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Evaluation of alternative nonlinear mixed effects models for estimating pig growth parameters

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ABSTRACT

Animal growth models are used to identify alternative strategies to improve the efficiency of livestock production and to estimate daily nutrient requirements for the animal of different age and sex group. In this study, the efficiency of nonlinear mixed effects models were explored and a comparison was made between the predictive power of fixed effects models and the mixed effects models. Three hundred body weight (BW) data including male and female pigs were used for model fitting. One pig specific random effect was included in each of the models. The random function was a random deviation of mature BW of the individual from average mature BW of its genotype. Logistic, Gompertz and Von-Bertalanffy fixed and mixed models were explored for these data. It was found that Logistic mixed effects model performed better than the other nonlinear mixed effects models based on mean square error (MSE) and root mean square error (RMSE).

Key words: Fixed effects model, Mixed effects model, MSE, Nonlinear growth model, Pig growth, RMSE

Non-linear models are important tool in different domains of research work. Fitting of nonlinear model is not a single-step procedure. Fitting of these types of models solely depend on the objective and application domains.The animal growth pattern is associated with longitudinal measurements. With longitudinal data, the variance of observations may increase with time (age), and repeated measurements of an individual over time are correlated. The non-independence of data violates a key assumption underlying many statistical procedures and has been ignored in most traditional non-linear fixed effect models. A solution to this problem is the use of non-linear mixed effect models (NLMM) (Aggrey 2002, Wang and Zuidhof 2004). In case of longitudinal growth data, there are within and between individuals variation. The variability between individuals are not included in fixed effect model (Craig and Schinckel 2001). In case of longitudinal data, there are a correlation between the variance of observations along with increment of time (age). In case of repeated measurements, body weight (BW) from the same animal are likely to be more closely correlated than measures made on different animals, and measures made close in time on the same individual are likely to be more highly correlated than measures made further apart time (Littell *et al*. 2000). This relation between the BW and age in longitudinal data contradict the assumption that errors are independent of each other's with a constant variance overs the ages. For longitudinal data, there are two types variability, i.e.within and between individuals variation.The ignorance of this relationship affects the statistical property of data. Mixed model accounts the variance-covariance structure of serial BW data and has led to the development of a stochastic pig growth model (Schinckel *et al.* 2003, 2006). Karaman *et al*. (2013) applied Nonlinear mixed effects model in Japanese quail. Das *et al*. (2016) applied non-linear mixed effect models for estimation of growth parameters in Goats. Fitting of different growth model may help to get optimal growth pattern for the different ages as well as both male and female pigs.The basic objective of study was to find out the best fitted nonlinear mixed effects model and estimating the parameters.

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MATERIALS AND METHODS

Body weight (BW) data (300) including male and female pigs were used for model fitting. The pig data were collected from Piggery farm of IVRI, Izatnagar, for time interval of 1994 to 2001. The pig data were taken from the time period from birth to 8 week after birth, viz. W1, observations of body weights at birth; W2, observations of body weights after 1st week; W3, observations of body weights after 2nd week; W4, observations of body weights after 3rd week; W5, observations of body weights after 4th week; W6, observations of body weights after 5th week; W7, observations of body weights after 6th week; W8, observations of body weights after 7th week; and W9, observations of body weights after 8th week. All the observations were taken in kg. One pig specific random effect was included in each of the models. The random function was a random deviation of mature BW of the individual from average mature BW of its genotype. This function transformed the fixed effects models into mixed effects models. In the present study, 3 models, viz. i.e. Logistic, Gompertz and Von-Bertalanffy were used. The mathematical forms of the fixed effects models are:

1. Gompertz model: $W_{it} = W_m \exp^{(-\exp(b(t-t^*)))} + e_{it}$

2. Logistic model: $W_{it} = \dfrac{W_m}{1+e^{-(\frac{t-t^*}{b})}} + e_{it}$

3. Von-Bertalanffy model: $W_{it} = W_m(1-Be^{-b(t-t^*)})^3 + e_{it}$

where, W_m is the average mature BW of all individuals in the same group; b is rate of maturing; t* is the time in days at which growth rate is maximum; e_{it} is the residual BW of individual i at age t. e_{it} is assumed to be normally distributed with mean 0 and constant variance σ_{ij}. W_{it} is the expected BW of individual i at age t days. B is the integrating function.

