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PLANS FOR DIALLEL CROSSES AND THEIR ANALYSES

By

K. SIVARAM

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K Sivaram

(K. SIVARAM)

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CHAPTER I

INTRODUCTION

To identify superior cross combinations, both in plant and animal breeding, testing programmes are undertaken. There are two measures which reflect the performance of the parental breeds, strains or lines and the crosses they give raise to. These are general and specific combining abilities which were originally defined by Sprague and Tatum (1942). The performance of the parental lines and their cross combinations as measured by g.c.a. and s.c.a. determines the genetic architecture of the population under consideration. General combining ability may be defined as the average performance of a breed, strain or a line in hybrid (i.e. cross) combination. Specific combining ability may be defined as the deviation in performance of a cross from that which would be expected on the basis of the average performance (g.c.a.) of the parental breeds, strains or lines involved.

According to these definitions and the genetic mechanisms involved, general combining ability is a consequence of the additive genetic effects while specific combining ability is a consequence of non-additive genetic effects. The latter may involve dominance, over-dominance and epistasis.

The mean performance among the female line progeny is a function of both general combining ability and maternal effects. Maternal effects are estimated from the deviation between the average performance of female-line progeny and male-line progeny.

Diallel crossing system is one of the testing programmes most successfully used for estimating combining ability among line crosses. It is also used to estimate the actual yielding capacities of the crosses. Diallel crosses are defined as a set of single crosses obtained by mating v inbred lines in all combinations so that v^2 single crosses are obtained. If $\overline{y}_{ij}$ denotes the mean yield (over a certain number of replications) of the cross between lines i and j, we may write

$$\overline{y}_{ij} = \mu + t_i + t_j + s_{ij} + m_j + \overline{e}_{ij}$$

where μ is the average effect , t_i and t_j are the general combining ability effects of the lines i and j (effects due to lines), s_{ij} is the specific combining ability of the cross i x j (effect due to cross), m_j is the maternal effect of the line j (whenever j is the female parent) and $\overline{e}_{ij}$ is the random error which may include error due to plot deviation and also due to segregation within the cross. If the lines are a random sample from a large population, we may assume that t_i, m_j and s_{ij} are independently normally distributed with zero means and variances σ_t^2, σ_m^2 and σ_s^2. Furthermore, when matdrnal effects are absent , it is known that

$$\sigma_t^2 = \text{Cov(Half sibs)} \quad \text{and}$$

$$\sigma_s^2 = \text{Cov(Full sibs)} - 2\text{Cov(Half sibs)}.$$

Because of the inbred nature of the lines involved in the diallel crosses, if epistasis is absent, we also have $\sigma_t^2 = \frac{1}{2}\sigma_A^2$ and $\sigma_s^2 = \sigma_D^2$ where σ_A^2 is the additive genetic variance and σ_D^2 is the dominance variance. Hence, while $2\sigma_t^2 / 2\sigma_t^2 + \sigma_s^2$ measures the ratio of additive to total genotypic variance, when the gene frequencies are equal $\sqrt{\sigma_s^2/\sigma_t^2}$ measures the average degree of dominance.

According to Griffing (1956 b), diallel crossing techniques may vary depending upon whether or not the parental inbreds or the reciprocal F_1's are included or both. He considered four different possibilities and they may be listed as below.

Case (i) : One set of F_1's is included without considering the reciprocals and selfings

Case (ii): One set of F_1's is included along with their reciprocals but without selfings

Case (iii): Case (i) with selfings

Case (iv) : Case (ii) with selfings.

Griffing gives detailed analysis, under two models, of the above four categories in which diallel crosses may be made.

With the increase in number of parental lines the number of crosses increases very rapidly leading to the problems of resources and organizational difficulties. In crops like wheat and linseed where the number of seeds per reproductive unit is very low, complete diallel set with larger number of parents becomes unmanagable. Especially the problem is all the more involved when reciprocal crosses are to be considered. Therefore, the alternatives left are either to limit the number of parents or base the study on a sample of the full diallel, that is, on a partial diallel. While discussing the advantages of using partial diallels among a large number of parents as against making all possible crosses among a smaller selected number of parents, Kempthorne and Curnow (1961) cited the following in v favour of the former:

(i) the general combining ability of the parents can be estimated more accurately ;

(ii) selection can be made among the crosses from a wider range of parents and

(iii) the genral combining abilities of a larger number of parents can be estimated. Each parent will be assessed with a relatively low precision but larger genetic gains may result from the more intense selection that can be applied to the parents.

Sampling the diallel crosses has been studied , among others , by Kempthorne, Curnow, Fife and Gilbert. Many incomplete block designs have been made use of for this purpose. Balanced incomplete block (BIB) designs in two-plot blocks have been used to obtain complete diallel sets while fractions of the diallel have been obtained through partially balanced incomplete block (PBIB) designs with two-plot blocks. Kempthorne (1957) gave a method of constructing designs for partial sets of the crosses. Kempthorne and Curnow (1961) improved upon this and provided the analysis of these designs. The above designs along with those proposed by Gilbert (1958) and Fife and Gilbert (1963) are based principally on two-associate PBIB designs with two-plot blocks. The main disadvantage with such designs providing plans of partial diallels is that the number of parental lines to be considered for investigation is conditioned to a great extent by the requirements the designs impose.

After selecting the plan of crosses, either for complete diallel or for partial diallel, the experimental data is provided by adopting an experimental design which has invariably been a randomised block design. It is not to gainsay the fact that even a fraction of the crosses bring-in enough heterogenity in the blocks so as to entail shortening the block-size.

The present investigation is aimed at studying some of the above aspects of complete and partial diallel crosses especially in the light of providing new plans for partial diallels and indicating methods to adopt incomplete block designs for field experimentation. Firstly, plans for partial diallel crosses have been provided for four different cases according to whether or not the parental inbreds or the reciprocal F_1's are included or both. Making use of partially balanced incomplete block designs with any block size (as against block size two so far used), any values of λ and any number of associate classes, the plans in the four cases give the methods of estimating general combining ability along with the expectation of mean squares for the g.c.a. effects. Expressions for the standard errors for comparing the g.c.a.'s of any two participating lines are given. It has been seen that a PBIB design with large number of replications is no drawback for obtaining plans for partial diallel crosses through them as in the case of agricultural (block) experiments. It has become possible to estimate the maternal effects whenever the data permit.

Secondly, methods for obtaining plans of complete and partial diallel crosses through BIB and PBIB designs respectively have been obtained so that the resulting crosses may be grown in incomplete blocks. This gives a unified treatment of estimating the general combining ability taking into consideration the blocking aspect of the field experimentation. Methods are indicated for similar blocking of partial diallels when reciprocal crosses are also performed. Estimates of g.c.a. and expectations of mean squares due to g.c.a. are also given.

CHAPTER II

REVIEW OF THE PREVIOUS WORK DONE

Genetic analysis of populations using covariances between relatives and the associated problems of partitioning the total genotypic variance into additive and non-additive genetic components have been tackled extensively ever since the early work of Fisher in 1918. The practical application of these concepts of quantitative inheritance to plant and animal breeding have increasingly been realised in recent years and techniques involving 'diallel crosses' are some of the means to that end. Sprague and Tatum, Henderson, Griffing, Hayman and Jinks are but some of the names of early workers associated with diallel crosses and their use. But an exact generalised treatment showing the relationship between diallel crossing method to Fisher's method of covariances between relatives as expressed in terms of general and specific combining ability variances seems to have been taken up only from 1956 on wards.

Griffing (1956 a, 1956 b) presents an extensive study of the major problems with the use of complete diallel crosses. Classifying the experimental methods utilising diallel crosses into 4 categories depending upon whether or not the inbred and/or the reciprocal F_1's are included, Griffing (1956 a) indicates the analyis of such methods. A unified theory of the same analysis under two different models (Model I and Model II of Eisenhart) has been further given by him (1956 b). Here under each model the above four methods have been discussed giving estimates and their standard errors of general and specific combining abilities and reciprocal effects. Under both the models he gives the expectations of mean squares of the various estimates. Discussion of diallel analysis in the presence of maternal effects can be found in the works of Hazel, Lamoreux, Nordskog, Topham and others (see the list of References).

But the idea of partial diallel crosses has been widely accepted of late and many sampling designs have been put forward with a view to minimize the resources and organizational difficulties a plant or animal breeder is faced with. Sampling the diallel crosses is reported to have been taken up for the first time by Dr. G. W. Brown (1948). The broad outlines of his method have formed the basis for circulant samples developed by Kempthorne (1957) which otherwise were not exploited by Brown himself. Yates (1947) presented a method of analysis for a partial set of the diallel. The partial set did not arise out of sampling in as much as it was a consequence of mutual incompatibilities within subgroups of the plants. It was Gilbert (1958) who suggested partial diallel crosses as it is understood today, and his scheme depends on using a Latin square. Accordingly, when the number of lines v is even the sample should be chosen by super-imposing on the 'diallel table' a $v \times v$ symmetric latin square with a single letter on the main diagonal. Crosses corresponding to suitable number of letters in the latin square are then sampled which ensures equal representation of the lines among crosses. If v is divisible by 4 the use of latin squares symmetrical about both the diagonals is recommended for achieving balance among the crosses. The very use of latin squares indicates the impracticability of obtaining crosses when there are several parental lines under consideration which is often the case in plant breeding. Moreover, Gilbert's samples have singular least square equations and great care is to be taken to avoid samples which lead to unsolvable equations for the general combining abilities.

With a view to estimate the genetic variance components Hinkelmann and Stern (1960) described the construction and analysis of some circulant samples in which line 1 is always crossed with line 2 and with those lines whose numbers form an arithmetic progression from 2 to n. They also showed how to construct and analyse a partial set of test crosses.

Circulant samples: Developing on the methods earlier indicated by Kempthorne, Kempthorne and Curnow (1961) discussed circulant samples and it was followed up further by Curnow (1963). When there are v parental lines arranged from 1 to v in a random order, the method of generating such samples may be described as follows. Let each of the v parents be involved in the same number of crosses s, so that the total number of crosses in the sample is $vs/2$. Clearly, s must at least be 2 and v and s cannot both be odd. Then line 1 is crossed with lines $k+1, k+2, \ldots\ldots, k+s$ where $k = \frac{1}{2}(v+1-s)$. Line 2 is crossed with lines $k+2, k+3, \ldots\ldots, k+s+1$ and so on, the line numbers being reduced modulo v when necessary. The least square equations for estimating general combining ability involve a circulant matrix which has s for its elements on the main diagonal and non-diagonal elements are either 1 or 0 according as the corresponding cross has been sampled or not. The crosses so generated may result in as many as $v/2$ different standard errors for comparison of g.c.a.'s. The expectation of mean squares for the estimates have been presented. Kempthorne and Curnow also give the comparison of the yielding capacities of the crosses.

Curnow (1963) identified the one-to-one correspondence between a partial diallel cross of the above type and partially balanced incomplete block design with two plots per block and two associate classes with values of $\lambda = 0$ and 1. Cross $i \times j$ occurring in the sample corresponds to treatments i and j occurring together in the same block of the PBIB design. In the notation of diallel crosses the parameters of the PBIB design are

$$v = v \qquad \lambda_1 = 1$$
$$b = vs/2 \qquad \lambda_2 = 0$$
$$r = s \qquad n_1 = s$$
$$k = 2 \qquad n_2 = v - s - 1$$
$$p_{11}^1 = \alpha$$
$$p_{11}^2 = \beta$$

where the above λ's correspond to whether a cross is sampled or not; α is the number of lines crossed to both lines i and j where cross $i \times j$ is in the sample and β is the number of lines crossed to both lines i and j where cross $i \times j$ is not in the sample. Making use of PBIB designs listed by Clatworthy (1955), Curnow was able to find certain two-variance circulant samples in addition to enumerating some more circulants.

Samples from Triangular and Factorial Designs: Fife and Gilbert (1963) have given two-variance sample designs called triangular and factorial designs.

The former designs are for $N = \frac{1}{2} n(n-1)$ parents, where n is an integer. The parents are numbered off into an $(n-1)(n-1)$ triangle. For example, if $n = 5$, the ten parents are numbered 54, 53, 52, 51, 43, 42, 41, 32, 31 and 21. Then each parent is denoted as ab, where a can take any value from 2 to n, and b can take any value from 1 to (a-1). Then the partial diallel consists of all crosses ab x cd, where a, b, c and d are all different. As each line is involved in $\frac{1}{2}(n-2)(n-3)$ of the crosses in this design there is a restriction that n must exceed 4. The complementary of the above is again a triangular design. We may note that this type of generating crosses is directly related to 2 plot block PBIB designs obtainable through a triangular association scheme (cf. Bose et al 1954).

The factorial designs are for $N = mn$ parents, where m and n are integers. The parents here are numbered off into an m x n rectangle. For example, if $m = 4$ and $n = 3$, the twelve parents are number 11, 21, 31, 41, 12, 22, 32, 42, 13, 23, 33 and 43. Each parent is then denoted as ab, where $1 \leqslant a \leqslant m$ and $1 \leqslant b \leqslant n$, and all the crosses ab x cd are selected such that $a \neq c$ and $b \neq d$; in this case every line is involved in $(m-1)(n-1)$ crosses so that m, n must both exceed 2.

The present work is an extension of the author's previous dissertation on "Incomplete block designs and Partial diallel crosses", which he submitted as partial fulfilment for the award of M. Sc. degree at the Institute of Agricultural Research Statistics, Delhi. The early part of chapter IV of this volume has a bearing on this dissertation work. (Also refer Das and Sivaram 1968) .

