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Two-level factorial designs in block size four

Anurag Rawat, Sukanta Dash and Kaushal Kumar Yadav

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Anurag Rawat
1) The Graduate School, ICAR-Indian Agricultural Research Institute, New Delhi, Delhi, India
2) ICAR-Indian Agricultural Statistics Research Institute, New Delhi, Delhi, India

Sukanta Dash
ICAR-Indian Agricultural Statistics Research Institute, New Delhi, Delhi, India

Kaushal Kumar Yadav
1) The Graduate School, ICAR-Indian Agricultural Research Institute, New Delhi, Delhi, India
2) ICAR-Indian Agricultural Statistics Research Institute, New Delhi, Delhi, India

Corresponding Author:
Anurag Rawat
1) The Graduate School, ICAR-Indian Agricultural Research Institute, New Delhi, Delhi, India
2) ICAR-Indian Agricultural Statistics Research Institute, New Delhi, Delhi, India

Abstract

Two-level factorial designs are widely used to identify significant effects in agriculture and allied fields. The occurrence of practical constraints, which can make it necessary to arrange runs in small block size like block size four, provide a major motivation for work in this area. Investigating the optimal number of replications needed to be carried out to estimate important factorial effects for factorial experiments with blocks of size four has attracted considerable interest from researchers. Estimating all main effects and two-factor interactions in a two-level factorial experiment in blocks setup, obtaining precise estimates generally requires a significant number of replications. The article presents methodologies aimed at obtaining efficient block designs for two-level factorial experiments in block size 4, which hold potential for improving the efficiency factor and effectiveness of experimental investigations.

Keywords: Block designs, 2^{n} Factorial experiments, confounding, efficiency factor

Introduction

In scientific experiment designs, managing heterogeneity in the experimental material is essential. While dealing with one-way heterogeneity present in the experimental material, block designs are useful in such scenarios. Factorial effects are crucial in experimental design because they offer comprehensive insights into the relationship between multiple factors and their combined impact on the outcome. Draper and Guttman (1997) [3] demonstrate that for basic 2^{k-p} designs, where $p \geq 0$, it is necessary to have $k-p$ replicates of block designs with size two that ensures estimation of all the standard (estimable) effects accurately. Yang and Draper (2003) [10] examines the combinations of designs and pairing arrangements that allows for the estimation of all main effects and two-factor interactions and presented the confounding patterns of various combined designs that integrate multiple 2^{k} designs with different pairings for different values of k while considering higher order interactions among the various factors to be negligible. Kerr (2006) [7] obtained block designs for 2^{n} factorial experiments in blocks of size 2 to estimate all main effects and two-factor interactions. Huang (2009) [6] provided effective techniques for building block designs for block sizes two in 2^{k} plans along with the techniques to accurately show the bare minimum of blocks required to estimate the essential effects and also provide a framework for building combination designs. Dash *et al.* (2013) [1] presented technique for creating row-column designs based on orthogonal parameterization to estimate main effects and two-factor interaction effects in 2^{n} factorial microarray studies. Dash *et al.* (2014) [2] also created a process for building row-column designs for estimating of odd order factorial effects and all main effects using orthogonal estimation. Gopinath et al. (2018) [5] gave construction designs with two approaches to creating row-column designs for factorial experiments. Godolphin (2021) [4] proposed construction of blocked factorial designs for estimating main effects and selected two-factor interactions. Yadav (2023) [9] developed a construction method for efficient block/row-column designs by minimizing the number of replicates for n (number of factors) $2 \leq n \leq 9$ for estimating all main effects and specific two-factor interactions for the two-level factorial experiment in two rows. All the works present in the literature mostly emphasize on the block designs for two-level factorial experiments with block of size two while considering minimum number of replications to estimate important factorial effects. However, in practice, experiments with block size three or four can be more useful than block size two.

For example, in agricultural research, especially in the post-harvest technology, experiments frequently employ block designs with a size of four due to practical constraints.

Another example, when aiming to improve crop yields, agricultural setups often use small test plots that can accommodate only four different treatments at once. Likewise, experimental configurations may include scenarios such as evaluating four wafers within a single lot, arranging four units on a tray, or conducting four experimental runs within a single day or production cycle. Such constraints are prevalent in real-world applications and are discussed in detail by Wang (2016).

Selecting unsuitable designs for experiments involving small block sizes can result in the need for a greater number of replicates, which in turn increases both the time and cost of the study. Scarcity of resources frequently pose substantial challenges, occasionally making it less feasible to conduct a design that estimates all the important factorial effects. As the number of factors increases, the demand for resources can grow significantly. Consequently, it is essential to choose an appropriate experimental design that estimates all the main effects and the two-factor interactions with greater efficiency while minimizing the number of replications, thereby optimizing resource allocation and reducing overall costs. Wang (2016) [8] investigated designs with the least number of replicates for factorial experiments in blocks of size four and presented a generation method using orthogonal arrays to obtain such designs up to 2^8 factorial experiments and other than this no such work related to block size 4 are available in literature. Hence, there is need to construct block designs in factorial replicates for the problem of obtaining efficient block designs in block size 4 which is easy to implement and capable of estimation of all the important factorial effects.

