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Row-Column Designs for Two Level Factorial Experiments

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SUMMARY

Row-column designs are useful for the experimental situations in which there are two cross classified sources of heterogeneity in the experimental material. Often it is desired to compare two or more factors in row-column set up where only two units can be accommodated in a single column. In this article, a general method of construction has been developed to generate row-column designs for factorial experiments with two rows which permit orthogonal estimation of all main effects and specific two factor interactions as per the choice of experimenters.

Keywords: Row-Column designs; Orthogonal; Factorial experimemnts.

1. INTRODUCTION

Two-level factorial designs are widely used to identify significant effects in industrial and agricultural processes. The occurrence of practical constraints, which can make it necessary to arrange runs in row-column designs in two rows, provide a major motivation for work in this area. For example, consider an experiment conducted for improving quality of products, experimental processes in the laboratory might involve using a small oven in which only two experimental units can be put. Practical situations involving blocking in agricultural experiments are described by several authors [see e.g. Yang and Draper (2003), Glonek and Solomon (2004), Kerr (2006), Choi and Gupta (2008), Dash *et al.* (2014) and Wang (2017)]. Since, two-level factorial and fractional factorial designs are widely used to identify significant effects in agricultural and industrial processes but more of the available factorial and fractional factorial designs in literature are in block set up. There is relatively little work on designs in fractional replicates in blocks of size two.

Sometimes due to lack of resources and unavailability of large number of experimental units, experimenters cannot run these types of experiments even in minimum number of columns given by Dash *et al.* (2013). In addition to this, it is not always the choice of experimenters to estimate all two factor interactions because experimenter may be interested in estimation of all main effects and some selected two factor interactions. Hence, there is a need to develop such row-column designs with a very few columns for estimating all the main effects and a subset of two factor interactions. A brief description about such type of designs in block set up is given by Godolphin (2019a, 2019b). For p two-level factors, designs comprising full replicates with runs in row-column designs in two rows are investigated. The minimum number of replicates for estimation of all main effects and two factor interactions is established and a construction method is developed based on column generator. Efficient row-column designs are recommended in the minimum number of replicates for n (number of factors) ≤ 9 to estimate main effects and two-factor interactions, and

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designs recommended are catalogued for users as a ready reckoner.

In this article, we propose a method of construction for row-column designs for two level factorial experiments for the estimation of main effects and specific two factor interactions of interest.

2. PRELIMINARIES

The factors of a 2^nfactorial experiment are denoted A_1, A_2, … , A_n. Factor A_i has levels $x_i \in \{0, 1\}$. If $x_i=0$, then A_i is described as being at low level and, if $x_i=1$, A_i is described as being at high level.A treatment combination is represented as $a_1^{x_1} a_2^{x_2} \cdots a_n^{x_n}$, where $a_i^0 = 1$ and $a_i^1 = a_i$ and equivalentlyas the vector $(x_1, x_2, \cdots, x_n)$.

The statistical model based on row-column setup is

$$y_{ijk} = \mu + \tau_i + \rho_j + \gamma_k + \varepsilon_{ijk}$$

Here, y_{ijk} is the observation resulting from application of the i^{th} treatment combination to the experimental unit in the j^{th} row and k^{th} column. The overall mean is μ and τ_i, ρ_j and γ_k are the effects of the i^{th} treatment combination, the j^{th} row and the k^{th} column. The error terms ε_{ijk} are assumed to be uncorrelated, all with variance σ^2.

In the constructions developed, treatment combinations are allocated to experimental units by means of generating sets. The first column and first rowof an array will be termed the principal column and principal row. These contain the treatment combinations that are generated by the factors not of interest. The construction methods for 2^n factorial experiments are described in details below.

3. METHODOLOGY

It is not always the choice of experimenters to estimate all two factor interactions because experimenter may be interested in estimation of all main effects and some specific two factor interactions. The experimental situations where the number of factors become large it is not possible to maintain homogeneity within a block hence in those situations fractional factorial experiments are suggested over factorial experiments. Also a little work based on row-column designs for factorial experiments in two rows are available in literature. Hence,Construction of efficient row-column designs with two rows have been investigated. Construction method for obtaining efficient row-column designs for factorial experiments in two rows is described in Section 3.1.

3.1 Row-column designs for two level factorial experiments in two rows

In this section, we propose a construction method to generate row-column designs for factorial experiments in two rows where the specific factors of interactions are not of interest to experimenter. Hence, the design obtainable from the method of construction can estimate all the main effects and specific two factor interactions of interest.

Method of construction

we propose a method of construction of row-column designs for estimation of all main effects and specific two factor interaction based on choice of experimenters in 2^n factorial experiments $(2 \le n \le 9)$ in minimum number of replications ($r = [\log_2 n]$, here [.] denotes greatest integer function and n denotes the number of factors). Here,we construct the row-column designs for 2^n factorial experiments when experimenter is interested for estimation of all main effects and two factor interactions upto $n - [\log_2 n]$ number of factor of not interest where $2 \le n<5$ and for $n \ge 5$ any two factor interactions which are not of interest for the experimenter. The step wise construction method is describe in details as below.

Step 1: Obtain a block design of block size 2 for a 2^n factorial experiment represented as $(2^n, 2)$. The block containing (1) as key element is the principal block. Once the treatment combination to be paired with (1) in the principal block has been chosen, the remaining blocks can be constructed easily by taking other sets of combination in the block. The estimability properties of the blocked replicate are obtained from the 2^n-1 estimable effects which are remain unconfounded.

