ण-संकरों द्वारा संभव है, वर्ण-संकरों में वांछनीय अनुवांशिक गुण अंतनिर्विष्ट करने में अधिक अनुरक्त होता है। लेकिन प्रजनक समान्यतः उच्च क्रम वर्ण-संकरों का प्रयोग करने में उनके बड़े आकार के कारण असमर्थ होता है और साहित्य में उपलब्ध पद्धतियाँ, वर्ण-संकरो की बड़ी संख्या वाली अभिकल्पनाएँ देती हैं जिनको संभालना प्रजनकों के लिए प्रायः असंभव होता है। अतः इस रिक्तता हेतु लघु एवं दक्ष अभिकल्पना की संरचना की सरल एवं सामान्य पद्धति तैयार की गयी है। कम विभाजन की मात्रा के साथ उच्च दक्षता वाली त्रि-पथ एवं चार-पथ वर्ण-संकरों हेतु अभिकल्पनाएँ एवं योजनाएँ तैयार की गयी हैं ताकि उनका उपयोग संसाधनों कि कमी होने कि स्थिति में किया जा सके। ये पद्धतियाँ प्रचलिक संयोगों के लगभग सभी समुच्चयों हेतु अभिकल्पनाएँ / योजनाएँ उपलब्ध कराती हैं।

किसी भी प्रजनन परीक्षण को करते समय प्रजनकों के सामने ऐसी परिस्थिति आ जाती है जब उनके पास नई परीक्षण लाइनों का विकसित समुच्चय हो जिनकी तुलना वे पहले से ही उपस्थित मानक नियंत्रण लाइनों से उनके सामान्य संयोजन क्षमता प्रभावों के सापेक्ष करना चाहते हैं। यदि प्रजनकों के पास परीक्षण लाइनों की एक बड़ी संख्या है, तो यह

संभव हो सकता है कि संचालित हो रहे प्रजनन कार्यक्रमों में इन सभी प्रारम्भिक विकसित लाइनों का बहुत अधिक महत्व न हो। इस प्रकार नई विकसित परीक्षण लाइनों की तुलना नियंत्रण लाइनों से करते हुए उनके प्रदर्शन के आधार पर केवल कुछ नयी विकसित परीक्षण लाइनों को लेकर आगे बढ़नें में संसाधनों कि बचत होगी। अतः परीक्षण लाइनों की तुलना एक नियंत्रण लाइन से करने हेतु आंशिक त्रि-पथ वर्ण-संकर योजना की संरचना पद्धति विकसित की गयी है। यह पद्धति बहुत ही सरल है और इनसे प्रचलिया संयोग की विस्तृत परास के लिए योजनाएँ प्राप्त की जा सकती हैं। इन योजनाओं की विस्तृत क्ष्रेणी के लिए विभाजन की मात्रा कम होने के कारण प्रयोक्ता इनका उपयोग सीमित साधनों की उपलब्धता की स्थिति मे कर सकते हैं।

एक अन्य स्थिति में, प्रजनक के सामने लाइनों के दो समुच्चय होते हैं और प्रजनक के लिए इन दोनों समुच्चयों का महत्व समान नहीं है। इस स्थिति में बेहतर गुणों वाली प्राथमिक लाइनों के समुच्चय से, प्रत्येक लाइन का वर्ण-संकर शेष अन्य लाइनों से किया जाना चाहिए और द्वितीयक लाइनों के समुच्चय से एक समुचित भाग लिया जा सकता है। इस प्रकार की परिस्थिति हेतु संसर्ग अभिकल्पनाओं को प्रायः उच्च वर्ण-संकरों हेतु संवर्धित अभिकल्पनाओं के नाम से जाना जाता है। अतः प्राथमिक बनाम प्राथमिक, प्राथमिक बनाम द्वितीयक एवं द्वितीयक बनाम द्वितीयक लाइनों की तुलना हेतु संवर्धित आंशिक चार-पथ वर्ण-संकर योजनाओं की संरचना पद्धति का विकास किया गया है। योजनाओं की विकसित श्रेणी की विभाजन की मात्रा भी कम होती है।

त्रि-पथ एवं चार-पथ वर्ण-संकर परीक्षण हेतु अभिकल्पनाओं की प्रत्येक विकसित श्रेणी के लिए सामान्य संयोजन क्षमता प्रभावों से संबन्धित मौलिक व्यतिरेकों के प्रामाणिक दक्षताओं एवं औसत प्रसरण घटकों के अनुमानों की गणना हेतु SAS कोड्स लिखे गए हैं। ये कोड्स परीक्षण करने हेतु योग्य दक्ष अभिकल्पनाओं के चयन करने में प्रजनकों के लिए सहायक एवं प्रयोक्ता अनूकूल हैं क्योंकि इसमें परिणाम प्राप्त करने हेतु प्रयोक्ता को केवल अभिकल्पना एवं कुछ प्राचलों की प्रविष्टि करनी होती है।

सभी लेखक, भा.कृ.अनु.प.–भारतीय कृषि सांख्यिकी अनुसंधान संस्थान के निदेशक का, कार्य सफलतापूर्वक करने हेतु सभी आवश्यक सुविधाएं उपलब्ध करवाने व उनके समर्थन के लिए हार्दिक धन्यवाद व्यक्त करतें हैं। प्रभागाध्यक्ष, परीक्षण अभिकल्पना प्रभाग, भा.कृ.अनु.प.–भारतीय कृषि सांख्यिकी अनुसंधान संस्थान के सभी वैज्ञानिकों, तकनीकी

कर्मियों एवं अन्य कर्मचारीगणों से प्राप्त सहयोग के लिए धन्यवाद सहित आभार व्यक्त करतें हैं। हम, श्री देवेंद्र कुमार का विशेष धन्यवाद करते हैं। हम आंतरिक निर्णायक के बहुमूल्य सुझावों के लिए धन्यवाद व्यक्त करते हैं जिससे हमें इस रिपोर्ट की विषय-वस्तु एवं प्रस्तुति को सुधारने में सहायता मिली।

[मोहम्मद हारून] **[अनिंदिता दत्ता]** **[सिनी वर्गीस]** **[सीमा जग्गी]**

Preface

Breeding techniques are used as a tool for the development of commercial hybrids for which a major objective of plant and animal breeders is to raise the genetic potential. Any breeding experiment centres on acquiring information regarding the general combining ability (gca) effects of the individual lines and the specific combining ability (sca) effects of the crosses. The information collected on gca and sca effects forms a basis of making correct choice of the best parental lines. Although small, but sca effects are of much importance for breeders besides the gca effects. Information on sca effects can form a basis for introducing various traits of genetical as well as economical importance.

In this investigatory study, higher order crosses like three-way and four-way crosses are considered as a researchable topic because breeders are much interested to collect more and more information on sca effects along with the gca effects, which higher order mating designs like three-way and four-way cross designs can provide, to inculcate desirable breeding qualities in the hybrids. But breeders generally avoid using higher order crosses due to increased size of experimentation and the methods available in literature are producing designs with a huge number of crosses which makes it almost impossible for the breeders to handle it. Thus easy and general methods of construction of small and efficient designs for the gap existing have been obtained. Three-way and four-way cross plans have also been obtained with low degree of fractionation and high efficiencies that be used when there is scarcity of resources. These methods provide designs for almost all sets of parametric combinations.

While performing any breeding experiment, breeders come across a situation where they have a set of newly developed test lines which they want to compare with an already existing standard control line with respect to their gca effects. Since a large number of test lines are developed by the breeders, it may be possible that all the initially developed lines are not of much worth to be carried along the breeding programme. Thus, comparing the newly developed test lines with the control line and then further proceeding with only few test lines on the basis of performance will be always economical in terms of resource utilization. Hence, method of constructing partial three-way cross plans for comparing test lines with single control line has been developed. The method of construction is very simple and these plans are available for wide range of parametric combination. The developed class of plans is having low degree of fractionation and hence experimenter can use them in case limited resource availability.

In another situation, the breeders come across with them two sets of lines. These two sets of lines are not of equal importance from the breeder's point of view. In this situation, from the set of primary lines of superior quality, every line should be crossed with the remaining other lines and from the other set of secondary lines suitable fraction can be taken. Mating designs for these types of conditions are often referred as augmented designs for higher order crosses. Thus, method of constructing Augmented Partial Four-way Cross (APFC) plans for comparing primary Vs. primary, primary Vs. secondary and secondary Vs. secondary lines has been developed. The developed class of plans is having low degree of fractionation.

SAS codes for calculating canonical efficiencies and average variance factor estimates of elementary contrasts pertaining to the gca effects have been written for every developed class of designs for three-way and four-way cross experiment. These codes will help the breeders to choose an efficient design suitable for the experimentation. The codes are user friendly as the user has to just enter the design along with few parameters to get the result.

The authors express their sincere thanks to the Director, ICAR-IASRI for his support and for providing all necessary facilities to carry out the research work successfully. The cooperation received from Head, Division of Design of Experiments, ICAR-IASRI, scientific, technical and other staff in the Division is thankfully acknowledged. We would like to mention our special thanks to Sh. Devendra Kumar. We express our thanks to the internal referee whose suggestions helped in improving the contents and presentation of this report.

[Mohd Harun] **[Anindita Datta]** **[Cini Varghese]** **[Seema Jaggi]**

CONTENTS

CHAPTER 1

INTRODUCTION AND BACKGROUND

1.1 Introduction

Breeding techniques are used as a tool for the development of commercial hybrids for which a major objective of plant and animal breeders is to raise the genetic potential. Any breeding experiment centres on acquiring information regarding the general combining ability (gca) effects of the individual lines involved as parents and the specific combining ability (sca) effects of the crosses. The information collected on gca and sca forms a basis of making correct choice of the best parental lines. The sca effects are of much importance for breeders besides the gca effects. Information on sca effects can form a basis for introducing various traits of genetical as well as economical importance.

To compare in a stretch, in case of two-way crosses, only one component of sca effects, which is first order sca effect can be studied whereas in three-way crosses two components of sca which are, first order and second order sca effects and in case of four-way crosses three components which are first, second and even third order sca effect can be studied. Thus we can see that the higher order mating designs like three-way and four-way crosses are useful in exploiting the epistatic gene action needed to inculcate desirable breeding qualities in the hybrids. However, two-way cross is the most simple and easily manageable mating design, but at the same time three-way and four-way cross based hybrids are found to be genetically more viable, stable and consistent in performance. Also, breeders can introduce and improve the performance of crops and animals using the additional information provided to them while performing experimentation with three-way and four-way crosses if they are provided with suitable designs.

In practice, while performing any breeding experiment, generally the breeders come across a situation where they have a set of newly developed and identified lines (called the test lines) which they want to compare with an already existing stable line or standard line (called the control line) with respect to their gca effects. Since a large number of test lines are developed by the breeders, it may be possible that a large number of initially developed lines are not of much worth to be carried along the breeding programme, as it will demand for huge amount or resources which is almost impossible to handle in the situation of resource scarcity. Thus, comparing the newly developed test lines with the control line and then further proceeding with only few test lines on the basis of performance will be always economical in terms of resource utilization.

While performing any breeding experiment, the breeders come across a unique type of situation in which they are having with them two sets of lines. These two sets of lines are not of equal importance from the breeder's point of view. In such cases the breeder will be always interested to give a higher weight and having a higher proportion of superior lines (also called as primary lines). In this situation, from the set of lines of superior quality, every line should be crossed with the remaining other lines (also called as secondary lines) and from the other set of lines suitable sample fraction can be taken. Mating designs for these types of conditions are often referred as augmented designs for higher order crosses. Thus an augmented three-way cross is a mixture of complete three-way cross and partial three-way cross. In a similar manner, one can define the augmented four-way cross designs.

Statistical techniques related to designing and analyses of experiments are uninterruptedly used by the breeders and are having indispensable role while going for any breeding related study or experimentation. Breeding programmes are conducted using suitable designed experiments. When the size of experiment is large, then in order to handle the variability in the experimental material it is necessary to group it in various categories which are maximum possible homogenous within themselves and heterogeneous to each other. This concept leads to blocking concept in the field of design of experiments. Higher order crosses, like three-way and four-way cross experiments involve larger number of crosses which makes it difficult to handle as a single group. Even if we use block designs, it may be possible that a complete block is of very large in size and thus becomes heterogeneous, which is not acceptable. In this situation incomplete block designs with smaller block sizes are desirable. Hence, it is important to have small and efficient designs for mating designs of higher order.

1.2 Genesis and Rationale of the Project

There is always a scope for plant and animal hybrids production area as both are increasingly gaining popularity among the breeders and also the industrial sector. Breeders avoid using higher order crosses due to increased size of experimentation because as the number of lines increases, the number of crosses in three-way and four-way crosses increases manifold and becomes unmanageably large for the investigator to handle. At the same time, breeders are much interested to collect more and more information sca effects along with the gca effects. Hence, developing small and efficient three-way and four-way cross designs will not only attract the breeders to use them but the information obtained on the higher order sca effects may also help to improve the quantitative traits which are of economical as well as nutritional importance in crops and animals.

Some methods of construction for both complete and incomplete higher order (three-way and four-way) genetic cross plans are available in the literature for much specified conditions in terms of specific number of lines, specific orthogonality conditions requirement, etc. Moreover, the methods available in literature are producing designs with a huge number of crosses which makes it almost impossible for the breeders to handle it. In this kind of situations breeders are not much interested in using these unmanageable number of crosses. Thus, developing easy and general methods of construction of designs for multi-way crosses, for the gap existing would be helpful. Besides, developing combined mating-environmental designs under blocked set up would be helpful for the breeders to obtain a crossing plan along-with the layout plan. Also, obtaining mating–environmental designs in smaller number of crosses per units would be advantageous for the breeders.

In many experimental situations, there is a need to develop designs involving higher order genetic crosses for the purpose of comparing test lines and control lines with respect to their general combining ability effects. In literature there are very few studies available for developing designs for three-way and four-way crosses which can be used for test lines versus control lines comparison. The designs available so far are either producing layouts with a large number of crosses are not efficient. So, there is a need to develop some designs involving three-way and four-way crosses which are small as well as efficient for making comparisons between test line group and control lines group with respect to their general combining ability effects.

There is a need to develop designs for the situation where the breeders are interested in comparing two groups of parental lines of unequal economical or breeding importance. Thus, augmented designs, more specifically augmented partial three-way and augmented partial four-way cross plans are most fitted in this type of situations. Hence, there is a need to develop suitable designs where the breeders can make comparisons among lines with different levels of accuracies as per interest and arrive at useful and valid conclusions regarding usefulness and worth of parental lines. Small and efficient designs with crosses arranged in blocks will be much useful for the breeders of interest.

The breeders always find it difficult to choose an appropriate breeding plan based on the literature available on breeding experiments or the mating-environmental designs developed by statisticians and moreover, if they choose a design then they face difficulty in later stages of the experiments. The breeders are always interested in readymade layout plans for three-way or four-way crosses so that they can make the comparisons about the gca effects of the lines in a simple and efficient way. Thus, a list of the designs developed will attract the breeders to adopt the multi-way genetic crosses for their breeding experiments.

Moreover, SAS codes for calculating canonical efficiency factors and variance estimates of elementary contrasts pertaining to the gca effects for designs involving multi-way genetic crosses will be helpful for the breeders to choose an efficient design within the parametric limitations. It will be helpful for the breeder in choosing an appropriate plan for the given experimental situation.

Experimental Situations: There are many cases of plant (like maize) and animal (like swine and chicken) breeding where three-way and four-way crosses are the commonly used techniques of producing commercial hybrids.

These techniques (three-way and four-way crosses) help the breeders to improve the quantitative traits which are of economical as well as nutritional importance in crops and animals (Shunmugathai and Srinivasan, 2012).

Most of the common commercial hybrids in corn are either three-way or four-way cross hybrids. It has been established that the three-way or four-way cross hybrids are more stable than the pure lines and the diallel cross hybrids.

Three-way and four-way cross hybrids exhibit individual as well as population buffering mechanism because of broad genetic base (Khawaja *et al.,* 2013).

Three-way crossbred chickens showed better egg traits than two-way crossbred chickens with lower mortality.

The following figure shows one such trial at Directorate of Poultry Research, Hyderabad.

Parents Involved	Promising Qualities
PD1	High no. of eggs.
IWI	Higher egg weight
PD3	Higher body weight

NEW INITIATIVES

library periodicals' directory, reports, old question papers, research highlights, manuals, technical reports, e books, reprints, newsletters, advances and e journals' database of all units of the SKNAU. To make IR information accessible to online user community, it has to be stored as a Repository to create Institutional Database for further use and sharing among all unit libraries. A digital library portal has been created to bring digital collections and services of all libraries at one platform for speedy and prompt use of e resource collections. The digital library facilities have also been provided to all unit libraries of different teaching campuses of the University. More than 400 question papers, 40 manuals, 280 MSc /PhD theses, newsletters, reports, Webopac are available online through weblink http://117.240.227.133/centrallibrary/and http://117.240.227.134/webopac/ on the university Website link. The online archiving e journals' database facility will be created by maintaining back-up files of the list of research contributions of faculty / scientists / students under the Institutional Repositories (IR) to serve as a meaningful indicator of the university's agriculture research contributions.

K.M. Krishna
Sri Karan Narendra Agriculture University
Jobner (Rajasthan) 303 329
e-mail: kmkskn@gmail.com

A promising 3-way cross for backyard poultry

Backyard poultry is for rearing birds with desirable plumage and high performance compared to local indigenous birds, with very little changes in husbandry practices, followed for indigenous fowl. Improvement in genetic make-up can be achieved either by pure-line selection or by crossbreeding to improve desirable traits through heterosis or combination of both. Different pure-lines are developed through selection and are crossed to develop crossbreds for backyard poultry farming. Higher shank length with moderate body weight helps birds to run faster; protecting themselves from predators.

One such line PD1 has been developed with higher shank length; to be used as a male parent for backyard poultry. PD1 was used for developing different 2 way crosses. However, there is a requirement of 3-way cross to supply easily parent stocks and for exploiting heterosis. Keeping this in view, a 3-way cross was produced using PD1 × IWI male and PD3 female. PD1 ×IWI × PD3 was evaluated for different traits under intensive system of rearing.

3-way cross grower

Females in cages

Body weight of male and female at 16 weeks of age was 1670±62 and 1096±14 g, respectively. Corresponding shank length was 122±2 and 99±0.5 mm. Weight at sexual maturity was 1702±20 g. Body weight recorded at 40 and 72 weeks of age was 1971±26 and 2182±33 g, respectively. The age at sexual maturity was 163±1 days. Egg production per bird up to 40, 64 and 72 weeks of age was 91.71±1.37, 204.48±2.50 and 233.28±3.18 eggs, respectively. The egg weight during early age was high, and at 28, 40 and 52 weeks of age was 51.25±0.37, 57.15±0.39 and 60.05±0.47 g, respectively. Eviscerated carcass yield (%) at 16 weeks of age for male was 66.12±0.36 %. The birds consumed on an average 120g feed per bird per day during the laying period. The performance of this cross has been very encouraging as it lays good number of eggs with high egg weight; indicating that this cross may be used as a potential egg-type backyard poultry variety. The multicolour plumage of this cross will be of great help for its popularization in the field. The male can be sold at 16 weeks of age for meat purpose. The higher body weight of the female at the end of a year cycle at 72 weeks of age may also fetch good price in the market for meat purpose. The brown colour of the eggs of this cross would be liked by farmers and consumers in the rural areas. Quality of eggs measured at different weeks of age indicated them of good quality. However, before large-scale supply of this cross, birds must be evaluated in the field under backyard system.