For further reference please see the Annexture at the end.

CHAPTER III

PRELIMINARIES

As most of the subject matter in the subsequent pages deals with the application of partially balanced incomplete block designs, a definition of them becomes necessary here. According to Bose and Nair (1939), and latter Bose and Shimamoto (1952), a PBIB design in two associate classes is an arrangement of v treatments in b blocks, such that:

1. Each of the v treatments occurs r times in the arrangement, which consists of b blocks each of which contains k experimental units. No treatment appears more than once in any block.

2. Every pair among the v treatments occurs together in either λ_1 or λ_2 blocks (and are said to be i th associates, if they occur together in λ_i blocks, $i = 1, 2$).

3. There exists a relationship of association between every pair of the v treatments satisfying the following conditions:

   a. Any two treatments are either first or second associates.

   b. Each treatment has n_1 first and n_2 second associates.

   c. Given any two treatments that are i th associates, the number of treatments common to the j th associates of the first and k th associates of the second is p^i_{jk}, and this number is independent of the pair of treatments with which we start. Furthermore, $p^i_{jk} = p^i_{kj}$ ($i, j, k = 1, 2$).

The eight parameters $v, b, r, k, \lambda_1, \lambda_2, n_1$ and n_2 are known as the primary parameters , and the parameters p^i_{jk} ($i, j, k = 1, 2$) are called the secondary parameters. The secondary parameters may be displayed as elements of two symmetric matricies,

$$P_1 = \begin{bmatrix} p^1_{11} & p^1_{12} \\ p^1_{21} & p^1_{22} \end{bmatrix} \quad \text{and} \quad P_2 = \begin{bmatrix} p^2_{11} & p^2_{12} \\ p^2_{21} & p^2_{22} \end{bmatrix}$$

The following parametric relations are true.

$$n_1 + n_2 = v - 1$$

$$n_1\lambda_1 + n_2\lambda_2 = r(k-1)$$

$$p^i_{jk} = p^i_{kj}$$

$$n_i p^i_{jk} = n_j p^j_{ik}$$

and $$\sum_k p^i_{jk} = n_j - 1 \quad \text{when } i = j$$

$$= n_j \quad \text{otherwise.}$$

We shall use these PBIB designs to generate plans for partial diallel crosses and give analyses to estimate the general combining abilities, specific combining abilities and maternal effects (when they occur) of the participating lines. We shall also use PBIB designs in m-associate classes, where $m > 2$, to generate plans which are more flexible and broadly indicate the method of analyses.

Let there be v parental lines under consideration. When it is contemplated to have a partial set of the complete diallel which has a total of v^2 number of crosses, four different cases arise according to the following being true.

<u>Case (i):</u> The experiment consists of a given partial set of the F_1's only but does not include its reciprocals or parental lines (i.e. selfings).

Case (ii): The experiment consists of a given partial set of the F_1's and also its reciprocals, but does not include parental lines.

Case (iii): The experiment consists of a given partial set of the F_1's only without its reciprocalsand includes the parental lines.

Case (iv): The experiment consists of a given partial set of the F_1's with reciprocals and also with the parental lines.

Each case necessiates a different form of analysis.

It is the plant breeders or geneticist who is left with the problem of choosing one of the above four experimental methods depending upon the aims of the experiment, the experimental material at hand and the knowledge he has about the material. If it is intended to estimate the yielding capacities of the lines, information regarding the performance of the parental inbreds may be useful. When the data under investigation provide good evidence for the existence of maternal effects reciprocal crosses may have to be undertaken. Knowledge about the cost of experimentation, in particular, cost per cross, may also determine to certain extent the type of crosses to be performed.

After the plan of crosses has been selected and the crosses grown, during the analysis the components of genetic variance are determined by equating the expectation of mean squares due to the different effects to those observed. Measures of heritability, proportion of additive genetic variance to the total genotypic variance and the average degree of dominance may be calculated thereafter under appropriate assumptions.

The subsequent four chapters pertain to the four different cases that arise as indicated above. The rest of the chapters deal with the blocking of the diallel crosses in suitable incomplete blocks and their analyses.

CHAPTER IV

1. P. B. I. B. DESIGNS IN TWO ASSOCIATE CLASSES AND PARTIAL DIALLELS WITHOUT SELFINGS OR RECIPROCALS

This is the case (i) of chapter III.

Let the v parental lines be numbered at random from 1 to v. We shall consider them as the treatments while refering to PBIB designs. The first associates (or second associates, whichever is convenient) of each line (treatment) are written beside the line (treatment) in an ascending order of magnitude. Any line i ($i = 1, 2, \ldots, v$) is then crossed with every line j ($j > i$), where j is the first (second) associate of i. With the usual notations, therefore, we get $vn_1/2$ or $vn_2/2$ number of crosses according as the line i is crossed with its first or second associate lines. We shall give below the method of estimating general combining abilities and specific combining abilities of the participating lines assuming that the reciprocal crosses are identical and that we are not interested in the performance of the parental inbreds themselves.

It may be stressed at this point that the above plans for partial diallels and their analysis are entirely independent of the values of λ's and also of the number of replications in the PBIB design. Thus PBIB designs even with large number of replications can be used with advantage for obtaining plans for partial diallel crosses which otherwise would be useless for block experimentation. A further advantage is achieved from the fact that for most of the numbers of lines PBIB designs are available and therefore plans for partial diallel crosses are also available for these numbers.

As there are no reciprocal crosses and selfings, we make use of the model

$$\bar{y}_{ij} = \mu + t_i + t_j + s_{ij} + \bar{e}_{ij}$$

with the usual assumptions and get the following normal equations. When the i th line has been crossed with its first associates, we have

$$n_1\mu + n_1 t_i + S_1(t_i) = T_i \qquad (i = 1, 2, \ldots v) \qquad (4.1)$$

where μ is the general mean, $S_1(t_i)$ denotes the sum of the g.c.a.'s of lines which are the *first associates* of i and with which the ith lines is crossed, and T_i denotes the total yield of all the crosses with the i th line.

Adding such equations for all the lines with which i th line is crossed, we get

$$n_1^2\mu + n_1 S_1(t_i) + n_1 t_i + p_{11}^1 S_1(t_i) + p_{11}^2 S_2(t_i) = S_1(T_i) \qquad (4.2)$$

where $S_2(t_i)$ is the sum of g.c.a.'s of lines not crossed with the i th line and $S_1(T_i)$ is the sum of the totals T_i's of those lines with which the i th line is crossed.

Assuming $\sum_{1}^{v} t_i = 0$, we get $\hat{\mu} = 2G/vn_1$, where G is the grand total of the yields of all the crosses, and the equation (4.2) reduces to

$$n_1^2\mu + (n_1 - p_{11}^2)\, t_i + (n_1 + p_{11}^1 - p_{11}^2)\, S_1(t_i) = S_1(T_i) \qquad (4.3)$$

Solving (4.1) and (4.3), we get

$$\hat{t}_i = \frac{(n_1 + p_{11}^1 - p_{11}^2)\, T_i - S_1(T_i) - \frac{2G}{v}(p_{11}^1 - p_{11}^2)}{(n_1 - 1)(n_1 - p_{11}^2) + n_1 p_{11}^1}, \qquad (4.4)$$

$$i = 1, 2, \ldots, v$$

The sum of squares due to the g.c.a.'s of the lines is

$$\sum_{i=1}^{v} \hat{t}_i T_i$$

The variance of the difference between g.c.a.'s of two lines

(i) which are not crossed is $$\frac{2(n_1 + p_{11}^1 - p_{11}^2)\sigma^2}{(n_1 - 1)(n_1 - p_{11}^2) + n_1 p_{11}^1}$$

(ii) which are crossed is $$\frac{2(n_1 + p_{11}^1 - p_{11}^2 + 1)\sigma^2}{(n_1 - 1)(n_1 - p_{11}^2) + n_1 p_{11}^1}$$

where σ^2 is the error variance.

Given any line the first expression above is used to compare the difference of its g.c.a. from the g.c.a.'s of each of n_2 lines and the second, from each of n_1 lines. Thus we can get a weighted average of the above two variances taking n_2 and n_1 as weights, and this average is given below.

$$\text{Average variance} = \frac{2\left[(v-1)(n_1 + p_{11}^1 - p_{11}^2) + n_1\right]\sigma^2}{(v-1)\left[(n_1 - 1)(n_1 - p_{11}^2) + n_1 p_{11}^1\right]}$$

The results of second associate crosses are obtainable from the above results by replacing n_1 by n_2, p_{11}^1 by p_{22}^2 and p_{11}^2 by p_{22}^1. The plan got by crossing the second associate lines will be complementary to the one got through the first associate lines.

Note: - If the plan of crosses is obtained from a Group Divisible PBIB design, at least one of the two associate crosses will yield non-singular least squares equations so that a solution for t_i exists.

Considering that the above experiment is laid out in a randomised block design with replications r such that each block constitutes all the $vn_1/2$ crosses, the estimate of the specific combining ability of cross i x j is simply the mean of the yields over the replications and the variance of the difference between any two s.c.a.'s is $2\sigma^2/r$. The sum of squares due to s.c.a. is got by

S.S. due to crosses - S.S. due to g.c.a. - S.S. due to error(rep x crosses).

Now we shall consider finding of expectations of mean squares due to g.c.a., s.c.a. and error.

The sum of squares due to t_i is, as indicated already, $\sum_{i=1}^{v} \hat{t}_i T_i$. Taking expectations,

$$E \Sigma \hat{t}_i T_i = \frac{1}{\triangle} \left[(n_1 + p^1_{11} - p^2_{11}) E \Sigma T_i^2 - E \Sigma T_i S_1(T_i) + \frac{4(p^1_{11} - p^2_{11})}{v} E G^2 \right]$$

where $\triangle$ is the denominator in the estimate of t_i, given in (4.4). Using the model in which t_i, s_{ij} and e_{ij} are random variables with $E(t_i) = E(s_{ij}) = E(e_{ij}) = 0$ and the t, s and e quantities are uncorrelated and $E(t_i^2) = \sigma_t^2$, $E(s_{ij}^2) = \sigma_s^2$ and $E(e_{ijk}^2) = \sigma_e^2$, the following is obtained. We shall write σ^2 for $\sigma_s^2 + \sigma_e^2$.

$$E \Sigma T_i^2 = v \left[n_1^2 \mu^2 + n_1(n_1 + 1)\sigma_t^2 + n_1\sigma^2 \right]$$

$$E \Sigma T_i S_1(T_i) = v \left[n_1^3 \mu^2 + \left\{ n_1(n_1 - p^2_{11}) + n_1(n_1 + p^1_{11} - p^2_{11}) \right\} \sigma_t^2 + n_1\sigma^2 \right]$$

and

$$EG^2 = (v^2 n_1^2/4)\mu^2 + v n_1^2 \sigma_t^2 + (v n_1/2)\sigma^2.$$

$\therefore$ E $\Sigma \hat{t}_i T_i$ is

$$\frac{1}{\Delta}\left[\left\{vn_1(n_1+1)(n_1+p_{11}^1-p_{11}^2)-vn_1(2n_1+p_{11}^1-2p_{11}^2)-4n_1^2(p_{11}^1-p_{11}^2)\right\}\sigma_t^2\right.$$
$$\left.+\left\{vn_1(n_1+p_{11}^1-p_{11}^2)-vn_1-2n_1(p_{11}^1-p_{11}^2)\right\}\sigma^2\right].$$

Hence the expectation of mean squares due to g.c.a. is

$$\frac{1}{\Delta(v-1)}\left[vn_1(n_1+1)(n_1+p_{11}^1-p_{11}^2)-vn_1(2n_1+p_{11}^1-2p_{11}^2)\right.$$
$$\left.-4n_1^2(p_{11}^1-p_{11}^2)\right]\sigma_t^2+\sigma^2 ,$$

or $$\left[\frac{vn_1}{(v-1)}-\frac{4n_1^2(p_{11}^1-p_{11}^2)}{\Delta(v-1)}\right]\sigma_t^2+\sigma^2 .$$

Since there are r replications, the expectation of mean squares due to s.c.a. is simply $\sigma_e^2+r\sigma_s^2$ and that of mean squares due to error (replications x crosses) is σ_e^2. Equating the observed sum of squares due to these effects to the above expected ones, we obtain the estimates of σ_e^2, σ_s^2 and σ_t^2.

When mean yields of the crosses, averaged over r replications, are analysed then the analysis of variance table is as shown below.

Analysis of variance of Partial Diallel Crosses 1.

Source	d.f.	Expected values of mean squares
Replicates	$(r-1)$	
G.C.A.	$(v-1)$	$\sigma_e^2 + r\sigma_s^2 + \frac{rvn_1}{(v-1)}\left[1-4n_1(p_{11}^1-p_{11}^2)/\Delta v\right]\sigma_t^2$
S.C.A.	$v(\frac{n_1}{2}-1)$	$\sigma_e^2 + r\sigma_s^2$
Rep.xcrosses	$(r-1)(\frac{vn_1}{2}-1)$	σ_e^2
Total	$rvn_1/2-1$	

Example

In order to show how a plan of crosses for partial diallel can be got we shall furnish below an illustration using the two-associate cyclic PBIB design given by Bose et al (1954). The design has the following parameters.