In this article, we propose method of construction for the situation of factorial experiments in block of size four for estimation of all the main effects and two-factor interactions in minimum number of replications for $n \geq 2$., allowing researchers to deal the situation with the minimum number of resources required to the study the objective. This method offer flexibility in experimental design, catering to various research goals and analytical needs.

Methodology

The statistical model is used in block design setup with replications as follows:

$$y_{ijk} = \mu + \tau_i + \beta_{jk} + \rho_k + \epsilon_{ijk}$$

Here, y_{ijk} is the observation from application of the i^{th} treatment combination to an experimental unit in the j^{th} block of the k^{th} replicate. The overall mean is μ and τ_i, β_{jk} and ρ_k are the effects of the i^{th} treatment combination, the j^{th} block nested in the k^{th} replicate and the k^{th} replicate. The error terms ϵ_{ijk} are uncorrelated, all with variance σ^2.

Method of construction

The proposed method of construction focused to estimate main effects and two factor interactions with greater efficiency factor made by confounding minimum number of two factor interactions i.e. generally higher orders are confounded. Hence, the designs obtained from these construction methods can estimate all main effects and two factor interactions in minimum number of replications. The details about construction method have been describe in two steps where step1 generate the key block of the required design and step 2 generate whole design from the developed key block.

Step 1: Construction of key block

- For $n = 2$, total number of treatment combination will be 4 (2^2) hence, all treatments are to be allotted in a block of size 4 and can be considered as a key block.
- For $n > 2$, i.e. 2^n treatments are to be allotted in 2^{n-2} blocks each of size 4. Here, key block will assign 1st and 4th treatment from the key block for $n - 1$ as 1st and 2nd treatment then add nth factor to the rest 2nd and 3rd treatment from the key block for n-1 and assigned as 3rd and 4th treatment of the key block for n.
- This process of constructing key block will be same for all replications with modification of assigning different factor for different replications.

Step 2: Construction of design

Once the key block has been generated, one can easily construct whole design from the generated key block.

Example: Consider 2^5 factorial experiment where experimenter wants to estimate all main effects and two factor interactions in minimum number of replications.

Step 1: Construction of key block

Here $n = 5$, hence we have to generate key block of $n = 2, 3, 4$

Replication 1

n=2, key block is (1), a, b, ab (all treatment combinations)

n=3, key block is (1), ab, ac, bc (Keeping (1) and ab as first and second treatment from 1st and 4th treatment of key block, n=2; add 'c' (nth factor) to treatment 'a' and 'b' then assigned as third and fourth treatment)

n=4, key block is (1), bc, abd, acd (Keeping (1) and bc as first and second treatment from 1st and 4th treatment of key block, n=3; add 'd' (nth factor) to treatment 'ab' and 'ac' then assigned as third and fourth treatment)

n=5, key block is (1), acd, bce, abde (Keeping (1) and acd as first and second treatment from 1st and 4th treatment of key block, n=4; add 'e' (nth factor) to treatment 'bc' and 'abd' then assigned as third and fourth treatment).

Replication 2

Similarly, for replication 2, key block (1), ace, bcd, abde (changing factor 'd' with 'e')

Step 2: Construction of block design for 2^5 factorial experiments in block size 4

Replication 1							
Block1	**Block2**	**Block3**	**Block4**	**Block5**	**Block6**	**Block7**	**Block8**
(1)	A	B	AB	C	AC	BC	ABC
ACD	CD	ABCD	BCD	AD	D	ABD	BD
BCE	ABCE	CE	ACE	BE	ABE	E	AE
ABDE	BDE	ADE	DE	ABCDE	BCDE	ACDE	CDE
Replication 2							
Block9	**Block10**	**Block11**	**Block12**	**Block13**	**Block14**	**Block15**	**Block16**
(1)	A	B	AB	C	AC	BC	ABC
ACE	CE	ABCE	BCE	AE	E	ABE	BE
BCD	ABCD	CD	ACD	BD	ABD	D	AD
ABDE	BDE	ADE	DE	ABCDE	BCDE	ACDE	CDE

Here, all the main effects and two-factor interactions are estimated in two replications.

Remark: While the proposed method is not superior to those suggested by Wang (2016) [8], we offer a simpler and easier alternative for constructing designs compared to the methods proposed by Wang (2016) [8].