M.K. Padhi, R.N. Chatterjee and M. Niranjan
ICAR-Directorate of Poultry Research
Hyderabad (Telangana) 500 030
e-mail: rnch65@gmail.com

5 JANUARY-MARCH 2015

Further, it has been found that three-way or four-way is the most practical and acceptable scheme for the production of slaughter pigs having fast growth rate, good feed efficiency, and carcass quality. Three-way or four-way is mostly used for exploitation of heterosis in case of commercial silkworm production.

Keeping the above in view, the following objectives have been formulated:

Objectives

- To obtain suitable designs for test line(s) versus test line(s) comparison for partial three-way and four-way cross experiments
- To obtain designs for test line(s) vs. control line(s) comparisons in three-way and four-way cross experiments
- To obtain suitable designs for augmented partial three-way and four-way cross experiments
- To obtain a web solution/SAS macro(s) for the generation and analysis of three-way and four-way cross experiments

1.3 Critical Review of the Technology at National and International Levels

National

Three-way and four-way cross hybrids are genetically more viable, stable and consistent in performance than two-way cross hybrids. Most of the common commercial hybrids in corn are either three-way or four-way cross hybrids (Shunmugathai and Srinivasan, 2012). It has been established that the three-way or four-way cross hybrids are more stable than the pure lines and the two-way cross hybrids. Three-way and four-way cross hybrids exhibit individual as well as population buffering mechanism because of broad genetic base.

Hinkelmann (1965) for the first time introduced the concept of partial three-way crosses and gave a method of construction using generalized partially balanced incomplete block (GPBIB) designs and gave its analysis. Even for moderate sizes of n, say 20, leads to quite unmanageable experiments, therefore he proposed a new type of design which he called the partial three-way crosses. Hinkelmann (1965) proposed that partial three-way crosses are related to incomplete block designs. This relationship is utilized for the purpose of constructing and analyzing appropriate plans. Ponnuswamy (1971) suggested methods of construction of the incomplete block designs for three-way crosses, using Latin square and Graeco Latin square.

Arora and Aggarwal (1984) discussed application of extended triangular design as the confounded three-way cross experiments. The total degrees of freedom were partitioned into three orthogonal sets, said to belong to gca, first order sca and second order sca effects. Arora and Aggarwal (1989) extended their previous work for three-way cross experiments with reciprocal effects, where, the total degrees of freedom were partitioned into four sets, said to belong to gca, first order sca effects, second order sca effects and reciprocal effects, respectively.

Kaistha (1982) showed that the analysis of two-way cross experiment suggested by Griffing (1956) does not appear to be appropriate in the sense that the partitioning of the total variation into mutually orthogonal components due to gca and sca is not feasible.

Venkatesan (1985) considered flexible type of PDC experiments wherein one group of lines was crossed with lines of another distantly placed group with the assumption that crosses within each group are carried out at an earlier stage.

A method of construction of PTC using a special class of BIB designs and PBIB designs which preserves the property of three-way mating design has been developed by Ponnuswamy and Srinivasan (1991). Ponnuswamy and Subbarayan (1991) have given the analysis of PTC plans.

The higher order mating designs like three-way and double crosses have been found to be useful in exploiting the epistatic gene action (Srinivasan and Ponnuswamy, 1995).

Srivastava *et al.* (2013) obtained two-way cross desins for test vs. control comparisions. Harun (2014) discussed various methods of constructing designs for three-way cross experiments using MOLS, association schemes and PBIB designs. The variance factor of contrasts pertaining to estimated gca effects of first and second kind has been given. Srivastava *et al.* (2015) have also obtained some classes of augmented partial diallel cross (APDC) plans using various types of association schemes of Partially Balanced Incomplete Block (PBIB) designs.

Harun *et al.* (2016 a) developed some methods of constructing designs for breeding trials involving complete/partial three-way crosses based on mutually orthogonal Latin squares, which yield three-way crosses arranged in blocks and based on two-associate class partially balanced incomplete block designs. The efficiency factor in terms of information per cross pertaining to general combining ability effects of half parents as well as full parents in comparison to a complete three-way cross plan, assuming the error variance to be same for both the plans, has been computed.

Harun *et al.* (2016 b & c) developed some methods for constructing classes of partial three-way cross designs for comparing a set of test lines with a control line. The proposed class of designs are variance balanced for estimating the contrasts pertaining to general combining ability effects of half as well as full parents. Harun *et al.* (2017, 2019) developed PTC plans based on BIB designs.

International

Rawlings and Cockerham (1962) gave the analysis for four-way cross hybrids. Analysis of variance was done and interpreted in terms of the variances of the effects defined in a linear model.

Optimal block designs for three-way cross experiments are investigated and several series of nested block designs, leading to optimal designs for three-way crosses have been reported by Das and Gupta (1997).

A systematic method of construction of PTC using Trojan Square Design has been developed by Dharmalingam (2002). Mating designs have been obtained using generalized incomplete Trojan type designs (Varghese and Jaggi, 2011).

Das *et al*. (2006) further investigated this problem and derived a sufficient condition for designs to be A-optimal. Using the lower bound to A-efficiency, type S_0 designs are shown to yield efficient designs. He considered the set-up of t (> 2) test lines and one control line.

Choi *et al*. (2004) studied two-way crosses for comparing a control line with test line under the model for completely randomized designs and listed designs that estimate control versus test comparisons with a minimum variance within a practical range of parameters. Type S designs with nested blocks were introduced and some series of type S block deigns were provided.

Hsu and Ting (2005) studied A-optimality of two-way cross experiments for comparing two or three test lines with a control line under the model for block designs. Families of A-optimal and efficient S_0 block designs were derived.

1.4 Scope of Present Study

Mating designs find an important place in the area of research done in the field of genetics as well as statistics with an idea of developing hybrids engrossed with desired traits of economical and genetical importance. A large number of breeders and statisticians working in this area and volume of research work published in this area show the importance of this topic.

In the present study, an attempt has been made to supplement and carry forward the work done in the area of designs involving higher order crosses, *i.e.*, three-way and four-way crosses. A general introduction and background to the topic along with the national and international review of literature related to the research topic is given in this chapter.

In Chapter II of this report development of general methods of constructing designs and plans for test versus test line comparison for three-way and four-way cross experiments has been discussed. Under a fixed effects model including sca effects for three-way crosses, the estimates of gca and sca have been obtained. Under a restricted model including lower order sca effects for four-way crosses, the orthogonal estimates of general and specific combining ability effects under a block design set up have been obtained. Various characterization properties including information matrices, eigenvalues, average variance factors, efficiency factors and degree of fractionation have been derived for developed class of designs. SAS codes for computation of information matrices, variance factors and canonical efficiency factor related to three-way cross experiments under blocked and unblocked setup.

Chapter III focuses on method of constructing designs for test versus control line comparison for three-way cross experiments. A class of partial three-way cross plans for comparing test-lines with single control-line has been developed. The method is based on incomplete block designs .The average variance factor pertaining to estimated gca effects of half parents as well as full parents for test versus test lines and test versus control line comparisons has been calculated. The method is illustrated through example and list of designs consisting of various parameters is also provided. SAS code has been written for computing average variance factors related to estimates of contrasts pertaining to gca effects of test Vs. test and test Vs. control lines for half parents as well as full parents under unblocked setup.

Chapter IV dedicates to augmented partial four-way cross designs. A method of constructing augmented partial four-way cross plans has been developed. The method is illustrated through an example. The information matrix of the developed class of augmented partial four-way cross plan has been derived by considering the usual fixed effect model for four-way crosses under unblocked setup. A list of augmented partial four-way cross plans has been prepared consisting of various parameters along with the degree of fractionation, and variance estimates of contrasts pertaining to gca effects of primary Vs. primary, primary Vs. secondary and secondary Vs. secondary lines. SAS code has been written for computing variance estimates of contrasts pertaining to gca effects of primary Vs. primary, primary Vs. secondary and secondary Vs. secondary lines for augmented partial four-way cross plans.

Finally, the report concludes with a brief summary (in English and Hindi), abstracts (in English and Hindi), references and annexures (I-IV) consisting of SAS codes.

CHAPTER 2

PARTIAL THREE-WAY AND FOUR-WAY CROSS DESIGNS

2.1 Introduction

Greater genetical viability, and stability along with consistency of higher order crosses as compared to two-way crosses, is the main attraction to breeders for adopting higher order mating plans like three-way and four-way crosses. Besides this, three-way and four-way cross hybrids exhibit higher individual as well as population buffering mechanism because of the broad genetic base. For breeders, sca effects are of much importance in addition to gca effects and these techniques provide ample amount of information on sca effects and help the breeders to improve various traits which are of economical as well as nutritional importance in crops and animals.

2.2 Partial three-way cross designs

Three-way cross hybrids find a vital role in the area of plant and animal breeding experiments due to their uniformity, stability and the relative simplicity of selecting and testing. Three-way crosses are intermediate between two-way and four-way crosses with respect to number of lines used, complexity of handling the crosses and the amount of information regarding combining abilities.

2.2.1 Model, estimates of combining abilities and information matrices

There are various methods of analyzing the data collected through a three-way cross experiment, based on the model considered. The model may include all types of sca effects, or only lower order sca effects or may not include sca effects, along with gca effects, as per the experimental objectives.

Full model with sca effects

Consider a three-way cross experiment involving N number of lines giving rise to T number of crosses. Let a cross of type $(i \times j) \times k$ is represented as (i, j, k) and the fixed effect of the three-way cross (i, j, k) by y_{ijk}, then the following model can be used for representing cross effect:

$$y_{ijk} = \bar{y} + h_i + h_j + g_k + s_{ij} + s_{ik} + s_{jk} + s_{ijk} + e_{ijk},$$

where $\bar{y}$ is the average effect of the crosses, $\{h_\alpha\}, \alpha = i, j$ and $\{g_k\}$ represents the gca effects half parents and full parents respectively, $\{s_{\alpha\beta}\}, (\alpha, \beta) \in (i, j, k)$ represents the first order sca effects, s_{ijk} represents the second order sca effects, e_{ijk} represents the random error component with the constraints

$$\sum_{i=1}^{N} h_i = 0 \text{ and } \sum_{i=1}^{N} g_i = 0,$$

$$\sum_{\alpha\beta} s_{\alpha\beta} = 0 \; \forall \; (\alpha, \beta) \in (i, j, k), i \neq j \neq k = 1,2, \dots, N \text{ and}$$

$$\sum_{ijk} s_{ijk} = 0 \; \forall \; i \neq j \neq k = 1,2, \dots, N.$$

It is important to note here that if a cross (i, j, k) is occurring in the experiment then the other two alternative forms (i, k, j) and (j, k, i) are also included in the experiment, to satisfy the structural symmetry property (SSP) of three-way crosses.

Restricted model with first order sca effects

Let us consider that the three-way crosses (i, j, k), are arranged in the order $(1, 2,3)$, $(1, 2,4)$,...,$(1,2, k)$,...,$(1,2, N)$,$(1,3,4)$,...,$(1, j, j + 1)$,...,$(1, (N - 1), N)$,$(2, 3,4)$,..., $(i, i + 1, i + 2)$,..., $\{(N - 2), (N - 1), N\} \; \forall \; i \neq j \neq k = 1,2, \dots, N$. It should be noted that along with every cross of the type (i, j, k) crosses of the types (i, k, j) and (j, k, i) are also included in the experiment simultaneously. Let $\boldsymbol{g} = (g_1, \; g_2, \dots, g_N)'$ be a $N \times 1$ vector of gca effects of full parents, $\boldsymbol{h} = (h_1, \; h_2, \dots, h_N)'$ be a $N \times 1$ vector of gca effects of half parents. $\boldsymbol{y}$ and $\boldsymbol{s}$ be $T \times 1$ vectors whose elements are $\{y_{ijk}\}$ and $\{s_{ijk}\}$, respectively. Let us define a matrix $\boldsymbol{W}$of order $2N \times T$ with rows indexed by the line numberings as $1,2, \dots, N$ repeatedly two times and, the columns by the T number of crosses, in same order as described previously, of the types (i, j, k), including the crosses of the types (i, k, j) and (j, k, i) also simultaneously. Then, the $\{t, (i, j, k)\}^{\text{th}}$ entry of $\boldsymbol{W}$ takes a value 0.5 if $t \in (i, j)$, takes a value 1 if $t \in k$ and 0, otherwise. Before arriving at the final model we have $\boldsymbol{W}\mathbf{1}_T = \frac{(N-1)(N-2)}{2}\mathbf{1}_{2N}, \boldsymbol{W}'\mathbf{1}_{2N} = 2\mathbf{1}_N$ and $\bar{y} = \frac{\mathbf{1}_T'\boldsymbol{y}}{T}$.

Now the model can be rewritten in matrix notation as follows:

$$\boldsymbol{y} = \bar{y}\,\mathbf{1}_T + \boldsymbol{W}' \begin{pmatrix} \boldsymbol{g} \\ \boldsymbol{h} \end{pmatrix} + \boldsymbol{s} + \boldsymbol{e},$$

where $\bar{y}$ is the average effect of the crosses, $\mathbf{1}_T$ is the $T \times 1$ vector of unity and $\boldsymbol{e}$ is the $T \times 1$ vector of random errors . The constraints

$$g_1 + g_2 + \cdots + g_N = 0 \text{ or } \textstyle\sum_{i=1}^{N} g_i = 0 \text{ or } \mathbf{1}_N'\boldsymbol{g} = 0,$$

$$h_1 + h_2 + \cdots + h_N = 0 \text{ or } \textstyle\sum_{i=1}^{N} h_i = 0 \text{ or } \mathbf{1}_N'\boldsymbol{h} = 0 \text{ and}$$

$$\textstyle\sum_j \sum_{k>j} s_{(jk)i} = 0 \;\forall\; 1 \le i \le N \text{ or } \boldsymbol{Ws} = \mathbf{0},$$

have been imposed to the model.

Before proceeding further it is important to derive the forms of $\boldsymbol{WW'}$ and $(\boldsymbol{WW}')^{+}$.

$$\boldsymbol{WW}' = \begin{bmatrix} \frac{(N-1)(N-2)}{2}\boldsymbol{I}_N & -\frac{(N-2)}{2}(\boldsymbol{I}_N - \boldsymbol{J}_N) \\ -\frac{(N-2)}{2}(\boldsymbol{I}_N - \boldsymbol{J}_N) & \frac{(N-2)^2}{4}\boldsymbol{I}_N + \frac{(N-2)}{4}\boldsymbol{J}_N \end{bmatrix} = \begin{bmatrix} \boldsymbol{A} & \boldsymbol{B} \\ \boldsymbol{B}' & \boldsymbol{D} \end{bmatrix} \text{ (say)}$$

Now, we can see that $rank(\boldsymbol{A}) = rank\,(\boldsymbol{B}) = rank\,(\boldsymbol{D}) = N$ and $rank\,(\boldsymbol{WW}') = 2N-1$. Thus, it is not possible to find the true inverse of $\boldsymbol{WW'}$. Also, we can see that the condition $rank\,(\boldsymbol{WW}') = rank(\boldsymbol{A})$ for finding the generalized inverse of a matrix by partitioning is not satisfied. So, in order to find a unique inverse of $\boldsymbol{WW'}$, it is better to proceed for finding the Moore Penrose inverse denoted as $(\boldsymbol{WW}')^{+}$, through full rank factorization method.

After substituting all the intermediate forms involved in the calculation, the final form of the Moore Penrose inverse $(\boldsymbol{WW}')^{+}$ is

$$\frac{2}{N(N-2)}\begin{bmatrix} \frac{(N-2)}{(N-3)}\left(\boldsymbol{I}_N - \frac{(3N-21N^3+68N^2-104N+64)}{4N(N-1)(N-2)(N^2-4N+8)}\boldsymbol{J}_{N-1}\right) & \frac{2}{(N-3)}\left(\boldsymbol{I}_N - \frac{\begin{matrix}(N^6-19N^5+128N^4-480N^3\\+1024N^2-1216N+512)\end{matrix}}{4N(N-1)(N-2)(N^2-4N+8)^2}\right)\boldsymbol{J}_N) \\ \frac{2}{(N-3)}\left(\boldsymbol{I}_N - \frac{\begin{matrix}(N^6-19N^5+128N^4-480N^3\\+1024N^2-1216N+512)\end{matrix}}{4N(N-1)(N^2-4N+8)^2}\right)\boldsymbol{J}_N) & \frac{2(N-1)}{(N-3)}\left(\boldsymbol{I}_N - \frac{\begin{matrix}(7N^6-69N^5+336N^4-928N^3\\+1536N^2-1344N+512)\end{matrix}}{8N(N-1)^2(N^2-4N+8)^2}\right)\boldsymbol{J}_N) \end{bmatrix}$$

Estimates of gca and sca: The previous results obtained can be used to derive the estimates of gca and sca. Thus, the joint estimates of gca effects of full and half parents can be simplified and expressed as:

$$\begin{pmatrix} \hat{\boldsymbol{g}} \\ \hat{\boldsymbol{h}} \end{pmatrix} = \boldsymbol{H}_1 \boldsymbol{y},$$

where $\boldsymbol{H}_1 = (\boldsymbol{W}\boldsymbol{W}')^{+}\boldsymbol{W} - \frac{\boldsymbol{1}}{2T}\boldsymbol{J}_{2N\times T}$.

Similarly, the combined estimate of sca effects is given as:

$$\hat{\boldsymbol{s}} = \boldsymbol{H}_2 \boldsymbol{y},$$

where $\boldsymbol{H}_2 = (\boldsymbol{I} - \boldsymbol{W}'(\boldsymbol{W}\boldsymbol{W}')^{+}\boldsymbol{W})$.

Also, it can be seen that $rank(\boldsymbol{H}_1) = 2(N-1)$, $rank(\boldsymbol{H}_2) = T - 2N + 1$, $\boldsymbol{H}_1\boldsymbol{1} = \boldsymbol{0}$, $\boldsymbol{H}_2\boldsymbol{1} = \boldsymbol{0}$ and $\boldsymbol{H}_1\boldsymbol{H}_2 = \boldsymbol{H}_2\boldsymbol{H}_1 = \boldsymbol{0}$.

It is clear that $\begin{pmatrix} \boldsymbol{g} \\ \boldsymbol{h} \end{pmatrix}$ and $\boldsymbol{s}$ represent orthogonal treatment contrasts having $2(N-1)$ and $(T - 2N + 1)$ degrees of freedom, respectively and can be used for obtaining orthogonal estimates of functions of gca and sca effects.