$v = 17,\ b = 34,\ r = 8,\ k = 4,\ n_1 = 8,\ n_2 = 8,\ \lambda_1 = 1,\ \lambda_2 = 2,$

$$P_1 = \begin{bmatrix} 3 & 4 \\ 4 & 4 \end{bmatrix} \text{ and } P_2 = \begin{bmatrix} 4 & 4 \\ 4 & 3 \end{bmatrix}.$$

The association scheme of this PBIB design has the property that the first associates of a treatment i are obtained by adding $(i-1)$ mod 17 to each of the first associates of treatment 1. Hence it is sufficient to indicate the first associates of the treatment numbered 1, and they are given below in an ascending order.

Treatment No.	First Associates
1	(4, 6, 7, 8, 11, 12, 13, 15)

Identifying the treatments with the inbred lines, line 1 is crossed with all its first associates j such that $j > 1$, giving us the crosses

(1 x 4) (1x 6) (1 x 7) (1 x 8) (1 x 11) (1 x 12) (1 x 13) (1 x 15)

Repeating the procedure by writing down the first associates of all the other treatments and making crosses as indicated above we shall get the total number of crosses for the partial diallel.

For analysis we make use of the data pertaining to a diallel involving 17 lines of bajra (Pennisetum typhoides). The character under study is productivity and is measured by giving scores ranging from 3 to 10 with reference to the yield of C.M.S. 24A as the standard with a score of 5. The observations were collected by G. Harinarayana, Genetics Division, I.A.R.I., New Delhi. The analysis is based on means.

Table 1

Lines	$\bar{T}_i$	$S_1(\bar{T}_i)$	t_i
1	26.0	229.0	-0.354072
2	27.5	228.5	-0.142533
3	31.5	226.0	0.444004
4	33.0	228.0	0.607466
5	29.0	227.5	0.078619
6	25.5	223.5	-0.315610
7	31.0	224.5	0.405542
8	30.0	225.0	0.261312
9	27.5	230.0	-0.171380
10	30.0	242.0	-0.065610
11	26.0	229.0	-0.354072
12	25.0	224.5	-0.402149
13	28.5	232.5	-0.084841
14	26.5	234.5	-0.392533
15	30.0	232.0	0.126696
16	31.0	227.0	0.357466
17	28.0	224.5	0.001696

Table 1 shows the values of $T_i, S_i(T_i)$ and the estimates t_i of the g.c.a. for all the 17 lines. The analysis of variance is as shown below and it indicates that the g.c.a. effects are insignificant.

ANALYSIS OF VARIANCE TABLE

Source	D.F.	S.S.	M.S.	F
g.c.a.	16	11.71889140	0.73	0.6 *
s.c.a.	51	61.91346153	1.21	
Total	67	73.63235294		

* Not significant.

2. P.B.I.B. DESIGNS IN m-ASSOCIATE CLASSES AND PARTIAL DIALLELS WITHOUT SELFINGS OR RECIPROCALS

In the previous section we discussed the method of obtaining plans for partial diallel through P.B.I.B. designs in two associate classes. The sample was so formed that each of the lines was crossed with those appearing in one of the two associate classes. This concept may be generalised by using PBIB designs with m-associate classes. In this case, each line is crossed with each of the lines present in r of the m associate classes.

Let us consider a PBIB design with v treatments (lines numbered in some random order) in m-associate classes. Let n_j $(j = 1, 2, \ldots, m)$ denote the number of lines present in the j th associate class. For each line i $(i = 1, 2, \ldots, v)$ a given r associate classes among its m classes are chosen and all the lines in them are pooled. The line i is then crossed with every line j in the pool such that $j > i$. Thus we will have a sample of crosses of size $vN/2$, where $N = \sum_{(j)} n_j$, $\sum_{(j)}$ implying the sum over the selected r associate classes. For analysis we shall assume, without loss of generality, that each line is crossed with its first r $(r < m)$ associate lines. The normal equations for estimating the g.c.a. of the lines through least squares technique taking the usual model comes out as below.

$$N\mu + Nt_i + \sum_{(j)} S_j(t_i) = \sum_{(j)} T_j = Q_i \qquad (4.5)$$

$$(i = 1, 2, \ldots, v)$$

where $S_j(t_i)$ is the sum of the g.c.a.'s of lines which are j th associates of the i th line and T_j is the total yield of the j th associate crosses involving the i th line. Adding such equations over the first associate lines of i, we get

$$Nn_1\mu + NS_1(t_i) + \sum_{(j)} p^1_{1j}\, S_1\,(t_i) + \sum_{(j)} p^2_{1j}\, S_2\,(t_i) + \dots$$

$$+ \sum_{(j)} p^m_{1j}\, S_m\,(t_i) = S_1(Q_i)$$

where $S_j\,(Q_i)$ is the sum of the Q's of lines which are j th associates of the i th line.

In general, adding such equations over the k th associates of i, we have

$$Nn_k\mu + NS_k(t_i) + \sum_{(j)} p^1_{kj}\, S_1(t_i) + \sum_{(j)} p^2_{kj}\, S_2(t_i) + \dots$$

$$+ \sum_{(j)} p^m_{kj}\, S_m(t_i) \quad = S_k\,(Q_i)$$

$$k = 1, 2, \dots\dots, (m-1) \qquad (4.6)$$

Assuming that $\sum_{i=1}^{v} t_i = 0$, equations (4.5) and (4.6) can be solved for t_i and the analysis completed in the usual lines.

<u>A particular case:</u> <u>Use of 3-associate designs</u>

When m = 3 and r = 1, we arrive at the simple case of generating plans for partial diallel crosses from a 3 associate PBIB design by crossing any line with its first associate lines. The normal equations for estimating general combining ability of the lines comes out as below from (4.5) and (4.6).

$$n_1\mu + n_1 t_i + S_1(t_i) = T_i \qquad (i = 1, 2, \dots\dots, v) \qquad (4.7)$$

$$n_1^2\mu + n_1 t_i + n_1 S_1(t_i) + p^1_{11} S_1(t_i) + p^2_{11} S_2(t_i) + p^3_{11} S_3(t_i) = S_1(T_i) \qquad (4.8)$$

and

$$n_1n_2\mu + n_1S_1(t_i) + p^1_{12}\, S_1(t_i) + p^2_{12}S_2(t_i) + p^3_{12}\, S_3(t_i) = S_2(T_i) \qquad (4.9)$$

Assuming $\Sigma\ t_i = O$, we get $\mu = \dfrac{2G}{vn_1}$ and from equations (4.7), (4.8) we get

$$\hat{t}_i = \left[(A_2B_3 - A_3B_2)T_i - B_3S_1(T_i) + A_3S_2(T_i) - \frac{2G}{v}(A_2B_3 - A_3B_2 - n_1B_3 + n_1A_3)\right] \Big/ \triangle \qquad (4.10)$$

where

$A_1 = (n_1 - p^3_{11})$ $\qquad$ $B_1 = -p^3_{12}$

$A_2 = (n_1 + p^1_{11} - p^3_{11})$ $\qquad$ $B_2 = (p^1_{12} - p^3_{12})$

$A_3 = (p^2_{11} - p^3_{11})$ $\qquad$ $B_3 = (n_1 + p^2_{12} - p^3_{12})$ and

$$\triangle = n_1(A_2B_3 - A_3B_2) - (A_1B_3 - A_3B_1).$$

The sum of squares due to g.c.a. of the lines is $\sum_{i=1}^{v} \hat{t}_i\, T_i$. As $\Sigma\, t_i = O$, no correction factor need be subtracted.

The variance of the difference between g.c.a.'s of two lines is now given by

(i) $V(\hat{t}_i - \hat{t}_{i'}) = \dfrac{2\sigma^2}{\triangle}(A_2B_3 - A_3B_2 + B_3)$

when line i' is crossed with the i th line

(ii) $V(\hat{t}_i - \hat{t}_{i'}) = \dfrac{2\sigma^2}{\triangle}(A_2B_3 - A_3B_2 + A_3)$

when the line i' is not crossed with the i th line but is crossed to a line to which line i is crossed

(iii) $V(\hat{t}_i - \hat{t}_{i'}) = \dfrac{2\sigma^2}{\triangle}(A_2B_3 - A_3B_2)$

when the line i' is neither crossed with the i th line nor with a line to which i is crossed.

The average variance of the above three is $\dfrac{n_1V_1 + n_2V_2 + n_3V_3}{n_1 + n_2 + n_3}$.

Through 3 associate PBIB designs we can reduce the total number of crosses when there is a large number of lines. This is so because the total number of lines is $\frac{1}{2}\ vn_i$ where n_i is the number of treatments in the i th associate class. Now if there be two classes the value of n_i is likely to be large. But with the same v if there be a design with three associate classes some of the values of n_i's are likely to be smaller and there is more flexibility in the choice of the associate classes.

CONNECTEDNESS

A plan of crosses is said to be connected if for every two lines, it is possible to pass from one to the other by forming a chain consisting of lines such that any two consecutive lines are crossed. While choosing a sample from a diallel it is better to choose a sample which is connected. When the plan of crosses is disconnected, the lines will fall into sets so that no two lines from different sets are crossed. But this need not be a serious draw-back as in the case of estimation of treatments through incomplete block designs, because the estimation of g.c.a. is still possible through such disconnected samples by following the method discussed by Curnow (1963). It may be pointed out that the method of analysis presented here holds even for disconnected samples, though depending on the parameters of specific designs the value of $\triangle$ may be zero in some cases and hence no solution is available in such cases.

CHAPTER V

1. P.B.I.B. DESIGNS IN TWO ASSOCIATE CLASSES AND PARTIAL DIALLELS WITHOUT SELFINGS BUT WITH RECIPROCALS

When maternal effects are present and so the reciprocal crosses are performed the diallel analysis differs significantly. Assuming that we are not interested in the performance of the parents themselves, we shall discuss this aspect which is cited as case (iii) in chapter III. The linear model for estimating the combining abilities of the lines is now given by

$$\bar{y}_{ij} = \mu + t_i + t_j + s_{ij} + m_j + \bar{e}_{ij}$$ model

where m_j stands for the maternal effect of the j th line appearing as the female parent.

To generate plans of partial diallel crosses PBIB designs with two associate classes are again used. As earlier every line is written beside its first associates. Any line i as male is then crossed with each of its first associate lines as females. Hence there will be a total of vn_1 crosses, when the first associate lines are crossed, and each line will be crossed n_1 times both as male and female.

The normal equations for estimating the g.c.a. and maternal effects through the least squares technique are as follows.

$$n_1\mu + n_1 t_i + S_1(t_i) + S_1(m_i) = T_i \quad (i = 1, 2, \ldots, v) \qquad (5.1)$$

and

$$n_1\mu + S_1(t_i) + n_1 t_i + n_1 m_i = T_i' \qquad (5.2)$$

where T_i is the yield total of those crosses where the i th line is present as male ; similarly T_i' is the total yield from crosses where the i th line occurs as female. $S_1(t_i)$ denotes the sum of g.c.a. effects of lines crossed with the i th line and $S_1(m_i)$ is the sum of maternal effects of all the females crossed to the i th male line. Adding equations (5.1) and (5.2) over all males and females respectively which are first associates of line i we obtain the equations (5.3) and (5.4), under the assumptions $\Sigma t_i = 0$ and $\Sigma m_i = 0$.

$$n_1^2\mu + (n_1 - p_{11}^2)t_i + (n_1 + p_{11}^1 - p_{11}^2)S_1(t_i) + (n_1 - p_{11}^2)m_i + (p_{11}^1 - p_{11}^2)S_1(m_i) = S_1(T_i) \qquad (5.3)$$

$$n_1^2\mu + (n_1 - p_{11}^2)t_i + (n_1 + p_{11}^1 - p_{11}^2)S_1(t_i) + n_1 S_1(m_i) = S_1(T_i') \qquad (5.4)$$

From (5.1) and (5.4) we have

$$n_1 T_i - S_1(T_i') = (n_1^2 - n_1 + p_{11}^2)t_i - (p_{11}^1 - p_{11}^2)S_1(t_i) \qquad (5.5)$$

Adding (5.5) over all male lines which are crossed with the i th line,

$$n_1 S_1(T_i) - n_1 T_i' - p_{11}^1 S_1(T_i') - p_{11}^2 S_2(T_i') = (n_1 - p_{11}^2)(p_{11}^1 - p_{11}^2)t_i + \left[(n_1^2 - n_1 + p_{11}^2) - (p_{11}^1 - p_{11}^2)^2\right]S_1(t_i) \qquad (5.6)$$

Solving (5.5) and (5.6) gives us

$$t_i = \frac{A\left[n_1 T_i - S_1(T_i')\right] + B\left[n_1 S_1(T_i) - \left\{(n_1 - p_{11}^2)T_i' + (p_{11}^1 - p_{11}^2)S_1(T_i') + p_{11}^2 G\right\}\right]}{CD}$$

$$(i = 1, 2, \ldots, v) \qquad (5.7)$$

where G denotes the grand total of all the yields and

$$A = (n_1^2 - n_1 + p_{11}^2) - (p_{11}^1 - p_{11}^2)^2$$

$$B = (p_{11}^1 - p_{11}^2)$$

$$C = (n_1 - 1)(n_1 - p_{11}^2) + n_1 p_{11}^1$$

$$D = n_1(n_1 - p_{11}^1 + p_{11}^2 - 1) + p_{11}^2.$$

Here $S_1(T_i)$ and $S_1(T_i')$ have similar meaning in terms of total yields as $S_1(t_i)$. Denoting $n_1 T_i - S_1(T_i')$ by Q_i, the sum of squares due to the g.c.a.'s of the lines adjusted for maternal effects, as each male line is not coming with the same set of females, is $\frac{1}{n_1} \sum_1^v \hat{t}_i Q_i$.