A random function u_i is incorporated in the model. The random function is a random deviation of mature BW of the individual from average mature BW of its genotype W_m. Now Mixed effect model can also be written as

1. Gompertz model: $W_{it} = f(t; \theta) + u_i g(t; \theta) + e_{it}$

where $\theta=\{W_m, b\}$, such that the random effect u_i is multiplied by $g(t; \theta) = \exp^{[-\exp\{b(t-t^*)\}]}$

2. Logistic model: $W_{it} = \dfrac{(W_m+u)}{1+e^{-(\frac{t-t^*}{b})}} + e_{it}$

3. Von-Bertalanffy model: $W_{it} = \dfrac{(W_m+u)}{1+e^{-(\frac{t-t^*}{b})}} + e_{it}$

All notations have same meaning as earlier defined.

All the fixed and mixed effects mode were fitted in this study. The analysis was done using SAS package version 9.3 and R console. NLMIXED procedure was used for analysis. Parameters were estimated using Levenberg-Marquardt method.

Model selection criteria: For selection of best fit models, the following criterion was used in this study

1. Root mean squared error (RMSE): Root mean squared error (RMSE) is defined as

$$\text{RMSE} = \sqrt{\sum_{j=1}^{N}\sum_{i=1}^{t_j}\{W(t)-\hat{W}(t)\}^2 \Big/ \sum_{j=1}^{N}(t_j-p)}$$

where p is number of parameters fitted; N is number of animals; t_j is the number of weights for the j^{th} animal.

2. Percent Prediction Error (PPE):

$$\text{PPE}= \left|\frac{W(t)-\hat{W}(t)}{W(t)}\right| \times 100$$

3. Mean absolute error (MAE):

$$\text{MAE} = \sum_{j=1}^{N}\sum_{i=1}^{t_j}\left|W(t)-\hat{W}(t)\right| \Big/ \sum_{j=1}^{N}(t_j-p)$$

4. AIC (Akaike's information criterion):

AIC= –ln L + p

where, L is the likelihood function for model with p parameters.

5. Bayesian information criterion (BIC):

BIC= $-2L_p$ + p ln n

where, n is the sample size; L_p is the maximized log-likelihood of the model and p is the number of parameters in the model.

For comparing the forecasting performance, the Diebold Mariano test was used. Diebold Mariano test utilize the following statistic (Diebold and Mariano 1995)

$$DM = \frac{\bar{d}}{\dfrac{\sqrt{2\pi \hat{f}_d(0)}}{T}}$$

Here d =L(e1t) – L(e2t), $\bar{d}$ =average distance between loss function, $2\pi \hat{f}_d(0)$ is a consistent estimator of the asymptotic variance of $\sqrt{Td}$.

For checking model adequacy, normality of the residuals was tested with the help of Shapiro–Wilk test and the residuals were found to be independent and normally distributed.

RESULTS AND DISCUSSION

In literature, there are number of growth models available to describe relation of body weights with the age of animals. In the present study, the non-linear growth model like Logistic, Gompertz and Von-Bertalanffymodels are used.

These models were fitted in pig body weight data. Weibull model did not fit well in pig body weight data. So this model was not incorporated in the study. The others models like Gompertz, Logistic and Von-Bertalanffy models had no such type problem in fitting the given data sets of data. For each animal, the growth parameters were estimated by using Levenberg-Marquardt method and also criteria of model selection, viz. mean square error (MSE), percentage prediction error (PPE) and root means square error (RMSE). The best model was selected. Linearization techniques were employed to find out initial estimates. The initial estimate values of the growth curve parameters are given in Table 1.

Table 1. Initial parameters

Initial values of pig	Female	Male
W_m	1	1
b	17	17
σ_e^2	22.74	35.23
σ_u^2	1.5	1.5

It was found that all parameters were same except error variances which were different. The above defined three models were fitted in this pig BW data.