Information matrix: Under the usual set up of a block design d, the joint information matrix regarding $\begin{pmatrix} \boldsymbol{H}_1 \\ \boldsymbol{H}_2 \end{pmatrix}\boldsymbol{y}$ is given by the expression:

$$\boldsymbol{C}_{d\_ca} = \begin{bmatrix} \boldsymbol{H}_1\boldsymbol{C}_d\boldsymbol{H}_1' & \boldsymbol{H}_1\boldsymbol{C}_d\boldsymbol{H}_2' \\ \boldsymbol{H}_2\boldsymbol{C}_d\boldsymbol{H}_1' & \boldsymbol{H}_2\boldsymbol{C}_d\boldsymbol{H}_2' \end{bmatrix},$$

where $\boldsymbol{C}_d = \boldsymbol{R}_d - \frac{1}{k}\boldsymbol{N}_d\boldsymbol{N}_d'$, $\boldsymbol{R}_d = diag(r_1, r_2, \ldots, r_T)$ is the diagonal matrix of replications of the crosses under the design d and $\boldsymbol{N}_d$ is the incidence matrix of crosses versus blocks. Here, $\boldsymbol{C}_d$ is the information matrix of the general block design d, where treatments are the T crosses with $\boldsymbol{C}_d\boldsymbol{1}_T = \boldsymbol{0}$. As discussed earlier regarding orthogonality, in order to estimate $\boldsymbol{H}_1\boldsymbol{C}_d\boldsymbol{H}_1'$ and $\boldsymbol{H}_2\boldsymbol{C}_d\boldsymbol{H}_2'$ orthogonally the off diagonal components must vanish and we must have $\boldsymbol{H}_2\boldsymbol{C}_d\boldsymbol{H}_1' = \boldsymbol{H}_1\boldsymbol{C}_d\boldsymbol{H}_2' = \boldsymbol{0}$. Thus, we have $\boldsymbol{C}_{\text{gca}} = \boldsymbol{H}_1\boldsymbol{C}_d\boldsymbol{H}_1'$ and $\boldsymbol{C}_{\text{sca}} = \boldsymbol{H}_2\boldsymbol{C}_d\boldsymbol{H}_2'$.

Model excluding sca effects

In this approach, gca effects of first and second kinds corresponding to half and full parents will be estimated for which it is assumed that the sca effects are contributing less to the total combining ability effects as compared to gca effects. The model can be written as

$$y_{ijk} = \bar{y} + h_i + h_j + g_k + e_{ijk} ,$$

where $\bar{y}$ is the average effect of the crosses, $\{h_\alpha\}, \alpha = i, j$, represents the gca effects of first kind corresponding to the lines occurring as half parents, $\{g_k\}$ represents the gca effects of second kind corresponding to the lines occurring as full parents, e_{ijk} is random error and

$$g_1 + g_2 + \cdots + g_N = 0 \text{ or } \sum_{i=1}^{N} g_i = 0,$$

$$h_1 + h_2 + \cdots + h_N = 0 \text{ or } \sum_{i=1}^{N} h_i = 0.$$

The model in matrix notation is expressed as:

$$\boldsymbol{y} = \bar{y}\,\mathbf{1}_T + \boldsymbol{W}_1'\boldsymbol{h} + \boldsymbol{W}_2'\boldsymbol{g} + \boldsymbol{e},$$

where $\boldsymbol{y}$ is the $T \times 1$ vector of responses due to crosses, $\bar{y}$ is the mean effect of crosses, $\boldsymbol{h}$ is the $N \times 1$ vector of gca effects due to half parents, $\boldsymbol{g}$ is the $N \times 1$ vector of gca effects due to full parents and $\boldsymbol{e}$ is the $T \times 1$ vector of random errors. $\boldsymbol{W}_1$ and $\boldsymbol{W}_2$ are $N \times T$ matrices with rows indexed by the line numbers $1,2, \dots N$ and columns by the three-way crosses arranged in the manner described earlier, such that the $\{t, (i, j, k)\}^{th}$ entry of $\boldsymbol{W_1}$ is 0.5 if $t \in (ij)$ and zero, otherwise and the $\{t, (i, j, k)\}^{th}$ entry of $\boldsymbol{W_2}$ is 1 if $t \in k$ and zero, otherwise. The normal equations are:

$$E(\boldsymbol{y}) = \bar{y}\,\mathbf{1}_T + \boldsymbol{W}_1'\boldsymbol{h} + \boldsymbol{W}_2'\boldsymbol{g}$$

$$\boldsymbol{W}_1 E(\boldsymbol{y}) = \bar{y}\,\boldsymbol{W}_1\mathbf{1}_T + \boldsymbol{W}_1\boldsymbol{W}_1'\boldsymbol{h} + \boldsymbol{W}_1\boldsymbol{W}_2'\boldsymbol{g}, \text{ and}$$

$$\boldsymbol{W}_2 E(\boldsymbol{y}) = \bar{y}\,\boldsymbol{W}_2\mathbf{1}_T + \boldsymbol{W}_2\boldsymbol{W}_1'\boldsymbol{h} + \boldsymbol{W}_2\boldsymbol{W}_2'\boldsymbol{g}.$$

Estimates of gca and sca: On solving these three normal equations, the estimate of gca effects of half parent is given as:

$$\widehat{\boldsymbol{h}} = [(\boldsymbol{W_1}\mathbf{W}_1')^{-}\boldsymbol{W_1} - (\boldsymbol{W_1}\mathbf{W}_1')^{-}\boldsymbol{W_1}\mathbf{J}_T/T)]\boldsymbol{y} = \boldsymbol{G}_1\boldsymbol{y},$$

and the estimate of gca effects of full parent is given as:

$$\begin{aligned}\hat{\boldsymbol{g}} &= [(\boldsymbol{W_2}\boldsymbol{W}_2')^{-}\boldsymbol{W_2} - (\boldsymbol{W_2}\boldsymbol{W}_2')^{-}\boldsymbol{W_2}\boldsymbol{J}_T/N)]\boldsymbol{y}\\ &= \boldsymbol{G}_2\boldsymbol{y}.\end{aligned}$$

It is clear that $\boldsymbol{h}$ and $\boldsymbol{g}$ represents orthogonal treatment contrasts, both having $(N-1)$ degrees of freedom and can be used for obtaining orthogonal estimates of function of gca effects of half and full parents. The restrictions being imposed in order to estimate the gca effect of half parents free from gca effect of full parents are as:

$$\boldsymbol{1}'\hat{\boldsymbol{h}} = \boldsymbol{1}'\hat{\boldsymbol{g}} = \boldsymbol{G}_1\boldsymbol{1} = \boldsymbol{G}_2\boldsymbol{1} = \boldsymbol{G}_1'\boldsymbol{G}_2 = \boldsymbol{0},\ rank(\boldsymbol{G}_1) = rank(\boldsymbol{G_2}) = N-1.$$

Information matrix: Now, under the usual set up of a block design d, the joint information matrix regarding $\begin{pmatrix}\boldsymbol{G}_1\\ \boldsymbol{G}_2\end{pmatrix}\boldsymbol{y}$ is given by:

$$\boldsymbol{C_{d\_gca}} = \begin{bmatrix}\boldsymbol{G}_1\boldsymbol{C}_d\boldsymbol{G}_1' & \boldsymbol{G}_1\boldsymbol{C}_d\boldsymbol{G}_2'\\ \boldsymbol{G}_2\boldsymbol{C}_d\boldsymbol{G}_1' & \boldsymbol{G}_2\boldsymbol{C}_d\boldsymbol{G}_2'\end{bmatrix},$$

where $\boldsymbol{C}_d = \boldsymbol{R}_d - \frac{1}{k}\boldsymbol{N}_d\boldsymbol{N}_d'$, $\boldsymbol{R}_d = diag(r_1, r_2, \dots, r_T)$ is the diagonal matrix of replications of the crosses under the design d and $\boldsymbol{N}_d$ is the incidence matrix of crosses versus blocks. Here, $\boldsymbol{C}_d$ is the information matrix of the general block design d where treatments are nothing but the T number of three-way crosses, hence we have $\boldsymbol{C}_d\boldsymbol{1}_T = \boldsymbol{0}$. In order to estimate $\boldsymbol{G}_1\boldsymbol{C}_d\boldsymbol{G}_1'$ and $\boldsymbol{G}_2\boldsymbol{C}_d\boldsymbol{G}_2'$ orthogonally the off diagonal components have to vanish and hence $\boldsymbol{G}_2\boldsymbol{C}_d\boldsymbol{G}_1' = \boldsymbol{G}_1\boldsymbol{C}_d\boldsymbol{G}_2' = \boldsymbol{0}$. Thus, we have $\boldsymbol{C}_{\text{gca\_half}} = \boldsymbol{G}_1\boldsymbol{C}_d\boldsymbol{G}_1'$ and $\boldsymbol{C}_{\text{gca\_full}} = \boldsymbol{G}_2\boldsymbol{C}_d\boldsymbol{G}_2'$.

2.2.2 Method of construction of partial three-way cross designs

General methods of constructing partial three-way cross plans are described in this section using various types of designs and association schemes.

Method 1: Partial three-way cross plans using triangular association scheme

Let there be $N = \frac{n(n-1)}{2}$ lines, where $n > 4$. Arrange these N lines in a two-associate triangular association scheme, *i.e.*, allot N lines to the off diagonal positions above the principal diagonal in a natural order and repeat the same below the diagonal such that the final arrangement is symmetrical about the diagonal. Diagonal positions are left empty.

Consider all possible pair of lines that can be made from each row of the array. Add a third line to each of these pairs to form triplets. Line that appears at the intersection of the second row containing the first line in the pair and column containing the second line in the pair is considered, and added to each pair to form triplets. Make three-way crosses from these triplets considering lines in the pairs as half parents and third added line in the triplet as full parent. This will result in a partial three-way cross design with parameters $N = \frac{n(n-1)}{2}$, $T = \frac{n(n-1)(n-2)}{2}$, $b = n$, $k = \frac{(n-1)(n-2)}{2}$, $r_h = 2(n-2)$ and $r_f = (n-2)$.

Remark: It can be seen that the above method of construction gives a layout in which the crosses are arranged in six groups. This excludes the necessity for an environmental design for laying out the crosses as these groups can be treated as blocks of a design. Hence, through this method one can get a combination of mating as well as environmental design, at one go.

Example : The method can be well understood by an example for $n = 6$ giving rise to $N = 15$.

*	A	B	C	D	E
A	*	F	G	H	I
B	F	*	J	K	L
C	G	J	*	M	N
D	H	K	M	*	O
E	I	L	N	O	*

The first cross of first block is obtained by considering the first pair of lines (*i.e.*, A & B) as half parents and then crossing it with the line present at the row-column intersection of lines A and B (*i.e.*, F), treating F as the full parent in the cross. Proceeding in the same manner, for all possible pairs of first row one can obtain the three-way crosses to be placed in first block. In a similar manner, from other rows, remaining blocks can be obtained. The design so obtained is:

Block 1	**Block 2**	**Block 3**	**Block 4**	**Block 5**	**Block 6**
(A×B)×F	(A×F)×B	(B×F)×A	(C×G)×A	(D×H)×A	(E×I)×A
(A×C)×G	(A×G)×C	(B×J)×C	(C×J)×B	(D×K)×B	(E×L)×B
(A×D)×H	(A×H)×D	(B×K)×D	(C×M)×D	(D×M)×C	(E×)N×C

(A×E)×I	(A×I)×E	(B×L)×E	(C×N)×E	(D×O)×E	(E×O)×D
(B×C)×J	(F×G)×J	(F×J)×G	(G×J)×F	(H×K)×F	(I×L)×F
(B×D)×K	(F×H)×K	(F×K)×H	(G×M)×H	(H×M)×G	(I×N)×G
(B×E)×L	(F×I)×L	(F×L)×I	(G×N)×I	(H×O)×I	(I×O)×H
(C×D)×M	(G×H)×M	(J×K)×M	(J×M)×K	(K×M)×J	(L×N)×J
(C×E)×N	(G×I)×N	(J×L)×N	(J×N)×L	(K×O)×L	(L×O)×K
(D×E)×O	(H×I)×O	(K×L)×O	(M×N)×O	(M×O)×N	(N×O)×M

Partial Three-way Cross Design

The parameters of this design are $n = 6,\ N = 15,\ T = 60,\ b = 6,\ k = 10,\ r_h = 8$ and $r_f = 4$.

Information matrices: Let $\mathbf{I}_N$ is an identity matrix of order N, $\mathbf{A}_N$ is a matrix of order N whose elements, $\{a_{ij}\}$ takes value 1 if i and j are first associates otherwise 0, and $\mathbf{B}_N$ is a matrix of order N whose elements, $\{b_{ij}\}$ takes value 1 if i and j are second associates, otherwise 0.

The general form of information matrix related to half parents ($\mathbf{C}_{\text{half}}$) is $a_0\,\mathbf{I}_N + a_1\,\mathbf{A}_N + a_2\,\mathbf{B}_N$, where $a_0 = \frac{2(n-3)(n-4)}{(n-2)}$, $a_1 = -\frac{2(n-3)(n-4)}{(n-2)^2}$ and $a_2 = \frac{4(n-4)}{(n-2)^2}$. The general form of information matrix related to full parents ($\mathbf{C}_{\text{full}}$) is $b_0\,\mathbf{I}_N + b_1\,\mathbf{A}_v + b_2\,\mathbf{B}_N$, where $b_0 = (n-4)$, $b_1 = -\frac{(n-4)}{(n-2)}$ and $b_2 = \frac{2(n-4)}{(n-2)(n-3)}$. For the example given earlier, $\mathbf{C}_{\text{half}} = 3\mathbf{I}_{15} - 0.75\mathbf{A}_{15} + 0.5\mathbf{B}_{15}$ and $\mathbf{C}_{full} = 2\mathbf{I}_{15} - 0.5\mathbf{A}_{15} + 0.33\mathbf{B}_{15}$.

Inverted information matrices: The general form of inverse of information matrix related to half parents ($\mathbf{C}_{\text{half}}^{-}$) is $c_0\,\mathbf{I}_N + c_1\,\mathbf{A}_N + c_2\,\mathbf{B}_N$, where $c_0 = \frac{(n-2)(n-3)}{2(n-1)^2(n-4)}$, $c_1 = -\frac{(n-3)}{2(n-1)^2(n-4)}$ and $c_2 = \frac{1}{(n-1)^2(n-4)}$. The general form of inverted information matrix related to full parents ($\mathbf{C}_{\text{full}}^{-}$) is $d_0\,\mathbf{I}_N + d_1\mathbf{A}_N + d_2\mathbf{B}_N$, where $d_0 = \frac{(n-3)^2}{(n-1)^2(n-4)}$, $d_1 = -\frac{(n-3)^2}{(n-1)^2(n-2)(n-4)}$ and $d_2 = \frac{2(n-3)}{(n-1)^2(n-2)(n-4)}$. For the example given earlier, $\mathbf{C}_{\text{full}}^{-} = 0.18\mathbf{I}_{15} - 0.045\mathbf{A}_{15} + 0.03\mathbf{B}_{15}$ and $\mathbf{C}_{\text{half}}^{-} = 0.12\mathbf{I}_{15} - 0.03\mathbf{A}_{15} + 0.02\mathbf{B}_{15}$.

Eigenvalues: The eigenvalues of $\mathbf{C}_{\text{half}}$ are $a_0 + (n-2)a_2$ and 0, whereas the eigenvalues of the $\mathbf{C}_{\text{full}}$ are $b_0 + (n-2)b_2$ and 0.

Variance factors: The general expression for variance factor of estimated contrasts for half parents ($V_{\text{half}}(\widehat{h_i - h_j})$) is $2(c_0 - c_1) = \frac{(n-3)}{(n-1)(n-4)}$, when i and $j(i \neq j)$ are first associates to each other, and $2(c_0 - c_2) = \frac{1}{(n-1)}$, when i and $j(i \neq j)$are second associates to each other. The general expression for average variance factor of estimated contrasts for half parents ($\overline{V}_{\text{half}}(\widehat{h_i - h_j})$) is $\frac{n(n-3)}{(n^2-1)(n-4)}$. The general expressions for variance factor of estimated contrasts for full parents ($V_{\text{full}}(\widehat{g_i - g_j})$) is $2(d_0 - d_1) = \frac{2(n-3)^2}{(n-1)(n-2)(n-4)}$, when i and $j(i \neq j)$ are first associates to each other, and $2(d_0 - d_2) = \frac{2(n-3)}{(n-1)(n-2)}$ when i and $j(i \neq j)$ are second associates to each other. The general expressions for average variance factor of estimated contrasts for full parents ($\overline{V}_{\text{full}}(\widehat{g_i - g_j})$) is $\frac{2n(n-3)^2}{(n^2-1)(n-2)(n-4)}$.

Degree of fractionation and efficiency factor: The degree of fractionation (f) for this series of designs involving three-way crosses is $\frac{8}{(n+1)(n^2-n-4)}$. The canonical efficiency factor for the developed class of designs pertaining to gca effects of half parents (E_h) is $\frac{(n-1)(n-4)}{(n-2)^2}$ and of full parents (E_f) is $\frac{(n-1)(n-4)}{(n-2)(n-3)}$.

List of designs: Considering the model under blocked setup, the canonical efficiency factor of the designs as compared to an orthogonal design with same number of replications has been calculated and listed along with other parameters in Table 1.

Table 1. List of designs using triangular association scheme for three-way crosses under blocked setup

n	N	b	k	T	f	r_h	r_f	$\overline{V}_{\text{half}}(\widehat{h_i - h_j})$	$\overline{V}_{\text{full}}(\widehat{g_i - g_j})$	E_h	E_f
5	10	5	6	30	0.08	6	3	0.42	0.56	0.44	0.67
6	15	6	10	60	0.04	8	4	0.26	0.39	0.63	0.83
7	21	7	15	105	0.03	10	5	0.20	0.31	0.72	0.90
8	28	8	21	168	0.02	12	6	0.16	0.27	0.78	0.93
9	36	9	28	252	0.01	14	7	0.14	0.23	0.82	0.95
10	45	10	36	360	0.01	16	8	0.12	0.21	0.84	0.96
11	55	11	45	495	0.01	18	9	0.11	0.19	0.86	0.97
12	66	12	55	660	0.01	20	10	0.10	0.17	0.88	0.98
13	78	13	66	858	< 0.01	22	11	0.09	0.16	0.89	0.98
14	91	14	78	1092	< 0.01	24	12	0.08	0.15	0.90	0.98

SAS code for three-way cross designs under blocked set-up

SAS code is written to compute the average variance factor of estimated contrast pertaining to gca effects of half parents as well as full parents for the developed three-way cross designs under blocked setup. Besides average variance factors, it also computes canonical efficiency factor pertaining to gca effects of half parents as well as full parents for the developed class of designs. The canonical efficiency factor is calculated relative to an orthogonal design with the same number of lines by working out the harmonic mean of $(1/r)$ times the non-zero eigenvalues of the information matrix. The efficiency factor along with the degree of fractionation can help the experimenter to choose proper design. The user has to just enter the three-way cross design in the data section of SAS code to get the result. The SAS code along with entered data is provided in Annexure I.

Method 2: Partial three-way cross designs based on Lattice designs

This method can be used to obtain block partial three-way cross designs for a wide range of parameters. In this method, lattice designs with standard parameters (v, b^*, r^*, k^*, s) is considered and the block contents are taken as lines. Now, from each block all possible three-way crosses are made such that the set of crosses made from all the blocks of a replication of lattice design constitute the block of the new design.