Also from (5.2) and (5.3)

$$n_1 T_i' - S_1(T_i) = (n_1^2 - n_1 + p_{11}^2)t_i - (p_{11}^1 - p_{11}^2)\, S_1(t_i)$$

$$+ (n_1^2 - n_1 + p_{11}^2)m_i - (p_{11}^1 - p_{11}^2)\, S_1(m_i) \qquad (5.8)$$

Adding (5.8) over all lines which are female and first associates of line i, we get equation similar to (5.6) but where (t_i+m_i) appears in place of t_i and $S_1(t_i) + S_1(m_i)$ in place of $S_1(t_i)$.

Then solving we obtain expression for (t_i+m_i) looking similar to t_i.

Hence

$$\hat{m}_i = A\left[\, n_1(T_i' - T_i) - \left\{S_1(T_i') - S_1(T_i)\right\}\right]$$
$$+ B\left[n_1 \left\{ S_1(T_i') - S_1(T_i)\right\} + (n_1 - p_{11}^2)(T_i' - T_i)\right.$$
$$\left. + (p_{11}^1 - p_{11}^2) \left\{ S_1(T_i') - S_1(T_i)\right\}\right] \Big/ CD \qquad (5.9)$$

The sum of squares due to m_i is $\frac{1}{n_1} \quad \Sigma\ \hat{m}_i M_i$, where

$$M_i = n_1 (T_i' - T_i) + S_1(T_i') - S_1(T_i)$$

For comparing any two lines, the variance of the difference between the g.c.a.'s is given by

$$V(\hat{t}_i - \hat{t}_m) = \frac{2n_1 A\sigma^2}{CD}$$ when the i th and m th lines are not crossed

$$= \frac{2n_1(A - B)\sigma^2}{CD}$$ when the i th and m th lines are crossed.

The average variance from the two expressions comes out to be

$$\frac{2n_1 \left[A(v - 1) - Bn_1 \right] \sigma^2}{CD\ (v - 1)}$$

The variance of the difference between maternal effects of two lines may similarly be written down.

The results of second associate crosses are obtained from the above results by replacing n_1 by n_2, p^1_{11} by p^2_{22} and p^2_{11} by p^1_{22}.

When the crosses are obtained from a GD design whose association scheme has say, n groups of m treatments each and when either m or n equals 2, then we get estimates of g.c.a. for one of the two associate crosses.

Making use of the corresponding random model in which $E(t_i) = E(m_i) = E(s_{ij}) = E(e_{ij}) = 0$ and the t, m, s and e quantities are uncorrelated and $E(t_i^2) = \sigma_t^2$, $E(m_i^2) = \sigma_m^2$, $E(s_{ij}^2) = \sigma_s^2$ and $E(e_{ijk}^2) = \sigma_e^2$, we obtain the expectation mean squares. When there are r replications, the expected mean squares for general combining ability is given by

$$\sigma_e^2 + r\sigma_s^2 + \frac{vr}{n_1(v-1)\,CD} \Bigg[A \Big\{ n_1^3(n_1+1) - 2n_1^2(2n_1+p^1_{11}-2p^2_{11}) + (n_1-p^2_{11})^2$$
$$+ n_1(n_1+p^1_{11}-p^2_{11})^2 \Big\}$$

$$+B\left[n_1^3(2n_1+p_{11}^1-p_{11}^2)-n_1^2(n_1+1)(n_1-p_{11}^2)\right.$$

$$-n_1^2(p_{11}^1-p_{11}^2)(2n_1+p_{11}^1-2p_{11}^2)-4n_1^3p_{11}^2-n_1^2(n_1+p_{11}^1-p_{11}^2)^2$$

$$+n_1(n_1-p_{11}^2)(n_1+p_{11}^1-p_{11}^2)+(p_{11}^1-p_{11}^2)\left\{(n_1-p_{11}^2)^2+n_1(n_1+p_{11}^1-p_{11}^2)^2\right\}$$

$$\left.\left.+2n_1p_{11}^2\left\{(n_1-p_{11}^2)+n_1(n_1+p_{11}^1-p_{11}^2)\right\}\right]\right]\sigma_t^2$$

The expectation of mean squares for m_i i.e. $\frac{1}{n_1(v-1)}\ E\sum_1^v \hat{m}_i\ M_i$ is got as

$$\sigma_e^2+r\sigma_s^2+\frac{vr}{n_1(v-1)CD}\left[\quad A\left\{n_1^2(n_1^2-3n_1+2p_{11}^1)+(n_1-p_{11}^2)^2+n_1(n_1-p_{11}^{11}+p_{11}^2)\right\}\right.$$

$$\left.+Bn_1^2\left\{(n_1+1)(n_1-p_{11}^2)-(n_1+p_{11}^1-p_{11}^2)(2n_1-p_{11}^1)\right\}\right]\sigma_m^2\,.$$

The expectations of mean squares for a.c.a. and error are obtained as usual.

2. USE OF 3 ASSOCIATE P.B.I.B. DESIGNS

Earlier analysis allows generalization in using three associate PBIB designs for obtaining plans in this case (ii). Further generalization is complicated. If each line is crossed with lines present in any two of the 3 associate lines, the normal equations for estimating the effects come out as shown below.

$$n_1\mu+n_1t_i+S_1(t_i)+S_1(m_i)=T_i \qquad (5.10)$$

Adding (5.10) over all male lines which are first and second associates of i th line we obtain (5.11) and (5.12).

$$n_1^2\mu+(n_1-p_{11}^3)t_i+(n_1+p_{11}^1-p_{11}^3)S_1(t_i)+(p_{11}^2-p_{11}^3)S_2(t_i)$$

$$+(n_1-p_{11}^3)m_i+(p_{11}^1-p_{11}^3)S_1(m_i)+(p_{11}^2-p_{11}^3)S_2(m_i)=S_1(T_i) \qquad (5.11)$$

$$n_1 n_2 \mu - p_{12}^3 t_i + (p_{12}^1 - p_{12}^3)\, S_1(t_i) + (n_1 + p_{12}^2 - p_{12}^3) S_2(t_i)$$

$$+ (p_{12}^1 - p_{12}^3) S_1(m_i) + (p_{12}^2 - p_{12}^3) S_2(m_i) = S_2(T_i) \qquad (5.12)$$

Also $n_1\mu + n_1 t_i + S_1(t_i) + n_1 m_i = T_i'$ (5.13)

Adding (5.13) over all female lines which are first and second associates of the line i, we get (5.14) and (5.15).

$$n_1^2\mu + (n_1 - p_{11}^3) t_i + (n_1 + p_{11}^1 - p_{11}^3)\, S_1(t_i) + (p_{11}^2 - p_{11}^3) S_2(t_i)$$

$$+ n_1 S_1(m_i) = S_1(T_i') \qquad (5.14)$$

$$n_1 n_2 \mu + (p_{12}^1 - p_{12}^3) S_1(t_i) + (n_1 + p_{12}^2 - p_{12}^3) S_2(t_i) + n_1 S_2(m_i) = S_2(T_i') \qquad (5.15)$$

It is obvious what the symbols stand for.

From (5.10) and (5.14)

$$n_1 T_i - S_1(T_i') = (n_1^2 - A_1) t_i + (n_1 - A_2)\, S_1(t_i) - A_3 S_2(t_i) \qquad (5.16)$$

where A_1, A_2, A_3 respectively stand for the coefficients of $t_i, S_1(t_i)$, $S_2(t_i)$ in (5.14).

Adding such equations (5.16) over all male lines which are the first and second associates of the i th line, we get two more equations involving $t_i, S_1(t_i)$ and $S_2(t_i)$. These two together with (5.16) can be solved for t_i. Also from (5.11) and (5.13) we get another equation as follows:

$$n_1 T_i' - S_1(T_i) = (n_1^2 - A_1) t_i + (n_1 - A_2) S_1(t_i) - A_3 S_2(t_i)$$

$$+ (n_1^2 - A_1) m_i + (n_1 - A_2) S_1(m_i) - A_3 S_3(m_i) \qquad (5.17)$$

This is exactly similar to (5.16) but for the fact that $(t_i + m_i)$ appears in place of t_i and $S_1(t_i) + S_1(m_i)$ appears in place of $S_1(t_i)$ etc. Solving this in similar lines we get the estimate of t_i+m_i. From the above two we can obtain the value of m_i by subtraction.

If Q_i stands for $[\, n_1 T_i - S_1(T_i') \,]$ then sum of squares due to t_i will be $\frac{1}{n_1} \Sigma \hat{t}_i Q_i$.

Also if M_i stands for $[\, n_1(T_i' - T_i) + S_1(T_i') - S_1(T_i') \,]$ the sum of squares due to m_i is $\frac{1}{n_1} \Sigma \hat{m}_i M_i$,

Example

The following example indicates the procedure of analysing the partial diallel crosses when reciprocal crosses have been made. In view of the fact that real data was not available at hand, ficticious data has been made use of for this purpose. Appendix I shows the small table of data , where observations along the rows correspond to those of female line progeny and observations along columns to male line progeny. Since the data may be assumed to consist of mean observations, only the estimates of g.c.a. and maternal effect of the lines are given together with their sum of squares.

A PBIB design with the following parameters and structure serves to provide a partial set of a 9 x 9 diallel cross.

$v = b = 9, \quad r = k = 3, \quad n_1 = 6 , \quad n_2 = 2, \quad \lambda_1 = 1, \quad \lambda_2 = 0,$

$$P_1 = \begin{bmatrix} 3 & 2 \\ & 0 \end{bmatrix} \quad \text{and } P_2 = \begin{bmatrix} 6 & 0 \\ & 1 \end{bmatrix} .$$

The Design			Treatment Number	Association Scheme
1	2	3	1	(2,3,4,5,6,7)
1	6	4	2	(1,3,4,5,8,9)
1	7	5	3	(1, 2,6,7,8,9)
6	8	3	4	(1, 2,6,7,8,9)
6	9	5	5	(1, 2,6,7,8,9)
7	8	4	6	(1, 3,4,5,7,9)
2	8	5	7	(1, 3,4,5,8,9)
7	9	3	8	(2,3,4,5,6,7)
2	9	4	9	(2,3,4,5,6,7)

Identifying treatments as parental lines, each line i ($i = 1,2,\ldots,9$) as male is crossed with every line j as female where j belongs to the first associate lines of i. Thus there will be 54 crosses in total, excluding selfings. Table 2 shows T_i, T_i' which are the yield totals of crosses having i th line as male and female parent respectively. It also gives $S_1(T_i)$, $S_1(T_i')$ and values of the g.c.a. estimates t_i, t_i' of males and females and m_i, the estimates of maternal effect. The analysis of variance is as shown below.

Analysis of Variance Table

Source	d.f.	S.S.	M.S.S.	F
g.c.a.	8	84.6666	10.5833	1.30
Mat.effect	8	93.1111	11.6389	1.43
Error(s.c.a.)	37	299.0549	8.0827	
Total	53	476.8333		

In this example both g.c.a. and maternal effects are not significant at 5 per cent level.

Table 2

Line No.	T_i	T'_i	$S_1(T_i)$	$S_1(T'_i)$	t_i	$t'_i = t_i + m_i$	m_i
1	128	133	845	865	-2.74074	-1.53703	1.20370
2	146	144	853	843	0.92592	0.40740	-0.51851
3	139	143	852	842	-0.18518	0.29629	0.48148
4	143	142	852	842	0.48148	0.12962	-0.35185
5	141	148	852	842	0.14814	1.12962	0.98148
6	133	149	853	843	- 1.24074	1.24074	2.48148
7	143	139	853	843	0.425925	-0.42592	-0.85185
8	156	141	845	865	1.92592	-0.20370	- 2.12962
9	146	136	845	865	0.25925	- 1.03703	- 1.29629

Grand Total = 1275.0

Correction Term = 30104.1667

CHAPTER VI

P.B.I.B. DESIGNS IN TWO ASSOCIATE CLASSES AND PLANS FOR PARTIAL DIALLELS WHEN SELFINGS ARE INCLUDED BUT NOT RECIPROCALS

It oftentimes becomes necessary that we study the performance of the parental inbreds also in order that a better comparison of the combining abilities among the hybrid combinations is effected. When the estimation of yielding capacities of the crosses, whether sampled or unsampled, is envisaged, information regarding the performance of selfings would greatly help in the selection. The yielding capacities of the crosses may be estimated either through their mean yields or by adding the g.c.a. effects to the grand mean for any given cross. The latter method is especially useful in dealing with the unsampled crosses. Assuming that reciprocal crosses are identical, the present situation - case (iii) of chapter III - necessiates that the plans of partial diallels discussed in chapter IV should only be supplemented by the selfings. The analysis of such plans when the first associate lines are crossed and selfing are included in addition, is the following.

With the usual notations, there are a total of $\frac{vn_1}{2} + v$ crosses in the diallel. The normal equation for estimating g.c.a. of a line is given below. It now consists of T_i which is the sum of yields of all intercrosses with line i and twice the yield of selfing of line i.

$$(n_1+2)\mu + (n_1+4)t_i + S_1(t_i) = T_i \qquad (i=1,2,\ldots,v) \qquad (6.1)$$

Adding such equations over all the n_1 first associates of the i th line, we get

$$n_1(n_1+2)\mu + (n_1-p^2_{11})t_i + (n_1+4+p^1_{11} - p^2_{11})S_1(t_i) = S_1(T_i) \qquad (6.2)$$

as $\sum_{i=1}^{v} t_i = 0$.