To estimate all the main effects in a single replication one can assign highest order interaction with key element in the principal block. But this is not the case here. We have to obtain the block design for the estimation of all the main effects and specific two factor interaction (interaction between interested factors).

Step 2: For obtaining a design in r replications, select r blocking types in $\binom{2^n - 1}{r}$ different ways.

Out of these $\binom{2^n-1}{r}$ different blocking types, select those combinations that estimate the desired factorial effects (all main effects and two factor interactions) in maximum number of the *r* replications. Let *s* be the number of blocking arrangements or replications in which a given factorial effect is not confounded. Then the efficiency factor of a factorial effect is *s*/*r*. Thus, we need to find out the specific interaction to be allotted with the key elements in individual replication.

- For the first replication select the interaction effect of all those factors for which two factor interaction is not of interest and any one factor from the remaining factors.
- Keep the selected interaction as the key element in the first block and rest of the 2^{n-1}-1 blocks can be constructed from the principal block.
- Similarly, for the second replication select the interaction effect of all those factors which two factor interaction is not of interest and any one from the remaining factors which is not used in first replication.
- Keep the selected interaction with the key element in the first block and rest of the 2^{n-1}-1 blocks can be constructed from the principal block. In selecting specific interaction if only one effect is left as interest (Say for 2^3 factorial experiment interaction with A and B are not of interest so remaining is only C) then for second replication select the interaction effect of all no interest (i.e AB) with the key elements in the principal block.
- Repeat this process for all r replications.

This would yield block design in minimum number of replications with estimation of all main effects and specific two factor interaction which are of interest to experimenter.

Step 3: Once a block design for 2^nfactorial experiments in block size 2 is obtained, the next step is to convert it into a row-column design in such a way that the treatment combinations become most balanced with respect to rows. For achieving this, we make use of the procedure given by Dash *et al.* (2013). According to Dash *et al.* (2013), let $D_R[D_C]$ respectively, denote the block designs obtained by ignoring column [row] classification and the confounding done in such a way that the factorial effect which is confounded in D_R is unconfounded in D_C and vice-versa. Then the factorial effects which are unconfounded in both D_R and D_C remain unconfounded in row-column design as well. Further, the factorial effects which are confounded separately for D_R and D_C, are also confounded in row-column design. We have to identify factorial effects (possibly higher order interactions) which are unconfounded in all the replications of block design obtained in Step 1 and 2. Now confound this factorial effect with row-component design.

Example 3.1: A row-column design in two rows for a 2^4 factorial experiment for the estimation of all main effects and two factor interactions except AB interaction may be the interest of an experimenter can be constructed in $\lceil \log_2 n \rceil$ (=2) replications. Consider the four factors are A, B, C and D. Suppose, factor A and B are of least interest in interacting with other factors like with C and D. i.e. experimenter is interested in all main effects (A, B, C and D) and the two factor interaction where factor A and B are not involved i.e CD. Hence, we have to construct a row-column design for estimating all main effects and one two factor interaction CD.

Step1: Find out the principal block i.e. block containing key elements. Then select a specific interaction to be allotted with key element in the principal block and rest of the 2^{4-1}-1=7 blocks can be constructed using principal block.

Block	**1**	**2**	**3**	**4**	**5**	**6**	**7**	**8**
unit1	(1)	a	b	ab	d	ad	bd	abd
unit2	abc	bc	ac	c	abcd	bcd	acd	cd
Block	**9**	**10**	**11**	**12**	**13**	**14**	**15**	**16**
unit1	(1)	a	b	ab	c	ac	bc	abc
unit2	abd	bd	ad	d	abcd	bcd	acd	cd

Step2: In this step one has to find the specific interaction to be allotted in the principal block with key elements. Here, factor A and B are of least interest in interacting with other factors like C and D, hence, interaction of principal block in first replication can be interaction between A, B with another remaining factor i.e. C or D. So in the first replication principal block is 1 with abc and in the second replication it is 1 with abd. After getting the principal block of replication 1, rest of the 7 blocks are constructed using principal block presented in block 2 to 8 in the table given below.

Similarly for second replication block number 9 to 16 can be generated as given below.

Step 3: The next step is to convert the above block design into a row-column design in such a way that the treatment combinations become most balanced with respect to rows. In the present example we can see that the factorial effect 'abcd' is unconfounded with block effects. Therefore, using step 3, effect 'abcd' was confounded with rows to convert the block design to row-column design where treatment combinations become most balanced with respect to rows. The row-column design obtained after confounding 'abcd' in each replication of block design and rearranging the column contents of each replication to obtained the final design as given below.

Col/Row	1	2	3	4	5	6	7	8
1	(1)	bc	ac	ab	abcd	ad	bd	cd
2	abc	a	b	c	d	bcd	acd	abd
Col/Row	**9**	**10**	**11**	**12**	**13**	**14**	**15**	**16**
1	abd	a	b	d	c	bcd	acd	abc
2	(1)	bd	ad	ab	abcd	ac	bc	cd

4. DISCUSSION

This article proposes a method of constructing row-column designs for factorial experiments in two rows. Several literatures on row-column designs are available for factorial experiments in two rows for estimation of all main effects and two factor interactions. But, it is not always the choice of experimenters to estimate all two factor interactions and may be interested in estimation of all main effects and some selected two factor interactions. The developed construction method is easy to implement, capable of estimating all main effects and selected two factor interaction accommodating any number of factors, and with less number of replication as comparing to available designs in literature. Considering the above situation the present investigation focus on construction of row column designs for factorial experiments in estimation of all main effects and specific two factor interactions.

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