The parameters of the resultant class of partial three-way cross designs are $N = v$, $b = r^*$, $k = \frac{sk^*(k^*-1)(k^*-2)}{2}$ and $T = \frac{bsk^*(k^*-1)(k^*-2)}{2}$. Different classes of partial three-way cross designs along with example is given. (It may be noted that along with every cross of the type (i, j, k), crosses of the types (i, k, j) and (j, k, i) are to be considered in the same block, but are not shown here in the design layouts).

Class I (Square lattices based partial three-way cross designs): Any square lattice with parameters, $v = s^2$, $b^* = s(s+1)$, $r^* = (s+1)$ and $k^* = s$, can be used to obtain partial three-way cross designs with parameters, $N = s^2, b = (s+1)$, $k = \frac{sk^*(k^*-1)(k^*-2)}{2}$ and $T = \frac{bsk^*(k^*-1)(k^*-2)}{2}$.

An example is illustrated here for $s = 3$ to construct a partial three-way cross design for 9 lines. Consider a square lattice design with parameters, $v = 9$, $b^* = 12$, $r^* = 4$, $k^* = 3$ and $s = 3$. The four replications of the lattice designs forms the four blocks of partial three-way cross designs with block contents as the three-way crosses formed by taking all the possible triplets from each block of a replication. Finally we get a partial three-way cross designs with parameters, $N = 9,\ b = 4,\quad k = 9, T = 36$ and $f = 0.142$.

Rep 1	**Blk 1**	**Blk 2**	**Blk 3**
	1,2,3	4,5,6	7,8,9
Rep 2	**Blk 1**	**Blk 2**	**Blk 3**
	1,4,7	2,5,8	3,6,9
Rep 3	**Blk 1**	**Blk 2**	**Blk 3**
	1,5,9	3,4,8	2,6,7
Rep 4	**Blk 1**	**Blk 2**	**Blk 3**
	1,6,8	2,4,9	3,5,7

Square Lattice design

Blk 1	(1×2)×3	(4×5)×6	(7×8)×9
Blk 2	(1×4)×7	(2×5)×8	(3×6)×9
Blk 3	(1×5)×9	(3×4)×8	(2×6)×7
Blk 4	(1×6)×8	(2×4)×9	(3×5)×7

Partial Three-way Cross Design

Class II (Rectangular lattices based partial three-way cross designs): Any rectangular lattice design with parameters $v^* = s(s-1),\ b^* = s^2,\ r^* = s$ and $k^* = (s-1)$, can be used to obtain partial three-way cross designs with parameters, $N = v = s(s-1), b = s$, $k = \frac{sk^*(k^*-1)(k^*-2)}{2}$ and $T = \frac{bsk^*(k^*-1)(k^*-2)}{2}$.

An example is illustrated here for $s = 4$, which can be used to construct partial three-way cross design for 12 lines. Consider a rectangular lattice design with parameters, $v = 12$, $b^* = 16,\ r^* = 4,\ k^* = 3$ and $s = 4$.

Rectangular Lattice Design	**Rep 1**	**Blk 1**	**Blk 2**	**Blk 3**	**Blk 4**
		1,5,9	2,6,10	3,7,11	4,8,12
	Rep 2	**Blk 1**	**Blk 2**	**Blk 3**	**Blk 4**
		1,6,11	2,5,12	3,8,9	4,7,10
	Rep 3	**Blk 1**	**Blk 2**	**Blk 3**	**Blk 4**
		1,8,10	4,5,11	2,7,9	3,6,12
	Rep 4	**Blk 1**	**Blk 2**	**Blk 3**	**Blk 4**
		1,7,12	3,5,10	4,6,9	2,8,11

Three-way crosses are made within blocks of each replication to construct a partial three-way cross design with parameters, $N = 12,\ b = 4,\ \ k = 12,\ T = 36$ and $f = 0.018$.

Partial Three-way Cross Design					
	Blk 1	(1×5)×9	(2×6)×10	(3×7)×11	(4×8)×12
	Blk 2	(1×6)×11	(2×5)×12	(3×8)×9	(4×7)×10
	Blk 3	(1×8)×10	(4×5)×11	(2×7)×9	(3×6)×12
	Blk 4	(1×7)×12	(3×5)×10	(4×6)×9	(2×8)×11

Class III (Circular Lattices based partial three-way cross designs): Any circular lattice design with parameters, $v = 2s^2,\ b^* = 2s,\ r^* = 2,$ and $k^* = 2s,$ can be used to obtain partial three-way cross designs with parameters, $N = v = 2s^2, b = r^*,\ k = \frac{sk^*(k^*-1)(k^*-2)}{2}$ and $T = \frac{bsk^*(k^*-1)(k^*-2)}{2}$.

An example is illustrated here for $s = 2$, which can be used to construct partial three-way cross design for 8 lines. Consider a circular lattice design with parameters, $v = 8,\ b^* = 4,$ $r^* = 2,\ k^* = 4$ and $s = 2$. All possible three-way crosses are made within each block of a replication of this lattice design to yield a partial three-way cross designs with parameters, $N = 8,\ b = 2,\ \ k = 24$ and $T = 16$.

Rep 1	**Blk 1**	**Blk 2**
	1,2,3,4	5,6,7,8
Rep 2	**Blk 1**	**Blk 2**
	1,3,5,7	2,4,6,8

Circular Lattice design

Blk 1	(1×2)×3	(1×2)×4	(1×3)×4	(2×3)×4
	(5×6)×7	(5×6)×8	(5×7)×8	(6×7)×8
Blk 2	(1×3)×5	(1×3)×7	(1×5)×7	(3×5)×7
	(2×4)×6	(2×4)×8	(2×6)×8	(4×6)×8

Partial Three-way Cross Design

Class IV (Cubic lattices based partial three-way cross designs): Any cubic lattice design with parameters, $v = s^3,\ b^* = 3s^2,\ r^* = 3,$ and $k^* = s,$ can be used to obtain partial three-way cross designs with parameters, $N = s^3, b = 3,\ \ k = \frac{sk^*(k^*-1)(k^*-2)}{2}$ and $T = \frac{bsk^*(k^*-1)(k^*-2)}{2}$.

An example is illustrated here for $s = 3$, which can be used to construct partial three-way cross design for 27 lines. Consider a circular lattice design with parameters, $v = 27,\ b^* = 27,\ r^* = 3,$ and $k^* = 3$.

Cubic Lattice design	Rep 1	**Blk 1**	**Blk 2**	**Blk 3**	**Blk 4**	**Blk 5**
		1,2,3	4,5,6	7,8,9	10,11,12	13,14,15
		Blk 6	**Blk 7**	**Blk 8**	**Blk 9**	
		16,17,18	19,20,21	22,23,24	25,26,27	
	Rep 2	**Blk 1**	**Blk 2**	**Blk 3**	**Blk 4**	**Blk 5**
		1,4,7	2,5,6	3,6,9	10,13,16	11,14,17
		Blk 6	**Blk 7**	**Blk 8**	**Blk 9**	
		12,15,18	19,22,25	20,23,26	21,24,27	
	Rep 3	**Blk 1**	**Blk 2**	**Blk 3**	**Blk 4**	**Blk 5**
		1,10,19	2,11,20	3,12,21	4,13,22	5,14,23
		Blk 6	**Blk 7**	**Blk 8**	**Blk 9**	
		6,15,24	7,16,25	8,17,26	9,18,27	

Treating the treatment numbers as line numbers, three-way crosses are made within block of each replication of this lattice design to give a partial three-way cross designs. The parameters, $N = 27,\ b = 3,\ \ k = 27,\ T = 81$ and $f = 0.003$ along with the design layout is shown below:

Partial Three-way Cross Designs	**Blk 1**	(1×2)×3	(4×5) ×6	(7×8)×9	(10×11)×12	(13×14)×15
		(16×17)×18	(19×20)×21	(22×23)×24	(25×26)×27	
	Blk 2	(1×4)×7	(2×5)×6	(3×6)×9	(10×13)×16	(11×14)×17
		(12×15)×18	(19×22) ×25	(20×23) ×26	(21×24) ×27	
	Blk 3	(1×10) ×19	(2×11) ×20	(3×12) ×21	(4×13) ×22	(5×14) ×23
		(6×15) ×24	(7×16) ×25	(8×17) ×26	(9×18) ×27	

List of designs

A list of parameters of partial three-way cross designs constructed using different types of lattice design along with degree of fractionation and efficiency factor has been given in the Table 2. This table can help the breeders to choose a suitable design for required parametric combination and availability of resources as these are readymade layouts of combined mating environmental designs.

Table 2. List of designs using lattice designs for three-way crosses under blocked setup

N	b	k	T	f	$\bar{V}_{\text{half}}(\widehat{h_i - h_j})$	$\bar{V}_{\text{full}}(\widehat{g_i - g_j})$	E_h	E_f	Type of lattices
8	2	24	48	0.29	0.31	0.47	0.54	0.71	Circular
9	4	9	36	0.14	0.30	0.52	0.84	0.96	Square
12	4	9	36	0.02	0.20	0.41	0.99	0.99	Rectangular
27	3	27	81	< 0.01	0.16	0.27	0.99	0.86	Cubic

Method 3:* Partial Three-way Cross *plans using Kronecker product

This method can be used to obtain partial three-way cross plans for composite number of lines. In this method we have to consider the incidence matrices of any two BIB designs. The Kronecker product of these two matrices is obtained and is considered as an incidence matrix of a block design. Now, from each block of this design all possible triplet combinations are considered to make three-way crosses. The process is carried out for all the blocks and hence a partial three-way cross plan is obtained. The method can be well understood through the example given below:

Example: A partial three-way cross plan for number of lines, $N = 12$ and number of crosses $T = 216$ can be obtained using two BIB designs, viz., Design 1 (3,3,2,2,1) and Design 2 (4,6,3,2,1). The layout of both the designs along with incidence matrix is

Blk 1	1	2
Blk 2	1	3
Blk 3	2	3

Design 1: BIBD (3, 3, 2, 2, 1)

	Trt 1	**Trt 2**	**Trt 3**
Blk 1	1	1	0
Blk 2	1	0	1
Blk 3	0	1	1

Incidence matrix

Blk 1	1	2
Blk 2	1	3
Blk 3	1	4
Blk 4	2	3
Blk 5	2	4
Blk 6	3	4

Design 2: BIBD (4, 6, 3, 2, 1)

	Trt 1	**Trt 2**	**Trt 3**	**Trt 4**
Blk 1	1	1	0	0
Blk 2	1	0	1	0
Blk 3	1	0	0	1
Blk 4	0	1	1	0
Blk 5	0	1	0	1
Blk 6	0	0	1	1

Incidence matrix

Now, write down the block versus treatment incidence matrices of order 3×3 and 6×4 respectively. The Kronecker product of these two matrices results in matrix of order 18×12, yielding the incidence matrix of a new incomplete design *i.e.,* a rectangular design with parameters, $v = 12,\ b = 18,\ r = 6,\ k = 4\ ,\ \lambda_1 = 3, \lambda_2 = 2, \lambda_3 = 1,\ n_1 = 2,\ n_2 = 3$ and $n_3 = 6$. This is shown ahead:

	TREATMENTS											
BLOCKS	1	1	0	0	1	1	0	0	0	0	0	0
	1	0	1	0	1	0	1	0	0	0	0	0
	1	0	0	1	1	0	0	1	0	0	0	0
	0	1	1	0	0	1	1	0	0	0	0	0
	0	1	0	1	0	1	0	1	0	0	0	0
	0	0	1	1	0	0	1	1	0	0	0	0
	1	1	0	0	0	0	0	0	1	1	0	0
	1	0	1	0	0	0	0	0	1	0	1	0
	1	0	0	1	0	0	0	0	1	0	0	1
	0	1	1	0	0	0	0	0	0	1	1	0
	0	1	0	1	0	0	0	0	0	1	0	1
	0	0	1	1	0	0	0	0	0	0	1	1
	0	0	0	0	1	1	0	0	1	1	0	0
	0	0	0	0	1	0	1	0	1	0	1	0
	0	0	0	0	1	0	0	1	1	0	0	1
	0	0	0	0	0	1	1	0	0	1	1	0
	0	0	0	0	0	1	0	1	0	1	0	1
	0	0	0	0	0	0	1	1	0	0	1	1

Incidence matrix

	TREATMENTS			
B-1	1	2	5	6
B-2	1	3	5	7
B-3	1	4	5	8
B-4	2	3	6	7
B-5	2	4	6	8
B-6	3	4	7	8
B-7	1	2	9	10
B-8	1	3	9	11
B-9	1	4	9	12
B-10	2	3	10	11
B-11	2	4	10	12
B-12	3	4	11	12
B-13	5	6	9	10
B-14	5	7	9	11
B-15	5	8	9	12
B-16	5	7	10	11
B-17	6	8	10	12
B-18	7	8	11	12

Design 3: PBIB Rectangular Design (12,18,6,4,3,2,1)

Now making all the possible three-way crosses from each block a partial three-way cross plan can be obtained for 12 lines involving 216 crosses with a degree of fractionation 0.27. The layout of the plan is given as: (it is to take care that to ensure SSP of the three-way cross plan along with each crosses the other two alternatives are also to added to given layout which are not given here in the layout)

(1×2)×5	(2×3)×10	(1×2)×6	(2×3)×11	(1×5)×6
(1×3)×5	(2×4)×10	(1×3)×7	(2×4)×12	(1×5)×7
(1×4)×5	(3×4)×11	(1×4)×8	(3×4)×12	(1×5)×8
(2×3)×6	(5×6)×9	(2×3)×7	(5×6)×10	(2×6)×7
(2×4)×6	(5×7)×9	(2×4)×8	(5×7)×11	(2×6)×8
(3×4)×7	(5×8)×9	(3×4)×8	(5×8)×12	(3×7)×8
(1×2)×9	(6×7)×10	(1×2)×10	(6×7)×11	(1×9)×10
(1×3)×9	(6×8)×10	(1×3)×11	(6×8)×12	(1×9)×11
(1×4)×9	(7×8)×11	(1×4)×12	(7×8)×12	(1×9)×12
(2×10)×11	(2×5)×6	(6×9)×10	(3×6)×7	(7×10)×11
(4×10)×12	(3×5)×7	(7×9)×11	(4×6)×8	(8×10)×12
(4×11)×12	(4×5)×8	(8×9)×12	(4×7)×8	(8×11)×12

Here, it should be noted that along with every cross of the type (i, j, k) crosses of the types (i, k, j) and (j, k, i) are not shown in the plan and hence there are only 72 crosses.

List of designs

A list of parameters partial three-way cross plans constructed using Kronecker product method along with the efficiency factors have been given in the Table 3.

Table 3 List of three-way cross plans using Kronecker product

N	T	f	r_h	r_f	$\bar{V}_{\text{half}}(\widehat{h_i - h_j})$	$\bar{V}_{\text{full}}(\widehat{g_i - g_j})$	E_h	E_f
9	108	0.43	24	12	0.1083	0.1833	0.9116	0.9427
12	180	0.27	36	18	0.0686	0.1206	0.9063	0.9382
15	360	0.26	48	24	0.0504	0.0903	0.8993	0.9321
18	540	0.22	60	30	0.0421	0.0795	0.8753	0.9124
20	720	0.21	72	36	0.0385	0.0654	0.8612	0.9009

SAS code for three-way cross designs under unblocked set-up

This SAS code is written to compute the average variance factor of estimated contrast pertaining to gca effects of half parents as well as full parents for the developed three-way cross plans under unblocked setup. The average variance factors computed are directly used along with number of crosses to calculate the information per cross of the developed three-way cross plan. Information per cross and degree of fractionation of a given three-way cross plan are used as measure to select appropriate plans by the breeder. The user has to just enter the three-way cross plan in the data section of SAS code to get the result. The SAS code along with entered data is provided in Annexure II.

2.3 Partial four-way cross designs

Four-way cross experiment provides us more information regarding the combining abilities and the hybrids developed based on them are found to be more stable and consistence in performance due to broad genetic base. The model for four-way cross experiments differs due to sca effects. A full model with sca involves estimation of sca effects up to third order along with gca effects. A restricted model may involve sca effects or may drop sca effects totally.

2.3.1 Fixed effects model with sca effects

Consider a four-way cross experiment in which the T crosses are regarded as treatments. Let a cross of type $(i \times j) \times (k \times l)$ be represented as (i, j, k, l), and the fixed effect of the cross $(i, j, k, l)$is denoted by y_{ijkl} , then the following model can be used:

$$y_{ijkl} = \bar{y} + g_i + g_j + g_k + g_l + s_{ij} + s_{ik} + s_{il} + s_{jk} + s_{jl} + s_{kl}$$

$$+ s_{ijk} + s_{ijl} + s_{ikl} + s_{jkl} + s_{ijkl} + e_{ijkl} ,$$

where $\bar{y}$ is the average effect of the crosses, $\{g_\alpha\}, \alpha = i, j, k, l,$ represents the gca effects, $\{s_{\alpha\beta}\}, (\alpha, \beta) \in (i, j, k, l)$ represents the first order sca effects, $\{s_{\alpha\beta\gamma}\}, (\alpha, \beta, \gamma) \in i, j, k, l,$ represents the second order sca effects, s_{ijkl} represents the third order sca effects, e_{ijkl} represents the random error, and with

$\sum_{i=1}^{N} g_i = 0, \sum_{\alpha\beta} s_{\alpha\beta} = 0, \sum_{\alpha\beta\gamma} s_{\alpha\beta\gamma} = 0$ and

$\sum_{ijkl} s_{ijkl} = 0, i \neq j \neq k \neq l, i < j, k < l , i, j, k, l = 1,2, \ldots, N,$

for every $(\alpha, \beta, \gamma) \in (i, j, k, l)$, for $i \neq j \neq k \neq l,\ i < j, k < l,\ i, j, k, l = 1,2, \ldots, N.$

Since complete four-way crosses are considered here, whenever a cross (i, j, k, l) is involved, it means that the other two alternative crosses of the types (i, k, j, l) and (i, l, j, k) are also included in the experiment, simultaneously.