Solving (6.1) and (6.2) for t_i, we have

$$\hat{t}_i = \frac{AT_i - S_1(T_i) - (n_1+2)(A-n_1)\mu}{A(n_1+4)-(n_1 - p_{11}^2)} , \quad (i = 1, 2, \ldots v) \qquad (6.3)$$

where $A = (n_1 + p_{11}^1 - p_{11}^2 + 4)$.

The estimate of μ is, in fact, $\sum_1^v T_i/v(n_1+2)$.

$$\text{i.e.} \quad \hat{\mu} = \frac{2G}{v(n_1+2)}$$

where G is the grand total of all the observations. Thus

$$\hat{t}_i = \frac{AT_i - S_1(T_i) - 2(p_{11}^1 - p_{11}^2 + 4)\frac{G}{v}}{A(n_1 + 4) - (n_1 - p_{11}^2)} \qquad (6.4)$$

The sum of squares due to g.c.a. is $\sum_{i=1}^v \hat{t}_i T_i$. Let the denominator in (6.4) be denoted by $\triangle$. Let the error variance be σ^2. Then the variance of the difference between g.c.a.'s of any two lines which are not crossed is $2A\sigma^2/\triangle$ and that of the difference between g.c.a.'s of any two lines which have been crossed is $2(A+1)\sigma^2/\triangle$. The average variance of the two comes out to be $\dfrac{2[(v-1)A+n_1]\sigma^2}{(v-1)\triangle}$.

The random model for analysis being

$$y_{ijk} = \mu+t_i+t_j+s_{ij}+e_{ijk}, \quad (i, j = 1, 2, \ldots, v)$$

we may assume here that $s_{ii} = 0$. As usual we have $E(t_i) = E(s_{ij}) = E(e_{ij}) = 0$ and the quantities t, s and e are uncorrelated having $E(t_i^2) = \sigma_t^2$, $E(s_{ij}^2) = \sigma_s^2$ and $E(e_{ijk}^2) = \sigma_e^2$. We shall now find the expectation of sum of squares due to g.c.a. which was given by $\sum_{i=1}^v \hat{t}_i T_i$.

$E\Sigma\hat{t}_iT_i$ is $\frac{1}{\Delta} E \left[A\Sigma T_i^2 - \Sigma T_iS_i(T_i) - 4(p_{11}^1 - p_{11}^2 + 4)G^2/v \right]$

Now

$$E\Sigma T_i^2 = v\left[(n_1+2)^2\mu^2 + \left\{(n_1+4)^2 + n_1\right\}\sigma_t^2 + (n_1+4)\sigma^2\right]$$

$$E\Sigma T_iS_i(T_i) = v\left[n_1(n_1+2)^2\mu^2 + \left\{(n_1+4)(n_1 - p_{11}^2) + n_1(n_1+p_{11}^1-p_{11}^2+4)\right\}\sigma_t^2 + n_1\sigma^2\right]$$

$$EG^2 = \frac{v^2(n_1+2)^2}{4}\mu^2 + v(n_1+2)^2\sigma_t^2 + \frac{v(n_1+2)}{2}\sigma^2$$

where $\sigma^2 = \sigma_s^2 + \sigma_e^2$.

Hence $E\,\Sigma\hat{t}_iT_i$ is

$$\frac{1}{\Delta}\left[v\left\{A(n_1^2+9n_1+16) - (n_1+4)(n_1-p_{11}^2) - n_1A - 4(p_{11}^1-p_{11}^2+4)(n_1+2)^2/v\right\}\sigma_t^2 + \left\{vA(n_1+4) - vn_1 - 2(n_1+2)(p_{11}^1-p_{11}^2+4)\right\}\sigma^2\right].$$

or the expectation mean squares due to g.c.a. is

$$\frac{v}{(v-1)\Delta}\left[A(n_1^2+9n_1+16) - (n_1+4)(n_1-p_{11}^2) - n_1A - 4(p_{11}^1-p_{11}^2+4)(n_1+2)^2/v\right]\sigma_t^2 + \sigma^2.$$

If the experiment is replicated r times, then

$\hat{s}_{ij} = \sum_r y_{ij}/r$ and the sum of squares due to s.c.a. is obtained by subtracting S.S. due to error and g.c.a. from the total S.S. of the mean yields. Then the expectation of mean squares due to g.c.a. is

$$\sigma_e^2 + r\sigma_s^2 + \frac{rv}{(v-1)\Delta}\left[A(n_1^2+9n_1+16) - (n_1+4)(n_1-p_{11}^2) - n_1A - 4(p_{11}^1-p_{11}^2+4)(n_1+2)^2/v\right]\sigma_t^2$$

where, as already indicated, $A = (n_1+p_{11}^1-p_{11}^2+4)$ and $\Delta = A(n_1+4)-(n_1-p_{11}^2)$.

Expectation of mean squares due to s.c.a. is $\sigma_e^2 + r\sigma_s^2$ and that due to error is σ_e^2.

The second associate crosses are analysed as usual.

The following example shows the analysis of the 17 x 17 diallel crosses discussed in chapter IV when the parental inbreds are included. Table 3 gives the totals T_i, $S_1(T_i)$ and the estimates of g.c.a., $\hat{t}_i$.

Example

This example is the same as the one already given with a plan for 17 x 17 diallel crosses using a PBIB design with parameters $v = 17$, $b = 34$, $r = 8$, $k = 4$ etc. But here the selfings are also included so that the total number of crosses is 85.

Analysis of Variance Table

Source	D.F.	S.S.	M.S.	F
g.c.a.	16	6.7741	0.42	0.3 *
s.c.a.	68	93.8023	1.37	
Total	84	100.5764		

* Not significant.

TABLE 3

Grand Total = 286.50

Correction Term = 965.67

T_i	$S_1(T_i)$	t_i
31.00	272.00	-0.250919
36.50	270.50	0.233455
33.50	270.00	-0.020450
36.00	266.00	0.225643
34.00	274.50	-0.012637
30.50	261.50	-0.211856
35.00	270.50	0.104549
37.00	267.00	0.303768
32.50	268.00	-0.090762
37.00	275.00	0.241268
31.00	265.00	-0.196231
32.00	266.50	-0.122012
35.50	274.50	0.116268
30.50	273.50	-0.305606
35.00	273.00	0.085018
34.00	273.00	-0.000919
32.00	263.50	-0.098575

2. USE OF 3 ASSOCIATE P.B.I.B. DESIGNS

With no further difficulty the above analysis for partial diallels when selfings are included may be generalized to the case of 3 associate PBIB designs. When the first associate lines are crossed, the normal equations for estimating g.c.a. are given by, assuming $\Sigma\ t_i = 0$,

$$(n_1+2)\mu+(n_1+4)t_i+S_1(t_i) = T_i \tag{6.5}$$

$$n_1(n_1+2)\mu+(n_1-p^3_{11})t_i+(n_1+4+p^1_{11}-p^3_{11})S_1(t_i)+(p^2_{11} - p^3_{11})S_2(t_i) = S_1(T_i) \tag{6.6}$$

$$n_2(n_1+2)\mu-p^3_{12}t_i+(p^1_{12} - p^3_{12})S_1(t_i)+(n_1+p^2_{12}-p^3_{12}+4)S_2(t_i) = S_2(T_i) \tag{6.7}$$

Putting

$$A_1 = (n_1 - p^3_{11}) \qquad B_1 = -p^3_{12}$$

$$A_2 = (n_1+p^1_{11}-p^3_{11}+4) \qquad B_2 = (p^1_{12} - p^3_{12})$$

$$A_3 = (p^2_{11} - p^3_{11}) \qquad B_3 = (n_1 + p^2_{12} - p^3_{12} + 4)$$

and solving for t_i, we have

$$\hat{t}_i = (A_2B_3-A_3B_2)T_i - B_3\ S_1(T_i)+A_3S_2(T_i)-(n_1+2)[A_2B_3-A_3B_2-n_1B_3+n_2A_3]\hat{\mu}$$

$$\div [(n_1+4)(A_2B_3-A_3B_2) - (A_1B_3 - A_3B_1)]$$

We may note that $\hat{\mu} = 2G/v(n_1+2)$, G being the grand total.

The sum of squares and standard errors are calculated in the usual fashion.

CHAPTER VII

1. CASE OF PARTIAL DIALLELS WITH PARENTAL INBREDS AND RECIPROCAL CROSSES

When maternal effects are present and reciprocal crosses are performed including selfings, the estimates of g.c.a. and maternal effects are given as shown below. This is the case (iv) of chapter III. Suppose each line i, occuring both as male and female, is crossed at each time with n_1 first associate lines and also line i is crossed with itself. This type of plan got through a PBIB design is different from the one presented in chapter V only in that this contains selfings too. There are $v(n_1+1)$ crosses that are sampled. Then, using the random model with maternal effect, we have

$$(n_1+1)\mu + (n_1+2)t_i+m_i+S_1(t_i)+S_1(m_i) = T_i \qquad (7.1)$$

where T_i is the sum of observations corresponding to crosses involving line i as male (and this includes the selfing of the i th line) and the other symbols stand for the usual quantities.

Adding (7.1) over all the male lines which are first associates of the i th line,

$$(n_1+1)^2\mu + \left\{2(n_1+1)-p_{11}^2\right\} t_i + (n_1+p_{11}^1-p_{11}^2+3)S_1(t_i)$$

$$+(n_1-p_{11}^2+1)\, m_i+(p_{11}^1 - p_{11}^2+2)S_1(m_i) = S_1(T_i), \qquad (7.2)$$

as $\sum\limits_{i}^{v} t_i = 0$. In this situation $S_1(T_i)$ includes T_i also.

Moreover, if T_i' indicates the sum of observations of crosses involving the i th line as female (and this sum includes the observation correspond-to selfing), we have

$$(n_1+1)\mu+(n_1+2)t_i+S_1(t_i)+(n_1+1)m_i = T_i' \qquad (7.3)$$

Adding (7.3) over all female lines which are first associates of the i th line,

$$(n_1+1)^2\mu + \left\{ 2(n_1+1)-p_{11}^2 \right\} t_i+(n_1+p_{11}^1-p_{11}^2+3)S_1(t_i) + (n_1+1)m_i$$

$$+ (n_1+1)S_1(m_i) = S_1(T_i'), \qquad (7.4)$$

where $S_1(T_i')$ includes T_i'.

Now from (7.1) and (7.4)

$$(n_1+1)T_i-S_1(T_i') = \left\{ (n_1+1)(n_1+2)-2(n_1+1)+p_{11}^2 \right\} t_i - (p_{11}^1-p_{11}^2+2)S_1(t_i)$$

$$= (n_1^2 + n_1 + p_{11}^1)t_i - (p_{11}^1 - p_{11}^2 +2)S_1(t_i) \qquad (7.5)$$

Adding (7.5) over all male lines which are crossed with line i,

$$(n_1+1)S_1(T_i) - (n_1 - p_{11}^1 + 1)T_i' - (p_{11}^1 - p_{11}^2 + 2)S_1(T_i') - p_{11}^2 G$$

$$= \left\{(n_1^2+n_1+p_{11}^2)-(n_1-p_{11}^2)(p_{11}^1-p_{11}^2+2) \right\} t_i$$

$$- \left\{ (p_{11}^1 - p_{11}^2+2)(p_{11}^1 - p_{11}^2 + 1) - (n_1^2+n_1+p_{11}^2) \right\} S_1(t_i) \qquad (7.6)$$

Equations (7.5) and (7.6) may be abbreviated as

$$A_1 t_i + B_1 S_1(t_i) = X_i$$

and $$A_2 t_i + B_2 S_1(t_i) = Y_i.$$

Hence $$\hat{t}_i = \frac{B_2 X_i - B_1 Y_i}{A_1 B_2 - A_2 B_1}, \qquad (i = 1,2,\ldots,v) \qquad (7.7)$$

where

$$A_1 = (n_1^2 + n_1 + p_{11}^2) \qquad B_1 = -(p_{11}^1 - p_{11}^2 + 2)$$

$$A_2 = (n_1 - p_{11}^2)B_1+A_1 \qquad B_2 = A_1+(p_{11}^1 -p_{11}^2+1)B_1$$

$$X_i = (n_1+1)T_i-S_1(T_i')$$

and $\quad Y_i = (n_1+1)S_1(T_i)-(n_1-p^1_{11}+1)T'_i-(p^1_{11}-p^2_{11}+2)S_1(T'_i) - p^2_{11}G.$

As usual G denotesthe grand total of all observations.

The estimate of maternal effects is given by

$$\hat{m}_i = \frac{B_2X'_i - B_1Y'_i}{A_1B_2 - A_2B_1}$$

where X'_i and Y'_i are obtained from X_i and Y_i by replacing

$(T'_i - T_i)$ for T_i, $(S_1(T'_i) - S_1(T_i))$for $S_1(T_i)$ and $(T_i - T'_i)$ for T'_i,

$(S_1(T_i)-S_1(T'_i))$ for $S_1(T'_i)$.

Sum of squares due to t_i is $\frac{1}{(n_1+1)}\sum_{i=1}^{v}\hat{t}_i Q_i$

where $Q_i = (n_1+1)T_i - S_1(T'_i) = X_i$; and sum of squares due to m_i is $\frac{1}{(n_1+1)}\sum_{i=1}^{v}\hat{m}_iX'_i$.