Concluding remarks

This article proposes a method of constructing block designs for factorial experiments in block size four. Several authors work on block designs for factorial experiments in block size 2 for estimation of all main effects and two-factor interactions. For estimation of important factorial effects i.e. main effects and two-factor interactions from a two-level factorial experiment in blocks, we might need many replicates in considering small block size. Choosing an inappropriate design for an experiment can significantly escalate the required resources, impacting both time and cost. When a design is not well-suited for the specific block size or number of factors, it often necessitates a greater number of replicates to ensure reliable results. This increased need for replicates can lead to a substantial rise in material costs, labour, and time needed for data collection and analysis. Wang (2016) [8] investigated block designs with the least number of replicates for factorial experiments in blocks of size four and presented a generation method using orthogonal arrays to obtain such designs up to 2^8 factorial experiments. No such work related to block size four is available in the literature. Hence, the present study focused to develop the construction method which is easy to implement, capable of estimating all main effects and two factor interaction accommodating any number of factors, and with a smaller or equal number of replications as compared to available designs in literature. Considering the above situation, the present investigation focuses on construction of block designs with block size four for factorial experiments in estimation of all main effects and two-factor interactions. Designs obtainable from this method of construction was catalogued for 2^n factorial experiments $n < 10$, where n is the number of factors. A catalogue of designs for $n < 10$ has been prepared along with main effects and two-factor interactions which are not estimable in different replications and given in Table 1. The efficiency factor of estimable main effects and two-factor interactions has also been given in Table 2.

Table 1: Factor effect which are not estimable in different replications in block designs for 2^n factorial experiments ($3 \leq n \leq 9$)

No. of factors	Replication number	Factorial effects confounded in different replications
3	R_1	-
4	R_1	AD
	R_2	BD
5	R_1	AD, BE
	R_2	AE, BD
6	R_1	AD, BE, CF
	R_2	AF, BD, CE
7	R_1	AD, AG, BE, CF, DG
	R_2	AE, BF, BG, CD, FG
8	R_1	AD, AG, BE, BH, CF, DG, EH
	R_2	AB, AC, BC, DE, DF, EF, GH
9	R_1	AD, AG, BE, BH, CF, CI, DG, EH, FI
	R_2	AB, AC, BC, DE, DF, EF, GH, GI, HI

Table 2: Efficiency factor of estimable main effects and 2-factor interactions in block designs for factorial experiments in block size four ($3 \leq n \leq 9$)

Factors	r	A	B	C	D	E	AB	AC	BC	AD	BD	CD	AE	BE	CE	DE
3	2	1.0	1.0	1.0			1.0	1.0	1.0							
4	2	1.0	1.0	1.0	1.0		1.0	1.0	1.0	0.5	0.5	1.0				
5	2	1.0	1.0	1.0	1.0	1.0	1.0	1.0	1.0	0.5	0.5	1.0	0.5	0.5	1.0	1.0
		A	B	C	D	E	F	AB	AC	BC	AD	BD	CD	AE	BE	CE
6	2	1.0	1.0	1.0	1.0	1.0	1.0	1.0	1.0	1.0	0.5	0.5	1.0	1.0	0.5	0.5
		DE	AF	BF	CF	DF	EF									
		1.0	0.5	1.0	0.5	1.0	1.0									
		A	B	C	D	E	F	G	AB	AC	BC	AD	BD	CD	AE	BE
7	2	1.0	1.0	1.0	1.0	1.0	1.0	1.0	1.0	1.0	1.0	0.5	1.0	0.5	0.5	0.5
		CE	DE	AF	BF	CF	DF	EF	AG	BG	CG	DG	EG	FG		
		1.0	1.0	1.0	0.5	0.5	1.0	1.0	0.5	0.5	1.0	0.5	1.0	0.5		
		A	B	C	D	E	F	G	H	AB	AC	BC	AD	BD	CD	AE
8	2	1.0	1.0	1.0	1.0	1.0	1.0	1.0	1.0	0.5	0.5	0.5	0.5	1.0	1.0	1.0
		BE	CE	DE	AF	BF	CF	DF	EF	AG	BG	CG	DG	EG	FG	AH
		0.5	1.0	0.5	1.0	1.0	0.5	0.5	0.5	0.5	1.0	1.0	0.5	1.0	1.0	1.0
		BH	CH	DH	EH	FH	GH									
		0.5	1.0	1.0	0.5	1.0	0.5									
		A	B	C	D	E	F	G	H	I	AB	AC	BC	AD	BD	CD
9	2	1.0	1.0	1.0	1.0	1.0	1.0	1.0	1.0	1.0	0.5	0.5	0.5	0.5	1.0	1.0
		AE	BE	CE	DE	AF	BF	CF	DF	EF	AG	BG	CG	DG	EG	FG
		1.0	0.5	1.0	0.5	1.0	1.0	0.5	0.5	0.5	0.5	1.0	1.0	0.5	1.0	1.0
		AH	BH	CH	DH	EH	FH	GH	AI	BI	CI	DI	EI	FI	GI	HI
		1.0	0.5	1.0	1.0	0.5	1.0	0.5	1.0	1.0	0.5	1.0	1.0	0.5	0.5	0.5

Conclusions

The proposed method for constructing block designs in factorial experiments with block size four offers a practical solution for efficiently estimating all main effects and two-factor interactions with minimal replication. While previous research primarily focuses on block designs with size two, this method addresses the limitations of smaller block sizes by allowing for more effective use of resources. It simplifies the design process compared to existing methods, reducing both time and cost while accommodating a greater number of factors. This approach ensures accurate estimation of crucial effects and provides a useful framework for experimental design in diverse research contexts.

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