2.3.2 Restricted model including lower order sca effects

Since the second order and third order sca effects are negligible as compared to the first order sca effects these can be dropped from the model. Then for y_{ijkl} , we can have the following representation with the conditions on g_i's as stated before:

$$y_{ijkl} = \bar{y} + g_i + g_j + g_k + g_l + s_{ij} s_{ik} + s_{il} + s_{jk} + s_{jl} + s_{kl} + e_{ijkl} ,$$

where $\bar{y}$ is the average effect of the treatments, $\{g_\alpha\}, \alpha = i, j, k, l,$ represents the gca effects, $\{s_{\alpha\beta}\}, (\alpha, \beta) \in (i, j, k, l)$ represents the first order sca effects, e_{ijkl} is the random errors, and

$$g_1 + g_2 + \cdots + g_N = 0 \text{ or } \sum_{i=1}^{N} g_i = 0,$$

$$s_{1i} + \cdots + s_{(i-1)i} + s_{i(i+1)} + \cdots + s_{iN} = 0, \text{ and } \sum_{\alpha\beta} s_{\alpha\beta} = 0,$$

for every $(\alpha, \beta) \in (i, j, k, l),\ i \neq j \neq k \neq l, i < j, k < l , i, j, k, l = 1,2, \dots, N.$

2.3.3 Estimates of combining abilities

Consider that the four-way crosses of the type, are arranged in the order $(1, 2, 3, 4)$, $\dots, (1, 2, 3, l), \dots, (1, 2, k, l), \dots, (1, j, k, l), \dots, (i, j, k, l),\ \dots ,\{(N-3), (N-2), (N-1),\ N\}$, $(i, j, k, l);\ i \neq j \neq k \neq l;\ i < j;\ k < l ;\ i, j, k, l = 1,2, \dots, N$. It should be noted that along with every cross of the type (i, j, k, l) crosses of the types (i, k, j, l) and (i, l, j, k) are also included in the experiment, simultaneously. Let $\boldsymbol{g} = (g_1,\ g_2, \dots, g_N)'$ be an $N \times 1$ vector of gca effects, $\boldsymbol{y}$ and $\boldsymbol{s}$ be $N \times 1$ vectors whose elements are $\{y_{ijkl}\}$ and $\{s_{\alpha\beta}\}$, respectively. Let us define a matrix $\boldsymbol{W}$ of order $N \times T$ with rows indexed by the line numberings as $1, 2, \dots m, \dots N$ and, the columns by the crosses, in same order as described earlier, of the types (i, j, k, l), including the crosses of the types (i, k, j, l) and (i, l, j, k) also, simultaneously. Then, the $\{m, (i, j, k, l)\}^{\text{th}}$ entry of $\boldsymbol{W}$ takes a value 1 if $m \in (i, j, k, l)$ and 0, otherwise. The followings results are obtained:

$$\boldsymbol{W}\boldsymbol{W}' = \frac{(N-2)(N-3)}{2}\{(N-4)\boldsymbol{I}_N + 3\boldsymbol{J}_N\},$$

$$(\boldsymbol{WW'})^{-1} = \frac{2}{(N-2)(N-3)(N-4)}\{\boldsymbol{I}_N - \frac{3}{4(N-1)}\boldsymbol{J}_N\},$$

$$\boldsymbol{W1}_T = \frac{(N-1)(N-2)(N-3)}{2}\boldsymbol{1}_N \text{ and } \boldsymbol{W}'\boldsymbol{1}_N = 4\boldsymbol{1}_T.$$

where $\boldsymbol{I}_N$ is an identity matrix of order N, $\boldsymbol{J_N} = \boldsymbol{1_N 1_N'}$ and $\boldsymbol{1_N}$ is an $N \times 1$ column vector of unities. Now, these results can be used to represent the cross (i, j, k, l) effect as

$$\boldsymbol{y} = \bar{y}\,\boldsymbol{1}_T + \boldsymbol{W}'\boldsymbol{g} + \boldsymbol{s} + \boldsymbol{e}, \text{ with}$$

$$\boldsymbol{1}_N'\boldsymbol{g} = 0, \boldsymbol{W's} = \boldsymbol{0},$$

and random error vector $\boldsymbol{e}$.

The estimate of gca effect is given as

$$\widehat{\boldsymbol{g}} = \boldsymbol{Ay},$$

where $\boldsymbol{A} = \frac{2}{(N-2)(N-3)(N-4)}\{\boldsymbol{W} - \frac{4}{N}\boldsymbol{J}_{N\times T}\}$, and

the estimate of sca effects is given as

$$\hat{\boldsymbol{s}} = \boldsymbol{By}, \text{ where}$$

$$\boldsymbol{B} = \boldsymbol{I}_T - \frac{2}{(N-2)(N-3)(N-4)}\{\boldsymbol{W}'\boldsymbol{W} - \frac{12}{(N-1)}\boldsymbol{J}_T\}.$$

Also, we have

$$\boldsymbol{A1}_T = \boldsymbol{B1}_T = \boldsymbol{0}, \boldsymbol{AB}' = \boldsymbol{BA}' = \boldsymbol{0}, rank(\boldsymbol{A}) = N-1 \text{ and } rank(\boldsymbol{B}) = T - N.$$

Hence, $\boldsymbol{g}$ represents contrasts pertaining to gca effect with $(N-1)$degrees of freedom and, $\boldsymbol{s}$ for sca effects with $(T-N)$ degrees of freedom. It can be verified that the contrasts representing $\boldsymbol{g}$ are orthogonal to those representing $\boldsymbol{s}$. It is important to note here that for number of lines $N = 5$, $\boldsymbol{s} = \boldsymbol{0}.$ Hence, it is considered that $N > 5$ throughout.

2.3.4 Class of four-way cross designs

A series of partial four-way cross designs which can be obtained using MOLS. Consider N, the number of lines as a prime or prime power. Out of total $(N-1)$ possible MOLS, consider any of the $\frac{(N-1)}{2}$ MOLS. Retaining the first four rows of each Latin square, N number of crosses corresponding to each column can be made from each MOLS. Thus, the parameters of the developed class of design is $T = \frac{N(N-1)}{2}$, $b = \frac{(N-1)}{2}$, r, $k = N$.

Example : The method can be well understood by an example for $N = 6$, $b = 3$, $k = 7$ and $T = 21$. Considering 3 MOLS of order 7 chosen at random out of the total 6 possible MOLS of order 7, and retaining only first 4 rows of each, based on the symbols A, B, C, D, E, F and G as given below.

MOLS I						
A	B	C	D	E	F	G
B	C	D	E	F	G	A
C	D	E	F	G	A	B
D	E	F	G	A	B	C

MOLS II						
A	B	C	D	E	F	G
C	D	E	F	G	A	B
E	F	G	A	B	C	D
G	A	B	C	D	E	F

MOLS III						
A	B	C	D	E	F	G
D	E	F	G	A	B	C
G	A	B	C	D	E	F
C	D	E	F	G	A	B

Now, considering the seven symbols as lines, from each LS a four-way crosses can be made by taking the four lines of each column. The crosses made from each LS will be constituting a block. The final layout of the design so obtained is given below.

Block 1	**Block 2**	**Block 3**
(A×B)×(C×D)	(A×C)×(E×G)	(A×D)×(G×C)
(B×C)×(D×E)	(B×D)×(F×A)	(B×E)×(A×D)
(C×D)×(E×F)	(C×E)×(G×B)	(C×F)×(B×E)
(D×E)×(F×G)	(D×F)×(A×C)	(D×G)×(C×F)
(E×F)×(G×A)	(E×G)×(B×D)	(E×A)×(D×G)
(F×G)×(A×B)	(F×A)×(C×E)	(F×B)×(E×A)
(G×A)×(B×C)	(G×B)×(D×F)	(G×C)×(F×B)

2.4 Discussion

Under a fixed effects model including sca effects for three-way crosses, the estimates of general and specific combining abilities have been obtained. A class of designs for partial three-way cross arranged in blocks has been obtained using triangular association scheme and information matrices, eigenvalues, variance factors, efficiency factors and degree of fractionation have been derived. Another method of constructing partial three-way cross designs has also been developed using various types of lattice designs viz., square, rectangular, circular and cubic lattice. This method gives designs for partial three-way crosses under blocked set up for a wide range of parameters. The main restriction of first series of designs is that these designs are available only for cases where the number of lines is of the particular form $N = \frac{n(n-1)}{2}$. The second series is available for many combinations and can be used in conjunction with the first method to fill the gaps of designs not available for particular parameters. The third method, of constructing partial three-way cross plans is based on Kronecker product of incidence matrices. The plans obtained are having low degree of fractionation and high efficiencies and can be used when there is scarcity of resources. These methods methods can provide designs for almost all sets of parametric combinations. With an adequate knowledge of block designs, all the proposed crossing plans can easily be constructed. Under a restricted model including lower order sca effects for four-way crosses, conditions of orthogonality have been derived for a block design such that the contrasts pertaining to the gca effects and sca effects are estimated free from each other, after eliminating the other nuisance factors. A method of construction of optimal block designs has been described and a class of optimal designs based on Mutually Orthogonal Latin Squares has been obtained. SAS codes are written to compute the average variance factor of estimated contrast pertaining to gca effects of half parents as well as full parents for the developed three-way cross designs. These SAS codes provided in Annexures I-II.

CHAPTER 3

PARTIAL THREE-WAY CROSS PLANS FOR TEST LINES VERSUS CONTROL LINE COMPARISON

3.1 Introduction

Three-way cross hybrids are found to be intermediate between two-way and four-way cross hybrids when yield, total number of crosses required in the cross plan, uniformity and stability are of main concern. When the number of lines are higher, then total number of crosses increases many folds and in this situation it is advisable to take a sample of complete three-way cross can be taken, known as partial three-way crosses. Partial three-way cross can also be used for the purpose of comparing test lines control line to study the genetic nature of qualitative traits so that further better breeding strategies can be identified and developed.

3.2 Test versus control comparison

In many experimental situations, a set-up for comparing newly developed lines (called test lines) with a standard line (called the control line) with respect to their combining ability effects is required. Suppose that several new lines are developed in the initial stage of an experiment and it is expected that only a few of the new lines are worthy of further investigation. Then the new lines may first be compared with a control line in order to screen out the lines that do not warrant further study. In this instance the greatest economy is obtained if the control is highly replicated than the new treatments under test.

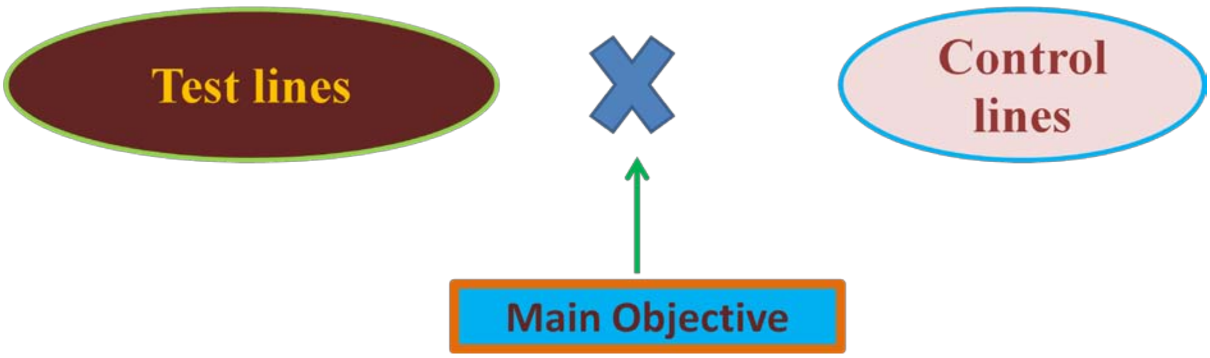

Figure: 1

The concept of test versus control comparisons can be well understood in Figure 1. Here we have two sets of lines namely the test lines and the control lines. We can achieve three types of comparisons *viz.* test versus test, control versus control and test versus control. Firstly consider comparisons among test lines, there may be a large number of lines developed by the experimenter, of which most probably few lines will outperform the existing standard lines, thus before performing all possible comparisons between test lines it is better to select the best few test lines and then proceed. One of the solutions to this situation is test versus control comparisons which will screen out those test lines which are of not much worth. Thus our main objective in this study is test versus control comparisons.

3.3 Partial three-way cross for test versus control comparisons

A lot of research work has been done for test versus control comparisons in the area of two-way crosses. But there is a need to develop three-way cross designs for test versus control comparisons because three-way cross based hybrids are genetically more viable, stable and consistent in performance and hence the information gained from this can lead to resultant product of good quality which is also economical. Here is an example of design for partial three-way cross for test versus control comparison for 7 test lines and 1 control line (0).

(1×2)×0	(4×5)×0	(7×1)×0	(3×0)×4	(6×0)×7	(3×0)×2	(6×0)×5
(2×3)×0	(5×6)×0	(1×0)×2	(4×0)×5	(7×0)×1	(4×0)×3	(7×0)×6
(3×4)×0	(6×7)×0	(2×0)×3	(5×0)×6	(2×0)×1	(5×0)×4	(1×0)×7
(1×3)×0	(4×6)×0	(7×2)×0	(3×0)×5	(6×0)×1	(4×0)×2	(7×0)×5
(2×4)×0	(5×7)×0	(1×0)×3	(4×0)×6	(7×0)×2	(5×0)×3	(1×0)×6
(3×5)×0	(6×1)×0	(2×0)×4	(5×0)×7	(3×0)×1	(6×0)×4	(2×0)×7
(1×4)×0	(4×7)×0	(7×3)×0	(3×0)×6	(6×0)×2	(5×0)×2	(1×0)×5
(2×5)×0	(5×1)×0	(1×0)×4	(4×0)×7	(7×0)×3	(6×0)×3	(2×0)×6
(3×6)×0	(6×2)×0	(2×0)×5	(5×0)×1	(4×0)×1	(7×0)×4	(3×0)×7

Partial three-way cross plan for test vs. control comparison (Harun *et al.*, 2016)

The parameters of the design given here are as: total number of lines $(N) = 8$ (7 test + 1 control), total number of crosses $(T) = 63$, number of times each test line occurring as full parent $(r_{ft}) = 6$, number of times the control line occurring as full parent $(r_{fc}) = 21$, number of times each test line occurring as half parent $(r_{ht}) = 12$, number of times control line occurring as half parent $(r_{hc}) = 42$ and degree of fractionation $(f) = 0.75$.

3.4 Model and experimental setup

In this approach, gca effects of first and second kind corresponding to half and full parents will be estimated for which it is assumed that the sca effects are contributing much less to the total combining ability effects as compared to gca effects and hence sca effects are negligible. The model can be written as

$$y_{ijk} = \bar{y} + h_i + h_j + g_k + e_{ijk},$$

where $\bar{y}$ is the average effect of the treatments, $\{h_\alpha\}, \alpha = i, j$, represents the gca effects of first kind corresponding to the lines occurring as half parents, $\{g_k\}$ represents the gca effects of second kind corresponding to the lines occurring as full parents, e_{ijk} is the random error component with

$$\sum_{i=1}^{N} g_i = 0 \text{ and } \sum_{i=1}^{N} h_i = 0.$$

The model in matrix notation is expressed as:

$$\boldsymbol{y} = \bar{y}\,\boldsymbol{1}_T + \boldsymbol{W}_1'\boldsymbol{h} + \boldsymbol{W}_2'\boldsymbol{g} + \boldsymbol{e},$$

where, $\boldsymbol{y}$ is the $T \times 1$ vector of responses due to crosses, $\bar{y}$ is the mean effect of crosses, $\boldsymbol{h}$ is the $N \times 1$ vector of gca effects due to half parent, $\boldsymbol{g}$ is the $N \times 1$ vector of gca effects due to full parent and $\boldsymbol{e}$ is the $N \times 1$ vector of random error component. $\boldsymbol{W_1}$ and $\boldsymbol{W_2}$ are $N \times T$ matrices with rows indexed by the line numbers $1, 2, \ldots N$ and columns by the three-way crosses arranged in the manner described earlier, such that the $\{t, (i, j, k)\}^{th}$ entry of $\boldsymbol{W_1}$ is 0.5 if $t \in (ij)$ and zero otherwise and the $\{t, (i, j, k)\}^{th}$ entry of $\boldsymbol{W_2}$ is 1 if $t \in k$ and zero otherwise.

Proceeding in the same way as in Chapter 2, the information matrices related to gca pertaining to half parent ($\boldsymbol{C}_{\text{gca\_half}}$) and full parent ($\boldsymbol{C}_{\text{gca\_full}}$) can be obtained. Further, the information matrices can be used to obtain variance factor of estimated contrasts for full parents for test vs. test line comparisons (V$_{\text{full_tvst}}$ $(\widehat{g_i - g_j})$, variance factor of estimated contrasts for full parents for test vs. control line comparisons (V$_{\text{full_tvsc}}$ $(\widehat{g_i - g_j})$), variance factor of estimated contrasts for half parents for test vs. test line comparisons (V$_{\text{half_tvst}}$ $(\widehat{h_i - h_j})$, variance factor of estimated contrasts for half parents for test vs. control line comparisons (V$_{\text{half_tvsc}}$ $(\widehat{h_i - h_j})$).

Since, the partial three-way cross plans obtained here are based on designs which may be partially variance balanced leading to more than one type of variance factors in each category discussed above. Thus, it is better to calculate average variance factor by taking weighted average of variance factors. Hence, the average variance factor of estimated contrasts pertaining to gca effects of, full parents for test vs. test line comparisons ($\overline{\mathrm{V}}_{\mathrm{full\_tvst}}\ (\widehat{g_i - g_j})$, full parents for test vs. control line comparisons ($\overline{\mathrm{V}}_{\mathrm{full\_tvsc}}\ (\widehat{g_i - g_j})$), half parents for test vs. test line comparisons ($\overline{\mathrm{V}}_{\mathrm{half\_tvst}}\ (\widehat{h_i - h_j})$ and half parents for test vs. control line comparisons ($\overline{\mathrm{V}}_{\mathrm{half\_tvsc}}\ (\widehat{h_i - h_j})$) has been calculated.

3.5 Method of construction of partial three-way cross design for test versus control comparison

Consider any Incomplete Block Design (IBD) with parameters v, b, r, k and λ_i. Add a control treatment (0) to each block. Each block is used to make a three way cross of the type $(i \times j) \times k$. Structural Symmetry Property (SSP) is satisfied by taking other two types of crosses $(i \times k) \times j$ and $(j \times k) \times i$ with the simultaneous occurrence of $(i \times j) \times k$. Three-way cross plan for v (=N) test lines and one control line can be obtained with total number of crosses, $T = 3b$.

Example 1: Consider an unreduced Balanced Incomplete Block Design (BIBD) with parameters $v = 4, b = 6, r = 3, k = 2$ and $\lambda = 1$. Add a control treatment (say 0) to each block of the BIBD as shown below:

Blocks	**Treatments**	
B-1	1	2
B-2	1	3
B-3	1	4
B-4	2	3
B-5	2	4
B-6	3	4

<u>BIBD (4, 6, 3, 2, 1)</u>

→

Blocks	**Treatments**		
B-1	1	2	0
B-2	1	3	0
B-3	1	4	0
B-4	2	3	0
B-5	2	4	0
B-6	3	4	0

<u>BIBD Added With An Extra Control Treatment To Each Blocks</u>

Now, a partial three-way cross plan for test versus control lines can be obtained by considering the treatments as lines and making three-way crosses from each blocks. In order to preserve the Structural Symmetry Property (SSP) of partial three-way cross, corresponding to each cross of the type $(i \times j) \times k$, other two types of crosses $(i \times k) \times j$ and $(j \times k) \times i$ are also included in the plan. The final layout of the partial three-way cross plan for 4 test lines versus a control line (0) is given below:

(1×2)×0	(1×0)×2	(0×2)×1
(1×3)×0	(1×0)×3	(0×3)×1
(1×4)×0	(1×0)×4	(0×4)×1
(2×3)×0	(2×0)×3	(0×3)×2
(2×4)×0	(2×0)×4	(0×4)×2
(3×4)×0	(3×0)×4	(0×4)×3

Partial Three-way Cross Plan for

Test Lines Vs. Control Line

The parameters of the given partial three-way cross plan for test lines vs. a control line are $N = 5$ (4 test + 1 control), $T = 18$, number $r_{ft} = 3$, $r_{fc} = 6$, $r_{ht} = 6$, $r_{hc} = 12$ and $f = 0.6$. Also, we have $V_{\text{full\_tvst}}\,(\widehat{g_i - g_j}) = 0.714$, $V_{\text{full\_tvsc}}\,(\widehat{g_i - g_j}) = 0.268$, $V_{\text{half\_tvst}}\,(\widehat{h_i - h_j}) = 0.429$ and $V_{\text{half\_tvsc}}\,(\widehat{h_i - h_j}) = 0.161$.