The expression for expectation of mean squares due to g.c.a. has a long form and comes out to be

$$\sigma^2_e + r\sigma^2_s + \frac{vr}{(n_1+1)(A_1B_2-A_2B_1)}\Big[(n_1+1)(n_1^2+5n_1+4)\left\{B_2(n_1+1)+B_1(n_1-p^1_{11}+1)\right\}$$

$$+ (3n_1^2+9n_1+n_1p^1_{11}-2n_1p^2_{11}-2p^2_{11}+4)\left\{2B_2(n_1+1)+B_1(n_1+1)^2\right.$$

$$\left.- B_1(n_1+1)(p^1_{11}-p^2_{11}+2)+B_1(n_1-p^1_{11}+1)\right\}$$

$$+ (5n_1^2+11n_1+n_1p^1_{11}-5n_1p^2_{11}-4p^2_{11}+p^{2^2}_{11}+4)\left\{B_2-B_1(p^1_{11}-p^2_{11}+2)\right\}$$

$$+ 4B_1p^2_{11}(n_1+1)^4$$

$$+ B_1(n_1+1)\left\{n_1^3+10n_1^2+17n_1+2n_1^2p^1_{11} - 2n_1^2p^2_{11}+6n_1p^1_{11}\right.$$

$$- 10n_1 p_{11}^2 + n_1 {p_{11}^1}^2 - 2n_1 p_{11}^1 p_{11}^2 + n_1 {p_{11}^2}^2 \Big\}$$

$$-B_1 p_{11}^2 \Big\{ 2n_1^3 + 12n_1^2 + 14n_1 + 2n_1^2 p_{11}^1 - 2n_1^2 p_{11}^2$$

$$+ 2n_1 p_{11}^1 - 4n_1 p_{11}^2 + 4 \Big\} \Big] \quad \sigma_t^2 \ .$$

CHAPTER VIII

Diallel crosses are generally grown in the field by following a randomised block lay-out . This being the practice so far, it is understandable that the blocks become heterogeneous when the number of crosses is considerably large. A complete diallel or even a partial diallel will involve enough number of crosses to be performed so that there is a necessity to shorten the size of blocks by using incomplete blocks. This chapter deals with this aspect of the problem and has three sections. In section 1 we give incomplete block plans for complete diallel crosses by using BIB designs along with their analysis. Section 2 pertains to generation of such plans for partial diallel crosses; their construction and analysis depend upon PBIB designs. This section has two parts. While Part (a) deals with the use of 'simple' PBIB designs, Part (b) makes use of PBIB designs in two associate classes with any values of λ's. The last section, section 3, indicates broadly the method of constructing incomplete block plans when reciprocal crosses are also present. In all the following cases we shall assume that there are v parental lines under investigation which have been numbered at random and that they have been identified with the number of treatments of the incomplete block designs used. The words 'lines' and 'treatments' are synonymous throughout the chapter just as in the earlier chapters.

1. INCOMPLETE BLOCKS FOR COMPLETE DIALLELS USING B.I.B.D.

Consider a balanced incomplete block design with parameters v, b, r, k, λ where v stands for the number of parental inbreds under consideration in complete diallel crosses. We shall assume that reciprocal crosses are identical and that we are not interested in studying selfings. It is required to generate a plan such that these available crosses may be grown on the field in incomplete blocks. The method is as follows.

The lines (treatments) in each of the blocks of the BIB design are arranged in an ascending order. Each line (treatment) i belonging to a particular block is then crossed with every line j in the same block such that $j > i$. Thus each block of the BIB design of size k will generate ${}^kC_2$ crosses which will form an incomplete set of all the crosses. Therefore there will be $b.{}^kC_2$ crosses in all generated by the blocks of the BIB design. In general, for a complete diallel of v parental lines a BIB design with parameters v, b, r, k, λ will provide a plan in b blocks each consisting of ${}^kC_2$ crosses such that every cross appears λ times in the plan and each line occurs in these crosses $r(k-1)$ times. In short a BIB design is first formed with the inbred lines as treatments. Next out of the lines in each such block a full diallel cross plan is obtained obtaining thereby b blocks each of size ${}^kC_2$. The analysis of such a plan is as follows.

We shall denote the yield of the cross between lines i and m in the j th block by y_{imj} and adopt the model

$$y_{imj} = \mu + t_i + t_m + s_{im} + b_j + e_{imj}$$

where μ is the general mean, t_i is g.c.a. of the i th line, s_{im} is the s.c.a. of the cross (i x m), b_j is the j th block effect and e_{imj} is the random error. Then the normal equations to estimate the g.c.a. of the lines by least squares technique are given by

$$T_i = r(k-1)\mu + r(k-1)t_i + \lambda(\sum_{m \neq i} t_m) + (k-1)\sum_{j(i)} b_j \qquad (8.1)$$

$$i, m = 1, 2, \ldots, v$$
$$j = 1, 2, \ldots, b$$

where T_i is the total yield from all the crosses involving the ith line and $\sum_{j(i)}$ indicates summation over all such blocks where the i th line occurs. Assuming $\sum_{i=1}^{v} t_i = 0$, we have

$$T_i = r(k-1)\mu + \left[r(k-1) - \lambda\right]t_i + (k-1)\sum_{j(i)} b_j \qquad (8.2)$$

Also, if B_j denotes the j th block total, we get

$$B_j = \frac{k(k-1)}{2}\mu + (k-1)\sum_{i(j)} t_i + \frac{k(k-1)}{2} b_j \qquad (8.3)$$

where $\sum_{i(j)} t_i$ is the sum of the g.c.a. effects of lines occuring in the j th block.

Adding (8.3) over all the blocks where the particular treatment i occurs,

$$\sum_{j(i)} B_j = \frac{rk(k-1)}{2}\mu + (k-1)(rt_i - \lambda t_i) + \frac{k(k-1)}{2}\sum_{j(i)} b_j \qquad (8.4)$$

$$i = 1, 2, \ldots, v.$$

Solving (8.2) and (8.4) we get

$$k/2\, T_i - \sum_{j(i)} B_j = \left[\frac{k}{2}\left\{r(k-1) - \lambda\right\} - (k-1)(r-\lambda)\right] t_i$$

$$\therefore \hat{t}_i = \frac{kT_i - 2\sum_{j(i)} B_j}{\lambda v(k-2)} \qquad (8.5)$$

The estimate of μ comes out to $\hat{\mu} = \dfrac{2G}{bk(k-1)}$, where G is the grand total of all the yields in the design.

The sum of squares due to the g.c.a. is $\sum_{i=1}^{v} \hat{t}_i T_i$; as $\sum \hat{t}_i = 0$, no correction factor need be subtracted.

The variance of difference between the g.c.a.'s of any two lines is given by

$$V(\hat{t}_i - \hat{t}_m) = \frac{2k\sigma^2}{\lambda v(k-2)} , \quad \text{where } \sigma^2 \text{ is the error variance.}$$

If the experiment had been in randomised blocks with the same number of replications of the crosses λ, the corresponding variance of the difference between two g.c.a.'s would have been $2\sigma_R^2/\lambda(v-2)$ where σ_R^2 is the error variance for randomised blocks. Hence the present design gives a more accurate experiment than that from randomised blocks if

$$\frac{\sigma^2}{\sigma_R^2} < \frac{v(k-2)}{k(v-2)} .$$

This is the efficiency factor of the design (plan of crosses).

Expectation of Mean Squares

We shall now find out the expectations of mean squares for the estimates of g.c.a..

The expectation of the sum of squares due to g.c.a. is given by

$$E\Sigma\,\hat{t}_i T_i = \frac{1}{\lambda v(k-2)} \left[kE\,\Sigma\, T_i^2 - 2E\Sigma \left(\sum_{j(i)} B_j\right) T_i \right],$$

$$i = 1, 2, \ldots . v.$$

Now $E\,\Sigma\, T_i^2 = v\left[\lambda^2(v-1)^2\mu^2 + \lambda^2(v-2)\sigma_t^2 + \lambda(v-1)(k-1)\sigma_b^2 + \lambda(v-1)\sigma^2\right]$

where $\sigma^2 = \sigma_s^2 + \sigma_e^2$.

$$E\Sigma \left(\sum_{j(i)} B_j\right) T_i = v\left[\frac{kr^2(k-1)^2}{2}\mu^2 + \lambda(v-2)(k-1)(r-\lambda)\sigma_t^2 + \frac{k(k-1)\lambda(v-1)}{2}\sigma_b^2 + \lambda(v-1)\sigma^2 \right]$$

$$\therefore E\, \Sigma\, \hat{t}_i T_i = \frac{(v-2)}{(k-2)} \left[k\lambda(v-2) - 2(k-1)(r-\lambda) \right] \sigma_t^2 + (v-1)\sigma^2 .$$

Hence, writing σ_s^2 separately, the expected value of mean squares due to g.c.a. is

$$\frac{(v-2)}{(v-1)(k-2)} \left[k\lambda(v-2) - 2(k-1)(r-\lambda) \right] \sigma_t^2 + \sigma_s^2 + \sigma_e^2 .$$

The analysis of variance for the plan of complete diallel crosses in incomplete blocks would be as shown below.

Analysis of Variance Table

Source	D.F.	S.S.	E.M.S.
Blocks	$(b-1)$	$\frac{2}{k(k-1)} \sum_{1}^{b} B_j^2 - CF$	
g.c.a.	$(v-1)$	$\sum_{1}^{v} t_i T_i$	E_g
Error			E_e
Total	$\frac{bk(k-1)}{2} - 1$	$\Sigma\, y_{imj}^2 - CF$	

Example

The example shows an incomplete block plan for a complete diallel cross experiment involving 6 parental lines. This plan is obtained by using a BIB design with the following parameters.

$b = 10,\ v = 6,\ r = 5,\ k = 3$ and $\lambda = 2$.

The blocks, both in BIB design and the plan of Diallel Crosses, are along the rows.

B.I.B.D.			Plan of Diallel Crosses		
1	4	3	1 x 3	1 x 4	3 x 4
1	2	5	1 x 2	1 x 5	2 x 5
1	6	4	1 x 6	1 x 4	4 x 6
3	2	1	1 x 2	1 x 3	2 x 3
1	5	6	1 x 5	1 x 6	5 x 6
2	6	3	2 x 6	2 x 3	3 x 6
4	5	2	2 x 4	2 x 5	4 x 5
2	4	6	2 x 4	2 x 6	4 x 6
3	4	5	3 x 4	3 x 5	4 x 5
5	3	6	3 x 5	5 x 6	3 x 6

Making use of some ficticious data the following analysis is carried out, Table 4 shows the treatment(line) totals, block totals where a particular treatment occurs and the estimates of g.c.a.

TABLE 4

Grand Total = 1730

Estimate of μ = 57.6666

No.	Treatment Total T_i	Block Total $\sum_{j(i)} B_j$	t_i
1	566	852	-0.5000
2	584	822	9.0000
3	579	850	3.0833
4	575	871	- 1.4166
5	582	936	-10.5000
6	574	859	0.3333

The analysis of variance for the example is as follows.

Analysis of Variance Table

Source	D.F.	S.S.	M.S.S.	F
Blocks	9	958.0000	106.44	0.5
g.c.a.	5	24.0000	4.80	0.0
Error	15	2858.6666	190.57	
Total	29	3840.6666		

The analysis indicates that the g.c.a. effects are not significantly different.

INCOMPLETE BLOCKS FOR PARTIAL DIALLELS USING P.B.I.B. DESIGNS

Part (a): Simple PBIB designs

When a partial set of the diallel crosses is to be arranged in incomplete blocks, then a similar method of generation of crosses may be adopted. In this case a partially balanced incomplete block design is used and in the present sub-section we shall discuss the use of a 'simple' PBIB design. A partially balanced design with two associate classes is said to be simple if either (i) $\lambda_1 \neq 0$, $\lambda_2 = 0$ or (ii) $\lambda_1 = 0$, $\lambda_2 \neq 0$ (Bose et al 1954). The case (ii) can be reduced to (i) by interchanging the designation of first and second associates. Hence case (i) will alone be taken.

A plan of crosses generated through a PBIB design of the above type provides in a neat form the blocking of the partial diallel crosses for which reciprocal crosses and selfings are absent. Identifying the treatments of the PBIB design as lines, all possible crosses among the lines in each block are made and these crosses form the blocks for the diallel cross plan. As $\lambda_1 \neq 0$, $\lambda_2 = 0$ the plan so generated will have b blocks each of size ${}^kC_2$ such that each cross is replicated λ_1 times. The analysis for estimating the general combining abilities of the lines is presented below.

Let the parameters of the standard simple type PBIB design be $b, v, r, k, \lambda_1, \lambda_2, n_1, n_2$,

$$P_1 = \begin{bmatrix} p^1_{11} & p^1_{12} \\ & p^1_{22} \end{bmatrix} \quad \text{and} \quad P_2 = \begin{bmatrix} p^2_{11} & p^2_{12} \\ & p^2_{22} \end{bmatrix}.$$

Using the same model as in section 1, the normal equations for estimating the g.c.a. effects of the lines are as shown below.

$$T_i = n_1\lambda_1\mu + n_1\lambda_1 t_i + \lambda_1 S_1(t_i) + (k-1)\sum_{j(i)} b_j \qquad (8.6)$$

$$i = 1, 2, \ldots, v$$

where T_i is the total yield of all crosses with line i and the other symbols are the usual ones.

If B_j denotes the total yield of the j th block, we also have

$$B_j = \frac{k(k-1)}{2}\mu + (k-1)\sum_{i(j)} t_i + \frac{k(k-1)}{2} b_j \qquad (8.7)$$

$$j = 1, 2, \ldots, b$$

where $\sum_{i(j)} t_i$ is the sum of all g.c.a.'s of lines in the j th block.