Mixed effect models performed better than the fixed effects models. Besides this, the logistic mixed effects model also fitted well. It had lowest MSE as well as RMSE. To check the performance of model, fit statistics of these models are computed. The details of fit statistics are given in Tables 2,3,4 and 5.

Fit statistics of the model provided an evidence about the good fitting of the logistic model. It was found that Logistic mixed effects model was best fit for both the gender. On the context of best fitting of model, logistic mixed effects model was used for parameter estimation. Table 6 gives details of parameter estimates.

Female had higher mature body weight (Wm) as compared to male. In this study, it was explored that male Pigs were growing fast.The accuracy of the fitted models (fixed and mixed) were evaluated to find the best model for both male and female. The results showed that the mixed effect Logistic models was more appropriate than the fixed effect models in both female and male body weight data. The Shapiro-Wilk test was used for checking normality of residuals from the fixed and mixed Logistic models in both female and male body weight data. It was found that the residuals from both the models were normally distributed.

Table 2. Model fitting criteria for female pigs

Female	Gompertz model		Logistic model		Von Bertalanffy model	
	Fixed effect	Mixed effect	Fixed effect	Mixed effect	Fixed effect	Mixed effect
MSE	7.684	2.683	1.999	0.453	4.629	3.811
RMSE	2.772	1.638	1.414	0.673	2.152	1.952

Table 3. Model fitting criteria for male pigs

Male	Gompertz model		Logistic model		Von-Bertalanffy model	
	Fixed effect	Mixed effect	Fixed effect	Mixed effect	Fixed effect	Mixed effect
MSE	10.651	4.892	5.122	3.677	7.828	6.217
RMSE	3.263	2.211	2.263	1.917	2.798	2.493

Table 4. Model fitting criteria for female pigs

Criterion	Gompertz model		Logistic model		VonBertalanffy model	
	Fixed effect	Mixed effect	Fixed effect	Mixed effect	Mixed effect	Fixed effect
-2 Log Likelihood	6093.6	6022.3	4766.0	3410.9	5899.9	5845.5
AIC (smaller is better)	6101.6	6030.3	4774.0	3418.9	5907.9	5853.5
AICC (smaller is better)	6101.7	6030.3	4774.0	3418.9	5907.9	5853.5
BIC (smaller is better)	6122.5	6042.3	4794.8	3430.9	5928.7	5865.6

Table 5. Model fitting criteria for male pigs

Criterion	Gompertz model		Logistic model		Von Bertalanffy model	
	Fixed effect	Mixed effect	Fixed effect	Mixed effect	Mixed effect	Fixed effect
-2 Log Likelihood	6388.3	6345.3	6036.5	5890.8	6609.0	6534.0
AIC (smaller is better)	6396.3	6353.3	6044.5	5898.8	6617.0	6542.0
AICC (smaller is better)	6396.3	6353.4	6044.5	5898.8	6617.1	6542.0
BIC (smaller is better)	6417.1	6365.4	6065.3	5910.9	6637.9	6554.0

Table 6. Parameter estimates with corresponding standard error in Logistic model

Parameter	Female				Male			
	Fixed effect		Mixed effect		Fixed effect		Mixed effect	
	Estimate	S.E.	Estimate	S.E.	Estimate	S.E.	Estimate	S.E.
b	3.992	0.056	3.953	0.027	5.328	0.149	5.221	0.125
Wm	28.770	0.435	29.038	0.582	21.499	0.519	21.828	0.591
	1.998	0.076	0.506	0.020	5.122	0.197	3.995	0.163
	-	-	43.905	5.296	-	-	20.693	3.460

The growth potential of a population of pig must be accurately characterized to get alternative management, nutrition, and marketing strategies. The variations in growth affects the total pig production. Nonlinear mixed effects models provide parameter estimates and their standard errors more precisely. It was observed that the Logistic model fitted well for present data. The accuracy of mixed logistic model was higher as compared to the fixed effect models. In general, the accuracy was more in female as compared to male as far as modelling body weight is concerned. The residual of the fitted models were examined for model adequacy by using Shapiro Wilk test. It was found that in all the cases residuals were normally distributed. Nonlinear mixed effects models provide more accurate and precise estimation of growth functions than the traditional fixed effects models.

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