Remark: Since method is based on BIBD which is a variance balanced design, the layout so obtained is also having the same characterization property and a single variance factor is obtained for each category. Thus, there is no need to calculate the average variance factor as it will be same.

Now, another example is illustrated here based on another class of IBD that is Partially Balanced Incomplete Block Design (PBIBD), which is not variance balanced as in the previous example and hence in this situation the average variance factors will be calculated for each category.

Example 2: Consider a PBIBD based on Group Divisible (GD) association scheme with parameters $v = 8, b = 16, r = 4, k = 2, \lambda_1 = 1$ and $\lambda_2 = 0$. Add a control treatment (say 0) to each block of the PBIBD as shown below:

Blocks	Treatments	
B-1	1	2
B-2	3	4
B-3	5	6
B-4	7	8
B-5	6	1
B-6	8	3
B-7	2	5
B-8	4	7
B-9	1	4
B-10	3	2
B-11	5	8
B-12	7	6
B-13	8	1
B-14	6	3
B-15	2	5
B-16	4	7

PBIBD (8, 16, 4, 2, 1, 0)

→

Blocks	Treatments		
B-1	1	2	0
B-2	3	4	0
B-3	5	6	0
B-4	7	8	0
B-5	6	1	0
B-6	8	3	0
B-7	2	5	0
B-8	4	7	0
B-9	1	4	0
B-10	3	2	0
B-11	5	8	0
B-12	7	6	0
B-13	8	1	0
B-14	6	3	0
B-15	2	5	0
B-16	4	7	0

PBIBD Added With An Extra Control Treatment To Each Blocks

A partial three-way cross plan for test versus control lines can be obtained by making three-way crosses from each blocks and adding crosses $(i \times k) \times j$ and $(j \times k) \times i$ corresponding to each cross of the type $(i \times j) \times k$ SSP of partial three-way cross. The final layout of the partial three-way cross plan for 8 test lines versus a control line (0) is given below:

(1×2)×0	(2×5)×0	(1×0)×2	(2×0)×5	(0×1)×2	(0×2)×5	(8×1)×0	(2×5)×0
(3×4)×0	(4×7)×0	(3×0)×4	(4×0)×7	(0×3)×4	(0×4)×7	(6×3)×0	(4×7)×0
(5×6)×0	(1×4)×0	(5×0)×6	(1×0)×4	(0×5)×6	(0×1)×4	(8×0)×1	(2×0)×5
(7×8)×0	(3×2)×0	(7×0)×8	(3×0)×2	(0×7)×8	(0×3)×2	(6×0)×3	(4×0)×7
(6×1)×0	(5×8)×0	(6×0)×1	(5×0)×8	(0×6)×1	(0×5)×8	(0×8)×1	(0×2)×5
(8×3)×0	(7×6)×0	(8×0)×3	(7×0)×6	(0×8)×3	(0×7)×6	(0×6)×3	(0×4)×7

Partial Three-way Cross Plan for Test Lines Vs. Control Line

The parameters of the given partial three-way cross plan for test lines vs. a control line are $N = 9$ (8 test + 1 control), $T = 48$, number $r_{ft} = 4$, $r_{fc} = 16$, $r_{ht} = 8$, $r_{hc} = 32$ and $f = 0.190$. Also, we have $\overline{\mathrm{V}}_{\mathrm{full\_tvst}}\left(\widehat{g_i - g_j}\right) = 0.428$, $\overline{\mathrm{V}}_{\mathrm{full\_tvsc}}\left(\widehat{g_i - g_j}\right) = 0.187$, $\overline{\mathrm{V}}_{\mathrm{half\_tvst}}\left(\widehat{h_i - h_j}\right) = 0.214$ and $\overline{\mathrm{V}}_{\mathrm{half\_tvsc}}\left(\widehat{h_i - h_j}\right) = 0.093$.

SAS code for three-way cross plans involving test Vs control comparisons: SAS code is written to compute the average variance factor of estimated contrast pertaining to gca effects of half parents as well as full parents for test vs. test line and for test vs. control line comparison through three-way cross plans. The user has to just enter the three-way cross plan in the data section of SAS code along with the number of test and control lines, and all four types of average variance factors will be computed. The SAS code along with entered data is provided in Annexure III.

List of plans: A list of partial three-way cross plans obtained using the introduced method of construction for comparing test lines with a control line along with parameters and average variance factors of estimated contrasts pertaining to gca effects of for test vs. test line comparisons of full parents ($\overline{\mathrm{V}}_{\mathrm{full\_tvst}}\left(\widehat{g_i - g_j}\right)$), test vs. test line comparisons of half parents ($\overline{\mathrm{V}}_{\mathrm{half\_tvst}}\left(\widehat{h_i - h_j}\right)$), test vs. control line comparisons of full parents ($\overline{\mathrm{V}}_{\mathrm{full\_tvsc}}\left(\widehat{g_i - g_j}\right)$) and test vs. control line comparisons for half parents ($\overline{\mathrm{V}}_{\mathrm{half\_tvsc}}\left(\widehat{h_i - h_j}\right)$) has been calculated and given in Table 3.4.

Table 3.4. Partial three-way cross plans for test lines versus control line comparisons

N	***T***	***f***	$\overline{\mathbf{V}}_{\mathbf{full\_tvst}}\left(\widehat{\boldsymbol{g_i - g_j}}\right)$	$\overline{\mathbf{V}}_{\mathbf{half\_tvst}}\left(\widehat{\boldsymbol{h_i - h_j}}\right)$	$\overline{\mathbf{V}}_{\mathbf{full\_tvsc}}\left(\widehat{\boldsymbol{g_i - g_j}}\right)$	$\overline{\mathbf{V}}_{\mathbf{half\_tvsc}}\left(\widehat{\boldsymbol{h_i - h_j}}\right)$
4+1	18	0.600	0.714	0.429	0.268	0.161
5+1	30	0.500	0.519	0.296	0.207	0.119
6+1	45	0.429	0.409	0.227	0.170	0.094
7+1	63	0.375	0.338	0.185	0.145	0.079
8+1	48	0.190	0.428	0.214	0.187	0.093
10+1	75	0.151	0.356	0.178	0.160	0.080

3.6 Discussion

Method of constructing partial three-way cross plans for comparing test lines with single control line has been developed. The method is based on Incomplete Block Designs (IBD). The method of construction is very simple and these plans are available for wide range of parametric combination. The developed class of plans is having low degree of fractionation and hence experimenter can use them in case limited resource availability. The average variance factor pertaining to estimated general combining ability (gca) effects of half parents as well as full parents for test versus test lines and test versus control line comparisons has been calculated and it is found that the precision for test versus control line comparison is higher as compared to test versus test lines comparison. The method is illustrated through an example based on both variance balanced and partially variance balanced IBD. SAS code written to compute the average variance factor of estimated contrast pertaining to gca effects is provided in Annexure III.

CHAPTER 4

AUGMENTED PARTIAL FOUR-WAY CROSS DESIGNS

4.1 Introduction

Four-way cross hybrids are found to be consistence in performance and provide valuable information in terms of study of combining abilities. When the number of lines are higher, it is difficult to accommodate the total number of crosses which increases many folds, and in this situation it is advisable to take a sample of complete four-way cross, known as partial four-way crosses. Partial four-way crosses can also be used for the purpose of constructing augmented designs which are used to study the genetic nature of qualitative traits so that further better breeding strategies can be identified and developed.

4.2 Augmented partial multi-way cross designs

Breeders came across a unique type of situation in which they are having with them two groups of lines. The two groups of lines are not of equal importance from the breeder's point of view. The situation can be well understood through Figure-1 well depicting the situation.

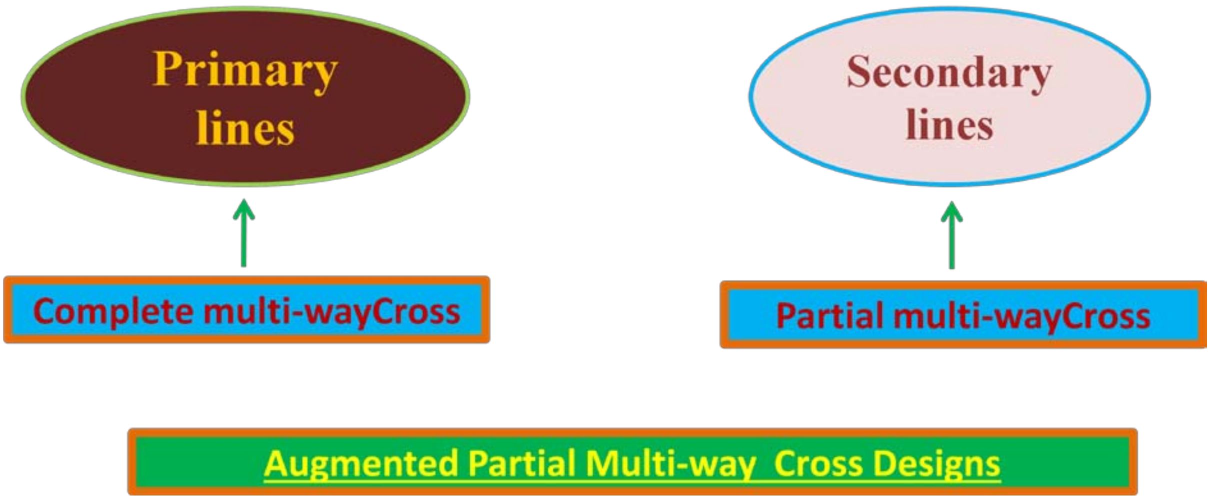

Figure: 1

In such cases the breeder will be always interested in having a higher proportion of superior lines known as primary lines. In this situation, from the set of primary lines, every line should be crossed with the remaining other lines referred as secondary lines and from the other set of lines suitable sample fraction can be taken. Mating designs for these types of conditions are often referred as augmented designs for multi-way crosses.

4.3 Augmented partial four-way cross designs

An Augmented Partial Four-way Cross (APFC) design is a combination of complete four-way cross and partial four-way cross. A complete four-way cross is obtained based on primary lines and a partial four-way cross is obtained based on secondary lines. The augmented setup also involves crosses based on mixture of primary and secondary lines, number of such crosses should be lying anywhere in between the other two categories.

Example 4.3.1: Here is an example of APFC plan for five primary lines (1, 2, 3, 4 and 5) and seven secondary lines (6, 7, 8, 9, 10, 11 and 12). (This peculiar setup is used to accommodate the plan in the available window space).

×	(1×2)	(1×3)	(1×4)	(1×5)	(2×3)	(2×4)	(2×5)	(3×4)	(3×5)	(4×5)	(6×7)	(7×8)	(8×9)	(9×10)	(10×11)	(11×12)	(12×1)
(1×2)	-	-	-	-	-	-	-	×	×	×	×	×	×	×	×	×	×
(1×3)	-	-	-	-	-	×	×	-	-	×	×	×	×	×	×	×	×
(1×4)	-	-	-	-	×	-	×	-	×	-	×	×	×	×	×	×	×
(1×5)	-	-	-	-	×	×	-	×	-	-	×	×	×	×	×	×	×
(2×3)	-	-	×	×	-	-	-	-	-	×	×	×	×	×	×	×	×
(2×4)	-	×	-	×	-	-	-	-	×	-	×	×	×	×	×	×	×
(2×5)	-	×	×	-	-	-	-	×	-	-	×	×	×	×	×	×	×
(3×4)	×	-	-	×	-	-	×	-	-	-	×	×	×	×	×	×	×
(3×5)	×	-	×	-	-	×	-	-	-	-	×	×	×	×	×	×	×
(4×5)	×	×	-	-	×	-	-	-	-	-	×	×	×	×	×	×	×
(6×7)	×	×	×	×	×	×	×	×	×	×	-	-	×	-	-	-	-
(7×8)	×	×	×	×	×	×	×	×	×	×	-	-	-	×	-	-	-
(8×9)	×	×	×	×	×	×	×	×	×	×	-	-	-	-	×	-	-
(9×10)	×	×	×	×	×	×	×	×	×	×	-	-	-	-	-	×	-
(10×11)	×	×	×	×	×	×	×	×	×	×	-	-	-	-	-	-	×
(11×12)	×	×	×	×	×	×	×	×	×	×	×	-	-	-	-	-	-
(12×1)	×	×	×	×	×	×	×	×	×	×	-	×	-	-	-	-	-

APFC plan for 5 primary and 7 secondary lines

It can be seen that for primary lines all possible four way crosses are included in the designs while for secondary lines only a fraction of total possible four-way crosses has been considered. The number of four-way crosses involving primary and secondary lines is intermediate between those involving primary and secondary lines.

4.4 Method of construction of APFC plan

Let n_p and n_s denote the number of primary and secondary lines respectively such that the total number of lines, n = n_p+ n_s. Let the total number of cross is N. Now, the design for APFC is obtained in three steps in which we have to prepare three sets of cross N_1, N_2 and N_3 respectively such that the total number of crosses in final design is$N$ = N_1 + N_2 + N_3.

Step 1: In this step we obtain a complete four-way cross based on primary lines. Thus the number of crosses prepared in the first step is given as

$$N_1 = \frac{n_p(n_p-1)(n_p-2)(n_p-3)}{8}.$$

Step 2: In this step firstly prepare two sets of complete two-way crosses based on n_p and n_s lines respectively. Now, all possible four-way crosses are obtained by considering every possible diallel pairs taken from different sets of complete two-way crosses. Thus, the number of crosses obtained in this step is

$$N_2 = \frac{n_p n_s(n_p-1)(n_s-1)}{4}.$$

Step 3: In the final step a partial four-way cross design is obtained based on secondary lines. This method is based on Mutually Orthogonal Latin Squares (MOLS). Consider n_s, the number of lines be a prime or prime power. Out of total (n_s-1) possible MOLS, consider any of the $\frac{(n_s-1)}{2}$ MOLS. Retaining the first four rows of each Latin square, n_s number of crosses corresponding to each column can be made from each MOLS. Since, there are n_s columns in each MOLS, the number of crosses obtained is

$$N_3 = \frac{n_s(n_s-1)}{2}.$$

Thus, the total number of crosses is

$$N = \frac{n_p(n_p-1)(n_p-2)(n_p-3) + \ 2n_p n_s(n_p-1)(n_s-1) + 4n_s(n_s-1)}{8}.$$

Degree of fractionation (f): The degree of fractionation of the constructed APFC plan is calculated as the ratio of number of crosses (N) involved in the APFC plan to the number of crosses (N_{CFC}) a complete four-way cross plan based on n lines. Thus,we have

$$f = \frac{n_p(n_p-1)(n_p-2)(n_p-3) + \ 2n_p n_s(n_p-1)(n_s-1) + 4n_s(n_s-1)}{(n_p+n_s)(n_p+n_s-1)(n_p+n_s-2)(n_p+n_s-3)}.$$

Example 4.4.1: The method can be well understood by an example for four primary lines (1, 2, 3 and 4) and five secondary lines (5, 6, 7, 8 and 9). The procedure is given stepwise as described earlier.

Step1: The four primary lines are considered here to obtain a complete set of four-way crosses. The crosses are as:

(1×2)×(3×4)
(1×3)×(2×4)
(1×4)×(2×3)

Complete four-way cross

based on 4 primary lines

Step 2: The four primary lines and five secondary lines are considered here to obtain two complete sets of two-way crosses. The two sets of two-way crosses are as:

(1×2)	(2×3)
(1×3)	(2×4)
(1×4)	(3×4)

Set 1

Complete two-way cross

based on 4 primary lines

(5×6)	(6×8)
(5×7)	(6×9)
(5×8)	(7×8)
(5×9)	(7×9)
(6×7)	(8×9)

Set 2

Complete two-way cross

based on 5 secondary lines

Now every possible combination of diallel cross from two sets is considered two obtain 60 four-way crosses as

(1×2) × (5×6)	(1×3) × (5×6)	(1×4) × (5×6)	(2×3) × (5×6)	(2×4) × (5×6)	(3×4) × (5×6)
(1×2) × (5×7)	(1×3) × (5×7)	(1×4) ×(5×7)	(2×3) ×(5×7)	(2×4) ×(5×7)	(3×4) ×(5×7)
(1×2) × (5×8)	(1×3) × (5×8)	(1×4) ×(5×8)	(2×3) ×(5×8)	(2×4) ×(5×8)	(3×4) ×(5×8)
(1×2) × (5×9)	(1×3) × (5×9)	(1×4) ×(5×9)	(2×3) ×(5×9)	(2×4) ×(5×9)	(3×4) ×(5×9)
(1×2) × (6×7)	(1×3) × (6×7)	(1×4) ×(6×7)	(2×3) ×(6×7)	(2×4) ×(6×7)	(3×4) ×(6×7)
(1×2) × (6×8)	(1×3) × (6×8)	(1×4) ×(6×8)	(2×3) ×(6×8)	(2×4) ×(6×8)	(3×4) ×(6×8)
(1×2) × (6×9)	(1×3) × (6×9)	(1×4) ×(6×9)	(2×3) ×(6×9)	(2×4) ×(6×9)	(3×4) ×(6×9)
(1×2) × (7×8)	(1×3) × (7×8)	(1×4) ×(7×8)	(2×3) ×(7×8)	(2×4) ×(7×8)	(3×4) ×(7×8)
(1×2) × (7×9)	(1×3) × (7×9)	(1×4) ×(7×9)	(2×3) ×(7×9)	(2×4) ×(7×9)	(3×4) ×(7×9)
(1×2) × (8×9)	(1×3) × (8×9)	(1×4) ×(8×9)	(2×3) ×(8×9)	(2×4) ×(8×9)	(3×4) ×(8×9)

Partial four-way cross based on both primary and secondary lines

Step 3: The five secondary lines are considered here to obtain a partial four-way cross plan based on MOLS of order five. Considering any of the 2 MOLS of order 5 chosen at random out of the total 4 possible MOLS of order 5 (constructed based on symbols 5, 6, 7, 8 and 9), and retaining only first 4 rows of each, as given below.

MOLS I (only four rows)				
5	6	7	8	9
6	7	8	9	5
7	8	9	5	6
8	9	5	6	7

MOLS II (only four rows)				
5	6	7	8	9
7	8	9	5	6
9	5	6	7	8
6	7	8	9	5

Now, considering each Latin Square, afour-way cross plan can be made by taking the four lines of each column. The final layout of the crosses so obtained is given below.

(5×6)×(7×8)	(5×7)×(9×6)
(6×7)×(8×9)	(6×8)×(5×7)
(7×8)×(9×5)	(7×9)×(6×8)
(8×9)×(5×6)	(8×5)×(7×9)
(9×5)×(6×7)	(9×6)×(8×5)

Partial four-way cross based on secondary lines

The final APFC plan obtained by taking all four-way crosses made in theses three steps together with parameters $n_p = 4$, $n_s = 5$, $n = 9$, $N_1 = 3$, $N_2 = 60$, $N_3 = 10$, $N = 73$ and $f = 0.19$.