Adding (8.7) over all the blocks where a particular treatment(line) appears,

$$\sum_{j(i)} B_j = \frac{rk(k-1)}{2}\mu + (k-1)\left\{ rt_i + \lambda_1 S_1(t_i)\right\} + \frac{k(k-1)}{2}\sum_{j(i)} b_j \qquad (8.8)$$

From equations (8.6) and (8.8) we get

$$kT_i - 2\sum_{j(i)} B_j = \lambda_1(k-2)\left[n_1 t_i - S_1(t_i)\right] \quad . \qquad (8.9)$$

Summing (8.9) over all first associates of the i th line and letting Z_i stand for the L.H.S. of (8.9), we get

$$S_1(Z_i) = \lambda_1(k-2)\left[(n_1-p^1_{11}+p^2_{11})S_1(t_i) - (n_1-p^2_{11})t_i\right] , \qquad (8.10)$$

as $\sum_{i=1}^{v} t_i = 0$.

$S_1(Z_i)$ is the sum of all those Z's corresponding to the first associate lines of the i th line. Solving (8.9) and (8.10) for t_i, we have

$$\hat{t}_i = \frac{(n_1 - p^1_{11} + p^2_{11})Z_i + S_1(Z_i)}{\lambda_1(k-2)\left[n_1(n_1-p^1_{11}+p^2_{11})-(n_1-p^2_{11})\right]} , \qquad (8.11)$$

$$i = 1, 2, \ldots, v$$

The sum of squares due to g.c.a. is $1/k\sum_{i=1}^{v}\hat{t}_i Z_i$.

The variance of difference between g.c.a.'s of any two lines is now given by

$$V(\hat{t}_i - \hat{t}_m) = \frac{2k(n_1 - p^1_{11} + p^2_{11} - 1)\sigma^2}{\triangle} ,$$

when the lines i and m are crossed

$$= \frac{2k(n_1 - p^1_{11} + p^2_{11})\sigma^2}{\triangle}$$

when the lines i and m are not crossed.

$\triangle$ denotes the denominator in (8.11) and σ^2 is the error variance.

We can find the average variance in the usual manner. It is given by

$$\frac{2k\left[(v-1)(n_1 - p_{11}^1 + p_{11}^2) - n_1\right]\sigma^2}{(v-1)\,\Delta} .$$

The estimate of mean is $\hat{\mu} = 2G/bk(k-1)$, G being the grand total of all the yields

Expectation of mean squares : - We shall now find out the expected value of the mean squares due to g.c.a. The sum of squares due to g.c.a. is known to be $1/k \sum_{i=1}^{v} \hat{t}_i Z_i$ where $Z_i = kT_i - 2\sum_{j(i)} B_j$ and

$$\hat{t}_i = \frac{(n_1 - p_{11}^1 + p_{11}^2)\,Z_i + S_1(Z_i)}{\lambda_1(k-2)\left[n_1(n_1 - p_{11}^1 + p_{11}^2) - (n_1 - p_{11}^2)\right]}$$

$$\therefore\ E\,1/k \sum \hat{t}_i Z_i = \frac{1}{k\,\Delta}\left[(n_1 - p_{11}^1 + p_{11}^2)\,E\sum Z_i^2 + E\sum Z_i S_1(Z_i)\right]$$

Now

$$E\sum Z_i^2 = vn_1\lambda_1^2\left[(n_1+1)k^2 - 4k(n_1+1) + 4n_1 + 4\right]\sigma_t^2 + vn_1\lambda_1 k(k-2)\sigma^2,$$

σ^2 indicating the error variance. And

$$E\sum Z_i S_1(Z_i) = vn_1\lambda_1^2\,(p_{11}^1 - 2n_1)(k-2)^2\sigma_t^2 - vn_1\lambda_1 k(k-2)\sigma^2,$$

so that from the above two expressions

$$E\,1/k \sum \hat{t}_i Z_i = \frac{vn_1\,\lambda_1(k-2)}{k}\,\sigma_t^2 + (v-1)\sigma^2$$

Therefore, the expectation of mean squares due to g.c.a. is given by

$$\frac{vn_1\lambda_1(k-2)}{k(v-1)}\,\sigma_t^2 + \sigma^2 .$$

The analysis of variance table for the plan of partiall diallel crosses in incomplete blocks would look like the following.

Analysis of Variance Table

Source	D.F.	S.S.	E.M.S.
Blocks	(b-1)	$\frac{2}{k(k-1)} \Sigma B_j^2$ -CF	
g.c.a.	(v-1)	$\frac{1}{k} \Sigma t_i Z_i$	Eg
Error			Ee
Total	$\frac{kb(k-1)}{2} - 1$	Σy_{imj}^2-CF	

Part (b): Use of PBIB designs with any values of λ's.

In this sub-section we shall discuss the case of using PBIB designs, in two associate classes having both values of λ's positive, in order to achieve an incomplete block plan for partial diallel crosses. Under two important assumptions regarding the parent PBIB design and the plan of crosses obtained therefrom we shall give the analysis for estimating the general combining ability.

Suppose that we are making use of a PBIB design with v treatments having both values of λ's greater than zero for generating a plan of partial diallel in v parental lines. We identify as usual the treatments with the lines. All treatment pairs in each of the block of the PBIB design are formed and are designated as crosses. These crosses arrange themselves into b blocks each of size k_{C_2} and they consist of both the first and second associate crosses.

Omitting all the second associate crosses we will be left with the first associate crosses alone arranged in the b incomplete blocks necessary for the partial set of the diallel. As a result of such omission of all second associate crosses from each of the blocks, the remaining block size need not be constant. But we restrict our investigation to those cases only where the remaining number of crosses is the same from each of the blocks. We thus constrain ourselves to plans got through crossing, say, the first associate lines and having a constant block size. Consider an i th line. Let c be the number of treatments common between the first associates of the i th treatment and treatments occuring with it in any given block of the PBIB design. We shall further assume that this number c is independent of i and also of the block in which it occurs. For partial diallel plans obtainable through such designs the analysis is as shown below.

Making use of the model

$$y_{imj} = \mu + t_i + t_m + s_{im} + b_j + e_{imj},$$

The normal equations for estimating the g.c.a. are as follows.

$$T_i = n_1\lambda_1\mu + n_1\lambda_1 t_i + \lambda_1 S_1(t_i) + c \sum_{j(i)} b_j \qquad (8.12)$$

$$i = 1, 2, \ldots . v$$

where T_i is the sum of all yields of crosses with the i th line and c is the constant defined earlier. Also, we get

$$B_j = kc/2\ \mu + c \sum_{i(j)} t_i + kc/2\ b_j \qquad (8.13)$$

$$j = 1, 2, \ldots, b$$

where B_j is the sum of all yields in the j th block.

Adding (8.13) over all the blocks where a particular line i occurs,

$$\sum_{j(i)} B_j = \frac{rkc}{2}\ \mu + c \left[rt_i + \lambda_1 S_1(t_i) + \lambda_2 S_2(t_i) \right] + \frac{kc}{2} \sum_{j(i)} b_j \qquad (8.14)$$

From (8.12) and (8.14) we have, assuming $\sum_{i=1}^{v} t_i = 0$,

$$Z_i = kT_i - 2\sum_{j(i)} B_j = k(n_1\lambda_1 - rc)\ \mu + \left[n_1\lambda_1 k - 2c(r - \lambda_2) \right] t_i + \left[\lambda_1 k - 2c(\lambda_1 - \lambda_2) \right] S_1(t_i) \qquad (8.15)$$

By adding (8.15) over all lines which are first associates of the i th line we get another equation, on the left hand side of which we have $S_1(Z_i)$. Solving this equation along with (8.15) will give us the estimate

$$\hat{t}_i = \frac{\left[B + D(p^1_{11} - p^2_{11}) \right] Z_i - DS_1(Z_i) - A\left[B + D\,(p^1_{11} - p^2_{11} - n_1) \right]\hat{\mu}}{B^2 + BD\,(p^1_{11} - p^2_{11}) - D^2(n_1 - p^2_{11})}, \qquad (8.16)$$

$$i = 1, 2, \ldots, v$$

where

$$A = k(n_1\lambda_1 - rc)$$

$$B = n_1\lambda_1 k - 2c\,(r - \lambda_2)$$

$$D = \lambda_1 k - 2c\,(\lambda_1 - \lambda_2) \quad \text{and}$$

$\hat{\mu} = 2G/vn_1\lambda_1$, G being the grand total of all yields.

Sum of squares due to t_i is $1/k \sum_{i=1}^{v} \hat{t}_i Z_i$, and

$$V(\hat{t}_i - \hat{t}_m) = \frac{2k\left[B + D(p_{11}^1 - p_{11}^2) + D\right]\sigma^2}{\Delta}$$

when the lines i and m are crossed

$$= \frac{2k\left[B + D(p_{11}^1 - p_{11}^2)\right]\sigma^2}{\Delta}$$

when the lines i and m are not crossed

where Δ is the denominator in (8.16).

If the second associate crosses have been made and they form the contents of the blocks, through similar arguments the analysis may be presented.

3. INCOMPLETE BLOCK PLANS WHEN RECIPROCAL CROSSES ARE ALSO PRESENT

This aspect of the problem is discussed only for the case of partial diallel crosses whose plans are obtainable through PBIB designs. Hence most of the succeeding arguments have a bearing on those of section 2 of this chapter.

When reciprocal crosses are also performed similar methods of generating incomplete block plans for the partial diallels as presented in section 2 may be used with slight modification to allow for the reciprocals. Two cases may arise. Firstly, the constant c may be even. That is, there are even number of treatments common between the first associate of a particular treatment, say i, and the other treatments occuring with it in any block of the PBIB design. To allow for the reciprocals in this ~~section~~ situation, the number of blocks in the plan will be doubled with the block-size remaining the same. Secondly, the constant c may be odd. In this case the block size of the plan is doubled, the number of blocks remaining constant. The analysis for the former is discussed using PBIB design with any values of λ's, while that for the latter case is discussed only for a 'simple' PBIB design. It may be seen that c is equivalent to (k-1) for the PBIB design of simple type.

Case 1: When c is even

The plan of crosses arranged in incomplete blocks and having reciprocal crosses also, is obtained from the plan for crosses without reciprocals by replicating each block twice. But here the replication of the block is effected in such a manner that in each of the identical blocks a line appears half the number of times as male and remaining half the number of times as female. An illustration would help understand the situation better. In the following illustration where the rows are blocks, the PBIB design is given along with the plan of crosses. Here the second associate crosses ($\lambda_2 = 1$) are omitted and the first associate crosses ($\lambda_1 = 2$) are retained and each block of the plan is replicated twice to accommodate the reciprocals.

Parameters of PBIB design
$v=b=9$, $r=k=4$,
$\lambda_1 = 2$, $\lambda_2 = 1$,
$n_1 = n_2 = 4$,
$$P_1 = \begin{bmatrix} 1 & 2 \\ & 1 \end{bmatrix} \text{ and }$$
$$P_2 = \begin{bmatrix} 2 & 2 \\ & 1 \end{bmatrix}.$$

Note: Each pair of letters below indicates a cross. The first letter stands for male line and the second for female line.

P.B.I.B.D.				The Plan of Crosses			
2	3	4	7	24	72	43	37
				42	27	34	73
1	3	5	8	15	81	38	53
				51	18	83	35
1	2	6	9	16	91	62	29
				61	19	26	92
1	5	6	7	15	61	75	67
				51	16	57	76
2	4	6	8	24	62	84	68
				42	26	48	86
3	4	5	9	34	53	94	59
				43	35	49	95
1	4	8	9	18	91	84	49
				81	19	48	94
2	5	7	9	27	92	75	59
				72	29	57	95
3	6	7	8	37	83	76	68
				73	38	67	86

Considering the model

$$y_{imj} = \mu + t_i + t_m + s_{im} + m_m + b_j + e_{imj}$$

$$i, m = 1, 2, \ldots\ldots v$$

$$j = 1, 2, \ldots\ldots, 2b$$

where m_m stands for the maternal effect of the m th parent (appearing as female in the cross) and the meaning of other symbols we already know, the following normal equations may be written down. In the general case for a plan of crosses from a PBIB design with the usual parameters we have,

$$T_i = n_1\lambda_1\mu + n_1\lambda_1 t_i + \lambda_1 S_1(t_i) + \lambda_1 S_1(m_i) + c/2 \sum_{j(i)} b_j \qquad (8.17)$$

where T_i is the yield total of all the crosses involving i th line as male.

$$T_i' = n_1\lambda_1\mu + n_1\lambda_1 t_i + n_1\lambda_1 m_i + \lambda_1 S_1(t_i) + c/2 \sum_{j(i)} b_j \tag{8.18}$$

where T_i' is the total yield of all the crosses involving i th line as female.

Also

$$B_j = \frac{kc}{2}\mu + c\sum_{i(j)} t_i + c/2 \sum_{i(j)} m_i + \frac{kc}{2} b_j \tag{8.19}$$

$$j = 1, 2, \ldots, 2b.$$

where B_j is the total yield in the j th block and $\sum\limits_{i(j)} m_i$ is defined similar to $\sum\limits_{i(j)} t_i$ for the maternal effects. Adding (8.19) over all the blocks where a particular line i occurs, we have

$$\begin{aligned} \sum_{j(i)} B_j &= rkc\mu + 2c\left[(r-\lambda_2)t_i + (\lambda_1-\lambda_2)S_1(t_i)\right] \\ &\quad + c/2\left[(r-\lambda_2)m_i + (\lambda_1-\lambda_2)S_1(m_i)\right] + kc/2 \sum_{j(i)} b_j \end{aligned} \tag{8.20}$$

From (8.17), (8.20) and (8.18) we get

$$\begin{aligned} X_i = kT_i - \sum_{j(i)} B_j &= k(n_1\lambda_1 - rc)\mu + \left[n_1\lambda_1 k - 2c(r-\lambda_2)\right] t_i \\ &\quad + \left[\lambda_1 k - 2c(\lambda_1-\lambda_2)\right] S_1(t_i) - c/2(r-\lambda_2)m_i \\ &\quad + \left[\lambda_1 k - c/2(\lambda_1-\lambda_2)\right] S_1(m_i) \end{aligned} \tag{8.21}$$

and

$$\begin{aligned} Y_i = kT_i' - \sum_{j(i)} B_j &= k(n_1\lambda_1 - rc)\mu + \left[n_1\lambda_1 k - 2c(r-\lambda_2)\right] t_i \\ &\quad + \left[\lambda_1 k - 2c(\lambda_1-\lambda_2)\right] S_1(t_i) + \left[n_1\lambda_1 k - c/2(r-\lambda_2)\right] m_i \\ &\quad - c/2(\lambda_1-\lambda_2) S_1(m_i) \end{aligned} \tag{8.22}$$

Then solving (8.21) and (8.22) we get

$$\hat{m}_i = \frac{(n_1 - p^1_{11} + p^2_{11})\,(Y_i - X_i) + S_1(Y_i - X_i)}{k\lambda_1\left[n_1(n_1 - p^1_{11} + p^2_{11}) - (n_1 - p^2_{11})\right]} ,$$

$$i = 1, 2, \ldots\ldots, v$$

where $S_1(Y_i - X_i)$ is the sum of the differences $(Y_i - X_i)$ corresponding to the first associate of the i th line. The estimate of t_i has the following form and is rather combursome.

$$\hat{t}_i = \frac{\left[e_2 + e_3(p^1_{11} - p^2_{11})\right] W_i - e_3 S_1(W_i) - e_1\left[e_2 + e_3(p^1_{11} - p^2_{11}) - n_1 e_1 e_3\right]\hat{\mu}}{e_2\left[e_2 + e_3\,(p^1_{11} - p^2_{11})\right] - e_3^2\,(n_1 - p^2_{11})}$$

$$i = 1, 2, \ldots\ldots, v.$$

where

$$e_1 = d_1 d_8 - d_4 d_5 \quad , \; e_2 = d_2 d_8 - d_4 d_6 , \quad e_3 = d_3 d_8 - d_4 d_7$$

$d_1 = a_1\left[a_5(p^1_{11} - p^2_{11}) - a_4 - n_1 a_1 a_5\right] -$ [illegible])

$$d_2 = a_2\left[a_5\,(p^1_{11} - p^2_{11}) - a_4\right] - a_3 a_5\,(n_1 - p^2_{11})$$

$$d_3 = -(a_3 a_4 + a_2 a_5)$$

$$d_4 = a_4\left[a_5\,(p^1_{11} - p^2_{11}) - a_4\right] + a_5^2\,(n_1 - p^2_{11})$$

$$d_5 = a_1\left[a_6 - a_7\,(p^1_{11} - p^2_{11}) + n_1 a_1 a_7\right]$$

$$d_6 = a_2\left[a_6 - a_7\,(p^1_{11} - p^2_{11})\right] + a_3 a_7\,(n_1 - p^2_{11})]$$

$$d_7 = a_3 a_6 + a_2 a_7$$

$$d_8 = a_6\left[a_6 - a_7\,(p^1_{11} - p^2_{11})\right] - a_7^2\,(n_1 - p^2_{11})$$

$$a_1 = k(n_1\lambda_1 - rc), \quad a_2 = n_1\lambda_1 k - 2c(r - \lambda_2)$$

$$a_3 = \lambda_1 k - 2c(\lambda_1 - \lambda_2) \quad , \; a_4 = c/2(r - \lambda_2)$$

$a_5 = k\lambda_1 - c/2(\lambda_1 - \lambda_2)$, $a_6 = n_1\lambda_1 k - c/2\,(r - \lambda_2)$,

$a_7 = c/2\,(\lambda_1 - \lambda_2)$ and

$W_i = d_8 Z_i + d_4 Z_i'$,

$Z_i = \left[a_5\,(p_{11}^1 - p_{11}^2) - a_4\right] X_i - a_5 S_1(X_i)$

$Z_i' = \left[a_6 - a_7\,(p_{11}^1 - p_{11}^2)\right] Y_i + a_7 S_1(Y_i)$, and the estimate of μ is

$\hat{\mu} = 2G/kc$, G denoting the grand total of all the yields.

Sum of Squares due to t_i is $\Sigma\,\hat{t}_i X_i/k$.

<u>Example</u>

This example will show all the computational steps need to analyse a partial diallel with reciprocal crosses which has been raised in an incomplete block plan. The plan is given in the next page. The normal equations for the analysis are as follows:

$$T_i = 6\mu + 6t_i + S_1(t_i) + S_1(m_i) + \sum_{j(i)} b_j \qquad \text{(E.1)}$$

$$T_i' = 6\mu + 6t_i + 6m_i + S_1(t_i) + \sum_{j(i)} b_j \qquad \text{(E.2)}$$

$$B_1 = 3\mu + 2\sum_{i(j)} t_i + \sum_{i(j)} m_i + 3b_1 \qquad \text{(E.3)}$$

$$\therefore \sum_{j(i)} B_j = 18\mu + 2\left[6t_i + 2S_1(t_i)\right] + \left[3m_i + S_1(m_i)\right] + 3\sum_{j(i)} b_j \qquad \text{(E.4)}$$

$$X_i = 3T_i - \sum_{j(i)} B_j = 6t_i + S_1(t_i) - 3m_i + 2S_1(m_i) \qquad \text{(E.5)}$$

$$Y_i = 3T_i' - \sum_{j(i)} B_j = 6t_i - S_1(t_i) + 15m_i - S_1(m_i) \qquad \text{(E.6)}$$

P.B.I.B.D			The Plan		
1	2	3	12	31	23
			21	13	32
1	6	4	14	61	46
			41	16	64
1	7	5	15	71	57
			51	17	75
3	8	6	36	83	68
			63	38	86
5	6	9	56	95	69
			65	59	96
4	7	8	47	84	78
			74	48	87
3	9	7	37	93	79
			73	39	97
2	5	8	25	82	58
			52	28	85
2	4	9	24	92	49
			42	29	94

Parameters of the PBIB design: $b = v = 9$, $r = k = 3$, $\lambda_1 = 1$, $\lambda_2 = 0$,
$n_1 = 6$, $n_2 = 2$, $P_1 = \begin{pmatrix} 3 & 2 \\ & 0 \end{pmatrix}$, $P_2\begin{pmatrix} 6 & 0 \\ & 1 \end{pmatrix}$.

$$S_1(X_i) = 9S_1(t_i) - 9S_1(m_i) \quad \text{(E.7)}$$

$$S_1(Y_i) = 9S_1(t_i) + 18S_1(m_i), \quad \text{as } \Sigma t_i = \Sigma m_i = 0. \quad \text{(E.8)}$$

$$\therefore 9X_i + 2S_1(X_i) = 54t_i - 27m_i + 9S_1(t_i) \quad \text{(E.9)}$$

$$18Y_i + S_1(Y_i) = 108t_i - 9S_1(t_i) + 270m_i \quad \text{(E.10)}$$

From (E.9) and (E.10)

$$Z_i = 90X_i + 20S_1(X_i) + 18Y_i + S_1(Y_i) = 648t_i + 81S_1(t_i) \quad \text{(E.11)}$$

$$\therefore S_1(Z_i) = 405S_1(t_i), \quad \therefore \hat{t}_i = \frac{405Z_i - 81\,S_1(Z_i)}{(405)\,(648)}$$

Also $(Y_i - X_i) = 18m_i - 3S_1(m_i)$

$$\therefore m_i = \frac{9(Y_i - X_i) + S_1(Y_i - X_i)}{162} .$$

For actual data handling the following table would help.

No.	T_i	T_i'	$\sum_{j(i)} B_j$	X_i	Y_i	$S_1(X_i)$	$S_1(Y_i)$	Z_i	$S_1(Z_i)$

Case 2 : When c is odd

In this case each line in a block of the PBIB design is crossed with all the other lines in it once as male and once as female and the second associate crosses are omitted. Thus the b blocks so generated in the plan are each of size kc. We give the analysis below for such a plan when a simple PBIB design is used for which c equals (k-1).

On the earlier model, the normal equations take the following form

$$T_i = n_1\lambda_1\mu + n_1\lambda_1 t_i + \lambda_1 S_1(t_i) + \lambda_1 S_1(m_i) + (k-1)\sum_{j(i)} b_j \qquad (8.23)$$

$$i = 1, 2, \ldots, v$$

where T_i is the total yield of crosses involving i th line as the male parent

$$T_i' = n_1\lambda_1\mu + n_1\lambda_1 t_i + \lambda_1 S_1(t_i) + n_1\lambda_1 m_i + (k-1)\sum_{j(i)} b_j \qquad (8.24)$$

where T_i' is the total yield of crosses involving i th line as the female.

$$B_j = k(k-1)\mu + 2(k-1)\sum_{i(j)} t_i + (k-1)\sum_{i(j)} m_i + k(k-1)b_j \qquad (8.25)$$

$$j = 1, 2, \ldots, b$$

$$\therefore \sum_{j(i)} B_j = rk(k-1)\mu + 2(k-1)\left[rt_i + \lambda_1 S_1(t_i)\right] + (k-1)\left[rm_i + \lambda_1 S_1(m_i)\right]$$
$$+ k(k-1)\sum_{j(i)} b_j \qquad (8.26)$$

From (8.23), (8.24) and (8.26) we have

$$X_i = kT_i - \Sigma B_j = r(k-1)(k-2)t_i - \lambda_1(k-2)S_1(t_i) - r(k-1)m_i + \lambda_1 S_1(m_i) \qquad (8.27)$$

and

$$Y_i = kT_i' - \Sigma B_j = r(k-1)(k-2)t_i - \lambda_1(k-2)S_1(t_i) + r(k-1)^2 m_i - \lambda_1(k-1)S_1(m_i) \qquad (8.28)$$

Adding (8.27) and (8.28) separately over the first associate lines of the i th line we obtain $S_1(X_i)$ and $S_1(Y_i)$.

Then, if w indicates the value $[\, n_1(n_1-p^1_{11} + p^2_{11}) - (n_1-p^2_{11}) \,]$, we have

$$Z_{1i} = (n_1-p^1_{11} + p^2_{11})X_i + S_1(X_i) = \lambda_1(k-2)wt_i - \lambda_1 wm_i \qquad (8.29)$$

and

$$Z_{2i} = (n_1 - p^1_{11} + p^2_{11})Y_i + S_1(Y_i) = \lambda_1(k-2)wt_i + (k-1)\lambda_1 wm_i \quad (8.30)$$

Solving (8.29) and (8.30), we obtain

$$\hat{t}_i = \frac{(k-1)Z_{1i} + Z_{2i}}{k(k-2)w}, \text{ and}$$

$$\hat{m}_i = (k-2)t_i - \frac{Z_{2i}}{w}, \quad i = 1, 2, \ldots, v$$

Sum of squares due to g.c.a. is $1/k\Sigma\hat{t}_i X_i$.

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APPENDIX I

Table showing data corresponding to 9 x 9 diallel including selfings and reciprocals.

♀ \ ♂	1	2	3	4	5	6	7	8	9
1	21	21	20	19	22	21	25		
2	24	20	22	28	25			24	23
3	19	25	19			28	23	20	24
4	24	26		22		21	24	28	20
5	24	25			21	24	19	28	21
6	19		28	19	24	25		20	23
7	23		24	27	23		25	21	25
8		25	28	27	30	26	20	23	
9		22	21	22	24	29	28		24

SUMMARY

The role of partial diallel crosses in plant and animal breeding experiments is wellknown. The plans for partial diallel crosses hitherto have been generated by using partially balanced incomplete block designs in two plot blocks and two associate classes. But these plans are not free from certain restrictions which the above two plot-block designs impose.

In the present investigation new methods of evolving plans for partial diallels are suggested which prove to certain extent more flexible than the earlier ones. Making use of PBIB designs with any block size and any number of associate classes, the plans have been generated to suit analysis when reciprocal crosses are also performed. Methods of analysis for estimating general and specific combining abilities and maternal effects are presented along with standard errors for comparison. Expectation of mean squares due to general and specific combining abilities and error have been calculated. Different cases according to whether parental inbreds with/without the reciprocal F_1's are included or not have been discussed in chapter IV through chapter VII.

The necessity for shorter blocks in the layout of the experiment becoming evident, methods for growing partial and complete diallel crosses in incomplete blocks are indicated. The analyses of such incomplete blocks of crosses are shown in chapter VIII and they include the case when reciprocal crosses are also present.

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Annexture to Chapter II

The author's attention has been brought to a paper by Hinkelmann and Kempthorne (Biometrica (1963), 50, p 281) who have considered the correspondence, in thd usual sense, between partial diallel crosses and PBIB designs with m associate classes. The block size of the PBIB's remains two as was the case earlier and theypresent analysis in a generalized fashion. Maternal effects and other related extensions have not been considered by them.