(1×2)×(3×4)	(1×3)×(2×4)	(1×4)×(2×3)	(2×3)×(5×6)	(2×4)×(5×6)	(3×4)×(5×6)
(1×2)×(5×6)	(1×3)×(5×6)	(1×4)×(5×6)	(2×3)×(5×7)	(2×4)×(5×7)	(3×4)×(5×7)
(1×2)×(5×7)	(1×3)×(5×7)	(1×4)×(5×7)	(2×3)×(5×8)	(2×4)×(5×8)	(3×4)×(5×8)
(1×2)×(5×8)	(1×3)×(5×8)	(1×4)×(5×8)	(2×3)×(5×9)	(2×4)×(5×9)	(3×4)×(5×9)
(1×2)×(5×9)	(1×3)×(5×9)	(1×4)×(5×9)	(2×3)×(6×7)	(2×4)×(6×7)	(3×4)×(6×7)
(1×2)×(6×7)	(1×3)×(6×7)	(1×4)×(6×7)	(2×3)×(6×8)	(2×4)×(6×8)	(3×4)×(6×8)
(1×2)×(6×8)	(1×3)×(6×8)	(1×4)×(6×8)	(2×3)×(6×9)	(2×4)×(6×9)	(3×4)×(6×9)
(1×2)×(6×9)	(1×3)×(6×9)	(1×4)×(6×9)	(2×3)×(7×8)	(2×4)×(7×8)	(3×4)×(7×8)
(1×2)×(7×8)	(1×3)×(7×8)	(1×4)×(7×8)	(2×3)×(7×9)	(2×4)×(7×9)	(3×4)×(7×9)
(1×2)×(7×9)	(1×3)×(7×9)	(1×4)×(7×9)	(2×3)×(8×9)	(2×4)×(8×9)	(3×4)×(8×9)
(1×2)×(8×9)	(1×3)×(8×9)	(1×4)×(8×9)	(5×6)×(7×8)	(5×7)×(9×6)	(6×7)×(8×9)
(6×8)×(5×7)	(7×8)×(9×5)	(7×9)×(6×8)	(8×9)×(5×6)	(8×5)×(7×9)	(9×5)×(6×7)
(9×6)×(8×5)					

APFC plan for 4 primary and 5 secondary lines

4.5 Information matrix

The information matrix of the developed class of APFC plan can be derived by considering the usual fixed effect model for four-way crosses under unblocked setup.

4.5.1 Model and experimental setup

The model can be written in matrix notation as

$$\boldsymbol{y} = \mu\mathbf{1} + \Delta^{'}\boldsymbol{\tau} + \boldsymbol{e},$$

where $\boldsymbol{y}$ is an $N \times 1$ vector of responses, $\mathbf{1}$ is a $N \times 1$ vector of ones, $\Delta^{'}$ is a $N \times n$ incidence matrix of crosses versus lines, $\boldsymbol{\tau}$ is a $N \times 1$ vector of treatment (cross) effect and $\mathbf{e}$ is a $N \times 1$ vector of errors. Now, the design matrix $\boldsymbol{X}$ be partitioned into parameters of interest ($\boldsymbol{X}_1$) and the nuisance parameters ($\boldsymbol{X}_2$).

Thus, the design matrix corresponding to the model given here can be partitioned as

$$\boldsymbol{X} = [\boldsymbol{X}_1\ \boldsymbol{X}_2] = [\Delta'\ \mathbf{1}].$$

The information matrix can be obtained as $\boldsymbol{C}_\tau = \boldsymbol{X}_1'\boldsymbol{X}_1 - \boldsymbol{X}_1'\boldsymbol{X}_2(\boldsymbol{X}_2'\mathbf{X}_2)^{-}\boldsymbol{X}_2'\boldsymbol{X}_1$, where $N \times N$ matrix $\boldsymbol{C}_\tau$ is symmetric, non-negative definite and doubly centered matrix with zero row and column sums.

Before arriving at final form of information matrix following intermediate results are needed to be obtained. Firstly, we will define and derive the important terms needed to derive the information matrix.

Let the number of times for which a primary line is participating in different four-way crosses of the APFC plan is denoted by r_p. The expression for r_p is obtained as:

$$r_p = r_p \in N_1 + r_p \in N_2.$$

Now, we can obtain

$$r_p \in N_1 = \frac{(n_p - 1)(n_p - 2)(n_p - 3)}{2}$$

and

$$r_p \in N_2 = \frac{n_s(n_s - 1)(n_p - 1)}{2}.$$

Thus, by adding we can have

$$r_p = \frac{(n_p - 1)}{2}\{(n_p - 2)(n_p - 3) + n_s(n_s - 1)\}.$$

Let the number of times for which a secondary line is participating in different four-way crosses of the APFC plan is denoted by r_s. The expression for r_s is obtained as:

$$r_s = r_s \in N_2 + r_p \in N_3.$$

In a similar way, we can find that

$$r_s \in N_2 = \frac{n_p(n_p - 1)(n_s - 1)}{2}$$

and

$$r_s \in N_3 = 2(n_s - 1).$$

Thus, we have

$$r_s = \frac{(n_s - 1)}{2}\{n_p(n_p - 1) + 4\}.$$

Now, let the number of times a given pair of primary lines appearing together in different four-way crosses of the APFC plan is denoted by r_{pp}. Then we have

$$r_{pp} = r_{pp} \in N_1 + r_{pp} \in N_2.$$

Now, we can calculate

$$r_{pp} \in N_1 = \frac{3(n_p - 2)(n_p - 3)}{2}$$

and

$$r_{pp} \in N_2 = \frac{n_s(n_s - 1)}{2}.$$

Finally, by adding both expressions we get

$$r_{pp} = \frac{3\left(n_p - 2\right)\left(n_p - 3\right) + n_s(n_s - 1)}{2}.$$

Now, let the number of times a given pair of secondary lines appearing together in different four-way crosses of the APFC plan is denoted by r_{ss}. Then we have

$$r_{ss} = r_{ss} \in N_2 + r_{ss} \in N_3.$$

Now,

$$r_{ss} \in N_2 = 6$$

and

$$r_{ss} \in N_3 = \frac{n_p(n_p - 1)}{2}.$$

Thus, we have

$$r_{ss} = \frac{12 + n_p(n_p - 1)}{2}.$$

Let the number of times a given pair of primary and secondary lines appearing together in different four-way crosses of the APFC plan is denoted by r_{ps}. Then we have

$$r_{ps} = \left(n_p - 1\right)(n_s - 1).$$

Now the information matrix related to gca effects of lines is given as

$$\boldsymbol{C}_{gca} = \begin{bmatrix} \boldsymbol{C}_{11} & \boldsymbol{C}_{12} \\ \boldsymbol{C}_{21} & \boldsymbol{C}_{22} \end{bmatrix},$$

where

$$\boldsymbol{C}_{11} = \left(r_p - r_{pp}\right)\boldsymbol{I}_{n_p,n_p} + \left(r_{pp} - \frac{\boldsymbol{r_p^2}}{\boldsymbol{N}}\right)\boldsymbol{J}_{n_p,n_p},$$

$$\boldsymbol{C}_{12} = \boldsymbol{C}_{21}' = (r_{ps} - \frac{r_p r_s}{\boldsymbol{N}})\boldsymbol{J}_{n_p,n_s}$$

and finally we have expression for

$$\boldsymbol{C}_{22} = (r_s - r_{ss})\boldsymbol{I}_{n_s,n_s} + \left(r_{ss} - \frac{\boldsymbol{r_s^2}}{\boldsymbol{N}}\right)\boldsymbol{J}_{n_s,n_s}.$$

Proceeding with thederived information matrices related to gca effects ($\boldsymbol{C}_{gca}$), the variance factor estimates of contrasts pertaining to gca effects of primary Vs. primary {$\mathrm{V}_{\mathrm{pvsp}}\left(\widehat{g_i - g_j}\right)$}, primary Vs. secondary {$\mathrm{V}_{\mathrm{pvss}}\left(\widehat{g_i - g_j}\right)$} and secondary Vs. secondary lines {$\mathrm{V}_{\mathrm{svss}}\left(\widehat{g_i - g_j}\right)$} can be obtained. Since, the plans so obtained may not be variance balanced leading to more than one type of variance estimates based on underlying association schemes for each category. Hence, a weighted average based on number of associates can be calculated. Thus, the average variance factor estimates of contrasts pertaining to gca effects of primary Vs. primary {$\overline{\mathrm{V}}_{\mathrm{pvs\,p}}\left(\widehat{g_i - g_j}\right)$}, primary Vs. secondary {$\overline{\mathrm{V}}_{\mathrm{pvss}}\left(\widehat{g_i - g_j}\right)$} and secondary Vs. secondary lines {$\overline{\mathrm{V}}_{\mathrm{svss}}\left(\widehat{g_i - g_j}\right)$} can also be obtained.

Remark: For the APFC plan illustrated in Example 4.4.1, the average variance factor estimates are $\overline{\mathrm{V}}_{\mathrm{pvsp}}\left(\widehat{g_i - g_j}\right) = 0.100$, $\overline{\mathrm{V}}_{\mathrm{pvss}}\left(\widehat{g_i - g_j}\right) = 0.098$, $\overline{\mathrm{V}}_{\mathrm{svss}}\left(\widehat{g_i - g_j}\right) = 0.1$, which are almost same. The results are not in tune with the standard results of augmented designs because for lower number of secondary lines the replications are almost same for primary and secondary lines. As the number of lines increases, the trend of variance factor estimates changes, which can be seen in results tabulated in the table in next section.

SAS code for APFC plans: SAS code is written to compute the average variance factor of estimated contrast pertaining to gca effects for primary Vs. primary, primary Vs. secondary and secondary Vs. secondary lines comparison through augmented partial four-way cross plans. The user has to just enter to enter the number of primary lines, number of secondary lines along with the three-way cross plan in the data section of SAS code and all three types of average variance factors will be computed. The SAS code along with entered data is provided in Annexure IV.

List of plans: A list of APFC plans for comparing primary Vs. primary, primary Vs. secondary and secondary Vs. secondary lines along with parameters and average variance factors for different categories of comparisons has been tabulated here.

Table 4.4. APFC plans for comparing primary Vs. primary, primary Vs. secondary and secondary Vs. secondary lines

N	T	f	$\overline{\mathrm{V}}_{\mathrm{pvsp}}\left(\widehat{g_i - g_j}\right)$	$\overline{\mathrm{V}}_{\mathrm{pvss}}\left(\widehat{g_i - g_j}\right)$	$\overline{\mathrm{V}}_{\mathrm{svss}}\left(\widehat{g_i - g_j}\right)$
4+5	73	0.193	0.100	0.098	0.100
4+7	150	0.152	0.056	0.053	0.048
4+11	388	0.095	0.029	0.025	0.018
4+13	549	0.077	0.024	0.019	0.013

4.6 Discussion

Method of constructing Augmented Partial Four-way Cross (APFC) plans for comparing primary Vs. primary, primary Vs. secondary and secondary Vs. secondary lines has been developed. The method is based on Mutually Orthogonal Latin Squares (MOLS). The method of construction is very simple and these plans are available for any number of primary lines and prime or prime power number of secondary lines. The developed class of plans is having low degree of fractionation and hence experimenter can use them in case limited resource availability. The information matrix related to general combining ability (gca) effects has been also derived. The average variance factor of contrasts pertaining to estimated general combining ability (gca) effects of for comparing primary Vs. primary, primary Vs. secondary and secondary Vs. secondary lines has been calculated. It is found that the precision for comparing primary Vs. primary lines is highest followed by primary Vs. secondary and secondary Vs. secondary lines comparison. The method is illustrated through an example and a list of plans is also tabulated. SAS code written to compute the average variance factor of estimated contrast pertaining to gca effects for primary Vs. primary, primary Vs. secondary and secondary Vs. secondary lines is provided in Annexure IV.

सार

त्रि-पथ एवं चार-पथ वर्ण-संकरों का दोहन अनुवांशिक लक्षणों के साथ-साथ विशिष्ट संयोजन क्षमता प्रभावों संबंधी सूचना प्राप्त कर आर्थिक महत्व सहित व्यवसायिक वर्ण-संकरो के विकास हेतु एक साधन के रूप में किया जा सकता है और यह केवल तभी संभव है यदि हम किसी तरह एक उचित पद्धति का विकास कर परीक्षण के आकार को कम करने में सक्षम हों। इस अध्ययन में, त्रि-पथ और चार-पथ वर्ण-संकरों हेतु विशिष्ट संयोजन क्षमता प्रभावों को सम्मिलित कर एक स्थायी प्रभाव मॉडल के अंतर्गत सामान्य एवं विशिष्ट संयोजन क्षमताओं के प्रभाव का आंकलन किया गया है। प्राचलिक संयोगों की विस्तृत परास के लिए त्रि-पथ एवं चार-पथ वर्ण-संकरों हेतु दक्ष अभिकल्पनाओं एवं योजनाओं की संरचना पद्धति प्राप्त की गयी है। सूचना आव्यूहों, औसत प्रसरण घटकों एवं विभाजन की मात्रा को व्युत्पन्न किया गया है। एक नियंत्रण लाइन के साथ परीक्षण लाइनों की तुलना हेतु कम विभाजन की मात्रा वाली, अपूर्ण खण्ड अभिकल्पनाओं पर आधारित दक्ष त्रि-पथ वर्ण-संकर योजनाओं की संरचना पद्धति भी विकसित की गयी है। परीक्षण बनाम परीक्षण लाइनों एवं परीक्षण बनाम नियंत्रण लाइनों की तुलना हेतु अर्ध-अभिभावकों के साथ-साथ पूर्ण-अभिभावकों की सामान्य संयोजन क्षमता प्रभावों के आंकलित व्यतिरेकों से संबन्धित औसत प्रसरण घटकों की गणना की गयी है। इसके अतिरिक्त, प्राथमिक बनाम प्राथमिक, प्राथमिक बनाम द्वितीयक एवं द्वितीयक बनाम द्वितीयक लाइनों हेतु संवर्धित आंशिक चार-पथ वर्ण-संकर योजना की संरचना पद्धति विकसित की गयी है। त्रि-पथ एवं चार-पथ अभिकल्पनाओं की सभी विकसित क्षेणियाँ और योजनाएं दक्ष एवं कमतर विभाजन की मात्रा वाली हैं जो सीमित संसाधनो वाले परीक्षणों हेतु उपयुक्त हैं। अंततः त्रि-पथ एवं चार-पथ वर्ण-संकर परीक्षण हेतु निर्मित अभिकल्पनाओं एवं योजनाओं से संबन्धित प्रामाणिक दक्षता कारकों, औसत प्रसरण घटकों इत्यादि की गणना के लिए प्रयोक्ता अनुकूल SAS कोड्स लिखे गए हैं।

ABSTRACT

Three-way and four way crosses can be exploited as a tool for the development of commercial hybrids with traits of genetical as well as economical importance by acquiring information regarding specific combining ability (sca) effects, and this is only possible if anyhow we are able to reduce the experimentation size by developing suitable methodology. In this study, under a fixed effects model including sca effects for three-way and four-way crosses, the estimates of general and specific combining abilities have been obtained. Methods of construction efficient designs and plans involving partial three-way and partial four-way crosses have been obtained for a wide range of parametric combinations. The information matrices, average variance factors, efficiency factors and degree of fractionation have been derived. Also, method of constructing efficient partial three-way cross plans based on Incomplete Block Designs (IBD) with low degree of fractionation for comparing test lines with single control line has been developed. The average variance factor pertaining to estimated contrasts of general combining ability (gca) effects of half parents as well as full parents for test versus test lines and test versus control line comparisons has been calculated. Moreover, method of constructing Augmented Partial Four-way Cross (APFC) plans for comparing primary Vs. primary, primary Vs. secondary and secondary Vs. secondary lines has been developed. The information matrix related to gca effects has been also derived. All the developed class of three-way and four-way designs as well as plans are efficient and having low degree of fractionation making those suitable for experimentation with limited resources. Finally, user friendly SAS codes are written for every class of designs and plans obtained here for three-way and four-way cross experiment to compute canonical efficiencies, average variance factors etc.

SUMMARY

Breeding techniques are used as a tool for the development of commercial hybrids for which a major objective of plant and animal breeders is to raise the genetic potential. Any breeding experiment centres on acquiring information regarding the general combining ability (gca) effects of the individual lines involved as parents and the specific combining ability (sca) effects of the crosses. The information collected on gca and sca forms a basis of making correct choice of the best parental lines. The sca effects are of much importance for breeders besides the gca effects. Information on sca effects can form a basis for introducing various traits of genetical as well as economical importance.

Three-way and four-way crosses have been considered for the study because, in case of two-way crosses, only one component of sca effects, which is first order sca effect can be studied whereas in three-way crosses two components of sca which are, first order and second order sca effects and in case of four-way crosses three components which are first, second and even third order sca effect can be studied. Thus we can see that the higher order mating designs like three-way and four-way crosses are useful in exploiting the epistatic gene action needed to inculcate desirable breeding qualities in the hybrids. However, two-way cross is the most simple and easily manageable mating design, but at the same time three-way and four-way cross based hybrids are found to be genetically more viable, stable and consistent in performance. Also, breeders can introduce and improve the performance of crops and animals using the additional information provided to them while performing experimentation with three-way and four-way crosses.

In this study under a fixed effects model including sca effects for three-way crosses, the estimates of general and specific combining abilities have been obtained. A class of designs for partial three-way cross arranged in blocks has been obtained using triangular association scheme and information matrices, eigenvalues, variance factors, efficiency factors and degree of fractionation have been derived. Another method of constructing partial three-way cross designs has also been developed using various types of lattice designs viz., square, rectangular, circular and cubic lattice. This method gives designs for partial three-way crosses under blocked set up for a wide range of parameters. The main restriction of first series of designs is that these designs are available only for cases where the number of lines is of the

particular form $N = \frac{n(n-1)}{2}$. The second series is available for many combinations and can be used in conjunction with the first method to fill the gaps of designs not available for particular parameters. The third method, of constructing partial three-way cross plans is based on Kronecker product of incidence matrices. The plans obtained are having low degree of fractionation and high efficiencies and can be used when there is scarcity of resources. These methods methods can provide designs for almost all sets of parametric combinations. With an adequate knowledge of block designs, all the proposed crossing plans can easily be constructed. Under a restricted model including lower order sca effects for tetra-allele crosses, conditions of orthogonality have been derived for a block design such that the contrasts pertaining to the gca effects and sca effects are estimated free from each other, after eliminating the other nuisance factors. A method of construction of optimal block designs has been described and a class of optimal designs based on Mutually Orthogonal Latin Squares have been obtained.

Method of constructing partial three-way cross plans for comparing test lines with single control line has been developed. The method is based on Incomplete Block Designs (IBD). The method of construction is very simple and these plans are available for wide range of parametric combination. The developed class of plans is having low degree of fractionation and hence experimenter can use them in case limited resource availability. The average variance factor pertaining to estimated general combining ability (gca) effects of half parents as well as full parents for test versus test lines and test versus control line comparisons has been calculated and it is found that the precision for test versus control line comparison is higher as compared to test versus test lines comparison. The method is illustrated through an example based on both variance balanced and partially variance balanced IBD.

Method of constructing Augmented Partial Four-way Cross (APFC) plans for comparing primary Vs. primary, primary Vs. secondary and secondary Vs. secondary lines has been developed. The method is based on Mutually Orthogonal Latin Squares (MOLS). The method of construction is very simple and these plans are available for any number of primary lines and prime or prime power number of secondary lines. The developed class of plans is having low degree of fractionation and hence experimenter can use them in case limited resource availability. The information matrix related to general combining ability (gca) effects has been also derived. The average variance factor of contrasts pertaining to estimated general combining ability (gca) effects of for comparing primary Vs. primary, primary Vs. secondary and secondary Vs. secondary lines has been calculated. It is found that the precision for comparing primary Vs. primary lines is highest followed by primary Vs. secondary and secondary Vs. secondary lines comparison. The method is illustrated through an example and a list of plans is also tabulated.

SAS codes are written for every class of designs obtained here for three-way and four-way cross experiment. These codes will help the breeders to choose an efficient design suitable for the experimentation by calculating the efficiencies of the designs along with the degree of fractionation. The codes are user friendly as the user has to just enter the design along with few parameters to get the result.

Finally, it may be noted that higher order crosses like three-way and four way crosses can be exploited as a tool for the development of commercial hybrids with traits of genetical as well as economical importance by acquiring information regarding specific combining ability (sca) effects. Here, the investigation has been done by taking only lower order sca effects in the model. A possible extension of the present study may be including sca effects of higher order in the model.

REFERENCES

Arora, B.S. and Aggarwal, K.R. (1984). Confounded triallel experiments and their applications. *Sankhya,* **B46**, 54-63.

Arora,B.S. and Aggarwal, K.R.(1989). Triallel experiments with reciprocal effects. *Journal of the Indian Society of Agricultural Statistics*, **41**, 91-103.

Choi, K.C., Gupta, S. and Kageyama, S. (2004). Designs for diallel crosses for test versus control comparisons. *Utilitas Mathematica*, **65**, 167-180.

Das, A. and Gupta, S. (1997). Optimal block designs for triallel cross experiments. *Communications in Statistics-Theory and Methods*, **26(7)**, 1767-1777.

Das, A., Gupta, S. and Kageyama, S. (2006). A-optimal diallel crosses for test versus control comparisons. *Journal of Applied Statistics,***33(6)**, 601-608.

Dharmalingam, M. (2002). Construction of partial triallel crosses based on Trojan square designs. *Journal of Applied Statistics*, **29(5)**, 695-702.

Harun, Mohd. (2014). Statistical designs involving three-way crosses. Unpublished M.Sc. Thesis, IARI, New Delhi.

Harun, Mohd., Varghese, C., Varghese, E. and Jaggi, S. (2016 a). Three-way cross designs for animal breeding experiments. *The Indian Journal of Animal Sciences,* **86(6),** 710–714.

Harun, Mohd., Varghese, C., Varghese, E. and Jaggi, S. (2016 b). Triallel cross designs for comparing a set of test lines with a control line. *The Indian Journal of Genetics and Plant Breeding,* **76(1)**, 84-87.

Harun, Mohd., Varghese, C., Varghese, E. and Jaggi, S. (2016 c). Three-way cross designs for test lines vs. control line comparisons. *Electronic Journal of Plant Breeding,* **76(1)**, 84-87.

Harun, M, Varghese, C., Jaggi, S., Varghese, E., Datta, A. and Bhowmik, A. (2017). Partial three-way cross plans for hybrid production. *Bharatiya Krishi Anusandhan Patrika,* **32(4),** 309-312.

Harun, M., Varghese, C., Jaggi, S., Datta, A. and Varghese, E. (2019). Efficient and Cost Effective Partial Three-Way Cross Designs for Breeding Experiments with Scarce Resources. *Journal of the Indian Society of Agricultural Statistics*, **73(1)**, 71-78.

Hinkelmann, K. (1965) . Partial triallel crosses. *Sankhya*, **27**, 173–196.

Hsu,Y.F. and Ting, C.P. (2005). A-optimal and efficient diallel cross experiments for comparing test treatments with a control. *Statistics and Probability Letters*, **71(1)**, 99-110.

Kaistha, B.L. (1982). Some investigations on the design anf analysis of diallel crosses, Unpublished Ph.D. Thesis, IARI, New Delhi.

Khawaja, T., Khan, S.H., Mukhtar, N., Parveen, A. and Fareed, G. (2013). Production performance, egg quality and biochemical parameters of three way crossbred chickens with reciprocal F_1 crossbred chickens in sub-tropical environment. *Italian Journal of Animal Science*, **12** (**1**), 178-189.

Ponnuswamy, K.N. (1971). Some contributions to design and analysis for diallel and triallel crosses. Unpublished Ph.D. Thesis, IARI, New Delhi.

Ponnuswamy, K.N. and Srinivasan, M.R. (1991). Construction of partial triallel crosses (PTC) using a class of balanced incomplete block designs (BIBD). *CommunicationsinStatistics - Theory and Methods*, **A20**, 3315-3323.

Rawlings, J.O. and Cockerham, C.C. (1962). Analysis of double cross hybrid populations, *Biometrika*, **18**, 229-244.

Shunmugathai, R and Srinivasan, M.R. (2012). Robustness of triallel cross experiments using NBIB mating design against interchange of a cross. *International Journal of Mathematics and Scientific Computing*, **2**(**2**), 86-92.

Srivastava S., Varghese C., Jaggi S. and Varghese E. (2013). Diallel cross designs for test versus control comparisons. *The Indian Journal of Genetics and Plant Breeding*. **73(2**), 186-193.

Srivastava S., Varghese C., Jaggi S. and Varghese E. (2015). Augmented partial diallel cross plans involving two sets of parental lines. *The Indian Journal of Genetics and Plant Breeding*.**75(1)**, 105-109.

Srinivasan, M.R. and Ponnuswamy, K.N. (1995). Estimation of variance componentsbased on partial triallel mating design. *Biometrical Journal*,**37(2)**, 161-171.

Varghese, C. andJaggi, S. (2011). Mating designs using generalized incomplete Trojan-type designs. *Journal of Statistics and Applications*,**6(3-4)**, 85-93.

Venkatesan, R. (1985). Some aspects of multi-way crosses. Unpublished Ph. D. Thesis, IARI, New Delhi.

ANNEXURE I

SAS code for computing canonical efficiency factor of the design involving three-way crosses for estimating contrast pertaining to gca effects of half parents as well as full parents under blocked set-up

```
%let r1=6;/*replication of half parents*/
%let r2=3;/*replication of full parents*/
data Triallel;
input Block line1 line2 line3;
cards;
1	1	2	3
1	4	5	6
1	7	8	9
;
run;
proc iml;
use triallel;
read all into xx;
/*print xx;*/
cross=xx[ ,2]||xx[ ,3]||xx[ ,4];
m=j(nrow(cross),1,1);
/*print cross;*/
x=j(nrow(cross),max(cross),0);
k=1;
do i=1 to nrow(cross);
do j=1 to ncol(cross)-1;
if cross[i,j]>0 then
x[k,cross[i,j]]=1;
end;
k=k+1;
end;
*print x;
z=j(nrow(cross),max(cross),0);
k=1;
do i=1 to nrow(cross);
if cross[i,3]>0 then
z[k,cross[i,3]]=1;
k=k+1;
end;
*print z;
block=j(nrow(xx[ ,1]),max(xx[ ,1]),0);
k=1;
do i=1 to nrow(xx[ ,1]);
if xx[i,1]>0 then
block[k,xx[i,1]]=1;
k=k+1;
end;
*print block;
x2=m||block;
c11=(x`*x)-(x`*x2)*ginv(x2`*x2)*(x2`*x);
```

```
c12=(x`*z)-(x`*x2)*ginv(x2`*x2)*(x2`*z);
c22=(z`*z)-(z`*x2)*ginv(x2`*x2)*(x2`*z);
c_mat=(c11||c12)//(c12`||c22);
*print c_mat;
c_halfparent=c11-c12*ginv(c22)*c12`;
c_fullparent=c22-c12`*ginv(c11)*c12;
*print c_halfparent;
*print c_fullparent;
l=nrow(c_halfparent);
ll=comb(nrow(c_halfparent),2);
contrast=j(ll,l,0);
k=1;
do i=1 to l-1;
do j=i to l-1;
contrast[k,i]=1;
contrast[k,j+1]=-1;
k=k+1;
end;
end;
*print contrast;
ginv_hp=ginv(c_halfparent);
ginv_fp=ginv(c_fullparent);
varcov_halfparent=contrast*ginv(c_halfparent)*contrast`;
varcov_fullparent=contrast*ginv(c_fullparent)*contrast`;
var_halfparent=j(ll,1,0);
do i= 1 to ll;
var_halfparent[i,1]=varcov_halfparent[i,i];
end;
ave_var_halfparent=var_halfparent[+, ]/nrow(var_halfparent);
*print var_halfparent;
*print ave_var_halfparent;
var_fullparent=j(ll,1,0);
do i= 1 to ll;
var_fullparent[i,1]=varcov_fullparent[i,i];
end;
*print var_fullparent;
ave_var_fullparent=var_fullparent[+, ]/nrow(var_fullparent);
*print ave_var_fullparent;
eigH=eigval(c_halfparent);
print eigH;
eigF=eigval(c_fullparent);
print eigF;
eigH1=eigH[loc(eigH>0.0000001),];/*positive eigen values*/
eigF1=eigF[loc(eigF>0.0000001),];/*positive eigen values*/
*print eigH1;
*print eigF1;
eigH2=eigH1/&r1;
eigF2=eigF1/&r2;
eigH3=1/eigH2;
eigF3=1/eigF2;
CanEffFacH=nrow(eigH3)/sum(eigH3);
CanEffFacF=nrow(eigF3)/sum(eigF3);
print CanEffFacH;
print CanEffFacF;
quit;
```

ANNEXURE II

SAS code for computing average variance factor of estimated contrasts pertaining to gca effects of test Vs. test lines for half parents as well as full parents through partial three-way cross plans under unblocked set-up

```
data Triallel;
input line1 line2 line3;
cards;
1       2       5
1       3       6
1       4       7
;
run;
proc iml;
use triallel;
read all into cross;
print xx;
k=max(cross[ ,1]);
kk=max(cross[ ,2]);
l=max(k,kk);
ll=comb(l,2);
print l;
print ll;
m=j(nrow(cross),1,1);
print cross;
x=j(nrow(cross),max(cross),0);
k=1;
do i=1 to nrow(cross);
do j=1 to ncol(cross)-1;
if cross[i,j]>0 then
x[k,cross[i,j]]=1;
end;
k=k+1;
end;
x=x/2;
print x;
z=j(nrow(cross),max(cross),0);
k=1;
do i=1 to nrow(cross);
if cross[i,3]>0 then
z[k,cross[i,3]]=1;
k=k+1;
end;
z=z;
print z;
x2=m;
c11=(x`*x)-(x`*x2)*ginv(x2`*x2)*(x2`*x);
c12=(x`*z)-(x`*x2)*ginv(x2`*x2)*(x2`*z);
c22=(z`*z)-(z`*x2)*ginv(x2`*x2)*(x2`*z);
c_mat=(c11||c12)//(c12`||c22);
*print c_mat;
```

```
c_halfparent=c11-c12*ginv(c22)*c12`;
c_fullparent=c22-c12`*ginv(c11)*c12;
print c_halfparent;
print c_fullparent;
l=nrow(c_halfparent);
ll=comb(nrow(c_halfparent),2);
contrast=j(ll,l,0);
k=1;
do i=1 to l-1;
do j=i to l-1;
contrast[k,i]=1;
contrast[k,j+1]=-1;
k=k+1;
end;
end;
*print contrast;
varcov_halfparent=contrast*ginv(c_halfparent)*contrast`;
varcov_fullparent=contrast*ginv(c_fullparent)*contrast`;
print var_halfparent;
print var_fullparent;
var_halfparent=j(ll,1,0);
do i= 1 to ll;
var_halfparent[i,1]=varcov_halfparent[i,i];
end;
ave_var_halfparent=var_halfparent[+, ]/nrow(var_halfparent);
print var_halfparent;
print ave_var_halfparent;
var_fullparent=j(ll,1,0);
do i= 1 to ll;
var_fullparent[i,1]=varcov_fullparent[i,i];
end;
print var_fullparent;
ave_var_fullparent=var_fullparent[+, ]/nrow(var_fullparent);
print ave_var_fullparent;
quit;
```

ANNEXURE III

SAS code for computing variance factors of estimated contrasts pertaining to gca effects of test Vs. test and test Vs. control lines for half parents as well as full parents through partial three-way cross plans for test Vs control comparisons

```
%let t=10;/*number of tests*/
%let cc=1;/*number of controls*/
data Triallel;
input line1 line2 line3;
cards;
11      1       2
1       2       3
2       3       4
;
run;
proc iml;
use triallel;
read all into cross;
/*print xx;*/
k=max(cross[ ,1]);
kk=max(cross[ ,2]);
l=max(k,kk);
ll=comb(l,2);
m=j(nrow(cross),1,1);
/*print cross;*/
x=j(nrow(cross),max(cross),0);
k=1;
do i=1 to nrow(cross);
do j=1 to ncol(cross)-1;
if cross[i,j]>0 then
x[k,cross[i,j]]=1;
end;
k=k+1;
end;
z=j(nrow(cross),max(cross),0);
k=1;
do i=1 to nrow(cross);
if cross[i,3]>0 then
z[k,cross[i,3]]=1;
k=k+1;
end;
x2=m;
c11=(x`*x)-(x`*x2)*ginv(x2`*x2)*(x2`*x);
c12=(x`*z)-(x`*x2)*ginv(x2`*x2)*(x2`*z);
c22=(z`*z)-(z`*x2)*ginv(x2`*x2)*(x2`*z);
c_mat=(c11||c12)//(c12`||c22);
c_halfparent=c11-c12*ginv(c22)*c12`;
c_fullparent=c22-c12`*ginv(c11)*c12;
tcont=j(&t*&cc,(&t+&cc),0);
k=1;
```

```
do i=1 to &cc;
do j=1 to &t;
tcont[k,j]=1;
tcont[k,&t+i]=-1;
k=k+1;
end;
end;
k=1;
if &cc>1 then do;
cccont=j(comb(&cc,2),(&t+&cc),0);
do i=&t+1 to (&t+&cc);
do j=i+1 to (&t+&cc);
cccont[k,i]=1;
cccont[k,j]=-1;
k=k+1;
end;
end;
end;
else do;
cccont=j(1,(&t+&cc),0);
end;
k=1;
totcont=j(comb(&t,2),(&t+&cc),0);/*test versus test*/
do i=1 to &t;
do j=i+1 to &t;
totcont[k,i]=1;
totcont[k,j]=-1;
k=k+1;
end;
end;
var_t_half=vecdiag(tcont*ginv(c_halfparent)*tcont`);
if &cc>1 then do;
var_c_half=vecdiag(cccont*ginv(c_halfparent)*cccont`);
end;
else do;
var_c_half=0;
end;
var_tot_half=vecdiag(totcont*ginv(c_halfparent)*totcont`);/*test versus test*/
avar_t_half=sum(var_t_half)/nrow(var_t_half);
avar_c_half=sum(var_c_half)/nrow(var_c_half);
avar_tot_half=sum(var_tot_half)/nrow(var_tot_half);/*test versus test*/
print avar_t_half avar_c_half avar_tot_half;
var_t_full=vecdiag(tcont*ginv(c_fullparent)*tcont`);
if &cc>1 then do;
var_c_full=vecdiag(cccont*ginv(c_fullparent)*cccont`);
end;
else do;
var_c_full=0;
end;
var_tot_full=vecdiag(totcont*ginv(c_fullparent)*totcont`);/*test versus test*/
avar_t_full=sum(var_t_full)/nrow(var_t_full);
avar_c_full=sum(var_c_full)/nrow(var_c_full);
avar_tot_full=sum(var_tot_full)/nrow(var_tot_full);/*test versus test*/
print avar_t_full avar_c_full avar_tot_full;
quit;
```

ANNEXURE IV

SAS code for computing average variance factor of estimated contrasts pertaining to gca effects of primary Vs. primary, primary Vs. secondary and secondary Vs. secondary lines for augmented partial four-way cross plans

```
%let t=5;/*number of secondary lines*/
%let cc=4;/*number of primary lines*/
data Tetrallele;
input line1 line2 line3 line4;
cards;
1       2       3       4
2       3       4       5
3       4       5       1
;
run;
proc iml;
use Tetrallele;
read all into cross;
m=j(nrow(cross),1,1);
x1=j(nrow(cross),max(cross),0);
k=1;
do i=1 to nrow(cross);
do j=1 to ncol(cross);
if cross[i,j]>0 then
x1[k,cross[i,j]]=1;
end;
k=k+1;
end;
x2=m;
print x1 x2;
c_mat=(x1`*x1)-(x1`*x2)*ginv(x2`*x2)*(x2`*x1);
print c_mat;
a= x1`*x1;
b=(x1`*x2);
c= ginv(x2`*x2);
d=(x2`*x1);
print a b c d;
/*l=nrow(c_mat);
print l;
ll=comb(nrow(c_mat),2);
print ll;
contrast=j(ll,l,0);
k=1;
do i=1 to l-1;
do j=i to l-1;
contrast[k,i]=1;
contrast[k,j+1]=-1;
k=k+1;
end;
end;
```

```
varcov=contrast*ginv(c_mat)*contrast`;
ginv=ginv(c_mat);
print ginv;
var=j(ll,1,0);
do i= 1 to ll;
var[i,1]=varcov[i,i];
end;
ave_var=var[+, ]/nrow(var);
tcont=j(&t*&cc,(&t+&cc),0);
k=1;
do i=1 to &cc;
do j=1 to &t;
tcont[k,j]=1;
tcont[k,&t+i]=-1;
k=k+1;
end;
end;
k=1;
if &cc>1 then do;
cccont=j(comb(&cc,2),(&t+&cc),0);
do i=&t+1 to (&t+&cc);
do j=i+1 to (&t+&cc);
cccont[k,i]=1;
cccont[k,j]=-1;
k=k+1;
end;
end;
end;
else do;
cccont=j(1,(&t+&cc),0);
end;
k=1;
totcont=j(comb(&t,2),(&t+&cc),0);
do i=1 to &t;
do j=i+1 to &t;
totcont[k,i]=1;
totcont[k,j]=-1;
k=k+1;
end;
end;
var_t=vecdiag(tcont*ginv(c_mat)*tcont`);
if &cc>1 then do;
var_c=vecdiag(cccont*ginv(c_mat)*cccont`);
end;
else do;
var_c=0;
end;
var_tot=vecdiag(totcont*ginv(c_mat)*totcont`);
avar_t=sum(var_t)/nrow(var_t);
avar_c=sum(var_c)/nrow(var_c);
avar_tot=sum(var_tot)/nrow(var_tot);
quit;
quit;
quit;
```